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BITRATE ALLOCATION FOR MULTIPLE VIDEO STREAMS AT COMPETITIVE EQUILIBRIA

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ABSTRACT

Current methods for multiplexing video streams often rely on identifying the relative complexity of the video streams to improve the combined overall quality. In such methods, not all the videos benefit from the multiplexing process. Typically, the quality of high motion videos is improved at the expense of reduction in the quality of low motion videos. In our approach, we use a competitive equilibrium allocation of bitrate to simultaneously improve the quality of all the video streams by finding trades between videos across time. The proposed method not only uses information about the differing complexity of the video streams at every moment but also the differing complexity of each stream over time.

1. INTRODUCTION

Applications where multiple compressed video streams are transmitted simultaneously through a shared channel include direct broadcast satellite, cable TV, video-on-demand service, and video surveillance. In existing methods for transmitting multiple video streams [1–3], improving the overall quality is the goal. The overall quality improvement is achieved by exploiting the relative complexity of the video streams at every moment. However, not all video streams benefit from multiplexing processes. Generally, the quality of high complexity videos improve at the expense of reduced quality of low complexity videos.

In this paper, we constrain that no video stream will suffer quality degradation by participating in the multiplexing process, compared to independent encoding. The method selects an expected efficient, Pareto optimal (PO), allocation of bitrate for multiple videos. By computing the expected competitive allocation in the Edgeworth box, a common tool in economics for equilibrium analysis, we find a point where all users perform better or at least as well as what they could achieve independently. This method exploits gains in quality that can be achieved by trading bits across time-slots rather than merely reallocating bits within each time-slot, as is done with current methods of multiplexing.

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The rest of the paper is organized as follows: Section 2 describes the Edgeworth box for illustrating competitive equilibria. Section 3 describes competitive equilibrium bitrate allocation methods for multiple video streams. Results and conclusions are given in Section 4.

2. EDGEWORTH BOX FOR COMPETITIVE EQUILIBRIUM

The Edgeworth (EW) box [4] is a graphical tool for exhibiting PO allocations and illustrating a competitive (Walrasian) equilibrium in a pure exchange economy [5], in which no production is possible and the commodities that are ultimately consumed are those that individual users possess as initial endowments. The users trade these endowments among themselves in a market for mutual advantage.

Consider two users ($i = 1, 2$) and two goods ($j = 1, 2$). User i's consumption vector is $x_i = (x_i^1, x_i^2)$, i.e., user i's consumption of good j is $x_i^j \geq 0$. Each user i is initially endowed with an amount $c_i^j \geq 0$ of good j. The total endowment of good j in the economy is denoted by $\bar{c}^j = c_1^j + c_2^j$, assumed strictly positive. An allocation $x \in \mathbf{R}_+^4$ is an assignment of a non-negative consumption vector to each user: $x = \{x_1, x_2\} = \{(x_1^1, x_1^2), (x_2^1, x_2^2)\}$. We say that an allocation is nonwasteful and feasible if $x_1^j + x_2^j = \bar{c}^j$ (the total consumption of each good is equal to the economy's aggregate endowment of it).

In the EW box, user 1's quantities are measured with the southwest corner as the origin (O_1), shown in Figure 1. User 2's quantities are measured using the northeast corner as the origin (O_2). For both users, the horizontal dimension measures quantities of good 1 and the vertical dimension measures quantities of good 2. The width and height of the box are $\bar{c}^1$ and $\bar{c}^2$, the economy's total endowment of goods 1 and 2. Any point in the box represents a division of the total endowment between users 1 and 2. Given $\bar{c}_i = (c_i^1, c_i^2)$, user i can calculate his utility $U_i(\bar{c}_i)$. The locus of all x_i yielding the same utility $U_i(x_i) = u_i$ is called an "indifference curve" of user i. The map of all indifference curves are the level sets of the utility function $U_i(x_i)$, as shown in Figure 1. We assume these curves are convex. For any user, the utility increases as we move away from its origin. If we draw indifference curves for both users in the box, the points where the

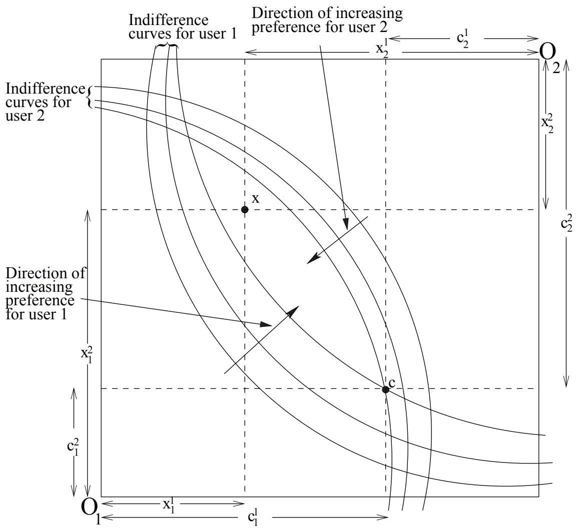


Fig. 1. An Edgeworth box for two users and two goods

indifference curves for both users are tangential to each other are PO allocations [5]. The set of all PO allocations is the *Pareto set*. The part of the Pareto set where both users do at least as well as at their initial endowments is called the *contract curve* (Figure 2). Bargaining between the users should result in some point on the contract curve as these are the only points at which both users do at least as well as at their initial endowments and for which there is no alternative trade that can make both users better off [5].

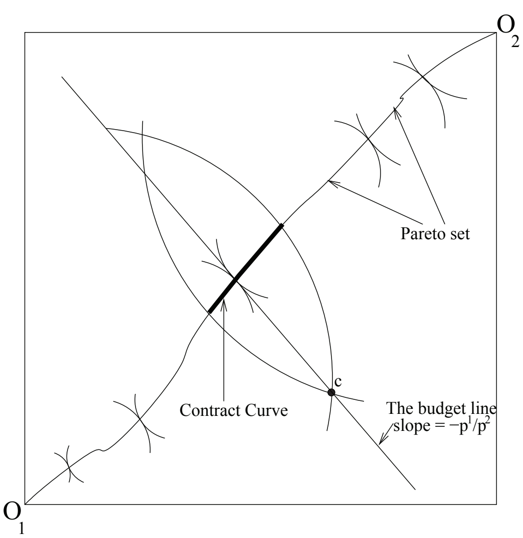


Fig. 2. The Pareto set and the contract curve in the EW box

Suppose users can buy or sell these goods in the market for prices p^1 and p^2. For any price system $p = (p^1, p^2)$ and initial endowments, the budget set for user i is:

$$B_i(p) = \{x_i \in \mathbf{R}^2_+ : p.x_i \leq p.c_i\} \tag{1}$$

A competitive equilibrium for an EW box economy is a price vector p^* and an allocation $x^* = (x_1^*, x_2^*)$ such that

$$U_i(x_i^*) \geq U_i(x_i') \qquad \forall\ x_i' \in B_i(p^*), \quad \forall\ i = 1, 2$$

and

$$\sum_{i=1}^{2} x_i^* = \sum_{i=1}^{2} c_i \tag{2}$$

At an equilibrium, each user i's demanded bundle at price vector p^* is x_i^* and one user's net demand for a good is exactly matched by the other's net supply. The intersection of a budget line and contract curve, where the budget line is also tangential to the indifference curve for both the users on the contract curve, defines a competitive equilibrium. At an equilibrium point, both users are better off compared to their initial endowment. This is shown in Figure 2. Under our assumptions, at least one competitive equilibrium will exist for every initial endowment allocation. More details about the EW box and competitive equilibrium can be found in [5].

3. VIDEO MULTIPLEXING USING THE EW BOX

We extend the concept of the EW box from two users to N video users. The two goods are the bits available in two time-slots (TS). We consider one TS as one GOP; however, one can choose TS at any level. The complexity of the EW box increases with box dimension and there are many TS in each video stream. So, we reduce the problem to two TS for each user. We will use the terms GOP and TS interchangeably. We generate the rate distortion (RD) curve for each TS by calculating the mean squared error (MSE) at different bitrates. Note that the complexity of generating the RD curve can be further reduced by using the method described in [6]. Suppose that 1000 bits are available in TS 1 and in TS 2. Suppose also that user 1 and user 2 each has an initial endowment of 500 bits in each TS. If the RD curves for the two users are such that giving user 1 600 bits in TS 1 and 400 in TS 2 (and vice versa for user 2) produces a more favorable total MSE than the equal initial endowment, then the EW box approach would favor this allocation over the initial one.

While trading across TS is the basic idea behind our approach, often adjacent TS have similar RD curves. Therefore, little benefit can be gained by trading bits between adjacent TS for two users. One would like to trade between the current encoding TS and some other TS widely separated in time. Since the specific RD curve for some distant TS is typically not known, we consider trades between the current encoding TS and an expected or approximate RD curve for the future.

We sequentially process each TS in the same manner. The RD curve for user i in TS j is fitted by

$$D_i^j(R_i^j) = a_i^j + \frac{b_i^j}{R_i^j} \tag{3}$$

where R_i^j is the number of bits and D_i^j is the MSE distortion for TS j in video stream i. We use the least squares approach to find a_i^j and b_i^j, the coefficients for generating this curve-fitting model. Note that the above function is convex. Other curve-fitting models are available in the literature [7]. Suppose the utility for user i is the convex function:

$$U_i(x_i^1, x_i^2) = -(a_i^1 + \frac{b_i^1}{x_i^1} + a_i^2 + \frac{b_i^2}{x_i^2}) \quad (4)$$

that is, the negative sum of the MSE in both TS. Let the initial endowment for user i be $\bar{c}_i$. Then the indifference curve through the initial endowment for user i can be derived as

$$-(a_i^1 + \frac{b_i^1}{x_i^1} + a_i^2 + \frac{b_i^2}{x_i^2}) = -(a_i^1 + \frac{b_i^1}{c_i^1} + a_i^2 + \frac{b_i^2}{c_i^2}) \quad (5)$$

for different combinations of x_i. A competitive equilibrium is found by solving

$$\max_{x_i^1, x_i^2} U_i(x_i^1, x_i^2) \ s.t.\ p^1 x_i^1 + p^2 x_i^2 = p^1 c_i^1 + p^2 c_i^2,\ \forall i = 1 to N \quad (6)$$

and

$$\sum_{i=1}^{N} x_i^j = \sum_{i=1}^{N} c_i^j \qquad \forall j = 1, 2 \quad (7)$$

The Lagrangian expression for user i is

$$L_i = U_i(x_i^1, x_i^2) + \lambda_i (p^1 c_i^1 + p^2 c_i^2 - p^1 x_i^1 - p^2 x_i^2) \quad (8)$$

By differentiating L_i with respect to x_i^1, x_i^2, and λ_i, equating the results to 0 and solving for x_i, and substituting in Eq. 7, we get

$$\sum_{i=1}^{N} \sqrt{\frac{b_i^j}{p^j}} \frac{p^1 c_i^1 + p^2 c_i^2}{\sqrt{p^1 b_i^1} + \sqrt{p^2 b_i^2}} = \sum_{i=1}^{N} c_i^j \qquad \forall j = 1, 2 \quad (9)$$

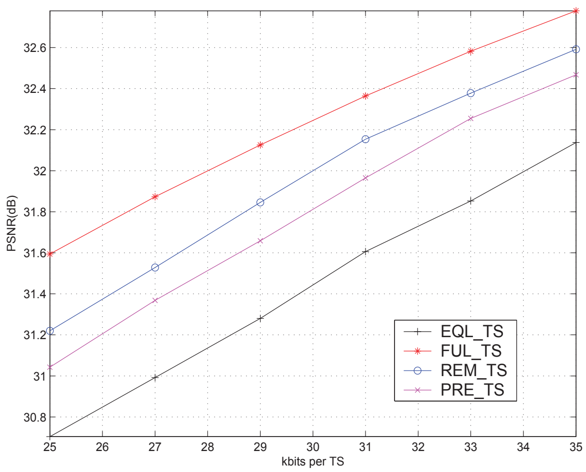
To determine the competitive equilibrium, we need to find the equilibrium prices p. Since the solution of Eq. 9 is homogeneous of degree 0 in prices, we only need to find an equilibrium price ratio p^2/p^1. Therefore, without loss of generality, we may take $p^1 = 1$ and solve Eq. 9 numerically for p^2. With p^2, we find x_i^j which comprise a competitive equilibrium.

To compare the improvement in video quality of the various multiplexing schemes, we first, we consider the constant bit allocation for each TS (**EQL_TS**). Here each user in every TS receives an equal number of bits to encode its video. This rate control method is applied in many video standards such as H.264/AVC [8]. We then compare the competitive equilibrium bit allocation for various video streams. The first TS is always considered to be the current TS that we are encoding. If we assume that we have some information about the future, such as the average RD curves for future TS, then we can use such information for trading bits for the current TS with the average of remaining TS (**REM_TS**). This is an *ex ante* approximation model where we assume some information about the future. If we have no future information then we predict the future by looking at the previous TS (*ex post*) with the assumption that the average RD curve of previous TS will be similar to that of the future. We trade bits for the current TS with the average of previous TS (**PRE_TS**). The performance of PRE_TS depends on how well the past represents the future. Both REM_TS and PRE_TS are solved for a competitive equilibrium. For comparison, we consider a method in which each user has full information about its RD curves in all TS, and proposes to divide the bits among all the TS based on their relative complexity. All video streams use this criteria for bit allocation for their TS independently. Since the total number of bits is constant for each TS, we normalize the number of bits allocated to each video stream by the total available bits for a TS (**FUL_TS**). Note that for this method each user attempts to allocate bits across TS but does not trade with other users. In this paper, these four bit allocation methods are compared for video multiplexing.

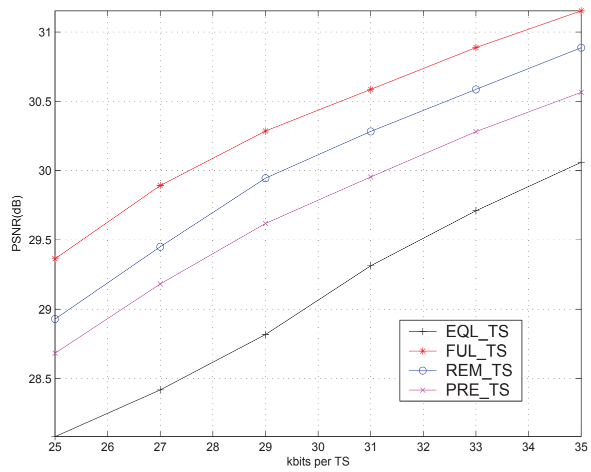
4. RESULTS

The simulation was performed using the baseline profile of H.264/AVC [9] reference software JM 11.0 [10]. The 30-second test videos containing varying types of scenes and motion were taken from a 72 minute travel documentary at a resolution of 176×120 pixels and 30 frames per second. The GOP size is 15 frames (I-P-P-P). The frames inside a TS are encoded using H.264 rate control [8]. The coding parameters such as resolution or GOP size can be changed for any appropriate application.

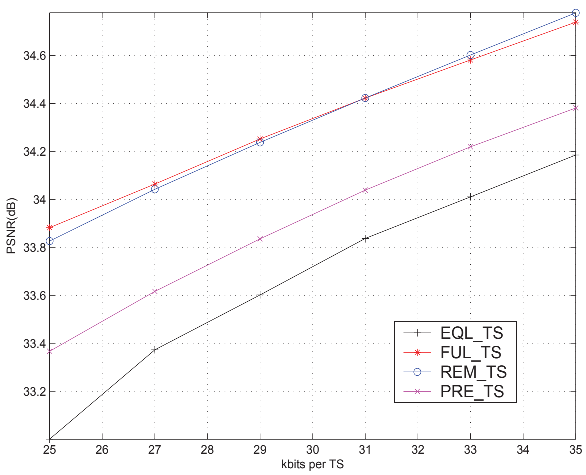
Figure 3 shows the results of multiplexing four video streams. The four curves in each plot represent the multiplexing methods described previously. Each plot shows PSNR versus bitrate (ranging from 25-35 kbits per TS (50-70 kbps)). We calculate the MSE of each frame and average across all frames of a video then convert to PSNR. The performance of EQL_TS is worst in all videos. This is the method used in most video standards for GOP level rate control. For archived video we know RD curves for all TS and we see that FUL_TS performs the best. The PSNR gain over EQL_TS varies from 0.25-0.43 dB for g12 to 1.10-1.50 dB for g9. However, this method cannot be used for real-time video multiplexing. If a user knows the average RD curve for future TS, then this is sufficient to improve the video quality as shown by REM_TS. This method finds a competitive equilibrium point for the current TS when compared to its average of remaining TS. This method improves the quality of each video stream from 0.18-0.34 dB for g12 to 0.82-1.13 dB for g9 over the EQL_TS method. Finally, we assume that we have no prior knowledge about the video and we predict the future RD curves by looking at the previous TS. Again we compute the competitive equilibrium for the current TS and the predicted future TS based on the average of the previous TS (PRE_TS curve in the figure). This method improves the PSNR from 0.11-0.23 dB for g12 to 0.50-0.80 dB for g9. Similarly, Figure 4 shows the results for two videos (the two most extreme cases) when



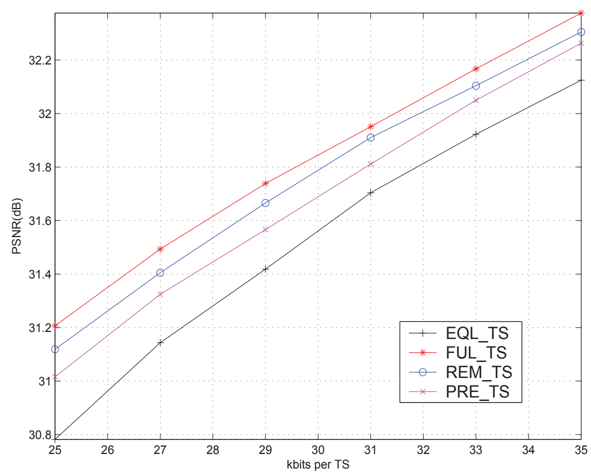
(a) g1 video stream



(b) g9 video stream



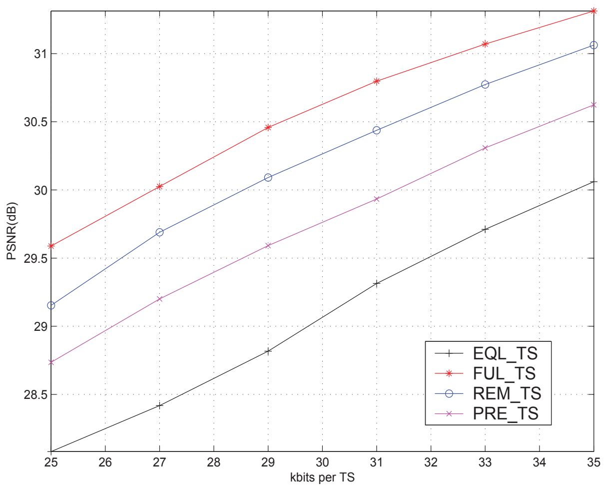
(c) g10 video stream



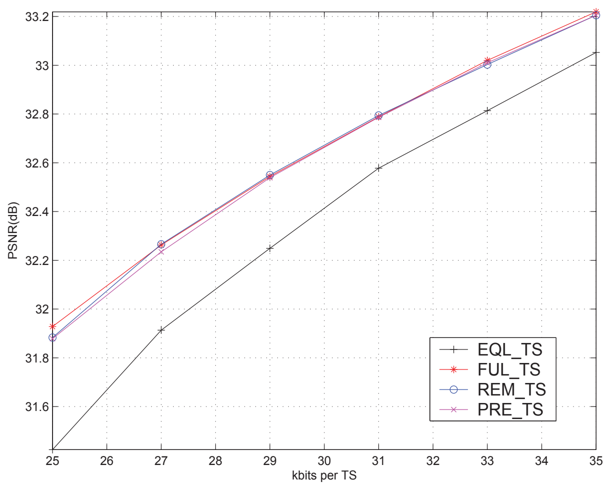
(d) g12 video stream

Fig. 3. PSNR variation with bitrate for four multiplexed video streams

six video streams are multiplexed together using the methods described above. The PSNR of all the six videos improves from these multiplexing methods for a wide range of bitrates. The g9 video again performs the best while the performance for REM_TS, PRE_TS, and FUL_TS are the same for the g5. We note that the largest PSNR gain is achieved by finding the competitive equilibrium when there is a lot of fluctuation in the video motion, for example g9. Conversely, the PSNR gain is low if the motion fluctuation in a video stream is low, for example g12. Most of the video streams have a lot of motion fluctuation and scene changes, so multiplexing them by computing the competitive equilibrium should improve their quality. The performance of PRE_TS depends on how accurate is the representation of future TS from past TS. As can be seen from Figure 3, all the video streams gain from the multiplexing process. The PSNR gain varies from one video to another, depending on the content. The multiplexing method using the competitive equilibrium borrows bits from a low motion TS of a video and gives these bits to another video in the same TS with the promise of taking it back when the need arises. So, the multiplexing method exchanges bits between video streams as well as across the TS. This leads to another observation that the quality fluctuation for each video stream is reduced because the high motion TS gets more bits than the low motion TS instead of getting the same number of bits for all TS. Figure 5 shows the PSNR fluctuation for g9 for all the multiplexing methods. The EQL_TS method has the highest fluctuation and FUL_TS method has the lowest. In the end, all the videos receive equal numbers of bits in the multiplexing method unlike previous methods for video multiplexing where some videos get more bits than the other videos. By changing the encoding technique inside a GOP (e.g., using multiple reference frame prediction or using hierarchical B-frames), along with these multiplexing methods, the overall video quality can be expected to further improve.



(a) g9 video stream



(b) g5 video stream

Fig. 4. PSNR variation with bitrate for six multiplexed video streams

In conclusion, we discussed four methods for multiplexing video streams. We proposed two novel methods of multiplexing video streams using the EW box solution for finding competitive equilibrium. The results show PSNR improvement for all videos unlike previous methods [1–3] where the quality of some videos is improved while degrading the quality of other videos. The PSNR gain is greater for videos with higher motion fluctuation. In future, we will examine alternative methods for estimating future frames from information of previous frames and *a priori* information about the stationarity of the video stream.

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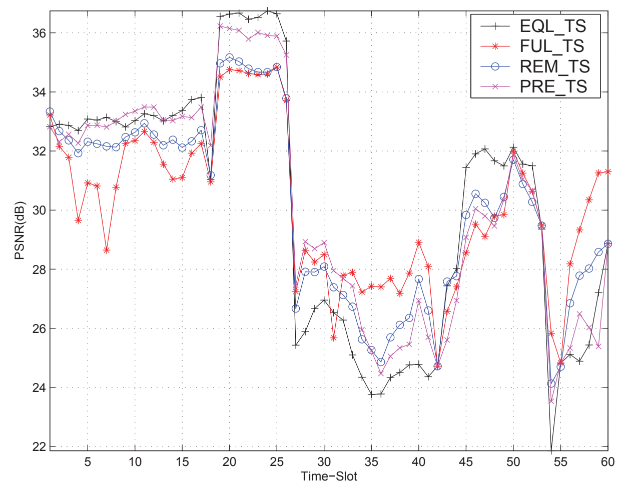

Fig. 5. Variation of PSNR with TS for g9 video at 50 kbps