

# Secure Wireless Communications via Cooperation

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**Abstract**—The feasibility of physical-layer-based security approaches for wireless communications in the presence of one or more eavesdroppers is hampered by channel conditions. In this paper, cooperation is investigated as an approach to overcome this problem and improve the performance of secure communications. In particular, a decode-and-forward (DF) based cooperative protocol is considered, and the objective is to design the system for secrecy capacity maximization or transmit power minimization. System design for the DF-based cooperative protocol is first studied by assuming the availability of global channel state information (CSI). For the case of one eavesdropper, an iterative scheme is proposed to obtain the optimal solution for the problem of transmit power minimization. For the case of multiple eavesdroppers, the problem of secrecy capacity maximization or transmit power minimization is in general intractable. Suboptimal system design is proposed by adding an additional constraint, i.e., the complete nulling of signals at all eavesdroppers, which yields simple closed-form solutions for the aforementioned two problems. Then, the impact of imperfect CSI of eavesdroppers on system design is studied, in which the ergodic secrecy capacity is of interest.

## I. INTRODUCTION

Due to the broadcast nature of wireless channels, the issues of privacy and security in wireless networks have taken on an increasingly important role, especially in military and homeland security applications. Physical (PHY) layer based security using an information-theoretic point of view is attracting much attention in this context. The basic idea of PHY-based security is to exploit the physical characteristics of the wireless channel. In the real world, signals transmitted over physical channels experience impairments such as channel fading and additive noise. While channel fading and thermal noise have traditionally been viewed as impediments, PHY layer security approaches can exploit these channel characteristics in order to enhance the security of digital communication systems. This line of work was pioneered by Wyner, who introduced the wiretap channel and established the possibility of creating almost perfectly secure communication links without relying on private (secret) keys [1]. Wyner showed that when the eavesdropper channel is a degraded version of the main channel, the source and destination can exchange perfectly secure messages at a non-zero rate, while the eavesdropper can learn almost nothing about the messages from its observations. The maximal rate of secrecy information from the source to its intended destination is defined by the term

*secrecy capacity*. Follow-up work by Leung-Yan-Cheong and Hellman characterized the secrecy capacity of scalar Gaussian wire-tap channel [2]. In a further paper, Csiszár and Körner generalized Wyner's approach by considering the transmission of confidential messages over broadcast channels [3]. Recently, there have been considerable efforts devoted to generalizing these studies to the wireless channel and multi-user scenarios (see [4]-[6] and references therein).

The feasibility of traditional PHY-based security approaches based on single antenna systems is hampered by channel conditions: if the channel between source and destination is worse than the channel between source and eavesdropper, the secrecy capacity is typically zero [1],[2]. Some recent work has been proposed to overcome this limitation by taking advantage of multiple antenna systems, e.g., multiple-input multiple-output (MIMO) [7],[8], single-input multiple-output (SIMO) [9] and multiple-input single-output (MISO) [10],[11]. However, due to cost and size limitations, multiple antennas may not be available at network nodes. Under such scenarios, node cooperation is an effective way to enable single-antenna nodes to enjoy the benefits of multiple-antenna systems [12].

In this paper, we consider a situation in which each network node is equipped with only a single omni-directional antenna and there are one or more eavesdroppers in the network. Secure communication is achieved via node cooperation in a decode-and-forward (DF) fashion. We assume that source and relays are located in the same cluster, while destination and eavesdropper(s) are at faraway locations outside this cluster. We propose a two-stage cooperative protocol. In Stage 1, the source node broadcasts its message locally to other nodes within the cluster. These local transmissions typically require a small amount of power only, and the information rate at faraway eavesdropper(s) can be ignored. Thus, transmissions in Stage 1 can be considered to be secure. In Stage 2, relay nodes decode the received messages. Then, the source node and relay nodes cooperatively transmit a weighted version of the message signal to the destination.

Our focus is on secret communications in Stage 2. We are interested in two optimization problems: (1) design node weights to maximize the secrecy capacity for a fixed transmit power; and (2) design node weights to minimize the transmit power for a fixed secrecy capacity. We assume that the global channel state information (CSI) is available for weight design. Cooperation is here used in place of multiple transmit antennas in MISO systems. Since there is a step involved

before transmission, during which the information is made available to the relays, the corresponding secrecy capacity is half of that corresponding to a MISO system. We should also point out that existing results for system design for a centralized MISO system can be also applied in system design for DF-based cooperative protocols. For example, in the case of one eavesdropper, the closed-form expression for weights that maximize the secrecy capacity subject to a transmit power constraint has been studied in [10], [11]. Beyond existing results in [10],[11], we here propose the following new results for the DF-based cooperative protocol: (1) For the case of one eavesdropper, we study system design to minimize the transmit power for a fixed secrecy capacity. We propose an iterative algorithm to reach the optimal solution, by using the solution for the problem of maximizing the secrecy capacity for a fixed transmit power. (2) Prior work considered the presence of one eavesdropper only. For the case of multiple eavesdroppers, the aforementioned optimization problems are in general intractable. We obtain a suboptimal (in terms of secrecy capacity or transmit power) but simple closed-form solution, by introducing an additional constraint, i.e., complete nulling of signals at all eavesdroppers. (3) Prior work assumed either complete knowledge of the eavesdroppers' channels, or only the channel statistics. In this paper, we investigate the weight design for the more practical case in which only imperfect estimates of eavesdroppers' channels are available.

This paper is organized as follows. In Section II, the system model and the DF-based cooperative protocol is described. In Section III, single and multiple eavesdroppers cases are investigated for the secrecy capacity maximization problem and the power minimization problem. The case of imperfect CSI of eavesdroppers is also studied. Simulations are described in Section IV, and conclusions are drawn in Section V.

We adopt the following notation. Bold uppercase letters denote matrices and bold lowercase letters denote column vectors. Transpose and conjugate transpose are represented by  $(\cdot)^T$  and  $(\cdot)^\dagger$  respectively;  $\mathbf{I}_M$  is the identity matrix of size  $M \times M$ ;  $\text{diag}\{\mathbf{a}\}$  denotes a diagonal matrix with the elements of vector  $\mathbf{a}$  along its diagonal;  $\mathbf{0}_{M \times N}$  denotes an all-zero matrix of size  $M \times N$ ;  $\mathcal{CN}(\mu, \sigma^2)$  denotes circularly symmetric, complex Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ ;  $\mathbb{E}\{\cdot\}$  denotes expectation.

## II. SYSTEM MODEL AND COOPERATIVE PROTOCOL

### A. System Model

We consider a wireless network model consisting of one source node (node index: 0),  $N - 1$  ( $N > 1$ ) trusted relay nodes (node index 1, 2,  $\dots$ ,  $N - 1$ ), a destination node, and  $J$  ( $J \geq 1$ ) eavesdroppers. We assume that the source and relays are located within the same cluster, while the destination and eavesdropper(s) are at faraway locations from this cluster. Each node is equipped with a single omni-directional antenna and operates in half-duplex mode.

A narrowband message signal  $s_0$  is to be transmitted from the source to the destination. The power of the message signal  $s_0$  is normalized to one, i.e.,  $\mathbb{E}\{|s_0|^2\} = 1$ . All channels are

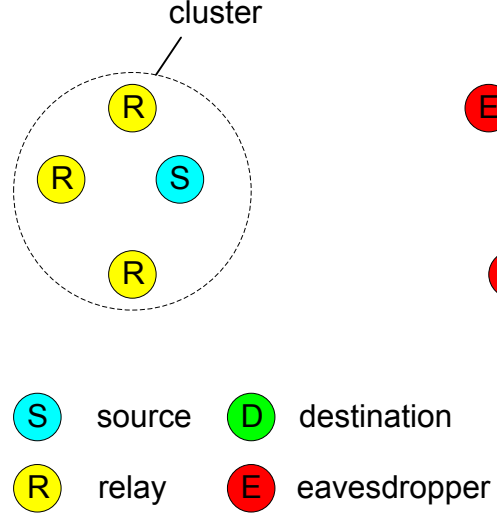


Fig. 1. System model in the presence of eavesdroppers.

flat fading. Let  $h_i$  denote the baseband complex channel gain between the  $i$ th cluster node and the destination, and  $g_{i,j}$  denote the channel gain between the  $i$ th cluster node and the  $j$ th eavesdropper. Thermal noise at all nodes is assumed to be zero-mean white complex Gaussian, i.e.,  $\mathcal{CN}(0, \sigma^2)$ . The configuration is illustrated in Fig. 1.

We assume that the global CSI is available for system design (the same assumption as in most of PHY-based security literature). In practice, destination-related CSI can be obtained by periodic pilots, and eavesdroppers-related CSI and the number of eavesdroppers may be obtained by monitoring the behavior of eavesdroppers. A cluster head (CH) then collects the global CSI, executes the weight computation algorithm and sends the weights back to cluster nodes for cooperative transmissions.

A DF-based cooperative protocol will be used. The number of relays with successful decoding is assumed to be known a priori (rather than being a random variable). To implement this in practice, each relay with successful decoding can send a non-interfering notification message to the CH.

### B. Cooperative Protocol

In this subsection, we describe the DF-based cooperative transmission protocol based on our system model.

**Stage 1:** The source broadcasts its message signal  $s_0$  locally to its trusted relays within the cluster. The transmit power is chosen so that the signal  $s_0$  can be decoded at the relays with high probability. In this paper, for simplicity we assume that the transmit power in Stage 1 is known a priori.

This stage usually requires a small amount of power only, and the information rate at the faraway eavesdropper(s) can be ignored. Thus, transmissions in Stage 1 can be considered to be secure.

#### Stage 2:

All the trusted relays that successfully decode the message  $s_0$ , together with the source, cooperatively transmit signal  $s_0$

to the destination. For convenience, we assume that all the  $N - 1$  relays successfully decode the message signal<sup>1</sup>. Then, totally  $N$  nodes ( $N - 1$  relays plus one source), indexed by  $i = 0, \dots, N - 1$ , participate in cooperative transmissions in Stage 2. Specifically, the  $i$ th node transmits a weighted signal of  $s_0$ , i.e.,  $w_i s_0$ ,  $i = 0, \dots, N - 1$ , where  $w_i$  is the weight of the  $i$ th node.

Let us define the  $N \times 1$  vectors  $\mathbf{w} = [w_0, \dots, w_{N-1}]^H$ ,  $\mathbf{h} = [h_0, \dots, h_{N-1}]^H$  and  $\mathbf{g}_j = [g_{0,j}, \dots, g_{N-1,j}]^H$ , and the  $N \times N$  matrices  $\mathbf{R}_h = \mathbf{h}\mathbf{h}^H$  and  $\mathbf{R}_g^j = \mathbf{g}_j\mathbf{g}_j^H$ .

At the destination, the received signal  $y_d$  equals

$$y_d = \mathbf{w}^H \mathbf{h} s_0 + n_d, \quad (1)$$

where  $n_d$  represents white complex Gaussian noise at the destination. Then, the capacity at the destination is

$$C_d = \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2} \right) \quad (2)$$

where the scalar factor  $1/2$  is due to the fact that two time units are required in the two-stage cooperative protocol.

At the  $j$ th eavesdropper, the received signal  $y_e^j$  equals

$$y_e^j = \mathbf{w}^H \mathbf{g}_j s_0 + n_e^j, \quad (3)$$

where  $n_e^j$  represents white complex Gaussian noise at the  $j$ th eavesdropper. The capacity at the  $j$ th eavesdropper is then

$$C_e^j = \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_g^j \mathbf{w}}{\sigma^2} \right). \quad (4)$$

Our objective is to design the node weights to maximize secrecy capacity for a fixed transmit power, or minimize transmit power for a fixed secrecy capacity. The secrecy capacity for  $J$  eavesdroppers is defined as [5]:

$$C_s = \max\{0, C_d - \max(C_e^1, \dots, C_e^J)\}. \quad (5)$$

### III. SYSTEM DESIGN FOR SECURE WIRELESS COMMUNICATIONS

In this section, we discuss the weight design for the DF-based cooperative protocol to achieve secure wireless communications, for the cases of one eavesdropper and multiple eavesdroppers, respectively.

#### A. One Eavesdropper

We first discuss the simple scenario of one eavesdropper. For notational convenience, the index of the eavesdropper is dropped. As long as  $\mathbf{h} \neq \mathbf{g}$ , we can always find a set of weights so that the secrecy capacity is non-zero. For example, one can completely null out the received signal at the eavesdropper. Thus, from (2) and (4), Eq. (5) can be written as

$$C_s = C_d - C_e = \frac{1}{2} \log_2 \left( \frac{\sigma^2 + \mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2 + \mathbf{w}^H \mathbf{R}_g \mathbf{w}} \right). \quad (6)$$

<sup>1</sup>the case in which  $M < N - 1$  relays successfully decode the message is equivalent to the case in which the total number of relays is  $M$ .

*1) Maximizing Secrecy Capacity for Fixed Transmit Power:* The problem of maximizing the secrecy capacity  $C_s$  for a fixed transmit power  $\mathbf{w}^H \mathbf{w} = P_0$  can be formulated as

$$\begin{aligned} \arg \max_{\mathbf{w}} & \frac{\sigma^2 + \mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2 + \mathbf{w}^H \mathbf{R}_g \mathbf{w}} \\ \text{s.t.} & \mathbf{w}^H \mathbf{w} = P_0. \end{aligned} \quad (7)$$

The solution of this Rayleigh quotient problem, reported in [10],[11], is the scaled eigenvector corresponding to the largest eigenvalue of the symmetric matrix  $\tilde{\mathbf{R}}_g^{-1} \tilde{\mathbf{R}}_h$ , where

$$\tilde{\mathbf{R}}_h \triangleq \frac{\sigma^2}{P_0} \mathbf{I}_N + \mathbf{R}_h \quad (8)$$

and

$$\tilde{\mathbf{R}}_g \triangleq \frac{\sigma^2}{P_0} \mathbf{I}_N + \mathbf{R}_g. \quad (9)$$

Also, the equality power constraint in (7) is equivalent to the inequality power constraint  $\mathbf{w}^H \mathbf{w} \leq P_0$  [10],[11]. As we will show in the next subsection, the solution of the problem in (7) can help solve another optimization problem of minimizing transmit power under a fixed secrecy capacity.

*2) Minimizing Transmit Power for Fixed Secrecy Capacity:* The problem of minimizing the transmit power  $\mathbf{w}^H \mathbf{w}$  for a fixed secrecy capacity  $C_s^0 > 0$  can be formulated as

$$\begin{aligned} \arg \min_{\mathbf{w}} & \mathbf{w}^H \mathbf{w} \\ \text{s.t.} & \frac{\sigma^2 + \mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2 + \mathbf{w}^H \mathbf{R}_g \mathbf{w}} = 4^{C_s^0}. \end{aligned} \quad (10)$$

However, the conventional method of Lagrange multipliers does not work for (10), as it yields a zero solution of  $\mathbf{w}$ . To solve (10), we first propose the following.

*Proposition 1:* The solutions of the following two optimization problems are identical:

(i) Find the weights that maximize  $C_s$  for a fixed transmit power  $P_0$ .

(ii) Find the weights that minimize the transmit power for a fixed  $C_s^{\max}$ , where  $C_s^{\max}$  is the maximal  $C_s$  of problem (i).

*Proof:* We prove Proposition 1 by contradiction. We assume that  $\mathbf{w}^{(1)}$  is the optimal solution that yields  $C_s^{(1)} = C_s^{\max}$  for fixed transmit power  $P_0$ , while a different weight vector  $\mathbf{w}^{(2)} \neq \mathbf{w}^{(1)}$  minimizes the transmit power for fixed  $C_s^{(2)} = C_s^{\max}$ . Thus, the transmit power  $(\mathbf{w}^{(2)})^H \mathbf{w}^{(2)}$  must be smaller than  $(\mathbf{w}^{(1)})^H \mathbf{w}^{(1)} = P_0$ . We can always find a scalar  $\rho > 1$  such that the weight vector  $\rho \cdot \mathbf{w}^{(2)}$  also achieves  $\rho^2 (\mathbf{w}^{(2)})^H \mathbf{w}^{(2)} = P_0$ .

Now, we prove that  $C_s$  based on the weight vector  $\rho \cdot \mathbf{w}^{(2)}$  is greater than  $C_s^{\max}$ . Let us define the function

$$F(z) = \frac{\sigma^2 + z^2 (\mathbf{w}^{(2)})^H \mathbf{R}_h \mathbf{w}^{(2)}}{\sigma^2 + z^2 (\mathbf{w}^{(2)})^H \mathbf{R}_g \mathbf{w}^{(2)}}. \quad (11)$$

We can equivalently prove  $F(\rho) > F(1)$  for  $\rho > 1$ . Taking the derivative of  $F(z)$  with respect to  $z$ , we obtain

$$\frac{dF(z)}{dz} \propto (\mathbf{w}^{(2)})^H \mathbf{R}_h \mathbf{w}^{(2)} - (\mathbf{w}^{(2)})^H \mathbf{R}_g \mathbf{w}^{(2)}. \quad (12)$$

As  $C_s > 0$ ,  $\frac{dF(z)}{dz} > 0$ . Thus,  $F(z)$  is a monotonically increasing function of  $z$  and it follows that  $F(\rho) > F(1)$  for  $\rho > 1$ . Hence, we have proved that  $C_s$  based on the weight vector  $\rho \cdot \mathbf{w}^{(2)}$  is greater than  $C_s^{\max}$ . In other words,  $C_s^{\max}$  is not the maximal value of  $C_s$  for transmit power  $P_0$ , which contradicts our assumption. Therefore,  $\mathbf{w}^{(1)}$  must be equal to  $\mathbf{w}^{(2)}$ , and thus Proposition 1 is proved. ■

Based on Proposition 1, we now propose the following iterative algorithm for finding the optimal solution of (10).

• **Initialization:**

S0) Set an initial value for the weights  $\rho^{(0)} \mathbf{w}^{(0)}$ , where  $\rho^{(0)}$  is a scalar such that  $C_s$  for  $\mathbf{w}^{(0)}$  equals  $C_s^0$ . Note that  $\mathbf{w}^{(0)}$  can be arbitrarily chosen but its corresponding secrecy capacity must be greater than zero. Then, compute the transmit power  $P^{(0)} = (\rho^{(0)})^2 (\mathbf{w}^{(0)})^H \mathbf{w}^{(0)}$ .

• **Iteration:**

S1) In the  $k$ th iteration, compute the weight vector  $\mathbf{w}^{(k)}$  that maximizes the secrecy capacity for fixed transmit power  $P^{(k-1)}$ , based on the method discussed in Section III-A.1.

S2) Compute the scalar  $\rho^{(k)}$ , such that  $C_s$  under  $\rho^{(k)} \mathbf{w}^{(k)}$  equals  $C_s^0$ . Calculate the updated transmit power  $P^{(k)} = (\rho^{(k)})^2 (\mathbf{w}^{(k)})^H \mathbf{w}^{(k)}$ .

S3) Iterate until  $P^{(k-1)} - P^{(k)}$  is smaller than a pre-defined threshold.

The objective function of (10) is convex and the updated power with each iteration is nonincreasing. Thus, the above algorithm eventually converges to the global minimum. In our simulations, the iteration always converged very rapidly.

## B. Multiple Eavesdroppers

In this subsection we discuss the scenario of  $J > 1$  eavesdroppers. From (5), the secrecy capacity for multiple eavesdroppers is related to the capacity at all eavesdroppers. Determining the weights that maximize secrecy capacity for fixed power, or minimize power for fixed secrecy capacity is in general intractable. In the following, we consider an additional constraint, i.e., completely nulling out signals at all eavesdroppers. The resulting secrecy capacity (transmit power) represents a lower (upper) bound of the optimal one.

1) *Minimizing Transmit Power for Fixed Secrecy Capacity:*

Let us define the  $N \times J$  matrix  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_J]$ . To null the signals at all eavesdroppers, we need

$$\mathbf{w}^H \mathbf{G} = \mathbf{0}_{1 \times J}. \quad (13)$$

To satisfy the fixed secrecy capacity  $C_s^0$ , we also need

$$C_s^0 = C_d = \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2} \right). \quad (14)$$

Eq. (14) can also be written as

$$\mathbf{w}^H \mathbf{h} = \sqrt{(4C_s^0 - 1)\sigma^2} \cdot e^{j\theta} \quad (15)$$

where  $\theta$  is an arbitrary angle within  $[0, 2\pi)$ .

Defining the  $(J+1) \times N$  matrix  $\tilde{\mathbf{G}} = [\mathbf{h}, \mathbf{G}]^H$  and the  $(J+1) \times 1$  vector  $\mathbf{e} = [1, \mathbf{0}_{1 \times J}]^T$ , we can rewrite the constraints in (13) and (15) as

$$\tilde{\mathbf{G}} \mathbf{w} = (\sqrt{(4C_s^0 - 1)\sigma^2} \cdot e^{j\theta}) \mathbf{e}. \quad (16)$$

To guarantee a non-zero solution for  $\mathbf{w}$ , we need  $N \geq J+1$ , which usually can be easily satisfied.

The optimal solution  $\mathbf{w}^{\text{opt}}$  that minimizes the transmit power corresponds to the least-squares solution of (16) produced by the pseudo-inverse of  $\tilde{\mathbf{G}}$  [13],[14], i.e.,

$$\mathbf{w}^{\text{opt}} = (\sqrt{(4C_s^0 - 1)\sigma^2} e^{j\theta}) \tilde{\mathbf{G}}^H (\tilde{\mathbf{G}} \tilde{\mathbf{G}}^H)^{-1} \mathbf{e}. \quad (17)$$

From (17), the transmit power  $(\mathbf{w}^{\text{opt}})^H \mathbf{w}^{\text{opt}}$  is independent of the selection of  $\theta$ . For convenience we can take  $\theta = 0$ .

2) *Maximizing Secrecy Capacity for Fixed Transmit Power:* The optimization problem can be formulated as

$$\begin{aligned} & \arg \max_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_h \mathbf{w} \\ \text{s.t. } & \mathbf{w}^H \mathbf{w} = P_0 \text{ and } \mathbf{w}^H \mathbf{G} = \mathbf{0}_{1 \times J}. \end{aligned} \quad (18)$$

The conventional method of Lagrange multipliers does not yield an insightful closed-form solution of (18). To solve (18), we propose the following.

*Proposition 2:* The solutions of the following two optimization problems are identical:

(i) Find the weights that maximize  $C_s$  for fixed transmit power  $P_0$ , and also meets the constraint that signals at all eavesdroppers are completely nulled. Let us denote the maximal  $C_s$  by  $C_s^{\max}$ .

(ii) Find the weights that minimize the transmit power for a fixed  $C_s^{\max}$  and also meets the constraint that signals at all eavesdroppers are completely nulled.

*Proof:* We follow arguments similar to those used in the proof of Proposition 1. We assume that weight vector  $\mathbf{w}^{(1)}$  achieves  $C_s^{\max}$  for the fixed transmit power  $P_0$ , while a different weight vector  $\mathbf{w}^{(2)} \neq \mathbf{w}^{(1)}$  achieves minimal transmit power for fixed  $C_s^{\max}$ . Thus, it holds that  $(\mathbf{w}^{(2)})^H \mathbf{w}^{(2)} < P_0$ . We can always find a scalar  $\rho > 1$  such that under the weights  $\rho \cdot \mathbf{w}^{(2)}$  the transmit power is  $\rho^2 (\mathbf{w}^{(2)})^H \mathbf{w}^{(2)} = P_0$ . However, the weight vector  $\rho \cdot \mathbf{w}^{(2)}$  achieves a secrecy capacity greater than  $C_s^{\max}$ . In other words,  $\mathbf{w}^{(1)}$  does not achieve the maximum of  $C_s$  for fixed power  $P_0$ , which contradicts our assumption. Therefore,  $\mathbf{w}^{(1)}$  must be equal to  $\mathbf{w}^{(2)}$ . ■

From Proposition 2, the optimization problem of (18) is equivalent to finding the weights that minimize the transmit power for fixed  $C_s^{\max}$ . From (17), the transmit power is proportional to  $4C_s^0 - 1$ . Thus, the solution of (18) is

$$\mathbf{w}^{\text{opt}} = \beta \tilde{\mathbf{G}}^H (\tilde{\mathbf{G}} \tilde{\mathbf{G}}^H)^{-1} \mathbf{e} \quad (19)$$

where  $\beta$  is a scalar and equals

$$\beta = \sqrt{\frac{P_0}{\mathbf{e}^H (\tilde{\mathbf{G}} \tilde{\mathbf{G}}^H)^{-1} \mathbf{e}}}. \quad (20)$$

Substituting (19) into the objective function of (18), one can see that the secrecy capacity is a monotonically increasing

function of the power budget  $P_0$ . Thus, the equality power constraint in (18) is equivalent to the inequality power constraint  $\mathbf{w}^H \mathbf{w} \leq P_0$ .

### C. Impact on Imperfect CSI of Eavesdroppers

The channels between cluster nodes and the destination can be estimated accurately, since they are trusted nodes. However, in practice there will be some certain estimation errors for the channels between cluster nodes and the eavesdroppers. In this subsection, we discuss weight design for such cases.

We model the perfect channels of the  $j$ th eavesdropper as  $\mathbf{g}_j = \hat{\mathbf{g}}_j + \Delta_j$ , where  $\hat{\mathbf{g}}_j$  is the imperfect channel estimate available for weight computation, and  $\Delta_j$  corresponds to the channel error. We further assume that the entries of  $\Delta_j$  are zero-mean random variables, and  $\mathbf{R}_\Delta \triangleq \mathbb{E}\{\Delta_j \Delta_j^H\}$  is known a priori and is independent of  $j$ . Thus, we obtain

$$\mathbf{R}_g^j \triangleq \mathbb{E}\{\mathbf{g}_j \mathbf{g}_j^H\} = \hat{\mathbf{R}}_g^j + \mathbf{R}_\Delta \quad (21)$$

where  $\hat{\mathbf{R}}_g^j = \hat{\mathbf{g}}_j \hat{\mathbf{g}}_j^H$ .

Note that we still assume the availability of perfect CSI of the destination.

1) *One Eavesdropper*: For one eavesdropper, the ergodic secrecy capacity is given by

$$\begin{aligned} \bar{C}_s &= \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2} \right) \\ &\quad - \mathbb{E} \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{g} \mathbf{g}^H \mathbf{w}}{\sigma^2} \right) \right\}. \end{aligned} \quad (22)$$

The optimization problem of maximizing ergodic secrecy capacity under a fixed power is in general difficult. To simplify the problem, we use Jensen's inequality to obtain

$$\begin{aligned} \bar{C}_s &\geq \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2} \right) \\ &\quad - \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_g \mathbf{w}}{\sigma^2} \right) \end{aligned} \quad (23)$$

in which the eavesdropper index is omitted for notational convenience. We now consider the problem of maximizing the lower bound on ergodic secrecy capacity in (23) under a fixed power  $\mathbf{w}^H \mathbf{w} = P_0$ . It is easy to see that this optimization problem is the same as (7), while the matrix  $\mathbf{R}_g$  is now given by (21). Also, the problem of minimizing the transmit power under a fixed lower bound on ergodic secrecy capacity can be solved by the iterative algorithm in section III-A.2.

2) *Multiple Eavesdroppers*: For  $J$  eavesdroppers ( $J > 1$ ), the lower bound on ergodic secrecy capacity is given by

$$\begin{aligned} \bar{C}_s &\geq \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2} \right) \\ &\quad - \max_j \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{\mathbf{w}^H \mathbf{R}_g^j \mathbf{w}}{\sigma^2} \right) \right\} \end{aligned} \quad (24)$$

where  $\mathbf{R}_g^j$  is given by (21).

To form nulls at all eavesdroppers, we need  $\mathbf{w}^H \mathbf{R}_g^j \mathbf{w} = 0$  for  $j = 1, \dots, J$ . A non-zero solution exists only if  $\mathbf{R}_\Delta$

is semi-positive definite. In case for which  $\mathbf{R}_\Delta$  is strictly positive definite, nulls cannot be formed at eavesdroppers, and  $\mathbf{w}^H \mathbf{R}_g^j \mathbf{w}$  is always greater than zero. To cover all cases, here we still consider the constraint  $\mathbf{w}^H \hat{\mathbf{R}}_g^j \mathbf{w} = 0$  or equivalently  $\mathbf{w}^H \hat{\mathbf{g}}_j = 0$ . The optimization problem of maximizing the lower bound on the ergodic secrecy capacity in (23) under a fixed power can be formulated as

$$\begin{aligned} &\arg \max_{\mathbf{w}} \frac{\sigma^2 + \mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\sigma^2 + \mathbf{w}^H \mathbf{R}_\Delta \mathbf{w}} \\ &\text{s.t. } \hat{\mathbf{G}} \mathbf{w} = \mathbf{0}_{J \times 1} \text{ and } \mathbf{w}^H \mathbf{w} = P_0 \end{aligned} \quad (25)$$

where  $\hat{\mathbf{G}} \triangleq [\hat{\mathbf{g}}_1, \dots, \hat{\mathbf{g}}_J]^H$ . Let us define the matrix  $\mathbf{T}$  containing all of the right singular vectors corresponding to zero singular values of  $\hat{\mathbf{G}}$ . To satisfy the first constraint in (25),  $\mathbf{w}$  shall be a linear combination of basis in the null space of  $\hat{\mathbf{G}}$ , i.e.,  $\mathbf{w} = \mathbf{T} \mathbf{v}$ , where  $\mathbf{v}$  is a column vector. Then, the optimization problem in (25) is equivalent to

$$\begin{aligned} &\arg \max_{\mathbf{v}} \frac{\sigma^2 + \mathbf{v}^H \mathbf{T}^H \mathbf{R}_h \mathbf{T} \mathbf{v}}{\sigma^2 + \mathbf{v}^H \mathbf{T}^H \mathbf{R}_\Delta \mathbf{T} \mathbf{v}} \\ &\text{s.t. } \mathbf{v}^H \mathbf{v} = P_0 \end{aligned} \quad (26)$$

which is a Rayleigh quotient problem similar to (7). The final solution of (25) is then  $\mathbf{w} = \sqrt{P_0} \mathbf{T} \mathbf{q}_{\text{unit}}$  where  $\mathbf{q}_{\text{unit}}$  is the unit-norm eigenvector of the matrix  $\mathbf{T}^H [\mathbf{R}_\Delta + (\sigma^2/P_0) \mathbf{I}]^{-1} [\mathbf{R}_h + (\sigma^2/P_0) \mathbf{I}] \mathbf{T}$  corresponding to its largest eigenvalue.

Due to the similarity between (26) and (7), and the duality as shown in Proposition 1, the problem of minimizing the transmit power under a fixed lower bound on secrecy capacity can be solved by the iterative algorithm in section III-A.2.

### D. Discussion

In the above analysis, for convenience we have assumed that the transmit power in Stage 1 is much smaller than the transmit power in Stage 2, and thus the information rates in Stage 1 at the faraway destination and eavesdropper(s) are ignored. In this subsection, we discuss the effects on weight design when the information rates in Stage 1 are also taken into account.

When both stages are taken into account, the destination or an eavesdropper combines the two received signal in both stages using maximal ratio combining (MRC) in order to maximize the signal-to-noise ratio (SNR). Suppose that transmit power in Stage 1 is  $\tilde{P}_0$ . The capacity at the destination is given by

$$C_d = \frac{1}{2} \log_2 \left( \alpha + \frac{\mathbf{w}^\dagger \mathbf{R}_a \mathbf{w}}{\sigma^2} \right) \quad (27)$$

where  $\alpha \triangleq 1 + \tilde{P}_0 |h_0|^2 / \sigma^2$ . Note that  $\tilde{P}_0 |h_0|^2 / \sigma^2$  is the received SNR in Stage 1 at the destination. Similarly, the capacity at the  $j$ th eavesdropper is

$$C_e^j = \frac{1}{2} \log_2 \left( \mu + \frac{\mathbf{w}^\dagger \mathbf{R}_b^j \mathbf{w}}{\sigma^2} \right) \quad (28)$$

where  $\mu \triangleq 1 + \tilde{P}_0 |g_{0,j}|^2 / \sigma^2$ . Note that  $\tilde{P}_0 |g_{0,j}|^2 / \sigma^2$  is the received SNR in Stage 1 at the  $j$ th eavesdropper. Here,  $\alpha$  and

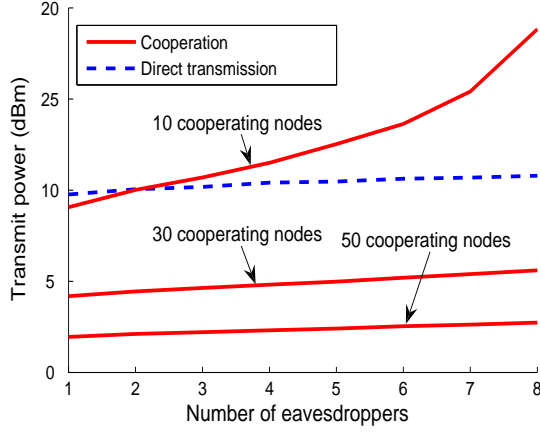


Fig. 2. Transmit power vs. number of eavesdroppers. Secrecy capacity is fixed at  $C_s^0 = 3$  b/s/Hz.

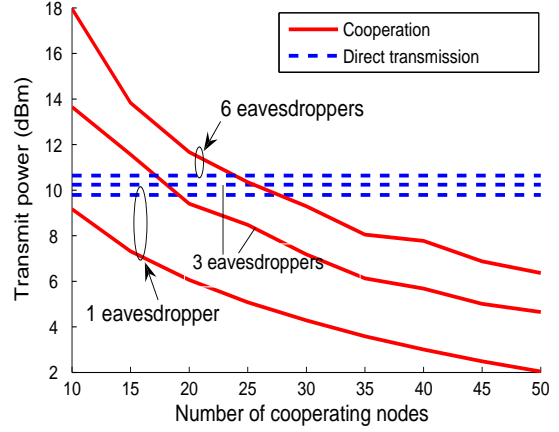


Fig. 3. Transmit power vs. number of cooperating nodes. Secrecy capacity is fixed at  $C_s^0 = 3$  b/s/Hz.

$\mu$  are considered to be constants, as  $\tilde{P}_0$  is assumed to be a priori.

Therefore, the only change on the capacity of the destination or eavesdropper is to replace the constant one in (2) or (4) by  $\alpha$  or  $\mu$ . It is easy to show that most of the proposed analysis (when ignoring Stage 1) can still be applied here, subject to minor changes only. The only exception is the power minimization problem for the case of one eavesdropper (see Section III-A.2). For this case, in order to guarantee the validation of Proposition 1, the fixed secrecy capacity  $C_s^0$  should be chosen to satisfy  $\mu \mathbf{w}^H \mathbf{R}_h \mathbf{w} > \alpha \mathbf{w}^H \mathbf{R}_g \mathbf{w}$  for every possible  $\mathbf{w}$ .

#### IV. SIMULATIONS

In this section, we investigate the performance of weight design algorithms via simulations. In these simulations, the carrier frequency is 900 MHz and the signal wavelength is  $\lambda = 0.33$  m. The noise power  $\sigma^2$  is  $-60$  dBm. The cluster is a disk with radius  $R = 5\lambda$ . The cluster nodes are uniformly located in the disk. For convenience, a simple line-of-sight channel model is used:  $h_i = d_i^{-\frac{\alpha}{2}} e^{j\phi_i}$  where  $d_i$  is the distance between the  $i$ th node and the destination,  $\alpha = 4$  is the path loss exponent and  $\phi_i$  denotes the phase offset.  $g_{ij}$  is defined in a similar way. All channel estimates are assumed to be perfect.

We try to compare the performance of DF-based cooperation with direct transmission (without cooperation). Based on the line-of-sight channel model, when the distance between any eavesdropper and the source is smaller than the distance between the destination and the source, the secrecy capacity of direct transmission without cooperation is always zero no matter how large the transmit power is. Thus, under such scenarios, cooperation always outperforms direct transmission. In the following simulations, we will focus on the case in which the distances between eavesdroppers and the source are greater than the distance between the destination and the source. The distances between the source and destination is  $20R$ . The distances between the source and eavesdroppers are

uniformly distributed within  $[40R, 100R]$ , and the azimuthal directions of eavesdroppers are uniformly distributed within  $[0, 2\pi)$ . We perform a Monte-Carlo experiment consisting of 1000 independent trials to obtain the average results. Locations of cluster nodes and eavesdroppers in one trial are chosen independently from those in other trials.

##### A. Fixed Secrecy Capacity

We first fix the secrecy capacity at  $C_s^0 = 3$  b/s/Hz and investigate the performance of transmit power. Fig. 2 shows the transmit power versus number of eavesdropper  $J$ . The number of cooperating nodes  $N$  is 10, 30 or 50. For a single eavesdropper, the transmit power with cooperation is obtained based on the iterative algorithm in Section III-A.2. For multiple eavesdroppers, the transmit power with cooperation is computed from (17). As observed, for both cooperation and direct transmission, more transmit power would be needed as the number of eavesdroppers increases. When the number of cooperating nodes is small, cooperation may not outperform direct transmission (see the curve for  $N = 10$  in Fig. 2), as its transmission time is longer. When the number of cooperating nodes is large, cooperation requires much less transmit power than direct transmission (see the curves for  $N = 30, 50$  in Fig. 2). Fig. 3 shows the transmit power versus number of cooperating nodes  $N$ . The number of eavesdroppers  $J$  is one, three or six. As expected, the transmit power for cooperation decreases as the number of cooperating nodes  $N$  increases, while the transmit power of direct transmission is independent of  $N$ .

##### B. Fixed Transmit Power

In this subsection, we investigate the performance of secrecy capacity by fixing the transmit power at  $P_0 = 5$  dBm. Fig. 4 shows the secrecy capacity versus number of eavesdroppers. For a single eavesdropper, the secrecy capacity with cooperation is obtained based on the result in Section III-A.1. For multiple eavesdroppers, the secrecy capacity with cooperation is computed based on the nulling weights of (19).

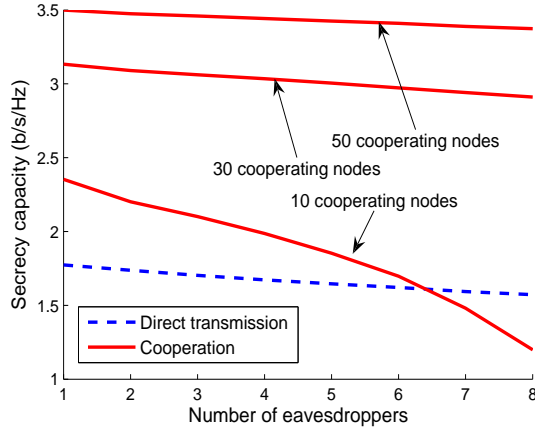


Fig. 4. Secrecy capacity vs. number of eavesdroppers. Transmit power is fixed at  $P_0 = 5$  dBm.

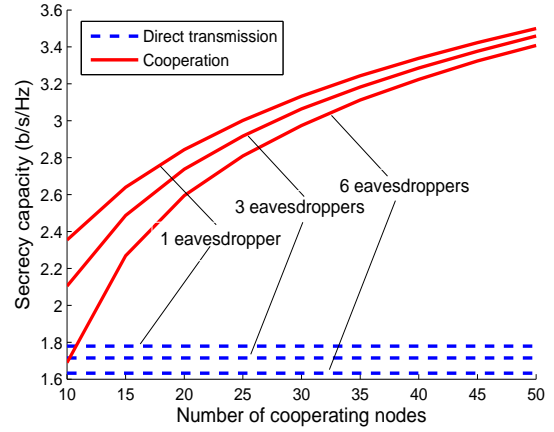


Fig. 5. Secrecy capacity vs. number of cooperating nodes. Transmit power is fixed at  $P_0 = 5$  dBm.

As expected, the secrecy capacity decreases as the number of eavesdroppers increases. A larger number of cooperating nodes yields higher secrecy capacity. Fig. 5 shows the secrecy capacity versus number of cooperating nodes  $N$ . The secrecy capacity for cooperation increases as  $N$  increases, while the secrecy capacity of direct transmission is independent of  $N$ .

## V. CONCLUSIONS

In this paper, we have considered a DF-based cooperative protocol to improve the performance of secure wireless communications in the presence of one or more eavesdroppers. For the case of one eavesdropper, we have considered the design problem of transmit power minimization and have proposed an iterative algorithm to reach the solution, by the help of existing results for another problem of secrecy capacity maximization. For the case of multiple eavesdroppers, we have derived suboptimal and closed-form solutions for the problems of transmit power minimization and secrecy capacity maximization by adding an additional constraint, i.e., the complete nulling of signals at all eavesdroppers. We have also investigated the impact of imperfect CSI of eavesdroppers on system design.

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