

Distributed Hypothesis Testing with Collaborative Detection

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Abstract—A detection system with a single sensor and two detectors is considered, where each of the terminals observes a memoryless source sequence, the sensor sends a message to both detectors and the first detector sends a message to the second detector. Communication of these messages is assumed to be error-free but rate-limited. The joint probability mass function (pmf) of the source sequences observed at the three terminals depends on an M -ary hypothesis ($M \geq 2$), and the goal of the communication is that each detector can guess the underlying hypothesis. Detector k , $k = 1, 2$, aims to maximize the error exponent *under hypothesis* i_k , $i_k \in \{1, \dots, M\}$, while ensuring a small probability of error under all other hypotheses. We study this problem in the case in which the detectors aim to maximize their error exponents under the *same* hypothesis (i.e., $i_1 = i_2$) and in the case in which they aim to maximize their error exponents under *distinct* hypotheses (i.e., $i_1 \neq i_2$). For the setting in which $i_1 = i_2$, we present an achievable exponents region for the case of positive communication rates, and show that it is optimal for a specific case of testing against independence. We also characterize the optimal exponents region in the case of zero communication rates. For the setting in which $i_1 \neq i_2$, we characterize the optimal exponents region in the case of zero communication rates.

I. INTRODUCTION

Consider the multiterminal hypothesis testing scenario shown in Figure 1, where an encoder observes a discrete memoryless source sequence $X^n \triangleq (X_1, \dots, X_n)$ and communicates with two remote detectors 1 and 2 over a common noise-free bit-pipe of rate $R_1 \geq 0$. Here, n is a positive integer that denotes the blocklength. Detectors 1 and 2 observe correlated memoryless source sequences $Y_1^n \triangleq (Y_{1,1}, \dots, Y_{1,n})$ and $Y_2^n \triangleq (Y_{2,1}, \dots, Y_{2,n})$, and Detector 1 can communicate with Detector 2 over a noise-free bit-pipe of rate R_2 . The sequence of observation triples $\{(X_t, Y_{1,t}, Y_{2,t})\}_{t=1}^n$ is independent and identically distributed (i.i.d) according to a joint probability mass function (pmf) that is determined by the hypothesis $\mathcal{H} \in \{1, \dots, M\}$. Under the hypothesis $\mathcal{H} = m$,

$$\{(X_t, Y_{1,t}, Y_{2,t})\}_{t=1}^n \text{ i.i.d. } \sim P_{XY_1Y_2}^{(m)}. \quad (1)$$

Detector 1 decides on a hypothesis $\hat{\mathcal{H}}_1 \in \{1, \dots, M\}$ with the goal to maximize the exponential decrease of the probability of error *under hypothesis* $\mathcal{H} = i_1 \in \{1, \dots, M\}$ (i.e., guessing $\hat{\mathcal{H}}_1 \neq i_1$ when $\mathcal{H} = i_1$), while ensuring that the probability of error under $\mathcal{H} = m$ with $m \neq i_1$ (i.e., guessing $\hat{\mathcal{H}}_1 \neq m$) does not exceed some prescribed constant value $\epsilon_1 \in (0, 1)$ for all sufficiently large blocklengths n . Similarly,

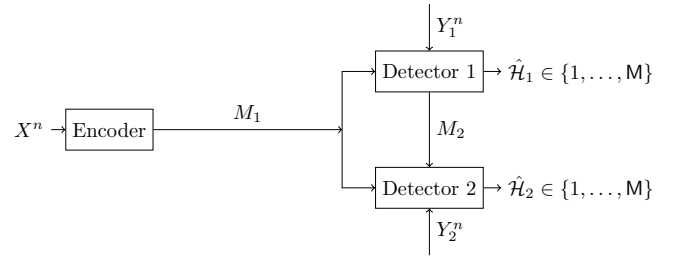


Fig. 1. A Heegard-Berger type source coding model with unidirectional conferencing for multiterminal hypothesis testing.

Detector 2 decides on a hypothesis $\hat{\mathcal{H}}_2 \in \{1, \dots, M\}$ with the goal to maximize the exponential decrease of the probability of error *under hypothesis* $\mathcal{H} = i_2 \in \{1, \dots, M\}$ (i.e., guessing $\hat{\mathcal{H}}_2 \neq i_2$ when $\mathcal{H} = i_2$), while ensuring that the probability of error under $\mathcal{H} = m$ with $m \neq i_2$ (i.e., guessing $\hat{\mathcal{H}}_2 \neq m$) does not exceed a constant value $\epsilon_2 \in (0, 1)$ for all sufficiently large blocklengths n .

In this paper, we study the problem of how cooperation among the two detectors can be used to improve the largest error exponents. We investigate this problem in both settings, the one in which the detectors aim at maximizing the error exponents under the *same* hypothesis (i.e., $i_1 = i_2$) and the one in which they aim at maximizing the error exponents under *different* hypotheses (i.e., $i_1 \neq i_2$).

A. Related Works

Problems of distributed hypothesis testing are strongly rooted in both statistics and information theory. In particular, the problem described above but without Detector 2 was studied in [1], [2]. In [1], Ahlswede and Csiszár presented a single-letter lower bound on the largest possible error exponent. It is optimal in the special case of *testing against independence*, but for the general case was improved by Han in [2]. Extensions of these results to networks with multiple encoders, multiple detectors, or interaction between terminals, can be found, e.g., in [3]–[11]. In particular, [5] studies the model considered here but without cooperation. [3] and [4] study the model of [5] in the specific case of testing against independence and conditional independence, respectively.

Han [2] also introduced a two-terminal hypothesis testing problem where the encoder sends a single bit to the decoder. He established a single-letter characterization of the optimal error exponent of this system. Subsequently, Shalaby and

Papamarcou [12] showed that Han's error exponent remains optimal even when the transmitter can send a sublinear number of bits to the receiver. Key to the derivation of the converse proof in [2] is an ingenious use of the "Blowing-Up" lemma [13, Theorem 5.4]. This lemma plays a similar crucial role for establishing converse parts for more general zero-rate hypothesis testing systems with exponential-type constraint on all errors [14].

B. Focus and Main Contributions

As we already mentioned, one major goal in this paper is the study of the role of cooperation link between the two detectors in improving the error exponents, i.e., *collaborative* decision making. On this aspect, we mention that the presence of the cooperation link makes the problem depart significantly from the aforementioned works. To see this, observe for example that even the seemingly easy case in which i) the rate R_1 is zero and ii) $M = 2$ and both detectors making guesses on the same hypothesis about whether (Y_1, Y_2) is independent of X or not, which is solved fully in [3] in the case without cooperation, seems to become of formidable complexity in the presence of such a cooperation link. Partly, this is because *binning* on the cooperation link may now be helpful in this scenario (See Remark 1).

Another goal in this paper is to investigate the effect of the detectors aiming at maximizing their error exponents under *distinct* hypotheses. Such a scenario was already studied in [5], but here we investigate it in a collaborative setup.

The contributions of this paper are as follows. For the case in which the detectors aim to maximize their error exponents under the same hypothesis (i.e., $i_1 = i_2$), we propose a coding and testing scheme for positive rates $R_1 \geq 0$ and $R_2 \geq 0$. Based on this scheme, we present an achievable exponents region for the case of positive communication rate, and we characterize the optimal exponents region for the case of zero communication rate. We also specialize the results to some specific cases of testing against independence for which we find the optimal error exponents. For the setting in which the detectors aim to maximize their error exponents under distinct hypotheses, we characterize the optimal exponents region for the case of zero communication rates.

C. Outline and Notation

The reminder of this paper is organized as follows. Section II contains a description of the system model. In Section III we study the model in which the two detectors aim at maximizing the error exponents under the same hypothesis, and in Section IV we study the model in which the two detectors aim at maximizing the error exponents under the different hypotheses. Throughout, we use the following notations. The set of all possible types of n -length sequences over \mathcal{X} is denoted $\mathcal{P}^n(\mathcal{X})$. For $\delta > 0$, the set of sequences x^n that are δ -typical with respect to the pmf P_X is denoted $\mathcal{T}_\delta^n(P_X)$. For random variables X and \bar{X} over the same alphabet \mathcal{X} with pmfs P_X and $P_{\bar{X}}$ satisfying $P_X \ll P_{\bar{X}}$ (i.e., for every $x \in \mathcal{X}$, if $P_X(x) > 0$ then also $P_{\bar{X}}(x) > 0$), both $D(P_X \| P_{\bar{X}})$ and $D(X \| \bar{X})$ denote the Kullback-Leiber divergence between X and \bar{X} .

II. SYSTEM MODEL

Let (X^n, Y_1^n, Y_2^n) be distributed i.i.d. according to one of $M \geq 2$ possible pmfs $\{P_{XY_1Y_2}^{(m)}\}_{m=1}^M$. The encoder observes a source sequence X^n and applies encoding function

$$\phi_{1,n}: \mathcal{X}^n \rightarrow \mathcal{M}_1 \triangleq \{1, \dots, \|\phi_{1,n}\|\} \quad (2)$$

to it. It then sends the resulting index

$$M_1 = \phi_{1,n}(X^n) \quad (3)$$

to both decoders.

Besides M_1 , Decoder 1 also observes the source sequence Y_1^n . It applies two functions to the pair (M_1, Y_1^n) : an encoding function

$$\phi_{2,n}: \mathcal{M}_1 \times \mathcal{Y}_1^n \rightarrow \mathcal{M}_2 \triangleq \{1, \dots, \|\phi_{2,n}\|\}, \quad (4)$$

and a decision function

$$\psi_{1,n}: \mathcal{M}_1 \times \mathcal{Y}_1^n \rightarrow \{1, \dots, M\}. \quad (5)$$

It sends the index

$$M_2 = \phi_{2,n}(M_1, Y_1^n) \quad (6)$$

to Decoder 2, and decides on the hypothesis

$$\hat{\mathcal{H}}_1 \triangleq \psi_{1,n}(M_1, Y_1^n). \quad (7)$$

Decoder 2 observes (M_1, M_2, Y_2^n) and applies the decision function

$$\psi_{2,n}: \mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{Y}_2^n \rightarrow \{1, \dots, M\} \quad (8)$$

to this triple. It thus decides on the hypothesis

$$\hat{\mathcal{H}}_2 \triangleq \psi_{2,n}(M_1, M_2, Y_2^n). \quad (9)$$

Let $(i_1, i_2) \in \{1, \dots, M\}^2$ be given. The probabilities at Decoder 1 and Decoder 2 are given by

$$\alpha_{1,n} \triangleq \max_{m \neq i_1} \Pr\{\hat{\mathcal{H}}_1 \neq m | \mathcal{H} = m\}, \quad (10)$$

$$\beta_{1,n} \triangleq \Pr\{\hat{\mathcal{H}}_1 \neq i_1 | \mathcal{H} = i_1\} \quad (11)$$

$$\alpha_{2,n} \triangleq \max_{m \neq i_2} \Pr\{\hat{\mathcal{H}}_2 \neq m | \mathcal{H} = m\},$$

$$\beta_{2,n} \triangleq \Pr\{\hat{\mathcal{H}}_2 \neq i_2 | \mathcal{H} = i_2\}.$$

Definition 1: Given rates $R_1, R_2 \geq 0$ and small positive numbers $\epsilon_1, \epsilon_2 \in (0, 1)$, an error-exponents pair (θ_1, θ_2) is said achievable, if for each blocklength n there exist functions $\phi_{1,n}$, $\phi_{2,n}$, $\psi_{1,n}$ and $\psi_{2,n}$ as in (2), (4), (5), and (8) so that the following limits hold:

$$\lim_{n \rightarrow \infty} \overline{\alpha}_{1,n} \leq \epsilon_1, \quad \lim_{n \rightarrow \infty} \overline{\alpha}_{2,n} \leq \epsilon_2, \quad (12)$$

$$\theta_1 \leq \lim_{n \rightarrow \infty} \frac{1}{n} \log \beta_{1,n}, \quad \theta_2 \leq \lim_{n \rightarrow \infty} \frac{1}{n} \log \beta_{2,n}, \quad (13)$$

and

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log \|\phi_{1,n}\| \leq R_1, \quad \lim_{n \rightarrow \infty} \frac{1}{n} \log \|\phi_{2,n}\| \leq R_2. \quad (14)$$

Definition 2: Given rates $R_1, R_2 \geq 0$ and numbers $\epsilon_1, \epsilon_2 \in (0, 1)$, the closure of the set of all achievable

exponent pairs (θ_1, θ_2) is called the *error-exponents region* $\mathcal{E}(R_1, R_2, \epsilon_1, \epsilon_2)$.

The main interest of this paper is on characterizing the set of all achievable error-exponent pairs. To do so, we distinguish the case in which the two detectors aim at maximizing the error exponents under the *same* hypothesis, i.e., $i_1 = i_2$, and the one in which they aim at maximizing the error exponents under *different* hypotheses, i.e., $i_1 \neq i_2$.

III. COOPERATIVE DETECTION

In this section, we study the setting in which the two detectors aim at maximizing the error exponents under the *same* hypothesis, i.e., $i_1 = i_2$. For simplicity, we first consider the case of simple null hypothesis, i.e., $M = 2$. For simplicity we set $i_1 = i_2 = 2$, and replace $P^{(1)}$ by P and $P^{(2)}$ by \bar{P} . For convenience, we assume that $\bar{P}(x, y_1, y_2) > 0$ for all $(x, y_1, y_2) \in \mathcal{X} \times \mathcal{Y}_1 \times \mathcal{Y}_2$.

Our first result is an inner bound on the error-exponents region $\mathcal{E}(R_1, R_2, \epsilon_1, \epsilon_2)$. To state the results, we make the following definitions. For given rates $R_1 \geq 0$ and $R_2 \geq 0$, define the following set of auxiliary random variables:

$$\mathcal{S}(R_1, R_2) \triangleq \left\{ (U, V) : \begin{array}{l} U \oplus X \oplus (Y_1, Y_2) \\ V \oplus (Y_1, U) \oplus (Y_2, X) \\ I(U; X) \leq R_1 \\ I(V; Y_1|U) \leq R_2 \end{array} \right\}.$$

Further, define for each $(U, V) \in \mathcal{S}(R_1, R_2)$:

$$\mathcal{L}_1(U) \triangleq \left\{ (\tilde{U}, \tilde{X}, \tilde{Y}_1) : P_{\tilde{U}\tilde{X}} = P_{UX}, P_{\tilde{U}\tilde{Y}_1} = P_{UY_1} \right\},$$

and

$$\mathcal{L}_2(UV) \triangleq \left\{ (\tilde{U}, \tilde{V}, \tilde{X}, \tilde{Y}_1, \tilde{Y}_2) : \begin{array}{l} P_{\tilde{U}\tilde{X}} = P_{UX} \\ P_{\tilde{U}\tilde{V}\tilde{Y}_1} = P_{UVY_1} \\ P_{\tilde{U}\tilde{V}\tilde{Y}_2} = P_{UVY_2} \end{array} \right\}.$$

Also, let $(\bar{X}, \bar{Y}_1, \bar{Y}_2) \sim P_{\bar{X}\bar{Y}_1\bar{Y}_2}$ and define the random variables (\bar{U}, \bar{V}) so as to satisfy $P_{\bar{U}|\bar{X}} = P_{U|X}$ and $P_{\bar{V}|\bar{Y}_1\bar{U}} = P_{V|Y_1U}$ and the Markov chains

$$\bar{U} \oplus \bar{X} \oplus (\bar{Y}_1, \bar{Y}_2) \quad (15)$$

$$\bar{V} \oplus (\bar{Y}_1, \bar{U}) \oplus (\bar{X}, \bar{Y}_2). \quad (16)$$

Theorem 1 (Positive Rates): Given rates $R_1 \geq 0$ and $R_2 \geq 0$ and numbers $\epsilon_1, \epsilon_2 \in (0, 1)$, the exponents region $\mathcal{E}(R_1, R_2, \epsilon_1, \epsilon_2)$ contains all nonnegative pairs (θ_1, θ_2) that for some $(U, V) \in \mathcal{S}(R_1, R_2)$ satisfy:

$$\theta_1 \leq \min_{\tilde{U}\tilde{X}\tilde{Y}_1 \in \mathcal{L}_1(U)} D(\tilde{U}\tilde{X}\tilde{Y}_1 \| \bar{U}\bar{X}\bar{Y}_1) \quad (17)$$

$$\theta_2 \leq \min_{\tilde{U}\tilde{V}\tilde{X}\tilde{Y}_1\tilde{Y}_2 \in \mathcal{L}_2(UV)} D(\tilde{V}\tilde{U}\tilde{X}\tilde{Y}_1\tilde{Y}_2 \| \bar{V}\bar{U}\bar{X}\bar{Y}_1\bar{Y}_2). \quad (18)$$

Proof: See Section V-A. ■

Theorem 1 characterizes an inner bound on the exponent rate region $\mathcal{E}(R_1, R_2, \epsilon_1, \epsilon_2)$. The following results indicate that in some scenarios the inner bound coincides with $\mathcal{E}(R_1, R_2, \epsilon_1, \epsilon_2)$.

Proposition 1: If

$$R_1 \geq H(X) \quad \text{and} \quad R_2 \geq H(Y_1|X), \quad (19)$$

for any $\epsilon_1, \epsilon_2 \in (0, 1)$ the exponents region $\mathcal{E}(R_1, R_2, \epsilon_1, \epsilon_2)$ coincides with the set of all non-negative pairs (θ_1, θ_2) satisfying

$$\theta_1 \leq D(XY_1 \| \bar{X}\bar{Y}_1) \quad (20)$$

$$\theta_2 \leq D(XY_1Y_2 \| \bar{X}\bar{Y}_1\bar{Y}_2). \quad (21)$$

Proof: Achievability follows by specializing Theorem 1 to $U \triangleq X$ and $V \triangleq Y_1$ and by noting that this choice is feasible, i.e., in $\mathcal{S}(R_1, R_2)$ because of (19). The converse holds because the right-hand side of (20) coincides with the error exponent when Decoder 1 can directly observe the source sequences X^n and Y_1^n , and the right-hand side of (21) coincides with the error exponent when Decoder 2 can directly observe the source sequences X^n , Y_1^n , and Y_2^n . ■

Consider now the case of zero cooperation rate $R_2 = 0$. For given rate $R_1 \geq 0$, define

$$\mathcal{S}(R_1) \triangleq \{U : I(U; X) \leq R_1, U \oplus X \oplus (Y_1, Y_2)\}. \quad (22)$$

Theorem 2 (Zero Cooperation Rate): If the pmfs $P_{XY_1Y_2}$ and $P_{\bar{X}\bar{Y}_1\bar{Y}_2}$ satisfy

$$P_{Y_1Y_2} = P_{Y_1}P_{Y_2}, \quad P_{\bar{X}\bar{Y}_1\bar{Y}_2} = P_{\bar{X}}P_{\bar{Y}_1}P_{\bar{Y}_2}, \quad (23)$$

then the asymptotic region

$$\bigcap_{\epsilon_1, \epsilon_2 > 0} \mathcal{E}(R_1, 0, \epsilon_1, \epsilon_2) \quad (24)$$

coincides with the set of all nonnegative pairs (θ_1, θ_2) that for some $U \in \mathcal{S}(R_1)$ satisfy

$$\theta_1 \leq I(U; Y_1) \quad (25)$$

$$\theta_2 \leq I(U; Y_1) + I(U; Y_2). \quad (26)$$

Proof: Achievability follows by specializing Theorem 1 to $R_2 = 0$. The form in (25) and (26) is then obtained through algebraic manipulations and by using the log-sum inequality. For the converse, see Section V-B. ■

Notice that the Theorem remains valid if only a single bit can be sent over the cooperation link. In fact it suffices that Detector 1 sends its decision to Detector 2. The latter then declares the null hypothesis $\mathcal{H} = 0$, if and only if, the message from Detector 1 indicates the null hypothesis and also its own observation combined with the message from the encoder indicate the null hypothesis.

Remark 1: Theorem 2 requires that $R_2 = 0$ and the observations Y_1 and Y_2 are independent under both null and alternative hypotheses. The reader may wonder whether a similar optimality result can be obtained when these assumptions are relaxed, e.g., both detectors making a guess on whether (Y_1, Y_2) is independent of X or not with $R_2 \geq 0$ and Y_1 and Y_2 arbitrarily correlated. Such a result however can certainly not be obtained from Theorem 2, because the communication over the cooperation link does employ binning to exploit Detector 2's side-information Y_2^n about the source Y_1^n .

Remark 2: For the model of Theorem 2 without the cooperation link, the optimal error exponent at Detector 2 is $I(U; Y_2)$ only [3]. The $I(U; Y_1)$ -increase of this exponent is made possible by the cooperation on the link of rate R_2 .

We now consider the case of zero rates $R_1 = R_2 = 0$. Define the following sets

$$\mathcal{L}_1 = \left\{ (\tilde{X}, \tilde{Y}_1) : P_{\tilde{X}} = P_X, P_{\tilde{Y}_1} = P_{Y_1} \right\} \quad (27)$$

$$\mathcal{L}_2 = \left\{ (\tilde{X}, \tilde{Y}_1, \tilde{Y}_2) : P_{\tilde{X}} = P_X, P_{\tilde{Y}_1} = P_{Y_1}, P_{\tilde{Y}_2} = P_{Y_2} \right\}. \quad (28)$$

Theorem 3 (Zero Rates): If all $(x, y_1, y_2) \in \mathcal{X} \times \mathcal{Y}_1 \times \mathcal{Y}_2$ have positive probabilities under $\mathcal{H} = 1$, $P_{\tilde{X}\tilde{Y}_1\tilde{Y}_2}(x, y_1, y_2) > 0$, then for any $\epsilon_1, \epsilon_2 \in (0, 1)$ the exponents region $\mathcal{E}(0, 0, \epsilon_1, \epsilon_2)$ coincides with the set of all nonnegative pairs (θ_1, θ_2) satisfying

$$\theta_1 \leq \min_{\tilde{X}\tilde{Y}_1 \in \mathcal{L}_1} D(\tilde{X}\tilde{Y}_1 \| \tilde{X}\tilde{Y}_1) \quad (29)$$

$$\theta_2 \leq \min_{\tilde{X}\tilde{Y}_1\tilde{Y}_2 \in \mathcal{L}_2} D(\tilde{X}\tilde{Y}_1\tilde{Y}_2 \| \tilde{X}\tilde{Y}_1\tilde{Y}_2). \quad (30)$$

Proof: Achievability follows by specializing Theorem 1 to $R_1 = R_2 = 0$. The form in (29) and (30) is then obtained through algebraic manipulations and application of the log-sum inequality. The converse can be proved by invoking a slight variation of [12, Theorem 3] in which the distributions are trivariate, instead of bivariate. ■

Notice that Theorem 3 is a strong converse, i.e., it holds for any values of $\epsilon_1, \epsilon_2 \in (0, 1)$. Moreover, it can be achieved with only a single bit of communication from the encoder to the Detectors 1 and 2, and with only a single bit of communication from Detector 1 to Detector 2. It suffices that the encoder and Detector 1 send a single bit that simply indicates whether their observed sequences X^n and Y_1^n are δ -typical according to the marginal laws P_X and P_{Y_1} . Detector 1 decides on the null hypothesis if both these tests are successful, and Detector 2 decides on the null hypothesis if both tests are successful and also its own observation Y_2^n is δ -typical according to the marginal P_{Y_2} . The analysis of this scheme is similar to the analysis in [2]. From this simple coding scheme, one can conclude that the optimal error exponent can be attained even if the encoder and both detectors are only told whether their sequences are typical with respect to the marginals under \mathcal{H} ; there is no need for them to observe the exact sequences.

IV. CONCURRENT DETECTION

We now turn to the setting in which the two detectors aim at maximizing the error exponents under *different* hypotheses, i.e., $i_1 \neq i_2$. Without loss of generality, let

$$i_1 = 1 \quad \text{and} \quad i_2 = 2. \quad (31)$$

For convenience, assume that for all $(x, y_1, y_2) \in \mathcal{X} \times \mathcal{Y}_1 \times \mathcal{Y}_2$ both probabilities $P^{(1)}(x, y_1)$ and $P^{(2)}(x, y_1, y_2)$ are positive.

Proposition 2: For any pair ϵ_1, ϵ_2 , the zero-rates exponents region $\mathcal{E}(0, 0, \epsilon_1, \epsilon_2)$ is given by the set of all nonnegative pairs (θ_1, θ_2) satisfying

$$\theta_1 \leq \min_{m \neq 1} \min_{\substack{\tilde{P}_{XY_1}: \\ \tilde{P}_X = P_X^{(m)}, \tilde{P}_{Y_1} = P_{Y_1}^{(m)}}} D(\tilde{P}_{XY_1} \| P_{XY_1}^{(1)}) \quad (32)$$

$$\theta_2 \leq \min_{m \neq 2} \min_{\substack{\tilde{P}_{XY_1Y_2}: \\ \tilde{P}_X = P_X^{(m)}, \tilde{P}_{Y_1} = P_{Y_1}^{(m)}, \\ \tilde{P}_{Y_2} = P_{Y_2}^{(m)}}} D(\tilde{P}_{XY_1Y_2} \| P_{XY_1Y_2}^{(2)}). \quad (33)$$

Proof: We first propose a coding scheme achieving this performance. Fix $\mu'' > \mu' > \mu > 0$.

$$M_1 = \begin{cases} m, & \text{if } x^n \in \mathcal{T}_\mu^n(P_X^{(m)}) \text{ for } m \in \{1, \dots, M\}, \\ M+1, & \text{otherwise.} \end{cases}$$

Given that Detector 1 observes message $M_1 = m_1$ and source sequence $Y_1^n = y_1^n$, it does the following. If $m_1 = M+1$, it declares $\hat{\mathcal{H}}_1 = 1$ and sends $M_2 = M+1$ over the cooperation link to Detector 2. Otherwise, it checks whether

$$y_1^n \in \mathcal{T}_{\mu'}^n(P_{Y_1}^{(m_1)}). \quad (34)$$

If successful, Detector 1 declares $\hat{\mathcal{H}}_1 = m_1$ and sends $M_2 = m_1$. Otherwise, it declares $\hat{\mathcal{H}}_1 = 1$ and sends $M_2 = M+1$.

Given that Detector 2 observes messages $M_1 = m_1$ and $M_2 = m_2$ and source sequence $Y_2^n = y_2^n$, it does the following. If $m_2 = M+1$, Detector 2 declares $\hat{\mathcal{H}}_2 = 2$. Otherwise, it checks whether

$$y_2^n \in \mathcal{T}_{\mu''}^n(P_{Y_2}^{(m)}). \quad (35)$$

If successful, it declares $\hat{\mathcal{H}}_2 = m$. Otherwise it declares $\hat{\mathcal{H}}_2 = 2$.

To summarize, Detector 1 declares $\hat{\mathcal{H}}_1 = m$, for $m \in \{2, \dots, M\}$ if and only if $(x^n, y_1^n) \in \mathcal{T}_\mu^n(P_X^{(m)}) \times \mathcal{T}_{\mu'}^n(P_{Y_1}^{(m)})$, and Detector 2 declares $\hat{\mathcal{H}}_2 = m$, for $m \in \{1, \dots, M\} \setminus \{2\}$, if and only if $(x^n, y_1^n, y_2^n) \in \mathcal{T}_\mu^n(P_X^{(m)}) \times \mathcal{T}_{\mu'}^n(P_{Y_1}^{(m)}) \times \mathcal{T}_{\mu''}^n(P_{Y_2}^{(m)})$. The analysis of the scheme is standard and omitted.

The converse can be proved by invoking a slight variation of [12, Theorem 3] in which the distributions are trivariate, instead of bivariate therein. ■

Remark 3: In the previously studied scenario where $i_1 = i_2$, the optimal exponents region with zero-rate communication can be achieved with single bits of communication. That means, it suffices to send $M_1 \in \{1, 2\}$ and $M_2 \in \{1, 2\}$. It can be shown that this is not the case in the scenario considered here, where $i_1 \neq i_2$. Clearly, in above scheme, both M_1 and M_2 take on value in $\{1, \dots, M+1\}$.¹ If M_1 is valued in an alphabet of size $\|\phi_{1,n}\| \leq M$, then the performance in Proposition 2 is generally not achievable. In particular, while the optimal exponents region in Proposition 2 is a rectangle, this is not true anymore when $\|\phi_{1,n}\| \leq M$. For the special case where M_2 is deterministic (i.e., no cooperation is possible), this was already observed in [5].

V. PROOFS

A. Proof of Theorem 1

1) *Preliminaries:* Choose a small positive number $\delta > 0$ and a pair of auxiliary random variables (U, V) satisfying the Markov chains

$$U \dashv\dashv X \dashv\dashv (Y_1, Y_2) \quad (36)$$

$$V \dashv\dashv (Y_1, U) \dashv\dashv (Y_2, X). \quad (37)$$

Fix the rates

$$R_1 = I(U; X) + \xi(\delta) \quad (38)$$

$$R_2 = I(V; Y_1|U) + \xi(\delta), \quad (39)$$

where $\xi(\cdot) \rightarrow 0$ is a function that tends to 0 as its argument tends to 0. By this choice, $(U, V) \in \mathcal{S}(R_1, R_2)$.

¹The scheme could easily be changed to have $M_2 \in \{1, 2\}$. In fact, in the scheme either $M_2 = M_1$ or $M_2 = M+1$. So, it suffices that Detector 2 sends 1 to indicate that it agrees with M_1 and 2 to indicate that it disagrees.

2) *Codebook Generation*: Randomly generate the codebook $\mathcal{C}_U \triangleq \{u^n(m_1), m_1 \in \{1, \dots, \lfloor 2^{nR_1} \rfloor\}\}$ by drawing each entry of each codeword $u^n(m_1)$ i.i.d. according to P_U .

For each index $m_1 \in \{1, \dots, \lfloor 2^{nR_1} \rfloor\}$, randomly construct the codebook $\mathcal{C}_V(m_1) \triangleq \{v^n(m_2|m_1), m_2 \in \{1, \dots, \lfloor 2^{nR_2} \rfloor\}\}$ by drawing the j -th entry of each codeword $v^n(m_2|m_1)$ according to the conditional pmf $P_{V|U}(\cdot|u_j(m_1))$, where $u_j(m_1)$ denotes the j -th component of codeword $u^n(m_1)$.

Reveal all codebooks to all terminals.

3) *Encoder*: Given that it observes the source sequence $X^n = x^n$, the encoder looks for an index $m_1 \in \{1, \dots, \lfloor 2^{nR_1} \rfloor\}$ such that

$$(u^n(m_1), x^n) \in \mathcal{T}_{\delta/8}^n(P_{UX}). \quad (40)$$

If no such index m_1 is found, the encoder sends the index $m_1 = 0$ over the common noise-free pipe to both decoders. If one or more indices can be found, the encoder selects one of them uniformly at random and sends it to both decoders.

4) *Decoder 1*: Given that Decoder 1 receives an index $M_1 = m_1$ not equal to 0 and that it observes the source sequence $Y_1^n = y_1^n$, it checks whether

$$(u^n(m_1), y_1^n) \in \mathcal{T}_{\delta/4}^n(P_{UY_1}). \quad (41)$$

If the test is successful, Decoder 1 decides on the null hypothesis, i.e., $\hat{\mathcal{H}}_1 = \bar{\mathcal{H}}$. Otherwise, it decides on the alternative hypothesis $\hat{\mathcal{H}}_1 = \bar{\mathcal{H}}$.

If $m_1 \neq 0$ and (41) holds, Decoder 1 looks for an index $m_2 \in \{1, \dots, \lfloor 2^{nR_2} \rfloor\}$ such that

$$(u^n(m_1), v^n(m_2|m_1), y_1^n) \in \mathcal{T}_{\delta/2}^n(P_{UVY_1}). \quad (42)$$

If one or more such indices can be found, Decoder 1 selects one of them uniformly at random and sends it over the cooperation link to Decoder 2. Otherwise it sends $M_2 = 0$.

5) *Decoder 2*: Given that Decoder 2 observes the indices $M_1 = m_1$ and $M_2 = m_2$ and the source sequence $Y_2^n = y_2^n$, it checks whether

$$(u^n(m_1), v^n(m_2|m_1), y_2^n) \in \mathcal{T}_{\delta}^n(P_{UVY_2}). \quad (43)$$

If this check is successful, Decoder 2 decides on the null hypothesis, $\hat{\mathcal{H}}_2 = \mathcal{H}$. Otherwise, it decides on the alternative hypothesis $\hat{\mathcal{H}}_2 = \bar{\mathcal{H}}$.

6) *Analysis*: The analysis can be performed along similar lines as in [2]. The main difference is the analysis of the probability of error under $\mathcal{H} = i_2$ at Decoder 2, which is detailed out in the following.

Define for each $y_2^n \in \mathcal{Y}_2^n$ and each pair of indices $(i, j) \in \{1, \dots, \lfloor 2^{nR_1} \rfloor\} \times \{1, \dots, \lfloor 2^{nR_2} \rfloor\}$ the set

$$\mathcal{S}_{ij}(y_2^n) := \mathcal{Q}_i \times \{u^n(i)\} \times \{v^n(j|i)\} \times \mathcal{G}_{ij} \times \{y_2^n\},$$

where $\mathcal{Q}_i \subseteq \mathcal{X}^n$ is the set of all sequences x^n for which the encoder sends $M_1 = i$ to the two decoders, and $\mathcal{G}_{ij} \subseteq \mathcal{Y}_1^n$ is the set of all y_1^n sequences for which Decoder 1 sends $M_2 = j$ over the cooperation link when $M_1 = i$. Note that, by construction the sets $\{\mathcal{Q}_i\}$ are disjoint. Also, define

$$\mathcal{J}_n := \bigcup_{i=1}^{\lfloor 2^{nR_1} \rfloor} \bigcup_{j=1}^{\lfloor 2^{nR_2} \rfloor} \bigcup_{y_2^n: (u^n(i), v^n(j|i), y_2^n) \in \mathcal{T}_{\delta/2}^n(P_{UVY_2})} \mathcal{S}_{ij}(y_2^n).$$

Denote by $K(X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)})$ the number of all tuples $(x^n, u^n(i), v^n(j|i), y_1^n, y_2^n) \in \mathcal{J}_n$ that have joint type $X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}$. This number can be bounded as

$$\begin{aligned} K(X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}) &\leq \sum_{i=1}^{\lfloor 2^{nR_1} \rfloor} \sum_{j=1}^{\lfloor 2^{nR_2} \rfloor} \exp[nH(X^{(n)}Y_1^{(n)}Y_2^{(n)}|U^{(n)}V^{(n)})] \\ &\leq \exp[n(H(X^{(n)}Y_1^{(n)}Y_2^{(n)}|U^{(n)}V^{(n)}) \\ &\quad + I(U; X) + I(V; Y_1|U) + 2\xi(\delta))]. \end{aligned} \quad (44)$$

Notice also that for a given triple of sequences (x^n, y_1^n, y_2^n) of joint type $X^{(n)}Y_1^{(n)}Y_2^{(n)}$:

$$\begin{aligned} \Pr[(X^n, Y_1^n, Y_2^n) = (x^n, y_1^n, y_2^n) | \bar{\mathcal{H}}] &= \exp[-n(H(X^{(n)}Y_1^{(n)}Y_2^{(n)}) \\ &\quad + D(X^{(n)}Y_1^{(n)}Y_2^{(n)} \| \bar{X}\bar{Y}_1\bar{Y}_2))] \end{aligned} \quad (45)$$

Defining

$$\begin{aligned} k(X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}) &\triangleq \\ &H(X^{(n)}Y_1^{(n)}Y_2^{(n)}) + D(X^{(n)}Y_1^{(n)}Y_2^{(n)} \| \bar{X}\bar{Y}_1\bar{Y}_2) \\ &\quad - H(X^{(n)}Y_1^{(n)}, Y_2^{(n)}|U^{(n)}V^{(n)}Y_2^{(n)}) - I(U; X) \\ &\quad - I(V; Y_1|U), \end{aligned} \quad (46)$$

the error probability under $\mathcal{H} = i_2$ at Decoder 2 can then be upper bounded as:

$$\begin{aligned} \beta_{2,n} &\leq \sum_{X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}} K(X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}) \\ &\quad \times \exp[-n(H(X^{(n)}Y_1^{(n)}Y_2^{(n)}) \\ &\quad + D(X^{(n)}Y_1^{(n)}Y_2^{(n)} \| \bar{X}\bar{Y}_1\bar{Y}_2))] \\ &\leq \sum_{X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}} \exp[-n(k(X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}) - 2\xi(\delta))] \end{aligned} \quad (47)$$

where the sum ranges over all joint types $X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)} \in \mathcal{P}^n(\mathcal{X} \times \mathcal{U} \times \mathcal{V} \times \mathcal{Y}_1 \times \mathcal{Y}_2)$ encountered in \mathcal{J}_n . Since each of these types satisfies the following three inequalities

$$|P_{U^{(n)}X^{(n)}}(u, x) - P_{UX}(u, x)| \leq \delta/8 \quad (48)$$

$$|P_{U^{(n)}V^{(n)}Y_1^{(n)}}(u, v, y_1) - P_{UVY_1}(u, v, y_1)| \leq \delta/2 \quad (49)$$

$$|P_{U^{(n)}V^{(n)}Y_2^{(n)}}(u, v, y_2) - P_{UVY_2}(u, v, y_2)| \leq \delta \quad (50)$$

for all $(x, u, v, y_1, y_2) \in \mathcal{X} \times \mathcal{U} \times \mathcal{V} \times \mathcal{Y}_1 \times \mathcal{Y}_2$ and since the number of joint types is upper bounded by $(n+1)^{|\mathcal{U}||\mathcal{V}||\mathcal{X}||\mathcal{Y}_1||\mathcal{Y}_2|}$, one obtains

$$\begin{aligned} \beta_{2,n} &\leq (n+1)^{|\mathcal{U}||\mathcal{V}||\mathcal{X}||\mathcal{Y}_1||\mathcal{Y}_2|} \\ &\quad \times \max \exp[-n(k(X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}) - 2\xi(\delta))], \end{aligned}$$

where the maximization is over all types $X^{(n)}U^{(n)}V^{(n)}Y_1^{(n)}Y_2^{(n)}$ satisfying (48)–(50). Taking now the limits $n \rightarrow \infty$ and $\delta \rightarrow 0$, by the continuity of the entropy and relative entropy and because $\xi(\delta) \rightarrow 0$ as

$\delta \rightarrow 0$, one obtains that the error exponent of the described scheme satisfies

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{1}{n} \log \beta_2 \\ & \geq \min \left[D(\tilde{X}\tilde{Y}_1\tilde{Y}_2 \| \tilde{X}\tilde{Y}_1\tilde{Y}_2) + H(\tilde{X}\tilde{Y}_1\tilde{Y}_2) - \right. \\ & \quad \left. H(\tilde{X}\tilde{Y}_1\tilde{Y}_2 | \tilde{U}\tilde{V}) - I(U; X) - I(\tilde{V}; \tilde{Y}_1 | \tilde{U}) \right], \end{aligned}$$

where the minimization is over all joint types $\tilde{U}\tilde{V}\tilde{X}\tilde{Y}_1\tilde{Y}_2 \in \mathcal{L}_2(UV)$. Simple algebraic manipulations establish the desired result.

B. Proof of the Converse to Theorem 2

Fix $\epsilon_1, \epsilon_2 \in (0, 1)$. Let encoding functions $\phi_{1,n}$ and $\phi_{2,n}$ and decision functions $\psi_{1,n}$ and $\psi_{2,n}$ be given that satisfy (12) and (14) with $R_2 = 0$. Let $\alpha_{1,n}, \alpha_{2,n}, \beta_{1,n}$, and $\beta_{2,n}$ be the error probabilities corresponding to the chosen functions.

For $i \in \{1, 2\}$:

$$\begin{aligned} D(P_{\hat{\mathcal{H}}_i | \mathcal{H}} \| P_{\hat{\mathcal{H}}_i | \bar{\mathcal{H}}}) &= -h_2(\alpha_{i,n}) - (1 - \alpha_{i,n}) \log(\beta_{i,n}) \\ &\quad - \alpha_{i,n} \log(1 - \beta_{i,n}) \end{aligned} \quad (51)$$

where $h_2(p)$ denotes the entropy of a Bernoulli- (p) memoryless source. Since $\alpha_{i,n} \leq \epsilon_i$, for each $i \in \{1, 2\}$, Inequality (51) yields

$$\theta_{i,n} \triangleq -\frac{1}{n} \log(\beta_{i,n}) \leq \frac{1}{n} \frac{1}{1 - \epsilon_i} D(P_{\hat{\mathcal{H}}_i | \mathcal{H}} \| P_{\hat{\mathcal{H}}_i | \bar{\mathcal{H}}}) + \mu_{i,n}$$

with $\mu_{i,n} \triangleq \frac{1}{n} \frac{1}{1 - \epsilon_i} h_2(\alpha_{i,n})$. Notice that $\mu_{i,n} \rightarrow 0$ as $n \rightarrow \infty$.

Consider first $\theta_{1,n}$:

$$\begin{aligned} \theta_{1,n} &\leq \frac{1}{n} \frac{1}{1 - \epsilon_1} D(P_{\hat{\mathcal{H}}_1 | \mathcal{H}} \| P_{\hat{\mathcal{H}}_1 | \bar{\mathcal{H}}}) + \mu_{1,n} \\ &\stackrel{(a)}{\leq} \frac{1}{n} \frac{1}{1 - \epsilon_1} D(P_{Y_1^n M_1 | \mathcal{H}} \| P_{Y_1^n M_1 | \bar{\mathcal{H}}}) + \mu_{1,n} \\ &\stackrel{(b)}{=} \frac{1}{n} \frac{1}{1 - \epsilon_1} I(Y_1^n; M_1) + \mu_{1,n} \\ &\stackrel{(c)}{=} \frac{1}{n} \frac{1}{1 - \epsilon_1} \sum_{k=1}^n H(Y_{1k}) - H(Y_{1k} | M_1 Y_1^{k-1}) + \mu_{1,n} \\ &\stackrel{(d)}{\leq} \frac{1}{n} \frac{1}{1 - \epsilon_1} \sum_{k=1}^n H(Y_{1k}) - H(Y_{1k} | M_1 Y_1^{k-1} X^{k-1}) \\ &\quad + \mu_{1,n} \\ &\stackrel{(e)}{=} \frac{1}{n} \frac{1}{1 - \epsilon_1} \sum_{k=1}^n H(Y_{1k}) - H(Y_{1k} | M_1 X^{k-1}) + \mu_{1,n} \\ &\stackrel{(f)}{=} \frac{1}{n} \frac{1}{1 - \epsilon_1} \sum_{k=1}^n I(Y_{1k}; U_k) + \mu_{1,n} \\ &\stackrel{(g)}{=} \frac{1}{1 - \epsilon_1} I(Y_1; U_Q | Q) + \mu_{1,n} \\ &\stackrel{(h)}{=} \frac{1}{1 - \epsilon_1} I(Y_1(n); U(n)) + \mu_{1,n} \end{aligned}$$

where: (a) follows by the data processing inequality for relative entropy; (b) holds since M_1 and Y_1^n are independent under the alternative hypothesis $\bar{\mathcal{H}}$; (c) is due to

the chain rule for mutual information; (d) follows since conditioning reduces entropy; (e) is due to the Markov chain $Y_1^{k-1} \text{---} (M_1, X^{k-1}) \text{---} Y_{1k}$; (f) holds by defining $U_k \triangleq (M_1, X^{k-1})$; (g) is obtained by introducing a random variable Q that is uniform over the set $\{1, \dots, n\}$ and independent of all previously defined random variables; and (h) holds by defining $U(n) \triangleq (U_Q, Q)$ and $Y_1(n) \triangleq Y_{1Q}$.

Similarly, one obtains for $\theta_{2,n}$:

$$\begin{aligned} \theta_{2,n} &\stackrel{(i)}{\leq} \frac{1}{n} \frac{1}{1 - \epsilon_2} D(P_{Y_2^n M_1 M_2 | \mathcal{H}} \| P_{Y_2^n M_1 M_2 | \bar{\mathcal{H}}}) + \mu_{2,n} \\ &\stackrel{(j)}{=} \frac{1}{n} \frac{1}{1 - \epsilon_2} (I(Y_2^n; M_1 M_2) \\ &\quad + D(P_{M_1 M_2 | \mathcal{H}} \| P_{M_1 M_2 | \bar{\mathcal{H}}})) + \mu_{2,n} \\ &\stackrel{(k)}{\leq} \frac{1}{n} \frac{1}{1 - \epsilon_2} (I(Y_2^n; M_1) + I(Y_2^n; M_2 | M_1)) \\ &\quad + D(P_{Y_1^n M_1 | \mathcal{H}} \| P_{Y_1^n M_1 | \bar{\mathcal{H}}}) + \mu_{2,n} \\ &\stackrel{(\ell)}{\leq} \frac{1}{n} \frac{1}{1 - \epsilon_2} (I(Y_2^n; M_1) + \log \|\phi_{2,n}\| \\ &\quad + D(P_{Y_1^n M_1 | \mathcal{H}} \| P_{Y_1^n M_1 | \bar{\mathcal{H}}})) + \mu_{2,n} \\ &\stackrel{(m)}{=} \frac{1}{n} \frac{1}{1 - \epsilon_2} (I(Y_2^n; M_1) + I(Y_1^n; M_1)) + \tilde{\mu}_{2,n} \\ &\stackrel{(o)}{\leq} \frac{1}{1 - \epsilon_2} (I(Y_2(n); U(n)) + I(Y_1(n); U(n))) + \tilde{\mu}_{2,n}, \end{aligned} \quad (52)$$

where (i) follows by the data processing inequality for relative entropy; (j) holds by the independence of the pair (M_1, M_2) with Y_2^n under the alternative hypothesis $\bar{\mathcal{H}}$; (k) by the data processing inequality for relative entropy; (l) holds since conditioning reduces entropy; (o) follows by proceeding along the steps (b) to (h) above; and (m) holds by defining $\tilde{\mu}_{2,n} \triangleq \log \|\phi_{2,n}\| / (n(1 - \epsilon_2)) + \mu_{2,n}$. Notice that by the assumption $R_2 = 0$, the term $1/n \log \|\phi_{2,n}\| \rightarrow 0$ as $n \rightarrow \infty$. Thus, also $\tilde{\mu}_{2,n} \rightarrow 0$ as $n \rightarrow \infty$.

We next lower bound the rate R_1 :

$$\begin{aligned} nR_1 &\geq H(M_1) \\ &= H(M_1) - H(M_1 | X^n) \\ &= I(M_1; X^n) \\ &= \sum_{k=1}^n I(M_1; X_k | X^{k-1}) \\ &= \sum_{k=1}^n I(X_k | U_k) \\ &= nI(X_Q; U_Q | Q) \\ &= nI(U(n); X(n)) \end{aligned}$$

For any blocklength n , the newly defined random variables $X(n), Y_1(n), Y_2(n) \sim P_{XY_1Y_2}$ and $(U(n) \text{---} X(n) \text{---} Y_1(n)Y_2(n))$. Letting now the blocklength $n \rightarrow \infty$ and $\delta \rightarrow 0$, the asymptotic exponents

$$\theta_1 \triangleq \lim_{n \rightarrow \infty} \theta_{1,n} \quad (53)$$

$$\theta_2 \triangleq \lim_{n \rightarrow \infty} \theta_{2,n} \quad (54)$$

satisfy

$$\theta_1 \leq I(U; Y_1) \quad (55)$$

$$\theta_2 \leq I(U; Y_1) + I(U; Y_2); \quad (56)$$

for some $U \in \mathcal{S}(R_1)$. This completes the proof of Theorem 2.

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