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***Abstract*—This paper is concerned with an energy consumption scheduling problem for consumers in smart grid, based on a real time pricing strategy. Firstly, the energy consumption scheduling problem is cast into a non-cooperative energy consumption game, where consumers compete with each other in order to minimize their electricity usage cost. Secondly, we prove that the non-cooperative energy consumption game has a unique Nash equilibrium point, *i.e.*, optimal energy consumption solution. Thirdly, the energy consumption solution is obtained by a discrete iterative algorithm. Simulation results show that the energy consumption scheduling scheme is effective in matching the varying generation capacity in a day.**

Keywords*—*smart grid; energy consumption scheduling; non-cooperative game; Nash equilibrium; real time pricing

I. INTRODUCTION

Matching supply with demand has been a hot topic in operating electricity networks. Traditionally, we have to provide enough generation capacity to meet the peak load, requiring substantial infrastructure that is idle for all but a few hours a year. Smart grid applications are being developed to deal with steadily increasing future demand. Energy consumption scheduling can induce consumers to shift their loads away from peak times [1-3]. Recently, various pricing schemes have been discussed for implementing energy scheduling in smart grid, such as Time of Use (TOU), Critical Peak Pricing (CPP), Extreme Day CPP (ED-CPP), Extreme Day Pricing (EDP) and Real Time Pricing (RTP) [4]. Recently, with the development of smart metering technologies, which will enable reliable, real-time, two-way information exchange between consumers and electricity energy providers, RTP can be provided to consumers multiple times daily, hourly, or in even shorter intervals. Many economists are convinced that RTP programs are the most direct and efficient scheduling programs suitable for competitive electricity markets [5]. In an RTP program, the energy provider announces electricity prices on a rolling basis, i.e., the price for a given time period (e.g., an hour) is determined and announced before the start of that period (e.g., 15 minutes beforehand).

To handle the two-way information exchange and decision making, consumers will rely on energy management controllers (EMCs), which can modify electricity usage across a home or building based on electricity prices and consumer preferences. From the energy provider's perspective, providing high frequency pricing updates will enable better load shaping and thus result in better matching of supply and demand. For consumers, RTP will provide new opportunities to lower cost by making smart usage decisions.

Rich literature exists on energy scheduling schemes based on RTP, see e.g. [6-10]. In [6] and [7] the main objective is to reduce the total energy cost to consumers without directly addressing the matching of the distributed loads to the available generating capacity. Whereas, in order to match the loads to the supply, [8] proposed a novel RTP algorithm to obtain optimal energy consumption for each consumer by maximizing the social welfare. However, the consumers within a smart grid are selfish, thus they will not cooperate with each other in order to maximize the social welfare. To deal with this problem, [9] gives time-varying prices that can align individual optimality with social welfare maximization. That is to say, social welfare can be implemented by optimizing individual consumers' utilities. In addition, a distributed scheduling mechanism is presented to reduce peak demand within a neighborhood of households [10], without addressing an optimal scheduling strategy. Since the DR system is a distributed feedback system, a distributed energy consumption control mechanism and pricing are necessary. Then a question arises naturally: how should the consumers choose their energy consumption in a distributed dynamic fashion and how should the energy provider set the electricity price, such that the total energy consumption can match the available supply?

In this paper, we attempt to shed some lights on the above question, and present an energy consumption scheduling scheme based on a non-cooperative game, which is suitable for analyzing distributed strategy design problems in Economics. The main contributions are as follows. We formulate the energy scheduling problem as a non-cooperative game, in which consumers act as players, and then prove the existence and uniqueness of Nash equilibrium in the energy consumption

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game. Meanwhile, a distributed, discrete energy consumption control is presented.

The rest of the paper is organized as follows. In Section II, we describe the system model, and formulate the energy scheduling problem as a non-cooperative game. Section III presents the distributed energy consumption scheduling algorithm and numerical results are shown in section IV. Finally, we draw conclusions in section V.

II. PROBLEM FORMULATION

A. System Model

As shown in Fig. 1, we consider a residential power system consisting of one energy provider and N consumers. The energy provider buys the electricity from the wholesale market and sells it to the consumers. We assume there is an EMC in each household. The role of the EMC is to interact with the energy provider through two-way communication network and schedule the energy usage among the smart appliances, such as Dish washer, Clothes washer, and Air condition.

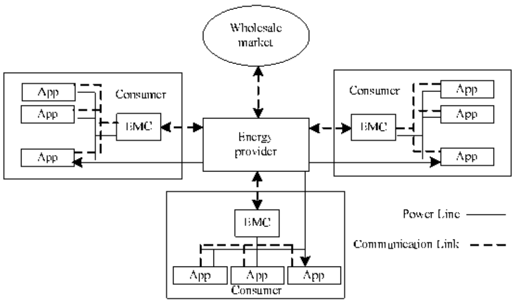

Fig. 1: A residential power system

Here, we assume that the intended operation cycle is divided into K time slots. In each time slot k ($k \in \{1,2,...,K\}$), energy provider receives energy consumption from consumers and sets the electricity price based on the total energy consumption, while consumers decides on their energy consumption by minimizing their electricity usage cost. In this paper, the set of consumers is denoted by $\mathrm{N} = \{1,2,...,N\}$, and the electricity price is

$$p(\boldsymbol{l}^k) = \frac{1}{\lambda L^k - \sum_{i\in\mathrm{N}} l_i^k} \tag{1}$$

where l_i^k is the energy consumption of consumer i in time slot k , $\boldsymbol{l}^k = (l_1^k,...,l_i^k,...,l_N^k)$, and λ is a constant parameter determined by energy provider to implement elastic pricing. L^k denotes the generating capacity so that

$$L^k > \sum_{i\in\mathrm{N}} l_i^k, k \in \{1,2,...,K\} \tag{2}$$

From (1), we see that the electricity price is increasing with the total energy consumption of consumers. Specifically, the energy provider will set a high price in order to reduce the peak energy consumption, and shift the energy consumption to the valley time by lowering the price. The role of electricity price is similar to the lever principle of economics.

Each consumer tends to minimize its electricity usage cost, consisting of two components. One is the payment for electricity usage; the other is the cost of dissatisfaction for deviating from nominal energy consumption d_i^k . In time slot k , the payment of consumer i is denoted by $p(\boldsymbol{l}^k)l_i^k$, and consumer i 's cost of dissatisfaction is $a_i(1-l_i^k/d_i^k)$, which is decreasing with the energy consumption of consumer i . Thus, the total cost of consumer i in time slot k is denoted by

$$C(\boldsymbol{l}^k) = p(\boldsymbol{l}^k)l_i^k + a_i(1-\frac{l_i^k}{d_i^k}) \tag{3}$$

The minimization of $C(\boldsymbol{l}^k)$ is equivalent to the maximization of $U(\boldsymbol{l}^k) = -C(\boldsymbol{l}^k) = a_i(l_i^k/d_i^k - 1) - p(\boldsymbol{l}^k)l_i^k$, which is the payoff of consumer i . From (1) and (3), we see that the payoff of each consumer is affected by the electricity price, which is the function of energy consumption of all consumers. That is to say, energy consumption of each consumer can affect the payoff of other consumers. It is noted that we omit the time slot index k for convenience in the following.

Here, each consumer is actually selfish and rational, and tries to maximize its own payoff, but not at the expense of others. In this situation, is the system able to reach an equilibrium wherein no consumer is interested in varying its parameters since each action it takes would lead to a decrease in its own payoff? Game theory provides the means to study these interactions and to solve the problem well.

B. Energy Consumption Game

In this section, we begin by introducing a non-cooperative energy consumption game in which each consumer aims to maximize its own payoff. Then, the competition among consumers can be cast into the following non-cooperative game.

Definition 1 A non-cooperative energy consumption game G is defined as a triple: $G := \{\mathrm{N}, (S_i)_{i\in\mathrm{N}}, (U_i(\boldsymbol{l}))_{i\in\mathrm{N}}\}$, where N is the set of active consumers participating in the game,

$$S_i := \left\{ l_i \middle| l_i \in \left[0, l^{max}\right] \right\} \tag{4}$$

is the set of possible strategies (energy consumption level) that consumer i can take, and

$$U_i(\boldsymbol{l}) = a_i(\frac{l_i}{d_i} - 1) - \frac{l_i}{\lambda L - \sum_{i\in\mathrm{N}} l_i} \tag{5}$$

is the payoff (utility) function.

Note that, $\boldsymbol{l}_{-i} := (l_1, l_2, ..., l_{i-1}, l_{i+1}, ..., l_N)$ denotes the set of strategies selected by all consumers, except for consumer i and the strategy profile is denoted by $(l_i, \boldsymbol{l}_{-i}) := (l_1, l_2, ..., l_{i-1}, l_i, l_{i+1}, ..., l_N)$.

Before proceeding further, we need to analyze the Nash equilibrium[1] of the non-cooperative energy consumption game G [11], which is based on the concept of a best response correspondence as follows.

Definition 2 For the non-cooperative energy consumption game G, the best response correspondence $r_i(\boldsymbol{l})$ is defined by,

$$r_i(\boldsymbol{l})=\{\boldsymbol{l}\in(S_i)_{i\in\mathrm{N}}|U_i(l_i^*,\boldsymbol{l}_{-i})\geq U_i(l_i',\boldsymbol{l}_{-i})\} \tag{6}$$

Then, a vector of energy consumption $\boldsymbol{l}^*=(l_1^*,l_2^*,...,l_N^*)$ is a Nash equilibrium of the non-cooperative energy consumption game if and only if $l_i^*\in r_i(\boldsymbol{l}^*)$ for all consumers $i\in\mathrm{N}$, that is to say, $U_i(l_i^*,\boldsymbol{l}_{-i}^*)\geq U_i(l_i',\boldsymbol{l}_{-i}^*)$ for any other $l_i'\in S_i$, where $U_i(l_i,\boldsymbol{l}_{-i})$ is the resulting payoff for the consumer i given the other consumers' energy consumption $\boldsymbol{l}_{-i}$.

We see that the Nash equilibrium is a set of strategies where no layer has an incentive to change his action strategy unilaterally given the strategies of the other players. In the following, we first analyze the existence and uniqueness of Nash equilibrium points in the non-cooperative energy consumption game. To prove the existence of Nash equilibrium, we first give the following lemma obtained from [12].

Lemma 1 A Nash equilibrium exists in game $G:=\{\mathrm{N},(S_i)_{i\in\mathrm{N}},(U_i(\boldsymbol{l}))_{i\in\mathrm{N}}\}$, for all $i\in\mathrm{N}$:

1) S_i is a nonempty, convex, and compact subset of some Euclidean space $\Re^N$,

2) $U_i(\boldsymbol{l})$ is continuous in $\boldsymbol{l}$ and quasi-concave in l_i,

then, we have the following theorem.

Theorem 1 There exists a Nash equilibrium point for non-cooperative energy consumption game G.

Proof: Given the strategy space defined by (4), S_i is a nonempty, convex and compact subset of the Euclidean space $\Re^N$. From (5), $U_i(\boldsymbol{l})$ is obviously continuous in $\boldsymbol{l}$. Now we take the second order derivative of $U_i(\boldsymbol{l})$ with respect to l_i, and we have

$$\partial^2U_i(\boldsymbol{l})\big/\partial l_i^2=-2\frac{\sum_{j\in\mathrm{N},j\neq i}l_j-\lambda L}{(\sum_{j\in\mathrm{N}}l_j-\lambda L)^3} \tag{7}$$

Since demand cannot exceed supply as expressed in (2), the second order derivative of $U_i(\boldsymbol{l})$ with respect to l_i is always less than 0, therefore, $U_i(\boldsymbol{l})$ is concave in l_i. □

According to *Lemma 1*, we conclude that non-cooperative energy consumption game G has a Nash equilibrium point, but its uniqueness needs to be addressed. Next we will prove the uniqueness of Nash equilibrium in non-cooperative energy consumption game G.

[1] Given than our space is constrained by the inequality depicted in Eq. (2), we will be concerned with an *inner* Nash Equilibrium point.

According to *Definition 2*, the best-response correspondence $r_i(\boldsymbol{l})$ is achieved when the first derivative of $U_i(\boldsymbol{l})$ with respect to l_i equals to 0, i.e.,

$$h_i(\boldsymbol{l})=\partial U_i(\boldsymbol{l})\big/\partial l_i=\frac{a_i}{d_i}+\frac{\sum_{j\in\mathrm{N},j\neq i}l_j-\lambda L}{(\sum_{i\in\mathrm{N}}l_i-\lambda L)^2}=0 \tag{8}$$

Denote a_i/d_i as b_i, and we have the following quadratic function of l_i:

$$b_il_i^2+2b_i(\sum_{j\in\mathrm{N},j\neq i}l_j-\lambda L)l_i+b_i(\sum_{j\in\mathrm{N},j\neq i}l_j-\lambda L)^2+\sum_{j\in\mathrm{N},j\neq i}l_j-\lambda L=0 \tag{9}$$

Solving (9), we have

$$l_i=\frac{2b_i(\lambda L-\sum_{j\in\mathrm{N},j\neq i}l_j)\pm\sqrt{4b_i(\lambda L-\sum_{j\in\mathrm{N},j\neq i}l_j)}}{2b_i} \tag{10}$$

To satisfy the constraints $\sum_{i\in\mathrm{N}}l_i\leq L$, the best-response correspondence is denoted by

$$r_i(\boldsymbol{l})=l_i=\frac{2b_i(\lambda L-\sum_{j\in\mathrm{N},j\neq i}l_j)-\sqrt{4b_i(\lambda L-\sum_{j\in\mathrm{N},j\neq i}l_j)}}{2b_i} \tag{11}$$

Next we employ a *standard* function similar to the definition in [13].

Definition 3 A function $r(\boldsymbol{l})$ is standard if for all $\boldsymbol{l}\geq\mathbf{0}$, the following properties are satisfied:

1) Positivity: $r(\boldsymbol{l})>\mathbf{0}$.

2) Monotonicity: If $\boldsymbol{l}\geq\boldsymbol{l}'$, then $r(\boldsymbol{l})\leq r(\boldsymbol{l}')$.

3) Scalability: For some $1<\beta<\lambda L\big/\sum_{j\in\mathrm{N},j\neq i}l_j$, there is $\beta r(\boldsymbol{l})<r(\beta\boldsymbol{l})$.

Lemma 2 The best-response correspondence $r_i(\boldsymbol{l})$ is standard if the following constraints are satisfied.

$$\sqrt{b_i(\lambda L-\sum_{j\in\mathrm{N},j\neq i}l_j)}>1 \tag{12}$$

Proof:

1) Positivity. In order to ensure $r_i(\boldsymbol{l})>0$, we have the following inequality,

$$2b_i(\lambda L-\sum_{j\in\mathrm{N},j\neq i}l_j)>\sqrt{4b_i(\lambda L-\sum_{j\in\mathrm{N},j\neq i}l_j)} \tag{13}$$

which is equivalent to (12).

Since $a>0$ and $\lambda L-\sum_{j\in \mathrm{N},j\neq i} l_j>0$, we can conclude that $r_i(\boldsymbol{l})>0$ under constraint (12). Then, we have $\boldsymbol{r}(\boldsymbol{l})=(r_1(\boldsymbol{l}),r_2(\boldsymbol{l}),...,r_N(\boldsymbol{l}))>\boldsymbol{0}$.

2) Monotonicity. Suppose $\boldsymbol{l}>\boldsymbol{l}'$ are different energy consumption vectors. Here, the vector inequality $\boldsymbol{l}>\boldsymbol{l}'$ means that $l_i>l_i'$, $\forall i\in \mathrm{N}$. If $\forall i\neq j, i,j\in \mathrm{N}$,

$$r_j\left(\left[l_1,...,l_i,...,l_N\right]\right)<r_j\left(\left[l_1,...,l_i',...,l_N\right]\right) \tag{14}$$

and

$$r_i\left(\left[l_1,...,l_i,...,l_N\right]\right)\leq r_i\left(\left[l_1,...,l_i',...,l_N\right]\right) \tag{15}$$

Therefore the problem reduces to proving $\partial r_j(\boldsymbol{l})/\partial l_i<0$ and $\partial r_i(\boldsymbol{l})/\partial l_i\leq 0$. The first order partial derivative of $r_j(\boldsymbol{l})$ with respect to l_i is

$$\partial r_j(\boldsymbol{l})/\partial l_i=\frac{1}{\sqrt{4b_i(\lambda L-\sum_{j\in \mathrm{N},j\neq i} l_j)}}-1 \tag{16}$$

From (11), we see $\partial r_j(\boldsymbol{l})/\partial l_i<0$.

It is easy to see that

$$\partial r_i(\boldsymbol{l})/\partial l_i=0 \tag{17}$$

Combining (15) and (16), the monotonicity is proved.

3) Scalability. Comparing $\beta r_i(\boldsymbol{l})>0$ and $r_i(\beta\boldsymbol{l})>0$ in an element-wise manner, we have

$$\begin{aligned} &f(\beta)=\beta r_i(\boldsymbol{l})-r_i(\beta\boldsymbol{l})\\ &=\frac{2b_i\lambda L(\beta-1)+\sqrt{4b_i(\lambda L-\beta\sum_{j\in \mathrm{N},j\neq i} l_j)}-\beta\sqrt{4b_i(\lambda L-\sum_{j\in \mathrm{N},j\neq i} l_j)}}{2b_i} \end{aligned} \tag{18}$$

The first order derivative of $f(\beta)$ is denoted by

$$\partial f(\beta)/\partial\beta=\lambda L-\frac{\sum_{j\in \mathrm{N},j\neq i} l_j}{2\sqrt{b_i(\lambda L-\beta\sum_{j\in \mathrm{N},j\neq i} l_j)}}-\sqrt{\frac{\lambda L-\sum_{j\in \mathrm{N},j\neq i} l_j}{b_i}} \tag{19}$$

where $1<\beta<\lambda L\Big/\sum_{j\in \mathrm{N},j\neq i} l_j$. There exists a threshold β', $\beta r_i(\boldsymbol{l})<r_i(\beta\boldsymbol{l})$ is satisfied when $\beta>\beta'$. Therefore, the scalability is proved. □

It has been pointed out in [13] that the fixed point of $\boldsymbol{l}=r_i(\boldsymbol{l})$ is unique if $r_i(\boldsymbol{l})$ is a standard function. Then, we have the following theorem.

Theorem 2 The non-cooperative energy consumption game G has a unique Nash equilibrium if (12) is satisfied.

Proof: Suppose $\boldsymbol{l}$ and $\boldsymbol{l}'$ are distinct fixed points. Since $\boldsymbol{r}(\boldsymbol{l})>0$, we must have $l_j>0$ and $l_j'>0$ for all j. Without loss of generality, we can assume there exists a j such that $l_j<l_j'$. Hence, there exists $\beta>\beta'$ such that $\beta\boldsymbol{l}>\boldsymbol{l}'$ and that for some j, $\beta l_j\geq l_j'$. The monotonicity and scalability properties imply

$$l_j'=r_j(\boldsymbol{l}')\geq r_j(\beta\boldsymbol{l})>\beta r_j(\boldsymbol{l}_j)=\beta l_j \tag{20}$$

Since $\beta l_j\geq l_j'$, we have found a contradiction, implying the fixed point must be unique. □

Remark 1: In practical, the condition (12) is equivalent to $p(\boldsymbol{l})<b_i$ when the number of consumers is very large, which indicates that the provider should set the electricity price larger than the willingness parameter of each consumer.

III. Energy Consumption Scheduling

In this section, we consider the dynamic energy consumption control algorithm, aiming to reach the unique Nash equilibrium. According to [14], we have the discrete-time iterative algorithm denoted by

$$\begin{aligned} l_i(m+1)&=[G_i(\boldsymbol{l})]_0^{l^{\max}}=[l_i(m)+\mu\,\partial U_i(\boldsymbol{l})/\partial l_i]_0^{l^{\max}}\\ &=[l_i(m)+\mu(\frac{a_i}{d_i}+\frac{\sum_{j\in \mathrm{N},j\neq i} l_j(m)-\lambda L}{(\sum_{i\in \mathrm{N}} l_i(m)-\lambda L)^2})]_0^{l^{\max}} \end{aligned} \tag{21}$$

Theorem 3 Suppose the energy consumption game G has a unique inner Nash equilibrium point $\boldsymbol{l}^*$, the gradient iteration of (21) converges to the unique Nash equilibrium, if the following condition is satisfied.

$$\mu<\frac{2(\lambda L-\sum_{j\in \mathrm{N},j\neq i} l_j)^3}{3\lambda L+l_i-3\sum_{j\in \mathrm{N},j\neq i} l_j} \tag{22}$$

Proof: Firstly, define a mapping $\Phi_i(\tau):[0,1]\to\Re$ for each consumer i by

$$\Phi_i(\tau)=\tau l_i+(1-\tau)l_i^*+\mu h_i(\tau\boldsymbol{l}+(1-\tau)\boldsymbol{l}^*) \tag{23}$$

where $h_i(\cdot)$ is defined in (8). By (21) and (23), we have

$$\begin{aligned} \left|l_i(m+1)-l_i^*\right|&\leq\left|G_i(\boldsymbol{l})-l_i^*\right|=\left|\Phi_i(1)-\Phi_i(0)\right|\\ &=\left|\int_0^1 d\Phi_i(\tau)/d\tau\, d\tau\right|\leq\int_0^1\left|d\Phi_i(\tau)/d\tau\right|d\tau\\ &\leq\max_{0\leq\tau\leq 1}\left|d\Phi_i(\tau)/d\tau\right| \end{aligned} \tag{24}$$

where the first inequality is due to the fact that $\left|\left[G_i(\boldsymbol{l})\right]_0^{l^{\max}}-l_i^*\right|\leq\left|G_i(\boldsymbol{l})-l_i^*\right|$, when $l_i^*\in\left[0,l^{\max}\right]$, for all i. Let $\hat{\boldsymbol{l}}=\tau\boldsymbol{l}+(1-\tau)\boldsymbol{l}^*$, and then $\left|d\Phi_i(\tau)/d\tau\right|$ can be further bounded by

$$|d\Phi_i(\tau)/d\tau| =$$
$$\left|(1+\mu)(l_i - l_i^*)\partial^2 U_i(\hat{\boldsymbol{l}})/\partial \hat{l}_i^2 + (l_j - l_j^*)\sum_{j\in \mathrm{N}, j\neq i} \partial^2 U_i\left(\hat{\boldsymbol{l}}\right)/\partial\hat{l}_i\partial\hat{l}_j\right|$$
$$\le \left|1+\mu(\partial^2 U_i(\hat{\boldsymbol{l}})/\partial\hat{l}_i^2 + \sum_{j\in \mathrm{N}, j\neq i} \partial^2 U_i(\hat{\boldsymbol{l}})/\partial\hat{l}_i\partial\hat{l}_j)\right| \cdot \left\|\boldsymbol{l}-\boldsymbol{l}^*\right\|_\infty \quad (25)$$

where $\|\boldsymbol{l}\|_\infty := \max_i |l_i|$. Note that

$$1+\mu(\partial^2 U_i(\boldsymbol{l})/\partial l_i^2 + \sum_{j\in \mathrm{N}, j\neq i} \partial^2 U_i(\boldsymbol{l})/\partial l_i \partial l_j)$$
$$= 1+\mu\frac{3\lambda L + l_i - 3\sum_{j\in \mathrm{N}, j\neq i} l_j}{(\sum_{j\in \mathrm{N}, j\neq i} l_j - \lambda L)^3} \quad (26)$$

To guarantee

$$0 < \left|1+\mu(\partial^2 U_i(\boldsymbol{l})/\partial l_i^2 + \sum_{j\in \mathrm{N}, j\neq i} \partial^2 U_i(\boldsymbol{l})/\partial l_i \partial l_j)\right| < 1,$$

there is

$$-1 < 1+\mu\frac{3\lambda L + l_i - 3\sum_{j\in \mathrm{N}, j\neq i} l_j}{(\sum_{j\in \mathrm{N}, j\neq i} l_j - \lambda L)^3} < 1 \quad (27)$$

Then, we obtain the condition in (23), under which, there is

$$\max_{\tau\in[0,1]}\left|\frac{d\Phi_i(\tau)}{d\tau}\right| \le \Psi\left\|\boldsymbol{l}-\boldsymbol{l}^*\right\|_\infty \quad (28)$$

where

$$0 < \Psi = \left|1+\mu(\partial^2 U_i(\boldsymbol{l})/\partial l_i^2 + \sum_{j\in \mathrm{N}, j\neq i} \partial^2 U_i(\boldsymbol{l})/\partial l_i \partial l_j)\right| < 1 \quad (29)$$

Combining with (24), it has been proved that the discrete-time iterative algorithm (21) converges to the unique inner Nash equilibrium as $m \to \infty$.□

Remark 2: One of the requirements in implementing the energy consumption scheduling strategy in (21) is to estimate the derivative of the payoff function $\partial U_i(\boldsymbol{l})/\partial l_i$. Based on (5), the derivative $\partial U_i(\boldsymbol{l})/\partial l_i$ is

$$\frac{a_i}{d_i} + \frac{\sum_{j\in \mathrm{N}, j\neq i} l_j(m) - \lambda L}{(\sum_{i\in \mathrm{N}} l_i(m) - \lambda L)^2}$$

where the term $\frac{a_i}{d_i}$ can be independently determined by each consumer. However, the term

$$\frac{\sum_{j\in \mathrm{N}, j\neq i} l_j(m) - \lambda L}{(\sum_{i\in \mathrm{N}} l_i(m) - \lambda L)^2} = \frac{\sum_{i\in \mathrm{N}} l_i(m) - \lambda L - l_i(m)}{(\sum_{i\in \mathrm{N}} l_i(m) - \lambda L)^2}$$

depends on the values of λL and $\sum_{i\in \mathrm{N}} l_i(m)$. The energy provider shall be able to collect all the load estimates by polling the consumers and it can then broadcast λL and $\sum_{i\in \mathrm{N}} l_i(m)$ to the consumers and iterate in order to schedule their strategies during each RTP period.

Remark 3: In a practical system, if the energy consumption of each consumer is much smaller than the total energy consumption of a group of consumers, i.e., $l_i(m) << \sum_{i\in \mathrm{N}} l_i(m)$, then, the term $\frac{\sum_{j\in \mathrm{N}, j\neq i} l_j(m) - \lambda L}{(\sum_{i\in \mathrm{N}} l_i(m) - \lambda L)^2}$ can be approximately estimated as $\frac{1}{\sum_{i\in \mathrm{N}} l_i(m) - \lambda L}$. The energy provider can broadcast this term to the consumers instead of broadcasting the terms λL and $\sum_{i\in \mathrm{N}} l_i(m)$. Please note that $\frac{1}{\sum_{i\in \mathrm{N}} l_i(m) - \lambda L} = -p(\boldsymbol{l})$ where $p(\boldsymbol{l})$ is the real-time electricity price as defined in (1).

IV. Numerical Results

In the simulations, the entire time cycle is divided into 24 time slots representing the 24 hours of the day, and a of all users vary from 0.8 to 1.3 in each time slot. Here, the number of consumers is 10, and the average nominal residential energy consumption d_i^k is obtained from [15]. The varying generation capacity values L^k are estimated from d_i^k.

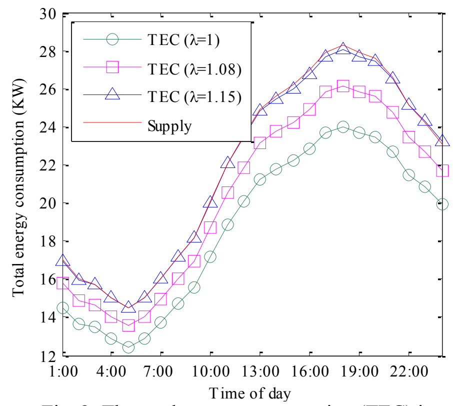

Fig. 2: The total energy consumption (TEC) in a day

Fig. 2 shows the varying generation capacity and total energy consumption with different λ across 24 hours in a day. It can be seen that the total energy consumption approaches the generation capacity when λ increases. The total energy consumptions can match the generation capacity when $\lambda = 1.15$.

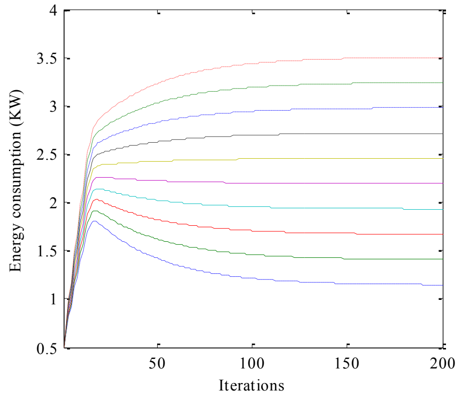

Fig. 3: Convergence of energy scheduling algorithm

Fig. 3 shows the convergence of the energy consumption scheduling algorithm (21) in time slot 24. Here, λ is set to be 1.15. All of the consumers get to the optimal energy consumption by participating in the non-cooperative energy consumption game.

From Fig. 2, we find that the total energy consumption is a little smaller than the generating capacity. To characterize the mismatch of the generating capacity and the total energy consumption, we calculate the supply surplus defined as follows

$$r = \frac{\sum_{k=1}^{24}(L^k - \sum_{i \in \mathrm{N}} (l_i^k)^*)}{\sum_{k=1}^{24} L^k} \tag{30}$$

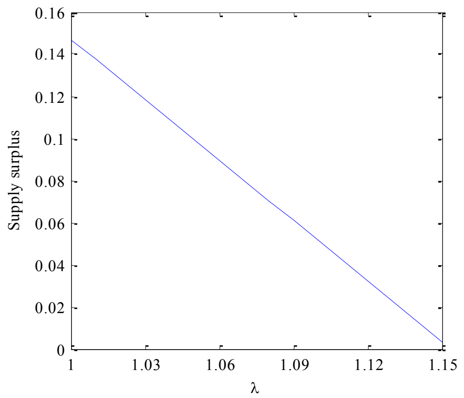

Fig. 4: Supply surplus v. s. λ

Then, we show the supply surplus versus λ in Fig. 4. It can be seen that the supply surplus is decreasing with the value of λ. That is to say, we can match the total energy consumption with the generating capacity by increasing λ, while minimizing the electricity usage cost.

V. Conclusions

In this paper, energy consumption scheduling based on non-cooperative game is considered, and an iterative energy consumption algorithm is presented. We find that the energy consumption scheduling with real time pricing can match with the supply across the time slots in a day, and the proposed energy consumption scheduling algorithm is stabilized at the Nash equilibrium.

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