Rational ABCD Modeling of High-Speed Interconnects

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Abstract

This paper introduces a new numerical approximation technique, called the *Differential Quadrature Method* (DQM), in order to derive the rational ABCD matrix representing the high-speed interconnect. DQM is an efficient differential equation solver that can quickly compute the derivative of a smooth function by estimating a weighted linear sum of the *function values* at few mesh points in the domain of the function. Using DQM, the s-domain Telegrapher's equations of interconnect are discretized as a set of easily solvable algebraic equations, which lead to the rational ABCD matrix. The entries of ABCD matrix take the form of *rational approximations* with respect to s, rather than the conventional ABCD matrix whose entries are complex transcendental functions in s. Although the rationalization result is comparable with Padé approximation of AWE, DQM does not require moment-generating or moment-matching. For both uniform and nonuniform interconnects, DQM-based rational ABCD matrices lead to high accuracy as well as high efficiency for transient analysis of high-speed interconnects.

1 Introduction

As the integrated circuits and systems are becoming larger in chip size and faster in operation, interconnect has become a dominating factor in determining circuit performance. If the interconnects are sufficiently long or the signal rise/fall times are comparable to the time of flight across the line, the interconnect delay will dominate gate delay, and the transmission line effects will make severe impact on circuit performance. The high-speed interconnects have been modeled as transmission lines with distributed parameters [1, 2]. As interconnects are generally terminated with nonlinear loads, the time domain models of interconnects are needed so that the transient response can be calculated and the signal integrity can be studied.

Asymptotic Waveform Evaluation (AWE) has been popularly used to approximate general linear networks [3, 4]. It generates moments by employing series expansion and matches the moments by employing Padé approximation. On one hand, AWE can generate reduced-order macromodels from linear network containing transmission lines, and on the other hand, this method has been used to model an individual transmission line as a device [5]. However, higher order moments lead to ill-condition so that increasing their number does not guarantee a better approximation. Furthermore, AWE may give a reduced macromodel which includes unstable poles although the original network was stable. In order to overcome these problems, the multipoint AWE techniques such as Complex Frequency Hopping (CFH) are developed as an extension to conventional AWE [6, 7, 8, 9].

An ABCD matrix can concisely describe transmission lines in the frequency domain. However, it is not easy to analytically obtain the time domain model from a conventional ABCD matrix, which consists of complex transcendental functions. AWE techniques and ABCD matrices have been used to build the macromodel of linear subnetwork containing interconnects [10]. In [11], AWE technique is applied to ABCD matrix for transient analysis of interconnects. The entries of an ABCD matrix are expanded as series to generate moments, then Padé approximations are obtained by moment-matching. By such a process, the entries that take form of transcendental functions are approximated by rational formulas, and then the time domain model can be analytically obtained by inverse Laplace transform.

All the methods mentioned above focus on uniform transmission lines. In order to analyze the time response of nonuniform transmission lines, a method of equivalent cascaded network chain is developed in [12]. The nonuniform transmission lines are segmented into a cascaded chain of subnetworks, each of which is approximated to be made of uniform lines. After the ABCD matrices of all subnetworks are evaluated, the overall ABCD matrix of the cascaded network chain can be obtained by multiplying all the ABCD matrices of subnetworks. However, the time domain response is finally calculated by fast Fourier transform and numerical convolution, which are very time-consuming and suffer from accuracy loss of aliasing.

In this paper, we develop the rational ABCD matrix of transmission line by using an efficient differential equation solver, called the Differential Quadrature method (DQM) [13]. The idea of DQM is to represent the derivative of a function with respect to a coordinate direction by a weighted linear sum of all the function values at all mesh points along that direction. Starting from the s-domain Telegrapher's equations, DQM discretizes the differential equations as a set of algebraic equations, which are then reduced to ABCD formation featuring rational approximations. Based on this process, a transmission line is described by ABCD matrix whose entries are rational formulas, and its time domain model is obtained by inverse Laplace transform. By means of recursive convolution, the model takes the form of companion model, whose computation complexity is linear with respect to the simulation time. This method can deal with uniform and nonuniform interconnects, and can accurately compute interconnect delays as well as voltage waveforms. Although the rational approximations in this method appear like the Padé approximations of AWE, it completely avoids the moment-matching process of AWE and does not generate the reduced order model. The rational matrix approach by DQM is naturally stable, and it guarantees passivity of interconnects.

2 Rational ABCD matrix of high-speed interconnect

Classical numerical techniques such as finite difference (FD) and finite element (FE) method have been fully developed to numerically solve differential equations. Despite their wide uses, they require computationally prohibitive time to solve large problems. Although the Differential Quadrature Method (DQM) is a numerical approximation, it can quickly calculate the derivative of a function by sampling a small set of grid points while keeping computational accuracy [13]. Therefore, it involves far fewer computing quantities than the FD and FE methods, nevertheless retaining the simple features of the direct numerical techniques.

2.1 Differential quadrature method

Assume a function u in a first order differential equation,

$$u_x = f(s, x, u) \quad x \in [0, 1] \tag{1}$$

with any given boundary conditions at $x = 0$ and $x = 1$, is sufficiently smooth, one can write the *differential quadrature* approximate relation [13],

$$u_x(x_j) = \sum_{i=1}^{N} a_{ij} u(x_i), \quad j = 1, 2, \ldots, N. \tag{2}$$

where a_{ij}s are *DQ coefficients*. Substitution of the above DQ approximation Eqn. 2 into the original Eqn. 1 yields a set of algebraic equations. The matrix of the weighting coefficients a_{ij} depends on the number of grid points, N, and the spacing between the grid points, x_js.

Bellman has employed the shifted Legendre polynomials and power functions to determine the DQ coefficients [13]. Other approaches, using Chebyshev polynomials and harmonic functions, have also been developed to determine DQ coefficients [14]. For example, the approach using harmonic functions requires that Eqn. 2 be exact when $u(x)$ take the following set of test functions:

$$\{1, \sin \pi x, \cos \pi x, \ldots, \sin \frac{N-1}{2}\pi x, \cos \frac{N-1}{2}\pi x\} \tag{3}$$

where N is the number of grid points that is normally an odd number. Substituting this set of test functions into the differential quadrature approximation Eqn. 2, a set of linear equations are obtained. The weighting coefficients are then determined by solving these equations.

Once the positions of selected points are fixed, each of the above mentioned approaches gives constant DQ coefficients, no matter in what applications the differential equations appear. In general, the grid points are selected to be symmetric with respect to the center of domain, or be equally spacing. All the Nth order DQ coefficients constitute a matrix $\mathbf{A}$, which is called the Nth order DQ coefficient matrix or the Nth order DQ operator. In the Appendix some DQ coefficient matrices are listed. Eqn. 2 represents a discretizing operator,

$$\frac{du}{dx} \Longrightarrow \mathbf{A}\mathbf{u} \tag{4}$$

where $\mathbf{u} = [u(x_1), u(x_2), \ldots, u(x_N)]^T$. Eqn. 4 indicates that a derivative is transformed into a vector.

2.2 Application to interconnect

In order to concisely express some manipulations of a matrix, we introduce the following definitions in advance. Let $\mathbf{X}$ be an $m \times n$ matrix.

Definition 1: A *Minus notation* applied as a suffix to a matrix denotes that a row/column of the matrix is eliminated from the matrix as in the following three cases: (a) $\mathbf{X}_{-i}$ the $(m-1) \times n$ submatrix of $\mathbf{X}$ whose ith row is eliminated, (b) $\mathbf{X}^{-j}$ the $m \times (n-1)$ submatrix of $\mathbf{X}$ whose jth column is eliminated, and (c) $\mathbf{X}_{-i}^{-j}$ the $(m-1) \times (n-1)$ submatrix of $\mathbf{X}$ whose ith row and jth column are both removed from $\mathbf{X}$.

Definition 2: A *Plus notation* applied as a suffix to a matrix denotes a specific column of the matrix: $\mathbf{X}^{+j}$ is the jth column of matrix $\mathbf{X}$.

In the frequency domain, an interconnect can be described by the Telegrapher's equations:

$$\frac{d}{dx}V(x,s) = -(sL(x) + R(x))I(x,s) \tag{5}$$

$$\frac{d}{dx}I(x,s) = -(sC(x) + G(x))V(x,s). \tag{6}$$

Applying operator 4 to Eqns. 5 and 6, the above equations are transformed into

$$\mathbf{AV} = -(s\mathbf{L} + \mathbf{R})\mathbf{I} \tag{7}$$

$$\mathbf{AI} = -(s\mathbf{C} + \mathbf{G})\mathbf{V} \tag{8}$$

where

$$\begin{aligned}
\mathbf{A} &= [a_{ij}] \in R^{N \times N} \\
\mathbf{V} &= [V(x_1,s), V(x_2,s), \ldots, V(x_N,s)]^T \\
\mathbf{I} &= [I(x_1,s), I(x_2,s), \ldots, I(x_N,s)]^T \\
\mathbf{L} &= diag\{L(x_1), L(x_2), \ldots, L(x_N)\} \\
\mathbf{R} &= diag\{R(x_1), R(x_2), \ldots, R(x_N)\} \\
\mathbf{C} &= diag\{C(x_1), C(x_2), \ldots, C(x_N)\} \\
\mathbf{G} &= diag\{G(x_1), G(x_2), \ldots, G(x_N)\}
\end{aligned}$$

and $x_1 = 0, x_N = 1$.

Therefore the Telegrapher's equations 5 and 6 (differential equations) are discretized to Eqns. 7 and 8 (algebraic equations). However, Eqns. 7 and 8 are themselves not independent and have no nontrivial solutions. By analogy with the finite-difference method, we can replace the equations at a proper point of $\{x_1, x_2, \ldots, x_N\}$ with the boundary conditions. Numerical experiments show that replacement of equations at the mid-point x_m ($m = N/2$ if N is even; $m = (N+1)/2$ if N is odd) gives the most accurate results. By means of the notation of *Definition 1*, the discretized Telegrapher's equations follow:

$$\begin{aligned} \mathbf{A}_{-m}\mathbf{V} + (s\mathbf{L}_{-m} + \mathbf{R}_{-m})\mathbf{I} &= 0 \quad (9) \\ \mathbf{A}_{-m}\mathbf{I} + (s\mathbf{C}_{-m} + \mathbf{G}_{-m})\mathbf{V} &= 0 \quad (10) \end{aligned}$$

Rewrite Eqns. 9 and 10 as below,

$$\left[\begin{array}{cc} \mathbf{A}_{-m} & s\mathbf{L}_{-m} + \mathbf{R}_{-m} \\ s\mathbf{C}_{-m} + \mathbf{G}_{-m} & \mathbf{A}_{-m} \end{array}\right] \left[\begin{array}{c} V_0 \\ \mathbf{V}_{inn} \\ V_1 \\ I_0 \\ \mathbf{I}_{inn} \\ I_1 \end{array}\right] = \mathbf{0} \quad (11)$$

where $V_0 = V(x_1, s) = V(0, s)$, $V_1 = V(x_N, s) = V(1, s)$ and $I_0 = I(x_1, s) = I(0, s)$, $I_1 = I(x_N, s) = I(1, s)$ are terminal voltages and currents, respectively, and

$$\begin{aligned} \mathbf{V}_{inn} &= [V(x_2, s), \ldots, V(x_{N-1}, s)]^T \\ \mathbf{I}_{inn} &= [I(x_2, s), \ldots, I(x_{N-1}, s)]^T \end{aligned}$$

are the vectors of voltages and currents at the inner grid points.

2.3 Rational matrix of interconnect

An ABCD model of single transmission line is as follows:

$$\left[\begin{array}{c} V_0 \\ I_0 \end{array}\right] = \left[\begin{array}{cc} A(s) & B(s) \\ C(s) & D(s) \end{array}\right] \left[\begin{array}{c} V_1 \\ I_1 \end{array}\right] \quad (12)$$

As $V_0 = A(s)V_1 + B(s)I_1$ and $I_0 = C(s)V_1 + D(s)I_1$, we can calculate $A(s)$, $B(s)$, $C(s)$ and $D(s)$ in this way:

$$\begin{aligned} A(s) = V_0|_{V_1=1, I_1=0}, B(s) = V_0|_{V_1=0, I_1=1} \\ C(s) = I_0|_{V_1=1, I_1=0}, D(s) = I_0|_{V_1=0, I_1=1} \end{aligned}$$

In Eqn.11, let $V_1 = 1, I_1 = 0$, we obtain:

$$\left[\begin{array}{cc} \mathbf{A}_{-m}^{-N} & s\mathbf{L}_{-m}^{-N} + \mathbf{R}_{-m}^{-N} \\ s\mathbf{C}_{-m}^{-N} + \mathbf{G}_{-m}^{-N} & \mathbf{A}_{-m}^{-N} \end{array}\right] \left[\begin{array}{c} V_0 \\ \mathbf{V}_{inn} \\ I_0 \\ \mathbf{I}_{inn} \end{array}\right] = - \left[\begin{array}{c} \mathbf{A}_{-m}^{+N} \\ s\mathbf{C}_{-m}^{+N} + \mathbf{G}_{-m}^{+N} \end{array}\right]. \quad (13)$$

where $\mathbf{A}_{-m}^{+N}$ and $\mathbf{C}_{-m}^{+N} + \mathbf{G}_{-m}^{+N}$ are column vectors (see *Definition 2*). By Cramer's rule, V_0, i.e. $A(s)$ in this case, can be solved from above equation.

$$A(s) = \frac{det\Delta_1}{det\mathbf{\Lambda}}$$

where

$$\mathbf{\Lambda} = \left[\begin{array}{cc} \mathbf{A}_{-m}^{-N} & s\mathbf{L}_{-m}^{-N} + \mathbf{R}_{-m}^{-N} \\ s\mathbf{C}_{-m}^{-N} + \mathbf{G}_{-m}^{-N} & \mathbf{A}_{-m}^{-N} \end{array}\right],$$

and $\mathbf{\Delta}_1$ is the matrix $\mathbf{\Lambda}$ whose first column is replaced by the rightside column of Eqn. 13. Obviously, $det\Delta_1$ and $det\mathbf{\Lambda}$ are polynomials with respect to s, and further analysis shows that both of the polynomials have the same $(2N-4)$th order. As a result, $A(s)$ is approximated by a rational formula. By Heaviside theorem, it can be expressed as:

$$A(s) = \frac{\sum_{i=0}^{2N-4} a_{1i}s^i}{\sum_{i=1}^{2N-4} b_{1i}s^i} = c_1 + \sum_{i=1}^{2N-4} \frac{q_{1i}}{s - p_{1i}} \quad (14)$$

where $c_1 = a_{1,2N-4}/b_{1,2N-4}$. In similar way, $B(s)$, $C(s)$ and $D(s)$ can be expressed by rational approximations with respected to s, and all their numerators and denominators have the same order $(2N-4)$, too. They also can be decomposed as:

$$B(s) = \frac{\sum_{i=1}^{2N-4} a_{2i}s^i}{\sum_{i=1}^{2N-4} b_{2i}s^i} = c_2 + \sum_{i=1}^{2N-4} \frac{q_{2i}}{s - p_{2i}} \quad (15)$$

$$C(s) = \frac{\sum_{i=1}^{2N-4} a_{3i}s^i}{\sum_{i=1}^{2N-4} b_{3i}s^i} = c_3 + \sum_{i=1}^{2N-4} \frac{q_{3i}}{s - p_{3i}} \quad (16)$$

$$D(s) = \frac{\sum_{i=1}^{2N-4} a_{4i}s^i}{\sum_{i=1}^{2N-4} b_{4i}s^i} = c_4 + \sum_{i=1}^{2N-4} \frac{q_{4i}}{s - p_{4i}} \quad (17)$$

After the ABCD matrix is built by the above process, the time domain model can be obtained by inverse Laplace transform. Since the time domain counterparts of $A(s)$, $B(s)$, $C(s)$ and $D(s)$ are the sum of exponential functions, the convolution integrals of the time domain model can be represented by recursive convolution [5], which keeps high computational efficiency.

For the multi-conductor transmission line, the frequency domain Telegrapher's equations are still like Eqns. 5 and 6, but the distributed voltage and current become vectors and PUL parameters matrices. Apply the DQ operator Eqn. 4 to every derivative, the ABCD matrix can be extracted by similar means to single transmission line.

3 Numerical results

A good comparison to DQM is the Chebyshev pseudospectral expansion [15]. According to [15], the maximum frequency of interest can be evaluated by

$$f_{max} = \frac{1}{t_r}, \quad (18)$$

where t_r is the rise time of the input waveform. The maximum frequency determines the minimum wavelength within the spectral range of interest. A heuristic rule for satisfying the resolution requirements of Chebyshev expansions is to use at

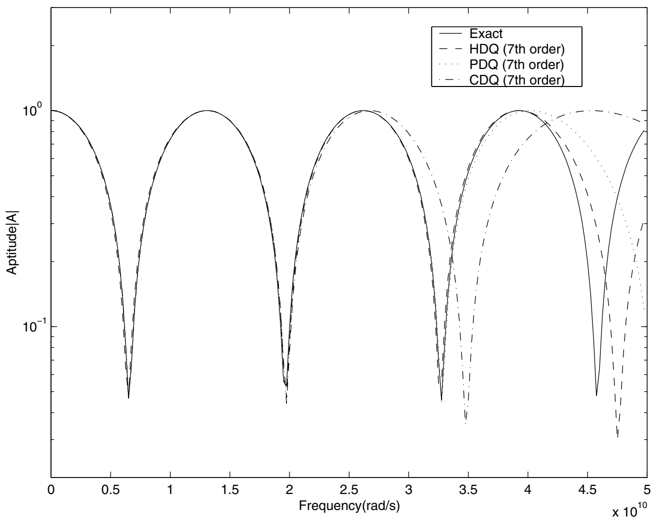

Figure 1: Comparison of frequency response of ABCD matrix.

least four collocation points per wavelength [15], and a resolution of two points per wavelength is sufficient for the modified Chebyshev method [16].

It is significant that the Chebyshev expansion method in [15] is in fact equivalent to the DQM in case of its coefficients are determined by Chebyshev polynomials (called CDQ) [14]. However, as the order of Chebyshev expansion increases, the collocation points (zeros of Chebyshev polynomials) tend to concentrate at two ends of the line. It leads to an oversampling of the voltages and currents at endpoints of the line, and an undersampling at the center portions, which causes great loss of accuracy.

On the other hand, DQM with coefficients determined by harmonic functions (called HDQ), or by power functions (called PDQ), overcomes this difficulty. The grid points of HDQ can be equally spacing or not, or, can be freely chosen by user. In general, we employ the equally spacing points to determine the HDQ coefficients. In this way, the coefficients can be completely determined as fixed constants which are applicable to any case.

In the *first example*, a single interconnect has PUL parameters as follows: $l = 360\ nH/m$, $c = 100\ pF/m$, $r = 100\ \Omega/m$, and $g = 0.01\ S/m$. The line length is $4\ cm$. By employing the above-mentioned procedure, the high-speed interconnect can be represented as an equivalent two-port network, whose properties are described by the ABCD matrix.

We use 7th order PDQ, CDQ and HDQ methods to build different ABCD matrices of this interconnect, respectively. The 7th order DQ operators used in this example can be found in Appendix. Fig. 1 shows the frequency domain responses of A in the different ABCD matrices. It is noted that the HDQ gives higher accuracy than any other DQ method does. A heuristic rule for the resolution of HDQ is to collect two points per minimum wavelength in the spectrum:

$$N = 2\frac{d}{\lambda_{min}} + 2. \tag{19}$$

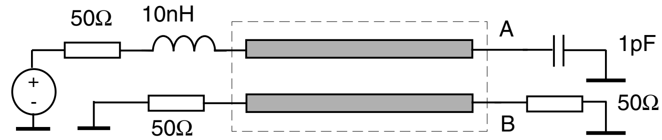

Figure 2: Circuit for coupled high-speed interconnects.

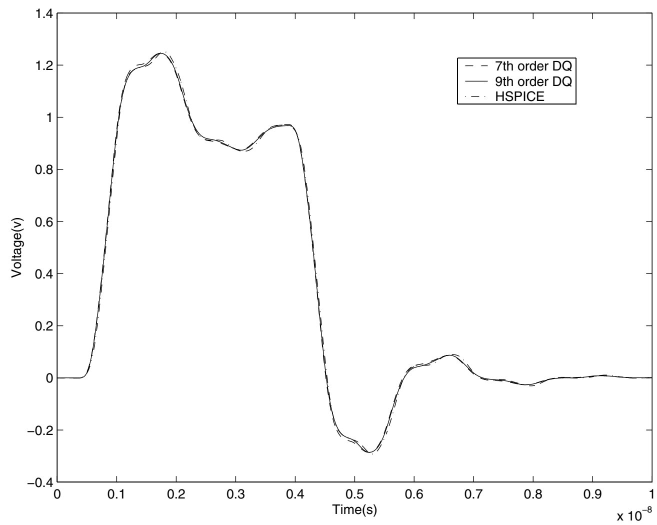

Figure 3: Transient response of uniform interconnect at A.

where λ_{min} is the minimum wavelength determined by 18. In order to ensure the basic discretization accuracy, we set 7 as the minimum number of grid points. That means, there will be at least 5 inner grid points along the interconnect. As a result, the order of DQM is the bigger of 7 or N in Eqn. 19, and the minimum order of any entry of ABCD matrix will be 10.

In the *second example* as shown in Fig. 2, the length of coupled transmission line is 8 cm, and the PUL parameters are as follows,

$$\mathbf{R} = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \Omega/m, \mathbf{L} = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} nH/m,$$

$$\mathbf{C} = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} pF/m, \mathbf{G} = \begin{bmatrix} 0.01 & -0.001 \\ -0.001 & 0.01 \end{bmatrix} S/m$$

We build the ABCD matrix models using Harmonic Differential Quadrature (HDQ) method, whose coefficients are determined by harmonic functions. In this example, 7th- and 9th-order HDQ operators are employed (see Appendix), respectively. The time domain models are obtained by inverse Laplace transform as well as recursive convolution [5]. The transient responses are shown in Fig. 3 and Fig. 4, altogether with the HSPICE results.

The waveforms of DQ methods are agreeable with those of HSPICE. Note that not only the delays are exactly modeled by using DQ methods, but also the crosstalk at the coupled line are well simulated. When applying AWE to interconnect modeling, it is known that the interconnect delay can be well simulated if the delay factor is separately extracted [5], other-

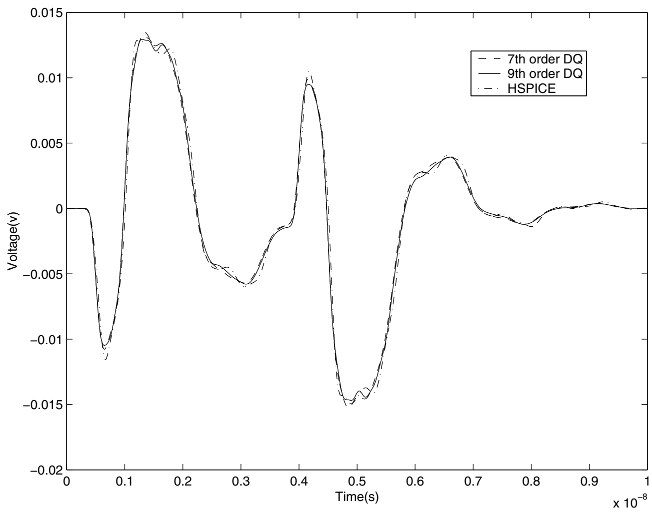

Figure 4: Transient response of uniform interconnect at B.

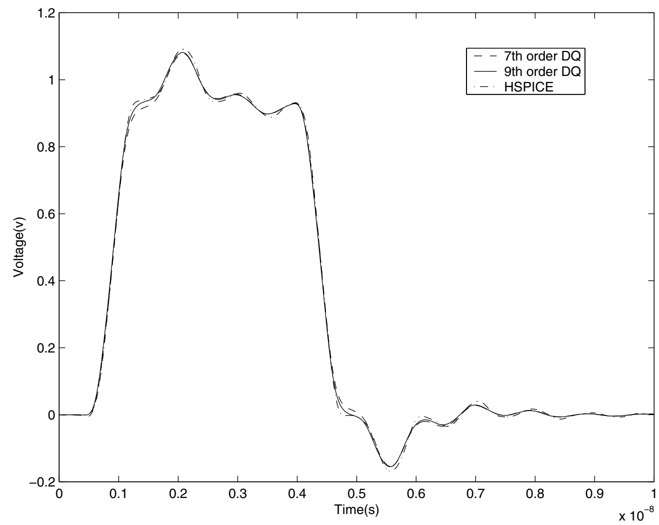

Figure 5: Transient response of nonuniform interconnect at A.

wise it will suffer from ringing even if higher order models are employed [4]. However, the interconnect delay can be accurately modeled by using the differential quadrature method.

HSPICE models the lossy transmission line as multiple lumped filter sections, and it computes the number of lumps from the line delay and the signal risetime [17]. There should be enough lumps in the high-speed interconnect model to ensure that each lump represents a length of line that is a small fraction of a wavelength at the highest frequency used. For the default number of lumps, HSPICE uses the smaller of 20 or $1+(20*T_d/t_r)$, where T_d is the interconnect delay. In this example, it takes more than 20 grid points for HSPICE to model the interconnects, while only 7 or 9 grid points are needed for DQ methods according to the criterion Eqn. 19, which give the same accuracy.

The *third example* includes nonuniform interconnects. The derivation process of rational ABCD matrix shows that this method can deal with nonuniform interconnects in the same way as it deals with uniform interconnects. The circuit is represented by Fig. 2, but the uniform interconnects are replaced with nonuniform coupled interconnects whose PUL parameters are as follows,

$$\mathbf{L} = \begin{bmatrix} L(x) & Lm(x) \\ Lm(x) & L(x) \end{bmatrix}, \mathbf{C} = \begin{bmatrix} C(x) & Cm(x) \\ Cm(x) & C(x) \end{bmatrix},$$
$$\mathbf{R} = \begin{bmatrix} R(x) & 0 \\ 0 & R(x) \end{bmatrix}, \mathbf{G} = \begin{bmatrix} G(x) & 0 \\ 0 & G(x) \end{bmatrix}$$

where

$$\begin{aligned} L(x) &= 494.6/((1+k(x))\, nH/m, \\ Lm(x) &= 63.3k(x)\, nH/m, \\ C(x) &= 62.8/(1-k(x))\, pF/m, \\ Cm(x) &= -4.9k(x)\, pF/m, \\ R(x) &= 50/(1+k(x))\, \Omega/m, \\ G(x) &= 0.01/(1-k(x))\, S/m, \end{aligned}$$

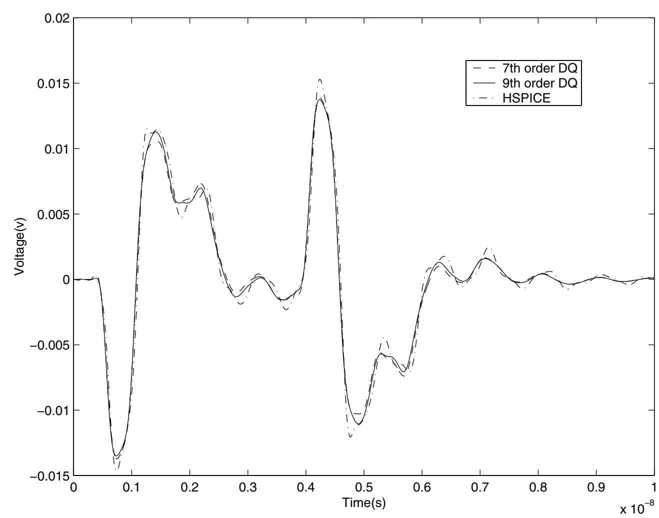

Figure 6: Transient response of nonuniform interconnect at B.

$$k(x) = 0.25(1+\sin(6.25\pi x + 0.25\pi)).$$

We respectively use HDQ with 7th and 9th-order coefficients (see Appendix) to compute the transient responses. Unlike the method of cascaded chain of segmented short lines [12], DQ method generates a rational ABCD matrix of the whole nonuniform line. The transient results are shown in Fig. 5 and 6. As comparison, the results of HSPICE are also shown. Since nonuniform transmission line cannot be handled by HSPICE directly, we segment the lines into 8 equal short sections, each of which is regarded as uniform line. The results of DQM and HSPICE are very agreeable. The computations are performed on Sun Ultra-1 workstation. It takes 3.6 s for HSPICE to obtain the indicated response using a time step 10 ps, while 9th-order DQM needs $1.2s$ and 7th-order DQM needs only $0.95s$ to generate the responses as shown.

4 Conclusions

A direct numerical technique, called the differential quadrature method (DQM), is used to develop rational ABCD matrix of high-speed interconnect. DQM discretize the interconnect into few grid points across the entire length, leading to a set of algebraic equations by computing the electrical parameters at those points. Based on the discretization, rational ABCD matrix can be obtained with its entries represented by rational approximations in the frequency domain. The time domain counterpart of rational ABCD matrix can be analytically obtained by inverse Laplace transformation. By means of recursive convolution, the time domain model retains high efficiency and accuracy. Although the rational approximations of ABCD matrix by the differential quadrature method are comparable with the results of Padé approximation of AWE, the DQM solution process differs significantly from the AWE algorithm. One notable feature of the differential quadrature method is that it is more stable than AWE, since it avoids the process of moment generation and moment matching. Despite its numerical operations, the differential quadrature method is an efficient solver to model transmission lines, and the DQM-based rational ABCD matrix leads to high efficiency for transient analysis of uniform and nonuniform interconnects.

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