# Detectability of distributed consensus-based observer networks: An elementary analysis and extensions

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*Abstract*— This paper continues the study of local detectability and observability requirements on components of distributed observers networks to ensure detectability properties of the network. First, we present a sketch of an elementary proof of the known result equating the multiplicity of the zero eigenvalue of the Laplace matrix of a digraph to the number of its maximal reachable subgraphs. Unlike the existing algebraic proof, we use a direct analysis of the graph topology. This result is then used in the second part of the paper to extend our previous results which connect the detectability of an observer network with corresponding local detectability and observability properties of its node observers. The proposed extension allows for nonidentical matrices to be used in the interconnections.

#### I. INTRODUCTION

The principle of distributed estimation can be traced back to the original work on decentralized estimation completed in the 90s [2], [5], while the more modern ideas are focused around the observer network design to allow node estimators to exchange information with the objective of improving their knowledge of the system state (or a part thereof) through reaching an agreement about the estimated quantity. A most recent development in this area is concerned with  $H_{\infty}$  distributed estimation in the presence of modelling uncertainties and perturbations [9], [10], [14], [12], [13].

Some efforts have recently been made to obtain a more detailed insight into the role played by the communication topology in distributed estimation problems. For example, in [6] an algebraic design of the communication topology for networked observers was considered using the structured systems theory. The objective in [7] was to maintain the collective detectability of the network while achieving a desired observer sparsity. Here, the term *collective detectability*, or distributed detectability, refers to the detectability property achieved by the entire network, in contrast to the local detectability which refers to the detectability of the plant from the measurements taken by individual nodes of the network. In [11] the property of distributed detectability of the observer network was related to the local detectability of the plant through measurements, observability of the node filters through interconnections, and the largest spanning trees of the underlying communication graph. In particular, it was shown that for the associated interconnected system of filtering error dynamics to be stabilizable via output injection, each network component spanned by a non-extendable tree must be collectively detectable. A similar requirement on the nodes with unobservable local error dynamics to have an incoming path connecting them with collectively observable subnetworks was discussed in [8].

In this paper, we revisit the distributed detectability problem for a network of interconnected state estimators observing a linear plant considered in [11]. The detectability conditions obtained in that reference are based on a relationship between the multiplicity of the zero eigenvalue of the graph Laplacian matrix and the number of maximal reachable clusters within the graph [1], [4]. In these references, the mentioned relationship was obtained as a special case of a more general theory, using the tools from the matrix algebra. Here we give an elementary self-contained proof of this relationship using a direct analysis of the graph topology. Furthermore, we use this result to present some extensions of the results in [11] which show that in the distributed estimation scenario, algebraic properties of the graph Laplacian must be complemented by the detectability and observability properties of the node filters through measurements and interconnections, respectively. The proposed extensions relax one of the limitations of these results in that we do not require all observers to use the same matrix for interconnections.

The paper is organized as follows. In Section II we provide the alternative proof of the above mentioned result about the relationship between the multiplicity of the zero eigenvalue of the Laplace matrix of a directed graph and the number of its maximal reachable components. In Section III this result is applied to establish the connection between collective detectability of the plant and its detectability by individual sensor nodes combined into certain clusters spanned by trees. This provides a natural way to analyse the collective detectability properties of the entire filter network from the corresponding properties of the nodes and interconnections within clusters, using the results in [11].

*Notation:* Throughout the paper,  $\mathbf{R}^n$  denotes the real Euclidean *n*-dimensional vector space, with the norm  $||x|| \triangleq (x'x)^{1/2}$ ; here the symbol ' denotes the transpose of a matrix or a vector. Ker *A* denotes the null-space of a matrix *A*.  $\mathbf{0}_k \triangleq [0 \dots 0]' \in \mathbf{R}^k, \mathbf{1}_k \triangleq [1 \dots 1]' \in \mathbf{R}^k$ , and  $I_k$  is the identity matrix; we will omit the subscript *k* when this causes no ambiguity. The symbol  $\otimes$  denotes the Kronecker product of matrices, or the tensor product of two vector spaces. Also we use the notation  $\prod_{l=1}^N \mathscr{P}_l$  to denote the Cartesian product of *N* vector spaces  $\mathscr{P}_1, \ldots, \mathscr{P}_N$ . dim  $\mathscr{X}$  denotes the dimension of a finite dimensional vector space  $\mathscr{X}$ . The symbol diag $[P_1, \ldots, P_N]$  denotes the block-diagonal matrix,

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# II. THE MULTIPLICITY OF THE ZERO EIGENVALUE OF THE DIGRAPH LAPLACIAN IS EQUAL TO TEST MANIMAL SUBGRAPHS SPANNED BY TREES

Consider a filter network with N nodes and a directed graph topology  $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ ;  $\mathbf{V} = \{1, 2, \dots, N\}$  and  $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$  are the set of vertices and the set of directed edges, respectively. The ordered pair (j, i) will denote the directed edge of the graph originating at node j and ending at node i. In accordance with the common convention, the graph  $\mathbf{G}$  is assumed to have no self-loops, i.e.,  $(i, i) \notin \mathbf{E}$ . The notation  $\mathbf{G}(i_1, \dots, i_M)$  will denote a subgraph of  $\mathbf{G}$  with the node set  $\{i_1, \dots, i_M\} \subseteq \mathbf{V}$  and an edge set  $\{(i_s, i_l) : i_s, i_l \in \{i_1, \dots, i_M\}\} \subseteq \mathbf{E}$ .

For each  $i \in \mathbf{V}$ , let  $\mathbf{V}_i = \{j : (j, i) \in \mathbf{E}\}$  be the neighbourhood of i. The cardinality of  $\mathbf{V}_{\mathbf{PS}}$  known as the in-degree of node i, is denoted  $p_i$ ; i.e.,  $p_i$  is equal to the number of incoming edges for node i.

Node *i* of a digraph is said to be reachable from node *j* if there exists a directed path originating at *j* and ending at *i*. The graph is connected if any two nodes are connected by an undirected path; the graph is strongly connected if its every node is reachable from any other node. A subgraph of **G** is a spanning tree if it has the same vertex set **V**, has no cycles, has N - 1 edges and contains a node from which every other node of **G** can be reached by traversing along the directed edges of **G** (the root node).

Throughout the paper,  $\mathscr{A}$ ,  $\mathscr{D}$  and  $\mathscr{L}$  will denote the adjacency matrix, the in-degree matrix and the Laplacian matrix of the graph G, respectively,

$$\mathscr{A} = [\mathbf{a}_{ij}]_{ij=1,\ldots,N}, \quad \mathbf{a}_{ij} = \begin{cases} 1 & \text{if } (j,i) \in \mathbf{E}, \\ 0 & \text{otherwise,} \end{cases}$$
$$\mathscr{D} = \text{diag} [p_1 \ \ldots \ p_N],$$
$$\mathscr{L} = \mathscr{D} - \mathscr{A}.$$

The eigenvalues of  $\mathscr{L}$  will be denoted  $\lambda_i$ , i = 1, ..., N. It is easy to check that zero is one of the eigenvalues of  $\mathscr{L}$ , and  $\mathbf{1}_N$  is the corresponding right eigenvector.

*Definition 1:* A subgraph of **G** is a *cluster*, if it satisfies the following requirements:

- (i) It contains a spanning tree; and
- (ii) It is a maximal subgraph in the sense that none of its spanning trees can be extended by adding nodes from the set V.

An example illustrating this definition is shown in Figure 1(a). It follows from Definition 1 that clusters have no outgoing connections to outside nodes. It also follows from Definition 1, that the nodes within a cluster which are roots of its spanning trees are not reachable from outside the cluster. These facts are now formally stated.

Proposition 1: Consider a cluster  $\mathbf{G}(i_1, \ldots, i_s) \subset \mathbf{G}$ . If  $j \notin \{i_1, \ldots, i_s\}$ , then it is not reachable from  $\{i_1, \ldots, i_s\}$ .

Proposition 2: Let  $i_1$  be the root node of a tree graph spanning a cluster  $\mathbf{G}(i_1, \ldots, i_s) \subset \mathbf{G}$ . If  $j \notin \{i_1, \ldots, i_s\}$ , then  $(j, i_1) \notin \mathbf{E}$ .

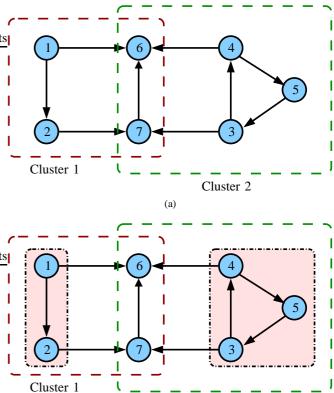


Fig. 1. An example of a directed graph consisting of two clusters (shown in Figure (a)), with inner subgraphs shown in Figure (b) (inside the shaded boxes).

(b)

Cluster 2

It follows from Proposition 1 that clusters are the largest subgraphs reachable from within themselves, i.e., they are reaches in the terminology of [4]. Here we call these subgraphs clusters, to acknowledge that our formal definition is different; unlike [4] it involves spanning trees. Also, it will be shown in the next section that these subgraphs form smallest collectively detectable clusters within collectively detectable networks, hence the name to reflect this.

We note the difference between clusters and strongly connected components of a digraph; cf. [8]. For example, the graph in Figure 1(a) consists of two clusters comprised of the vertex sets  $\{1, 2, 6, 7\}$  and  $\{3, 4, 5, 6, 7\}$  which have a nonempty intersection. On the other hand, strongly connected components of a digraph, being the largest components connected by directed paths, cannot overlap. As another important point of difference, by definition clusters cannot have outgoing edges connecting them to the outside nodes (but can have incoming edges) whereas strongly connected components of a digraph can have such outgoing edges. From the perspective of distributed estimation, this means that the observers that belong to a cluster do not share information with observers located outside this cluster, but can receive information from other clusters. However, there exist observer nodes within each cluster which do not receive such information. Such vertices form *isolated* subgraphs within the digraph [8]. In [8] isolated subgraphs were associated with irreducible (i.e., strongly connected) components of the underlying graph. In this paper, we adopt a somewhat different definition which does not require these subgraphs to be strongly connected; instead it emphasizes spanning trees within clusters. This allows to establish a natural relationship between the clusters and their inner isolated subgraphs; see Lemma 1 below.

Definition 2: A subgraph  $\mathbf{G}(i_1, \ldots, i_r)$  is inner, if

- (i) it contains a spanning tree; and
- (ii) it is a maximal subgraph with the property that its vertices  $i_1, \ldots, i_r$  are not reachable from any node  $j \in \mathbf{V} \setminus \{i_1, \ldots, i_r\}$ .

By definition, a single-vertex subgraph, which does not have incoming edges is inner.

*Lemma 1:* There is exactly one inner subgraph within any cluster of a connected graph.

*Remark 1:* It is easy to give an example of a graph containing a strongly connected component which has incoming edges and therefore does not contain isolated (inner) subgraphs. On the other hand, since each strongly connected component of a digraph contains a spanning tree, it is a subgraph of one of the digraph's clusters. This leads to the conclusion that the number of strongly connected components in a digraph is greater than or equal to the number of clusters.

We now present the main results of this section.

Theorem 1: The multiplicity of the zero eigenvalue of  $\mathscr{L}$  is equal to the number of clusters in the graph  $\mathscr{G}$ .

The proof of Theorem 1 relies on Lemma 1. According to this lemma, it suffices to show that the multiplicity of the zero eigenvalue of  $\mathscr{L}$  is equal to the number of inner subgraphs in the graph  $\mathscr{G}$ . The latter proof proceeds in a manner similar to proving a similar claim in Theorem 2 of [8] involving strongly connected components of the graph and its isolated subgraphs. The key observation underlying the proof is that by definition, for any inner subgraph  $\mathbf{G}(i_1, \ldots, i_s)$ , we have  $\mathscr{L}_{ij} = 0$ , if  $i \in \{i_1, \ldots, i_s\}$  and  $j \notin \{i_1, \ldots, i_s\}$ . This shows that by permuting rows and columns of  $\mathscr{L}$ , the Laplacian matrix can be represented in the following blockmatrix form

$$\mathscr{L} = \begin{bmatrix} \mathscr{L}_{11} \dots & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & \dots & \mathscr{L}_{ll} & 0 \\ F_1 & \dots & F_l & R \end{bmatrix},$$
(1)

where the first l rows of blocks correspond to inner subgraphs of **G**, and the remaining rows correspond to nodes that do not belong to any of the inner subgraphs. Furthermore, since by definition inner subgraphs do not have incoming edges, each of the diagonal blocks  $\mathscr{L}_{ii}$  is a Laplacian matrix of the corresponding subgraph. Also, it can be shown that R is nonsingular.

Theorem 1 allows to determine the basis of the null-space of  $\mathscr{L}$ . This result will be instrumental in the analysis of

the the unobservable subspace of system interconnections given in the next section. Without loss of generality, in the following corollary we assume that the vertices of the underlying digraph are ordered so that the Laplace matrix  $\mathscr{L}$  is of the form (1).

Corollary 1: Ker  $\mathscr{L} = \text{Span}(b_1, \ldots, b_l)$  where l is the multiplicity of the zero eigenvalue of  $\mathscr{L}$ , and

$$b_{1} = \begin{bmatrix} \mathbf{1}_{\dim \mathscr{L}_{11}} \\ \mathbf{0}_{\dim \mathscr{L}_{22}+\ldots+\dim \mathscr{L}_{ll}} \\ -R^{-1}F_{i}\mathbf{1}_{\dim \mathscr{L}_{11}} \end{bmatrix},$$
  
$$b_{i} = \begin{bmatrix} \mathbf{0}_{\dim \mathscr{L}_{11}+\ldots+\dim \mathscr{L}_{i-1,i-1}} \\ \mathbf{1}_{\dim \mathscr{L}_{ii}} \\ \mathbf{0}_{\dim \mathscr{L}_{i+1,i+1}+\ldots+\dim \mathscr{L}_{ll}} \\ -R^{-1}F_{i}\mathbf{1}_{\dim \mathscr{L}_{ii}} \end{bmatrix}, \quad i = 2, \ldots, l.$$

### III. THE DISTRIBUTED DETECTABILITY PROBLEM

The property of collective detectability refers to the ability of a network of consensus-based observers

$$\dot{\hat{x}}_i = A\hat{x}_i + L_i(y_i(t) - C_i\hat{x}_i) + K_i \sum_{j \in \mathbf{V}_i} H_i(\hat{x}_j - \hat{x}_i), \quad (2)$$
$$\hat{x}_i(0) = 0,$$

to provide an asymptotically accurate estimate of the state of a plant

$$\dot{x} = Ax + B\xi(t), \quad x(0) = x_0,$$
(3)

from their local measurements of the form

$$y_i(t) = C_i x(t) + D_i \xi(t) + \bar{D}_i \xi^i(t).$$
 (4)

Here  $x \in \mathbf{R}^n$  is the state of the plant,  $\hat{x}_i$  is its estimate calculated at node  $i, \xi \in \mathbb{R}^m, \xi \in \mathbb{R}^m, \xi^i \in \mathbb{R}^{m_i}$  represent the plant uncertainty and the measurement uncertainty at the local sensing node  $i, A, B, C_i, D_i, \overline{D}_i$  are given constant matrices of corresponding dimensions, and  $H_i$  are given matrices which describe the information shared by nodes  $j \in \mathbf{V}_i$  with i.

The matrices  $L_i$ ,  $K_i$  are parameters of the filters. In a distributed state estimation problem, they are to be determined to ensure that the size of the estimation error,  $\|\hat{x}_i(t) - x(t)\|$  reduces to 0 asymptotically, in an  $L_2$  sense or remains bounded in some sense, depending on the nature of the disturbance signals  $\xi$ ,  $\xi_i$ . For such matrices to exist the following detectability property is naturally expected to hold.

Define the matrices

$$\bar{A} = I_N \otimes A, \quad \bar{H} = \left[\bar{H}_{ij}\right]_{i,j=1,\dots,N}$$
  
where  $\bar{H}_{ij} = \begin{cases} p_i H_i & \text{if } j = i, \\ -\mathbf{a}_{ij} H_i & \text{if } j \neq i, \end{cases}$   
 $\bar{C} = \text{diag} \left[C_1, \dots, C_N\right].$ 

Definition 3: The system consisting of the plant (3), the measurements (4) and the observers (2) is said to be collectively detectable, if the matrix pair  $([\bar{C}', \bar{H}']', \bar{A})$  is detectable.

*Remark 2:* Detectability of the pair  $([\bar{C}', \bar{H}']', \bar{A})$  is necessary but is not sufficient for the existence of the set of observer gains  $K_i, L_i$  which ensure that the matrix  $\bar{A} - \text{diag}[L_1 \dots L_N]\bar{C} - \text{diag}[K_1 \dots K_N]\bar{H}$  is Hurwitz. Therefore, the collective detectability property is a necessary condition for the consensus-based filters (2) each to provide an estimate of the plant (3).

The pair  $([\bar{C}', \bar{H}']', \bar{A})$  can be detectable even if the individual pairs  $(C_i, A)$  are not detectable. As was shown in [11] in the case where  $H_i = H$ , i = 1, ..., N, for this to be true, each network node must be able to complement its local measurements with feedback it receives from its neighbours trough the interconnections. In this section, the condition  $H_i = H$  is relaxed.

Recall the definitions of undetectable and unobservable subspaces of a matrix pair (G, F),  $F \in \mathbf{R}^{n \times n}$ ,  $G \in \mathbf{R}^{m \times n}$ . Let  $\alpha_F(s)$  denote the minimal polynomial of F, i.e., the monic polynomial of least degree such that  $\alpha_F(F) = 0$  [15], factored as  $\alpha_F(s) = \alpha_F^-(s)\alpha_F^+(s)$ ; the zeros of  $\alpha_F^-(s)$  and  $\alpha_F^+(s)$  are in the open left and closed right half-planes of the complex plane, respectively. Note that  $\operatorname{Ker} \alpha_F^-(F) \cap \operatorname{Ker} \alpha_F^+(F) = \{0\}$ , and  $\operatorname{Ker} \alpha_F^-(F) \oplus \operatorname{Ker} \alpha_F^+(F) = \mathbf{R}^n$  [15]. The undetectable subspace of (G, F) is the subspace of (G, F) is the subspace of (G, F) is the subspace of  $(G, F^{l-1})$  [3].

Define the observability matrices associated with the matrix pairs  $(C_i, A)$  and  $(H_i, A)$ :

$$O_{C_i} \triangleq \begin{bmatrix} C_i \\ C_i A \\ \vdots \\ C_i A^{n-1} \end{bmatrix}, \quad O_{H_i} = \begin{bmatrix} H_i \\ H_i A \\ \vdots \\ H_i A^{n-1} \end{bmatrix}.$$

Also, consider the undetectable subspace of  $(C_i, A)$  and the unobservable subspace of  $(H_i, A)$ , which will be denoted  $\mathscr{C}_i$ ,  $\mathscr{O}_{H_i}$ . Furthermore, let  $\overline{\mathscr{O}}$  denote the unobservable subspace of  $(\overline{H}, \overline{A})$ ,

$$\bar{\mathscr{O}} \triangleq \bigcap_{l=1}^{nN} \operatorname{Ker}(\bar{H}\bar{A}^{l-1})$$

*Lemma 2:* The pair  $([\bar{C}', \bar{H}']', \bar{A})$  is detectable if and only if

$$\operatorname{Ker}\left(\operatorname{diag}[O_{H_1}, \ldots, O_{H_N}](\mathscr{L} \otimes I_n)\right) \cap \prod_{i=1}^N \mathscr{C}_i = \{0\}.(5)$$

The following necessary condition for collective detectability has been obtained in [11] for the special case where  $H_i = H$ ; see [11, Theorem 3]. Using Lemma 2, this requirement can be relaxed.

Theorem 2: Suppose the pair  $([\bar{C}', \bar{H}']', \bar{A})$  is detectable. Then, for every cluster  $\mathbf{G}(i_1, \ldots, i_s)$  the following statements hold:

(i) 
$$\bigcap_{i \in \{i_1, \dots, i_s\}} \mathscr{C}_i = \{0\};$$
  
(ii) for all  $i \in \{i_1, \dots, i_s\},$   
 $\left(\bigcap_{j:i \in \mathbf{V}_j} \mathscr{O}_{H_j}\right) \cap \mathscr{O}_{H_i} \cap \mathscr{C}_i = \{0\}.$ 

The interpretation of claims (i) and (ii) of Theorem 2 is as follows. Claim (i) states that every state of a collectively detectable plant is necessarily detectable by at least one observer within each cluster of the network. Also, condition (ii) states that communications between the observer nodes in a collectively detectable system must be designed so that each plant state x has at least one of the three properties at every node of every cluster: (a) it is detectable by the node from its measurements (i.e.,  $x \notin \mathcal{C}_i$ ), or (b) it is observable from the information the node receives from its neighbours (i.e.,  $x \notin \mathcal{O}_{H_i}$ ), or (c) it is observable by at least one of the neighbours with whom the node communicates (i.e, there exists j such that  $i \in V_j$  and  $x \notin \mathcal{O}_{H_j}$ ). In the case where  $H_i = H \ \forall i$ , we recover claim (ii) of [11, Theorem 3]: for all  $i \in \{i_1, \ldots, i_s\}, \ \mathcal{O}_H \cap \mathcal{C}_i = \{0\}.$ 

Next, a sufficient condition for collective detectability is presented which extends the corresponding condition in [11, Theorem 4] to the case where the interconnection protocol matrices  $H_i$  are not required to be identical. The proof of this result relies on Corollary 1, therefore we again assume that the vertices of the underlying digraph are ordered so that the Laplace matrix  $\mathscr{L}$  has the block structure (1).

Theorem 3: Suppose all the pairs  $(H_i, A)$  are observable. If every cluster in the network satisfies condition (i) of Theorem 2, then the pair  $([\bar{C}', \bar{H}']', \bar{A})$  is detectable.

*Remark 3:* The result of Theorem 3 remains valid in the case where  $H_i = 0$  for some of the nodes. Each such node represents the root node in the corresponding cluster, and also belongs to the inner subgraph of the cluster. Provided the remaining nodes have observable  $(H_j, A)$ , it is still true that  $\overline{\mathcal{O}} = \operatorname{Ker} \mathscr{L} \otimes \mathbf{R}^n$ . Hence the statement of the theorem holds in this case.

## **IV.** CONCLUSIONS

In this paper, we obtained necessary and sufficient conditions for distributed detectability of a linear plant via a network of state estimators, which were previously obtained under condition that all observers utilize the same matrix for communication. Our results show that in a collectively detectable system, each state in the plants' phase space must be detectable by every observer cluster spanned by a maximal tree. Furthermore, at every node of the network, every undetectable state of  $(C_i, A)$  must be observable through interconnections or must be transmitted to a neighbour who can observe it. Thus, the results of this paper elucidate the relationship between the network topology and detectability properties of the plant and observers. In particular, the paper makes explicit the role of spanning trees in ensuring collective detectability.

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