Generalized gramian based frequency interval model reduction for unstable systems

4
Author(s)

Ahmad Jazlan ; Victor Sreeram ; Roberto Togneri ; Ha Binh Minh

Abstract:
Frequency interval controllability and observability gramian matrices are important in order to understand the characteristics of systems which are inherently frequency dependent. Obtaining these frequency interval controllability and observability gramian matrices requires solving a pair of Lyapunov equations. However for certain systems these Lyapunov equations are not solvable. In addition the eigenvalues of the product of the frequency interval controllability and observability gramians may also be complex numbers and therefore these gramians are not applicable to used in the context of model reduction. To overcome these issues, generalized frequency interval controllability and observability gramians are introduced in this paper and the applicability of these generalized gramians to be used in model reduction is demonstrated.

Published in: Control Conference (AuCC), 2016 Australian

Date of Conference: 3-4 Nov. 2016

Date Added to IEEE *Xplore*: 02 March 2017

ISBN Information:
Electronic ISBN: 978-1-922107-90-9
Print on Demand(PoD) ISBN: 978-1-5090-5764-1

INSPEC Accession Number: 16709909

DOI: 10.1109/AUCC.2016.7868000

Publisher: IEEE

Conference Location: Newcastle, NSW, Australia

SECTION I.
Introduction

Controllability and observability gramians matrices have been applied in a broad range of applications such as for the input-output pairing methods in control configuration selection [1], sensitivity analysis of biochemical reaction networks [2] and the model order reduction of large

Generalized Gramian Based Frequency Interval Model Reduction for Unstable Systems

Ahmad Jazlan, Victor Sreeram, Roberto Togneri, and Ha Binh Minh

***Abstract*—Frequency interval controllability and observability gramian matrices are important in order to understand the characteristics of systems which are inherently frequency dependent. Obtaining these frequency interval controllability and observability gramian matrices requires solving a pair of Lyapunov equations. However for certain systems these Lyapunov equations are not solvable. In addition the eigenvalues of the product of the frequency interval controllability and observability gramians may also be complex numbers and therefore these gramians are not applicable to used in the context of model reduction. To overcome these issues, generalized frequency interval controllability and observability gramians are introduced in this paper and the applicability of these generalized gramians to be used in model reduction is demonstrated.**

***Index Terms*—Model Order Reduction, Controllability and Observability Gramians, Linear Systems, Unstable Systems, Lyapunov Equations.**

I. Introduction

Controllability and observability gramians matrices have been applied in a broad range of applications such as for the input-output pairing methods in control configuration selection [1], sensitivity analysis of biochemical reaction networks [2] and the model order reduction of large scale dynamical systems and systems with complex hyperbolic networks [3], [4]. Many problems which involve the computation of gramians are inherently frequency dependent [5]–[9]. Shaker had applied frequency limited controllability and observability gramians for input-output interactions for control configuration selection and also for the measurement of control reconfigurability in the context of fault tolerant control systems [10], [11].

For particular systems which are unstable or do not fulfill the solvability conditions of the standard Lyapunov equations, generalized controllability and observability gramians which are applicable to these systems have been described by Zhou, Salomon and Wu [12]. Inspired by this work, Shaker had introduced the generalized cross gramian to obtain the cross gramians for systems where the Sylvester equation is not solvable [13]. This generalization is based on the techniques by Zhou, Salomon and Wu and also the alternative definition of the gramians defined by Fernando and Nicholson [12], [14].

In this paper we propose a model reduction method based on solving a pair of generalized frequency interval controllability and observability gramians to deal with systems which do not have a solution to the standard frequency interval controllability and observability gramians [15]. This approach is inspired by the method by Zhou, Salomon and Wu [12] and also the method by Gawronski and Juang [15].

A. Jazlan is with the School of Electrical and Electronics Engineering, University of Western Australia and the Dept of Mechatronics Engineering, Faculty of Engineering, IIUM, Kuala Lumpur. E-mail: ahmadjazlan@iium.edu.my

V. Sreeram and R. Togneri are with the School of Electrical and Electronics Engineering, University of Western Australia. Email: victor.sreeram/roberto.togneri @uwa.edu.au

Ha Binh Minh is with Faculty of Management Information System, the Banking University Ho Chi Minh City, 39 Ham Nghi Street, District 1, Ho Chi Minh City, Vietnam

II. Preliminaries

A. Controllability and Observability Gramians for Continuous-Time Systems

Consider the following continuous-time system

$$G(s) = C(sI - A)^{-1}B + D \tag{1}$$

where $\{A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}, C \in \mathbb{R}^{q\times n}, D \in \mathbb{R}^{p\times q}\}$ is its minimal realization. The equivalent time and frequency domain controllability and observability gramians are given as follows

$$P = \int_0^\infty e^{A\tau}BB^*e^{A^*\tau}d\tau \tag{2}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}(j\omega I - A)^{-1}BB^*(-j\omega I - A^*)^{-1}d\omega \tag{3}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}H(\omega)BB^*H^*(\omega)d\omega \tag{4}$$

$$Q = \int_0^\infty e^{A^*\tau}C^*Ce^{A\tau}d\tau \tag{5}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}(-j\omega I - A^*)^{-1}C^*C(j\omega I - A)^{-1}d\omega \tag{6}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}H^*(\omega)C^*CH(\omega)d\omega \tag{7}$$

It is established that the gramians P and Q satisfy the following Lyapunov equations

$$AP + PA^* + BB^* = 0 \tag{8}$$

$$A^*Q + QA + C^*C = 0 \tag{9}$$

B. Frequency Interval Controllability and Observability Gramians for Linear Systems [15]

Definition 1 ([15]). The frequency interval controllability and observability gramians within the interval $\Omega = [\omega_1, \omega_2]$ for the stable linear system in (22) are defined as:

$$\hat{P}_\Omega = \hat{P}(\omega_2) - \hat{P}(\omega_1) \tag{10}$$

$$\hat{Q}_\Omega = \hat{Q}(\omega_2) - \hat{Q}(\omega_1), \tag{11}$$

where

$$\hat{P}(\omega) = \frac{1}{2\pi}\int_{-\omega}^{\omega} H(\omega)BB^*H^*(\omega)d\omega \tag{12}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\mu)\left[S(\omega)BB^* + BB^*S^*(\omega)\right]H^*(\mu)d\mu$$

$$= S(\omega)P + PS^*(\omega) \tag{13}$$

$$\hat{Q}(\omega) = \frac{1}{2\pi}\int_{-\omega}^{\omega} H^*(\omega)C^*CH(\omega)d\omega \tag{14}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H^*(\mu)\left[S(\omega)C^*C + C^*CS(\omega)\right]H(\mu)d\mu$$

$$= S^*(\omega)Q + QS(\omega) \tag{15}$$

and

$$S(\omega) = \frac{1}{2\pi}\int_{-\omega}^{+\omega} H(\omega)d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega}^{+\omega} (j\omega I - A)^{-1}d\omega \tag{16}$$

$$= \frac{j}{2\pi}\ln[(j\omega I + A)(-j\omega I + A)^{-1}] \tag{17}$$

It follows that $\hat{P}(\omega)$ and $\hat{Q}(\omega)$ are the solutions to the following Lyapunov equations [15]:

$$A\hat{P}(\omega) + \hat{P}(\omega)A^* + S(\omega)BB^* + BB^*S^*(\omega) = 0 \tag{18}$$

$$A^*\hat{Q}(\omega) + \hat{Q}(\omega)A + S^*(\omega)C^*C + C^*CS(\omega) = 0. \tag{19}$$

C. Controllability and Observability Gramians for Continuous-Time Unstable Systems [12]

The controllability and observability gramians defined in (2) and (5) are only valid for stable systems since for unstable systems the integrals will become unbounded. The Lyapunov equations in (8) and (9) however may still have solutions even though A is unstable provided that the matrix A does not have a pair of eigenvalues which are negatives of each other. If the matrix A has a pair of eigenvalues which are negatives of each other, then the Lyapunov equations in (8) and (9) will not have any solutions. For example consider the following system $G_1(s)$

$$G_1(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|c} -1 & 0 & 0.1 \\ 0 & 1 & 1 \\ \hline 0 & 1 & 0 \end{array}\right] \tag{20}$$

Equations (8) and (9) will not have any solutions for the this system $G_1(s)$ since the A matrix of this system has a pair of eigenvalues which are negatives of each other.

Another constraint of using equations (8) and (9) to obtain the controllability and observability gramians is that even if (8) and (9) have solutions, the eigenvalues of the product of the controllability and observability gramians may be complex numbers which prohibits the use of the controllability and observability gramians in applications such as model order reduction and control configuration selection which require real numbered Hankel singular values. An example of a system which exhibits such a property is as follows

$$G_2(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|cc} -1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \tag{21}$$

The eigenvalues of the product of the controllability and observability gramians calculated using (8) and (9) for this system $G_2(s)$ are a pair of complex conjugate numbers: $0.0625 + 0.2421i$ and $0.0625 - 0.2421i$

To overcome these constraints, Zhou, Salomon and Wu had defined the following generalized controllability and observability gramians for possibly unstable systems as follows [12]:

Suppose that (A, B) is stabilizable and (C, A) is detectable. Let X and Y be the stabilizing solutions to the following Riccati equations:

$$XA + A^*X - XBB^*X = 0 \tag{22}$$

$$AY + YA^* - XC^*CX = 0 \tag{23}$$

Let $F = -B^*X$ and $L = -YC^*$, the generalized controllability gramian P_F and observability gramian Q_F is the solution to the following modified Lyapunov equations:

$$A_F P_F + P_F A_F^* + BB^* = 0 \tag{24}$$

$$A_C^* Q_C + Q_C A_C + C^*C = 0 \tag{25}$$

where

$$A_F = A + BF \tag{26}$$

$$A_C = A + LC \tag{27}$$

In the form of integrals the solutions of (24) and (25) are:

$$P_F = \frac{1}{2\pi}\int_{-\infty}^{\infty} (j\omega I - A_F)^{-1}BB^*(-j\omega I - A_F^*)^{-1}d\omega \tag{28}$$

$$Q_C = \frac{1}{2\pi}\int_{-\infty}^{\infty} (-j\omega I - A_C^*)^{-1}C^*C(j\omega I - A_C)^{-1}d\omega \tag{29}$$

Remark 1: Suppose A is stable, it follows that $X = Y = 0$ and therefore P_F and Q_C in (24), (25), (28) and (29) are reduced to the standard controllability and observability gramians for stable systems as in (8),(9), (3) and (6).

III. Main Work

A. Generalized Frequency Interval Controllability and Observability Gramians for Unstable Systems

Definition 1: The frequency interval controllability gramian for an unstable system in the form of (1) is defined as follows

$$\hat{P}_\Omega = \hat{P}(\omega_2) - \hat{P}(\omega_1) \tag{30}$$

where $\hat{P}(\omega)$ satisfies

$$\hat{P}(\omega) = \frac{1}{2\pi}\int_{-\omega}^{\omega} (j\omega I - A_F)^{-1}BB^*(-j\omega I - A_F^*)^{-1}d\omega \tag{31}$$

Theorem 1: $\hat{P}(\omega)$ defined in (31) is the solution to the following Lyapunov equation

$$A_F\hat{P}(\omega)+\hat{P}(\omega)A_F^*+S_F(\omega)BB^*+BB^*S_F^*(\omega)=0 \quad (32)$$

where

$$\begin{aligned} S_F(\omega) &= \frac{1}{2\pi}\int_{-\omega}^{+\omega}(j\omega I-A_F)^{-1}d\omega \\ &= \frac{1}{2\pi}\int_{-\omega}^{+\omega}H_F(\omega)d\omega \end{aligned} \quad (33)$$

Proof: Let

$$(j\omega I-A)^{-1}B=NM^{-1}$$

be such that M is inner, i.e. $M^{\sim}(s)=M^{-1}$. The co-prime factorization can then be obtained by using the following property described in [16]

$$\begin{bmatrix} M \\ N \end{bmatrix}=\left[\begin{array}{c|c} A_F & B \\ \hline F & I \\ I & 0 \end{array}\right] \quad (34)$$

By using this property in (34), a generalized expression of $\hat{P}(\omega)$ in (12) can be written as follows

$$\begin{aligned} \hat{P}(\omega) &= \frac{1}{2\pi}\int_{-\omega}^{\omega}(j\omega I-A)^{-1}BB^*(-j\omega I-A^*)^{-1}d\omega \\ &= \frac{1}{2\pi}\int_{-\omega}^{\omega}N(j\omega)N^*(j\omega)d\omega \\ &= \frac{1}{2\pi}\int_{-\omega}^{\omega}(j\omega I-A_F)^{-1}BB^*(-j\omega I-A_F^*)^{-1}d\omega \end{aligned}$$

Taking into consideration that (24) can be re-written as

$$\begin{aligned} BB^* &= -A_FP_F-P_FA_F^* \\ &= (j\omega I-A_F)P_F+P_F(-j\omega I-A_F^*) \end{aligned}$$

Followed by substituting $BB^*=(j\omega I-A_F)P_F+P_F(-j\omega I-A_F^*)$ into (31) yields

$$\hat{P}(\omega)=\frac{1}{2\pi}\int_{-\omega}^{\omega}(j\omega I-A_F)P_F+P_F(-j\omega I-A_F^*)d\omega$$

Note that

$$S_F(\omega)=\frac{1}{2\pi}\int_{-\omega}^{+\omega}(j\omega I-A_F)^{-1}d\omega$$

Hence

$$\hat{P}(\omega)=S_F(\omega)P_F+P_FS_F^*(\omega) \quad (35)$$

Taking into consideration the following property [15], [17]

$$H_F(\omega_1)H_F(\omega_2)=H_F(\omega_2)H_F(\omega_1)$$

and substituting both (33) and (28) into (35) yields

$$\begin{aligned} &\hat{P}(\omega)= \\ &\frac{1}{2\pi}\int_{-\omega}^{+\omega}H_F(\omega)d\omega\cdot\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_F(\mu)BB^*H_F^*(\mu)d\mu+... \\ &\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_F(\mu)BB^*H_F^*(\mu)d\mu\cdot\frac{1}{2\pi}\int_{-\omega}^{+\omega}H_F^*(\omega)d\omega \\ &=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\frac{1}{2\pi}\int_{-\omega}^{+\omega}H_F(\omega)d\omega\right)H_F(\mu)BB^*H_F^*(\mu)d\mu+... \\ &\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_F(\mu)BB^*H_F^*(\mu)\left(\frac{1}{2\pi}\int_{-\omega}^{+\omega}H_F^*(\omega)d\omega\right)d\mu \\ &=\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_F(\mu)\left(\frac{1}{2\pi}\int_{-\omega}^{+\omega}H_F(\omega)d\omega\right)BB^*H_F^*(\mu)d\mu+... \\ &\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_F(\mu)BB^*\left(\frac{1}{2\pi}\int_{-\omega}^{+\omega}H_F^*(\omega)d\omega\right)H_F^*(\mu)d\mu \\ &=\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_F(\mu)\left[S_F(\omega)BB^*+BB^*S_F^*(\omega)\right]H_F^*(\mu)d\mu, \end{aligned} \quad (36)$$

Since A_F is stable, we can conclude that $\hat{P}(\omega)$ expressed in the form of (36) is the solution to the following Lyapunov equation

$$A_F\hat{P}(\omega)+\hat{P}(\omega)A_F^*+S_F(\omega)BB^*+BB^*S_F^*(\omega)=0.$$

■

Definition 2: The frequency interval observability gramian for an unstable system in the form of (1) is defined as follows

$$\hat{Q}_\Omega=\hat{Q}(\omega_2)-\hat{Q}(\omega_1) \quad (37)$$

where $\hat{Q}_\Omega$ satisfies

$$\hat{Q}_\Omega=\frac{1}{2\pi}\int_{-\omega}^{+\omega}(-j\omega I-A_C^*)^{-1}C^*C(j\omega I-A_C)^{-1}d\omega \quad (38)$$

Theorem 2: $\hat{Q}(\omega)$ defined in (38) is the solution to the following Lyapunov equation

$$A_C^*\hat{Q}(\omega)+\hat{Q}(\omega)A_C+S_C^*(\omega)C^*C+C^*CS_C(\omega)=0. \quad (39)$$

where

$$\begin{aligned} S_C(\omega) &= \frac{1}{2\pi}\int_{-\omega}^{+\omega}(j\omega I-A_C)^{-1}d\omega \\ &= \frac{1}{2\pi}\int_{-\omega}^{+\omega}H_C(\omega)d\omega \end{aligned} \quad (40)$$

Proof: The proof of Theorem 2 is similar to the proof of Theorem 1 and is therefore ommited for brevity. ■

B. Model Reduction Algorithm

The proposed frequency interval model reduction algorithm for unstable systems is therefore given as follows

Algorithm 1. Given the system matrices $\{A,B,C,D\}$.

(a) *Step 1:* Check the stabilizability of the pair of matrices (A,B) and the detectability of the pair of matrices (C,A)
(b) *Step 2:* Obtain the stabilizing solutions - X and Y for the Riccati equations in (22) and (23)
(c) *Step 3:* Obtain the frequency interval controllability gramian $\hat{P}_\Omega$ and frequency interval observability gramian $\hat{Q}_\Omega$ by solving (30) and (37) respectively.
(d) *Step 4:* Obtain the similarity transformation matrix which diagonalizes $\hat{P}_\Omega$ and $\hat{Q}_\Omega$ such that $T\hat{P}_\Omega T^*=(T^*)^{-1}\hat{Q}_\Omega T^{-1}$

(e) *Step 5:* Transform and partition to get a realization

$$\bar{A} = T^{-1}AT = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \bar{B} = T^{-1}B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$
$$\bar{C} = CT = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$$

(f) *Step 6:* The reduced order model is given by $A_r = A_{11}, B_r = B_1, C_r = C_1, D_r = D$

IV. Numerical Examples

Considering the following 8th order continuous-time unstable system

$$A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -0.2625 & -5.1234 & 0 & 0 \\ 5.1234 & 0 & 0 & 0 \\ -0.1679 & -3.2777 & -0.0594 & -2.4376 \\ 0 & 0 & 2.4376 & 0 \\ -0.1679 & -3.2777 & -0.0368 & -1.5084 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0076 & -0.8738 & 0 & 0 \\ 0.8738 & 0 & 0 & 0 \\ 0 & 1.1444 & 0 & 1.0000 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.1182 \end{bmatrix}, D = 0$$

The matrix A has a pair of eigenvalues which are negatives of each other (i.e. 1 and -1), therefore the frequency interval controllability and observability gramians of this system cannot be obtained by using the Lyapunov equations in (10) and (11). In addition many of the existing gramian based frequency weighted model reduction methods are not applicable for these type of systems [18]–[21].

In this section the original 8th order system is reduced to a 4th order system by using both the method by Zhou, Salomon and Wu [12] and also the proposed method. Figure 1 shows the magnitude responses of the original 8th order system, the magnitude response of the reduced 4th order system obtained using the method by Zhou, Salomon and Wu [12] and the magnitude response of the reduced 4th order system obtained using the proposed method (in which the range of the specified frequency interval is [0.2 rad/s, 1.5 rad/s]).

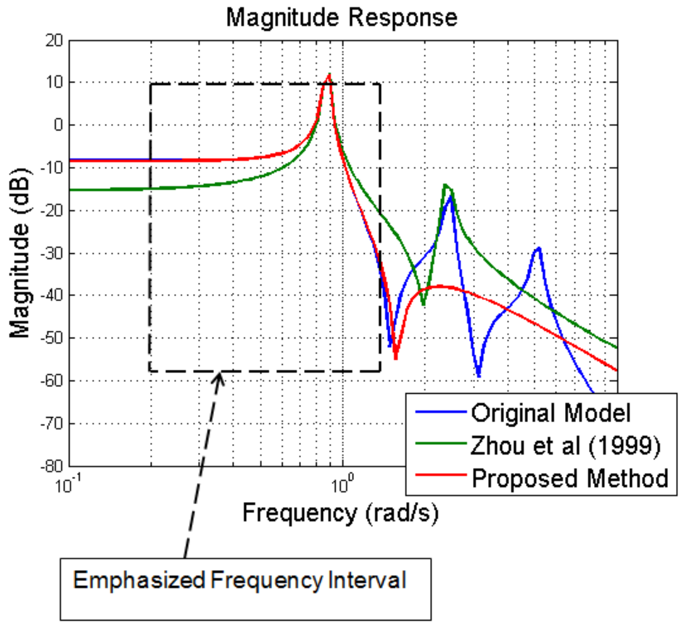

Fig. 1: Magnitude response plot for the original 8th order model, 4th order model obtained using the method by Zhou et al (1999) and 4th order model obtained using the proposed method

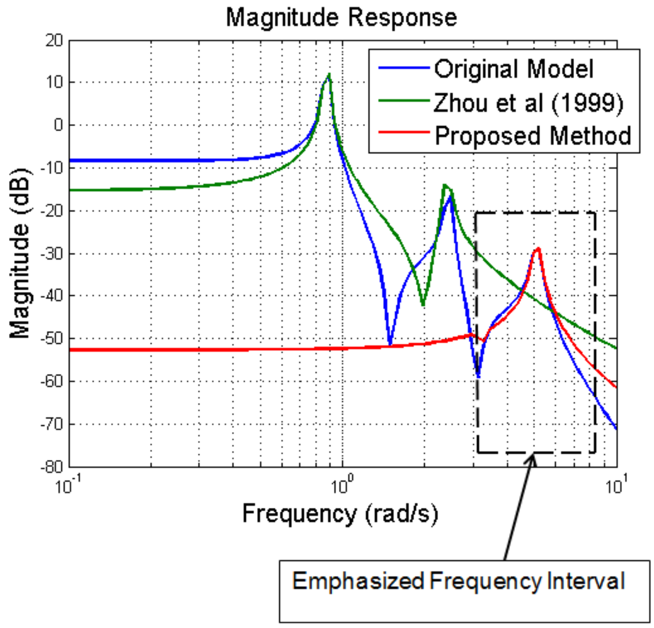

Fig. 2: Magnitude response plot for the original 8th order model, 4th order model obtained using the method by Zhou et al (1999) and 4th order model obtained using the proposed method

Similarly, Figure 2 shows the magnitude responses of the original 8th order system, the magnitude response of the reduced 4th order system obtained using the method by Zhou, Salomon and Wu [12] and the magnitude response of the reduced 4th order system obtained using the proposed method (in which the range of the specified frequency interval is [3 rad/s, 9 rad/s]) From Figure 1 and Figure 2 it can be observed that although the reduced order model obtained by using the method by Zhou, Salomon and Wu [12] gives a closer approximation to the original model for the entire frequency region, the reduced order model obtained using the proposed method gives a closer approximation to the original model in the specified frequency interval.

V. Conclusion

Generalized frequency interval controllability and observability gramians have been developed in order to obtain the frequency interval controllability and observability gramians for systems which do not have a solution to the standard Lyapunov equations. These gramians are used as part of the model reduction algorithm and numerical results demonstrate that the reduced order model obtained using the proposed method gives a closer approximation to the original model at the specified frequency interval.

References

[1] Moaveni, B., and A. Khaki-Sedigh , *Control Configuration Selection for Multivariable Plants, Lecture Notes in Control and Information Sciences*, Vol. 391, Springer-Verlag, Berlin, Heidelberg (2009).

[2] Streif, S., Findeisen, R. and Bullinger, E., Relating Cross Gramians and Sensitivity Analysis in Systems Biology, *Proc. 17th Int. Symp. Math. Theory of Networks and Systems*, pp. 24-28, (2006).

[3] Himpe, C. and Ohldberger, M., Cross Gramian Based Combined State and Parameter Reduction, *Mathematical Problems in Engineering*, vol.2014, no.843869, pp.1-13, (2014).

[4] Himpe, C. and Ohldberger, M., Model Reduction of Complex Hyperbolic Networks, *Proc. of the European Control Conference*, pp. 2739-2743, (2014).

[5] Du, X., Fan, F., Ding,D. and Liu, F., Finite-frequency model order reduction of discrete-time linear time-delayed systems, *Nonlinear Dynamics*, vol.83, no.49, pp.2485-2496, (2016).

[6] Imran, M. and A. Ghafoor, "Model reduction of descriptor systems using frequency limited gramians,"*J. Franklin Inst.*, Vol. 352, No. 1, pp. 33-51 (2015).

[7] Ding, D.W., X. Du, and X. Li , "Finite-frequency model reduction of two-dimensional digital filters,"*IEEE Trans. Autom. Control*, Vol. 60, No. 6, pp. 1624-1629 (2015).

[8] X.J. Li and G.H. Yang, Adaptive control in finite frequency domain for uncertain linear systems, *Information Sciences*, vol.314 no.1, pp.14-27, (2016).

[9] Li, X., C. Yu and H. Gao, "Frequency limited H_∞ model reduction for positive systems,"*IEEE Trans. Autom. Control*, Vol. 60, No. 4, pp 1093-1098 (2015).

[10] Shaker, H.R., "Frequency-interval interaction measure for control configuration selection for multivariable processes,"*Proc. IEEE Int. Conf. Control Appl.*, Hyderabad, India, pp. 889-893. (2013).

[11] Shaker, H.R., Frequency-Interval Control Reconfigurability for Automated Processes, *Natural Hazards*, vol.72, no.2, pp 1021-1027, (2014).

[12] Zhou, K., Salomon, G. and Wu, E., Balanced realization and model reduction for unstable systems, *International Journal of Robust and Nonlinear Control*, vol.9, no.3, pp. 183-198, (1999).

[13] Shaker, H., Generalized Cross Gramian for Linear Systems, *Proc. of the 7th IEEE Conference on Industrial Electronics and Applications (ICIEA)*, pp. 749-751, (2011).

[14] Fernando, K.V., and H. Nicholson, "On the fundamental property of the cross-gramian matrix,"*IEEE Trans. Circuits Syst.*, Vol. 21, No. 2, pp. 349-376 (1984).

[15] Gawronski, W., and J. Juang, "Model reduction in limited time and frequency intervals,"*Int. J. Syst. Sci.*, Vol. 21, No. 2, pp. 349-376 (1990).

[16] Zhou, K., Doyle, J.C. and Glover, K., *Robust and Optimal Control*, New Jersey: Prentice Hall, 1996.

[17] Gugercin, S. and A. C. Antoulas, "A survey of model reduction by balanced truncation and some new results,"*Int. J. Control*, Vol. 77, No. 8, pp.748-766 (2004).

[18] Sahlan, S., Ghafoor, A. and Sreeram, V., A new method for the model reduction technique via a limited frequency interval impulse response Gramian, *Mathematical and Computer Modelling*, vol.55, no.3-4, pp.1034-1040, (2012).

[19] Sreeram, V., Sahlan, S., Muda, W. M. W., Fernando, T. and Iu, H. H. C.,A generalised partial-fraction-expansion based frequency weighted balanced truncation technique, *International Journal of Control*, vol.86, no.5, pp.883-843, (2013).

[20] Sreeram, V. and Sahlan, S., Improved results on frequency-weighted balanced truncation and error bounds, *International Journal of Control*, vol.22, no.11, pp.1195-1211, (2012).

[21] Imran, M., Ghafoor, A. and Sreeram, V., A frequency weighted model order reduction technique and error bounds, *Automatica*, vol.50, no.12 pp.3304 - 3309, (2014).