Relaying Energy Allocation in Training-Based Amplify and Forward Relay Communications

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***Abstract*—We consider relay-assisted communication in a training-based transmission scheme. Each transmission block consists of a training phase and a data transmission phase. The relay node employs the amplify-and-forward protocol during all transmissions. We focus on the relay signaling design and investigate the benefit of allowing for different relaying power during the training phase and the data transmission phase. Specifically, the relaying energy allocation between the two phases is optimized for maximizing the average received signal-to-noise ratio at the destination node. We study this optimization problem for both single-antenna relay and multi-antenna relay and derive a simple closed-form relaying energy allocation strategy that achieves near-optimal performance. This closed-form strategy depends only on the length of the data transmission phase but not on other system parameters such as the relaying energy budget, the number of antennas at the relay, and the distances between the source, relay and destination nodes.**

I. Introduction

Relay-assisted communication has attracted considerable attention in the past few years. The use of relay increases both the service area and the quality of service (QoS) [1, 2], and was proposed to be incorporated into the WiMAX standard, IEEE 802.16m, as a cost-effective way to fulfill the requirements of the future generation of mobile communications [3]. Among various relaying strategies, the amplify-and-forward (AF) protocol has been extensively studied due to the low-complexity design at the relay [4].

One of the main assumptions in most existing studies on relay-assisted communication is the availability of perfect channel state information. This assumption does not hold in reality due to the time-varying nature of the fading channel. In order to obtain the channel state information (at least) at the receiver side, training-based transmission schemes are commonly used, which periodically insert pilot symbols into data transmission blocks [5]. For communications assisted by an AF relay, the dual-hop (source-relay-destination) channel needs to be estimated by the destination node. To this end, the linear minimum mean square error (LMMSE) estimator was studied in both autoregressive channel models [6, 7] and block-fading channel models [8, 9] to estimate the non-Gaussian dual-hop channel gain.[1]

In training-based transmission, one important design parameter is the energy allocation between the training phase and the data transmission phase. This design problem exists at both the source and the relay. For AF relay-assisted communication with single-antenna terminals, the authors in [11] focused on the energy allocation problem at the source, while the authors in [12] jointly optimized the transmission/relaying energy allocation between the two phases at both the source and the relay. The results in [11, 12] were obtained under a total energy budget between the source and the relay. In this work, we consider a separate energy constraint at each node, which is more practical, and focus on the energy allocation at the relay. Note that this problem was previously investigated for systems with a single-antenna relay in [13], where the authors presented analytical solutions in two special scenarios, *i.e.*, the relay is located either very close to the source or very close to the destination. In this work, we look at the signaling design for both single-antenna relay and multi-antenna relay.

Our main contribution is a closed-form design of the AF relaying energy allocation that achieves near-optimal performance, in terms of the average receive signal-to-noise ratio (SNR) at the destination. The main advantage of the proposed design, compared to the existing ones in the literature, is that it only depends on the block transmission structure but not on other system parameters such as the transmission energy budgets at the source and relay, and the distances between the three terminals. In other words, this design is robust to any changes in the energy budgets and positions of all terminals, hence is useful for systems aiming at simple design solutions.

II. System Model

We consider the communication between a source (S) and a destination (D) assisted by a relay (R) adopting the AF protocol. Both S and D have a single antenna for transmission and reception, while the number of antennas at R, denoted as N_R, can be more than one. An example of such system is shown in Fig. 1. The communications between the three

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[1] An alternative channel estimation scheme was considered in the literature, *e.g.*, [9, 10], which separately estimates the individual (source-relay and relay-destination) channel gains. This scheme requires extra transmission resource to reliably forward the estimate of source-relay channel to the destination. In this work, we do not consider this scheme and assume that the dual-hop channel gain is estimated at the destination.

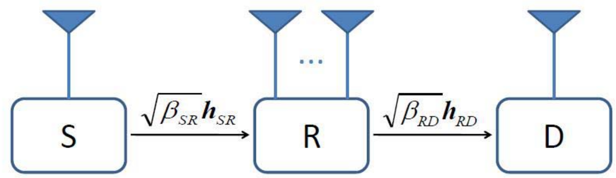

Fig. 1. The AF relay communication system considered in this work.

terminals are affected by path loss attenuation as well as Rayleigh fading. As shown in Fig. 1, we denote the channel gain between S and R as $\sqrt{\beta_{SR}}\boldsymbol{h}_{SR}$ and the channel gain between R and D as $\sqrt{\beta_{RD}}\boldsymbol{h}_{RD}$. In particular, β_{SR} and β_{RD} are scalar constants determined by the path loss, while $\boldsymbol{h}_{SR}$ and $\boldsymbol{h}_{RD}$ are independent fading variables having zero-mean circular-symmetric complex Gaussian (ZMCSCG) entries with unit variance. When R has multiple antennas, $\boldsymbol{h}_{SR}$ is a $N_R \times 1$ vector and $\boldsymbol{h}_{RD}$ is a $1 \times N_R$ vector.

A block-wise transmission scheme is considered, in which the channel gains remain constant during one transmission block and change to some independent values in the next block. In order to facilitate channel estimation at D, S first transmits L_t training symbols followed by L_d data symbols in each transmission block. Hence, D uses the channel estimates obtained from the training symbols to perform data detection. The transmit powers at S for training and data symbols are denoted as $\mathcal{P}_{St}$ and $\mathcal{P}_{Sd}$, respectively. Similarly, we denote the relaying powers at R for training and data symbols as $\mathcal{P}_{Rt}$ and $\mathcal{P}_{Rd}$, respectively. If R does not distinguish between training and data transmission, a constant relaying power should be used, *i.e.*, $\mathcal{P}_{Rt} = \mathcal{P}_{Rd}$. In this work, we aim to investigate the benefit from carefully designing the values of $\mathcal{P}_{Rt}$ and $\mathcal{P}_{Rd}$ under a constraint on the total relaying energy per transmission block.[2] In order to design $\mathcal{P}_{Rt}$ and $\mathcal{P}_{Rd}$, R needs to know the block transmission structure, *i.e.*, whether the current symbol is from training or data transmission. This information is usually easy to obtain at R with minimal cost.

In this work, we consider the following two scenarios:

1) **Single-Antenna Relay**: In this case, R does not need to know the channel gains and simply amplifies and forwards the received signal from S to D.
2) **Multi-Antenna Relay**: In this case, we consider that R has estimates of both $\boldsymbol{h}_{SR}$ and $\boldsymbol{h}_{RD}$. Based on these channel estimates, R applies receive and transmit beamforming to the received signal from S in order to maximize the signal strength arrived at D.

Since the relaying signaling is different in the above two scenarios, we will study them separately.

[2]As discussed in later sections, the design of relaying powers which maximizes the average received SNR at D is not affected by the presence of the direct link between S and D.

III. Single Antenna Relay

In this section, we consider that the relay has a single antenna. The received signal at R is given by

$$y_R = \sqrt{\mathcal{P}_S \beta_{SR}} h_{SR} x + n_R, \tag{1}$$

where x is the transmitted symbol with unit variance which can be either a training or data symbol, $\mathcal{P}_S = \mathcal{P}_{St}$ for training and $\mathcal{P}_S = \mathcal{P}_{Sd}$ for data transmission, n_R is the receiver noise at R which is a ZMCSCG random variable with variance σ_R^2. After AF relaying, the received signal at D is given by

$$y_D = \sqrt{\beta_{RD}} h_{RD} \alpha y_R + n_D, \tag{2}$$

where n_D is the receiver noise at D which is a ZMCSCG random variable with variance σ_D^2 and α is the amplification gain at R given by[3]

$$\alpha = \sqrt{\frac{\mathcal{P}_R}{\mathcal{P}_S \beta_{SR} + \sigma_R^2}}, \tag{3}$$

where $\mathcal{P}_R = \mathcal{P}_{Rt}$ for training and $\mathcal{P}_R = \mathcal{P}_{Rd}$ for data transmission. It is worthwhile to note that R only needs to know the average received signal strength $\mathcal{P}_S \beta_{SR}$, but not the individual values of $\mathcal{P}_S$ and β_{SR}, in order to design the amplification gain.

During the training phase, the LMMSE estimator [14] is used at D to estimate the dual-hop channel from S to D, *i.e.*, $h = h_{RD} h_{SR}$. Specifically, the received signal is

$$\begin{aligned} y_{D,i} = & \sqrt{\frac{\mathcal{P}_{Rt}\beta_{RD}\mathcal{P}_{St}\beta_{SR}}{\mathcal{P}_{St}\beta_{SR}+\sigma_R^2}} h x_i + \sqrt{\frac{\mathcal{P}_{Rt}\beta_{RD}}{\mathcal{P}_{St}\beta_{SR}+\sigma_R^2}} h_{RD} n_{R,i} \\ & + n_{D,i}, \qquad i = 1, 2, ..., L_t. \end{aligned} \tag{4}$$

The LMMSE estimate of h is obtained by combining the L_t training observations. We denote the estimate of h and the estimation error as $\hat{h}$ and $\tilde{h}$, respectively, *i.e.*, $h = \hat{h} + \tilde{h}$. The variances of $\hat{h}$ and $\tilde{h}$ are given by, respectively,

$$\sigma_{\hat{h}}^2 = \frac{\frac{L_t \mathcal{P}_{Rt}\mathcal{P}_{St}\beta_{RD}\beta_{SR}}{\mathcal{P}_{St}\beta_{SR}+\sigma_R^2}}{\frac{L_t \mathcal{P}_{Rt}\mathcal{P}_{St}\beta_{RD}\beta_{SR}}{\mathcal{P}_{St}\beta_{SR}+\sigma_R^2} + \frac{\mathcal{P}_{Rt}\beta_{RD}\sigma_R^2}{\mathcal{P}_{St}\beta_{SR}+\sigma_R^2} + \sigma_D^2}, \tag{5}$$

and

$$\sigma_{\tilde{h}}^2 = 1 - \sigma_{\hat{h}}^2. \tag{6}$$

During the data transmission phase, the received signal at D can be rewritten as

$$\begin{aligned} y_D = & \sqrt{\frac{\mathcal{P}_{Rd}\beta_{RD}\mathcal{P}_{Sd}\beta_{SR}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}} \hat{h} x + \sqrt{\frac{\mathcal{P}_{Rd}\beta_{RD}\mathcal{P}_{Sd}\beta_{SR}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}} \tilde{h} x \\ & + \sqrt{\frac{\mathcal{P}_{Rd}\beta_{RD}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}} h_{RD} n_R + n_D. \end{aligned} \tag{7}$$

As mentioned, the aim of this work is to design the relaying powers $\mathcal{P}_{Rt}$ and $\mathcal{P}_{Rd}$ to achieve good system performance.

[3]In this work, we assume that the relay's amplification gain is not adaptive to the instantaneous channel gain. Hence, the relaying powers, *i.e.*, $\mathcal{P}_{Rt}$ and $\mathcal{P}_{Rd}$, are the long-term average power during training and data transmission.

$$\rho_{\text{ave}} \approx \frac{\mathcal{P}_{Rd}\mathcal{P}_{Sd}\mathcal{P}_{Rt}\mathcal{P}_{St}\beta_{RD}\beta_{SR}L_t}{\mathcal{P}_{Rd}\mathcal{P}_{Sd}(\mathcal{P}_{Rt}\beta_{RD}\sigma_R^2 + \sigma_D^2\mathcal{P}_{St}\beta_{SR}) + \mathcal{P}_{Rt}\mathcal{P}_{St}L_t(\mathcal{P}_{Rd}\beta_{RD}\sigma_R^2 + \sigma_D^2\mathcal{P}_{Sd}\beta_{SR})}. \quad (10)$$

We focus on the long-term performance, which is usually measured in terms of the ergodic capacity or symbol error rate (SER). The non-Gaussian nature of the dual-hop channel from S to D makes it very difficult to find a mathematically trackable expression for the exact ergodic capacity or SER which takes channel estimation error into account [15]. In order to derive analytical solution to the relaying power optimization problem, we consider the average received SNR at D as the objective function. Note that if there exists a direct link from S to D, maximum ratio combining could be used [4] and the overall SNR at D is the sum of the SNRs of the direct link and the dual-hop link. Clearly, the choices of the relaying powers affect the SNR of the dual-hop link but not the SNR of the direct link. Hence, we focus on the dual-hop link only and define its average receive SNR based on (7) as

$$\begin{aligned}\rho_{\text{ave}} &= \frac{\mathbb{E}\left\{\left|\sqrt{\frac{\mathcal{P}_{Rd}\beta_{RD}\mathcal{P}_{Sd}\beta_{SR}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}}\hat{h}x\right|^2\right\}}{\mathbb{E}\left\{\left|\sqrt{\frac{\mathcal{P}_{Rd}\beta_{RD}\mathcal{P}_{Sd}\beta_{SR}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}}\tilde{h}x+\sqrt{\frac{\mathcal{P}_{Rd}\beta_{RD}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}}h_{RD}n_R+n_D\right|^2\right\}} \\ &= \frac{\frac{\mathcal{P}_{Rd}\beta_{RD}\mathcal{P}_{Sd}\beta_{SR}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}\sigma_{\hat{h}}^2}{\frac{\mathcal{P}_{Rd}\beta_{RD}\mathcal{P}_{Sd}\beta_{SR}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}\sigma_{\tilde{h}}^2 + \frac{\mathcal{P}_{Rd}\beta_{RD}}{\mathcal{P}_{Sd}\beta_{SR}+\sigma_R^2}\sigma_R^2 + \sigma_D^2}, \end{aligned} \quad (8)$$

where $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation, $\sigma_{\hat{h}}^2$ and $\sigma_{\tilde{h}}^2$ are given in (5) and (6). respectively. ρ_{ave} in (8) gives a long-term performance metric for the dual-hop link which admits a closed-form expression. Hence, we use it as the objective function to optimize $\mathcal{P}_{Rt}$ and $\mathcal{P}_{Rd}$.

A. Optimizing Relaying Energy Allocation

We consider a constraint on the total relaying energy per transmission block. This can also be interpreted as a constraint on the average relaying power, denoted by $\overline{\mathcal{P}}_R$. Hence, the total relaying energy per transmission block is given by $\overline{\mathcal{P}}_R(L_t+L_d)$. We denote the ratio of the total energy allocated to training as ϕ. Hence, we have

$$\mathcal{P}_{Rt} = \frac{\phi\overline{\mathcal{P}}_R(L_t+L_d)}{L_t}, \ \mathcal{P}_{Rd} = \frac{(1-\phi)\overline{\mathcal{P}}_R(L_t+L_d)}{L_d}. \quad (9)$$

The design problem can be written as $\max_\phi \rho_{\text{ave}}$. To obtain the optimal relaying energy allocation, one can numerically search for the value of $\phi \in (0,1)$ that maximizes ρ_{ave} in (8). The solution to the optimal ϕ depends on all system parameters, including the transmit powers at the source $\mathcal{P}_{St}$ and $\mathcal{P}_{Sd}$, the channel path loss factors β_{SR} and β_{RD}, and the receiver noise at the destination σ_D^2. Although these parameters are reasonably constant over time, it certainly incurs cost in obtaining them at R. For systems aiming at simple design solutions, it is desirable to have a closed-form solution of the relaying energy allocation that works well in most practical scenarios and depends on the least number of system parameters.

To obtain a closed-form design solution, we focus on the high SNR regime by assuming $\mathcal{P}_{St}\beta_{SR} \gg \sigma_R^2$, $\mathcal{P}_{Sd}\beta_{SR} \gg \sigma_R^2$, $\mathcal{P}_{Rt}\beta_{RD} \gg \sigma_D^2$ and $\mathcal{P}_{Rd}\beta_{RD} \gg \sigma_D^2$. The average receive SNR ρ_{ave} in (8) can be approximated as (10) on the top of this page. The optimal ϕ that maximizes ρ_{ave} in (10) can be found by first substituting (9) into (10), then set the first derivative of ρ_{ave} w.r.t. ϕ be zero and solve for the root. The solution is given as

$$\phi_o = \frac{1}{1+\sqrt{L_d}}. \quad (11)$$

We see that the above simple solution only depends on the data length L_d and does not depend on any other system parameters. As long as the block transmission structure is fixed, the relaying energy allocation given in (11) is the same regardless of the energy budgets at S and R as well as the distances between the three terminals. We will see in Section V that this simple solution gives near-optimal performance not only at high SNR but over a wide range of practical SNR values.

IV. Multi-Antenna Relay

Next, we consider that the relay has multiple antennas, *i.e.*, $N_R > 1$. In order to maximize the received signal strength at D, both receive and transmit beamforming are used at R based on its estimates of $\boldsymbol{h}_{SR}$ and $\boldsymbol{h}_{RD}$. Note that an additional training phase is required at the beginning of each transmission block to obtain these individual channel estimates at R. In practice, the individual channels can be either directly estimated by R, or first estimated by S and D, and then sent to R using feedback/feedforward links. Since the aim of this work is to design the relaying powers, we do not study the details of the additional training phase and consider that the individual channel estimates have already been obtained at R with certain error variances.

Let us denote the individual channel estimates as $\hat{\boldsymbol{h}}_{SR}$ and $\hat{\boldsymbol{h}}_{RD}$ and their estimation errors as $\tilde{\boldsymbol{h}}_{SR}$ and $\tilde{\boldsymbol{h}}_{RD}$. Furthermore, the variances of the estimation errors (*i.e.*, the variance of each element in $\tilde{\boldsymbol{h}}_{SR}$ and $\tilde{\boldsymbol{h}}_{RD}$) are denoted as σ_{e1}^2 and σ_{e2}^2, respectively. The relay precoding matrix, which consists of the maximum ratio combining vector and transmit beamforming vectors, is given by

$$\boldsymbol{W} = \frac{\hat{\boldsymbol{h}}_{RD}^\dagger}{\|\hat{\boldsymbol{h}}_{RD}\|}\frac{\hat{\boldsymbol{h}}_{SR}^\dagger}{\|\hat{\boldsymbol{h}}_{SR}\|}, \quad (12)$$

where $\|\boldsymbol{v}\| = \sqrt{\text{tr}\{\boldsymbol{v}^\dagger\boldsymbol{v}\}}$. Therefore, the received signal at D is given by

$$y_D = \alpha\sqrt{\beta_{RD}}\boldsymbol{h}_{RD}\boldsymbol{W}\boldsymbol{y}_R + n_D, \quad (13)$$

where $\boldsymbol{y}_R$ is the received signal at R given in (1) and α is the relay amplification gain given in (3).

The dual-hop channel from S to D is given by $h = \boldsymbol{h}_{RD}\boldsymbol{W}\boldsymbol{h}_{SR}$. Unlike the single-antenna case, the dual-hop channel in the multi-antenna case is not zero-mean due to the relay precoding. We denote its variance as σ_h^2, which can be expressed as

$$\begin{aligned}\sigma_h^2 &= \mathbb{E}\{|h|^2\} - (\mathbb{E}\{h\})^2\\ &= \mathbb{E}\{\|\hat{\boldsymbol{h}}_{RD}\|^2\}\mathbb{E}\{\|\hat{\boldsymbol{h}}_{SR}\|^2\} + \mathbb{E}\{\|\hat{\boldsymbol{h}}_{RD}\|^2\}\sigma_{e1}^2\\ &\quad+\mathbb{E}\{\|\hat{\boldsymbol{h}}_{SR}\|^2\}\sigma_{e2}^2+\sigma_{e1}^2\sigma_{e2}^2-(\mathbb{E}\{\hat{\boldsymbol{h}}_{RD}\})^2(\mathbb{E}\{\hat{\boldsymbol{h}}_{SR}\})^2.\end{aligned} \tag{14}$$

During the training phase, the LMMSE is used at D to estimate the dual-hop channel gain h. Specifically, we have

$$y_{D,i} = \alpha_t\sqrt{\mathcal{P}_{St}\beta_{RD}\beta_{SR}}hx_i + u_{t,i}, \quad i = 1, 2, ..., L_t, \tag{15}$$

where

$$\alpha_t = \sqrt{\frac{\mathcal{P}_{Rt}}{\mathcal{P}_{St}\beta_{SR} + \sigma_R^2}}, \tag{16}$$

and

$$u_{t,i} = \alpha_t\sqrt{\beta_{RD}}\boldsymbol{h}_{RD}\boldsymbol{W}\boldsymbol{n}_{R,i} + n_{D,i}. \tag{17}$$

We denote the variance of $u_{t,i}, \forall i$ as σ_{ut}^2, which can be computed as

$$\sigma_{ut}^2 = \mathbb{E}\{|u_{t,i}|^2\} = \alpha_t^2\beta_{RD}\sigma_R^2(\mathbb{E}\{\|\hat{\boldsymbol{h}}_{RD}\|^2\} + \sigma_{e2}^2) + \sigma_D^2. \tag{18}$$

The LMMSE estimate of h is obtained by combining the L_t training observations. Similar to the single-antenna case, we denote the estimate of h and the estimation error as $\hat{h}$ and $\tilde{h}$, respectively, *i.e.*, $h = \hat{h} + \tilde{h}$. The variances of $\hat{h}$ and $\tilde{h}$ are given by, respectively,

$$\sigma_{\hat{h}}^2 = \sigma_h^2\frac{\sigma_h^2\alpha_t^2\mathcal{P}_{St}\beta_{RD}\beta_{SR}L_t}{\sigma_h^2\alpha_t^2\mathcal{P}_{St}\beta_{RD}\beta_{SR}L_t + \sigma_{ut}^2}, \tag{19}$$

and

$$\sigma_{\tilde{h}}^2 = \sigma_h^2\frac{\sigma_{ut}^2}{\sigma_h^2\alpha_t^2\mathcal{P}_{St}\beta_{RD}\beta_{SR}L_t + \sigma_{ut}^2}. \tag{20}$$

During the data transmission phase, the received signal at D is given by

$$\begin{aligned}y_D &= \alpha_d\sqrt{\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}}hx + u_d\\ &= \alpha_d\sqrt{\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}}\hat{h}x+\alpha_d\sqrt{\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}}\tilde{h}x+u_d,\end{aligned} \tag{21}$$

where

$$\alpha_d = \sqrt{\frac{\mathcal{P}_{Rd}}{\mathcal{P}_{Sd}\beta_{SR} + \sigma_R^2}}, \tag{22}$$

and

$$u_d = \alpha_d\sqrt{\beta_{RD}}\boldsymbol{h}_{RD}\boldsymbol{W}\boldsymbol{n}_R + n_D. \tag{23}$$

We denote the variance of u_d as σ_{ud}^2, which can be computed as

$$\sigma_{ud}^2 = \mathbb{E}\{|u_d|^2\} = \alpha_d^2\beta_{RD}\sigma_R^2(\mathbb{E}\{\|\hat{\boldsymbol{h}}_{RD}\|^2\} + \sigma_{e2}^2) + \sigma_D^2. \tag{24}$$

Similar to the single-antenna case, we can define the average receive SNR of the dual-hop link based on (21) as

$$\begin{aligned}\rho_{\text{ave}} &= \frac{\mathbb{E}\Big\{\Big|\alpha_d\sqrt{\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}}\hat{h}x\Big|^2\Big\}}{\mathbb{E}\Big\{\Big|\alpha_d\sqrt{\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}}\tilde{h}x + u_d\Big|^2\Big\}}\\ &= \frac{\alpha_d^2\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}\mathbb{E}\{|\hat{h}|^2\}}{\alpha_d^2\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}\mathbb{E}\{|\tilde{h}|^2\} + \sigma_{ud}^2}\\ &= \frac{\alpha_d^2\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}\Big[\sigma_{\hat{h}}^2 + (\mathbb{E}\{h\})^2\Big]}{\alpha_d^2\mathcal{P}_{Sd}\beta_{RD}\beta_{SR}\sigma_{\tilde{h}}^2 + \sigma_{ud}^2},\end{aligned} \tag{25}$$

where $\mathbb{E}\{h\} = \mathbb{E}\{\|\hat{\boldsymbol{h}}_{RD}\|\}\mathbb{E}\{\|\hat{\boldsymbol{h}}_{SR}\|\}$ is the mean value of the dual-hop channel.

A closed-form expression of the average receive SNR can be obtained using the following identities:

$$\begin{aligned}\mathbb{E}\{\|\hat{\boldsymbol{h}}_{RD}\|^2\} &= N_R(1-\sigma_{e2}^2),\\ \mathbb{E}\{\|\hat{\boldsymbol{h}}_{SR}\|^2\} &= N_R(1-\sigma_{e1}^2),\\ (\mathbb{E}\{\|\hat{\boldsymbol{h}}_{RD}\|\})^2 &= \left[\frac{\Gamma(N_R+1/2)}{\Gamma(N_R)}\right]^2(1-\sigma_{e2}^2),\\ (\mathbb{E}\{\|\hat{\boldsymbol{h}}_{SR}\|\})^2 &= \left[\frac{\Gamma(N_R+1/2)}{\Gamma(N_R)}\right]^2(1-\sigma_{e1}^2).\end{aligned}$$

The derivations of the above equations are similar to those in [16, Eqs. (3-5)], hence are omitted for brevity.

A. Optimizing Relaying Energy Allocation

We formulate the same relaying energy allocation problem as in the single-antenna case described in (9). Specifically, we solve the optimization problem given by $\max_\phi \rho_{\text{ave}}$, subject to an average relaying power budget $\overline{\mathcal{P}}_R$. To obtain the optimal relaying energy allocation, one can numerically search for the value of $\phi \in (0, 1)$ that maximizes ρ_{ave} in (25). However, the optimal solution depends on all system parameters as well as the individual channel estimates. In the following, we look for a simple closed-form solution of the relaying energy allocation that depends on the least number of system parameters.

Applying the high SNR assumptions, *i.e.*, $\mathcal{P}_{St}\beta_{SR} \gg \sigma_R^2$, $\mathcal{P}_{Sd}\beta_{SR} \gg \sigma_R^2$, $\mathcal{P}_{Rt}\beta_{RD} \gg \sigma_D^2$ and $\mathcal{P}_{Rd}\beta_{RD} \gg \sigma_D^2$, the average receive SNR in (25) is approximated as

$$\begin{aligned}\rho_{\text{ave}} &\approx \frac{\mathcal{P}_{Rd}\beta_{RD}\Big[\sigma_h^4\mathcal{P}_{Rt}\beta_{RD}L_t+(\mathbb{E}\{h\})^2(\sigma_h^2\mathcal{P}_{Rt}\beta_{RD}L_t+\sigma_{ut}^2)\Big]}{\mathcal{P}_{Rd}\beta_{RD}\sigma_h^2\sigma_{ut}^2 + \sigma_{ud}^2(\sigma_h^2\mathcal{P}_{Rt}\beta_{RD}L_t + \sigma_{ut}^2)}\\ &\approx \frac{\mathcal{P}_{Rd}\beta_{RD}\Big[\sigma_h^4\mathcal{P}_{Rt}\beta_{RD}L_t + \sigma_h^2\mathcal{P}_{Rt}\beta_{RD}L_t(\mathbb{E}\{h\})^2\Big]}{\mathcal{P}_{Rd}\beta_{RD}\sigma_h^2\sigma_{ut}^2 + \sigma_h^2\mathcal{P}_{Rt}\beta_{RD}L_t\sigma_{ud}^2}\\ &\approx \frac{\mathcal{P}_{Rd}\mathcal{P}_{Rt}\beta_{RD}L_t\Big[\sigma_h^2 + (\mathbb{E}\{h\})^2\Big]/\sigma_D^2}{\mathcal{P}_{Rd}\Big(\frac{\mathcal{P}_{Rt}\beta_{RD}}{\mathcal{P}_{St}\beta_{SR}}\kappa + 1\Big) + \mathcal{P}_{Rt}L_t\Big(\frac{\mathcal{P}_{Rd}\beta_{RD}}{\mathcal{P}_{Sd}\beta_{SR}}\kappa + 1\Big)},\end{aligned} \tag{26}$$

where $\kappa = \frac{\sigma_R^2}{\sigma_D^2}(\mathbb{E}\{\|\hat{h}_{RD}\|^2\} + \sigma_{e2}^2) + 1$.

In this high SNR regime, the optimal value of ϕ that maximizes ρ_{ave} in (26) can be found by first substituting (9)

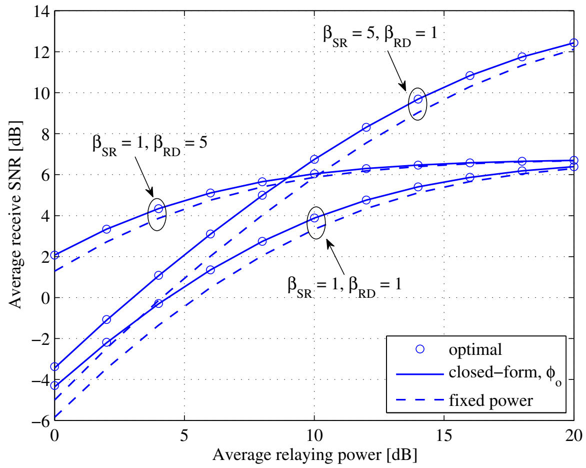

Fig. 2. The average receive SNR ρ_{ave} versus average relaying power $\overline{\mathcal{P}}_R$ in systems with a single-antenna relay. The markers show the maximum ρ_{ave} achieved with the optimal ϕ found numerically. The solid lines show the ρ_{ave} achieved using the proposed closed-form ϕ_o in (11). The dashed lines show the ρ_{ave} achieved with fixed power relaying, *i.e.*, $\mathcal{P}_{Rt} = \mathcal{P}_{Rd}$. The transmit power at S is set to $\mathcal{P}_{St} = \mathcal{P}_{Sd} = 10$ dB. The block structure is given by $L_t = 1$ and $L_d = 10$.

into (26), then set the first derivative of ρ_{ave} w.r.t. ϕ be zero and solve for the root. The solution is given as

$$\phi_o = \frac{1}{1+\sqrt{L_d}}. \tag{27}$$

This solution only depends on the data length L_d and does not depend on any other system parameters or the individual channel estimates. Hence, the designer can set the relaying energy allocation without knowing the quality of the channel estimates, and the energy budgets and locations of all terminals. Furthermore, this solution is independent of the number of antennas at R.

V. Numerical Results and Discussion

In this section, we present numerical results to illustrate the performance gain from optimizing the relaying energy allocation. We normalize the receiver noise variances as $\sigma_R^2 = \sigma_D^2 = 1$. Figs. 2 shows the average receive SNR ρ_{ave} in systems with a single-antenna relay, while Figs. 3 and 4 show ρ_{ave} for the case of a 4-antenna relay with different data lengths in the block-wise transmissions. In particular, we are interested in the performance of the proposed closed-form relaying energy allocation given in (11) or (27), which is derived from the high SNR regime. It is clear from the figures that this simple closed-form design achieves nearly the optimal performance over a wide range of relaying power budget for both a single-antenna relay and a multi-antenna relay.

Furthermore, the simple yet near-optimal design gives certain performance improvements over fixed power relaying (*i.e.*, $\mathcal{P}_{Rt} = \mathcal{P}_{Rd}$). As shown in Figs. 2, for achieving the same level of average receive SNR (which can be regarded as a target QoS), one saves around 1 dB of the relaying power by optimizing the relaying energy allocation instead of fixed

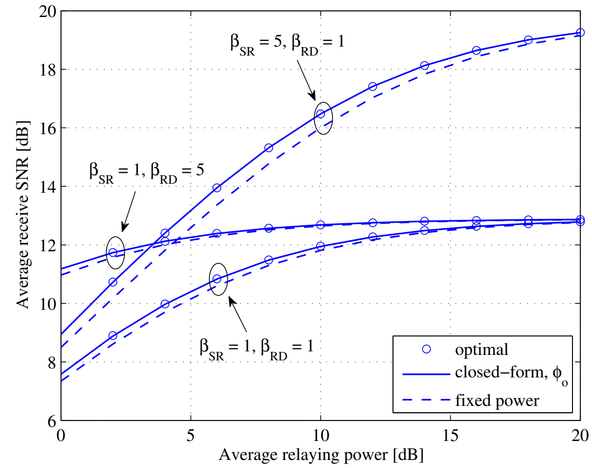

Fig. 3. The average receive SNR ρ_{ave} versus average relaying power $\overline{\mathcal{P}}_R$ in systems with a 4-antenna relay. The markers show the maximum ρ_{ave} achieved with the optimal ϕ found numerically. The solid lines show the ρ_{ave} achieved using the proposed closed-form ϕ_o in (27). The dashed lines show the ρ_{ave} achieved with fixed power relaying, *i.e.*, $\mathcal{P}_{Rt} = \mathcal{P}_{Rd}$. The transmit power at S is set to $\mathcal{P}_{St} = \mathcal{P}_{Sd} = 10$ dB. The estimation error of the individual channels has variance of $\sigma_{e1}^2 = \sigma_{e2}^2 = 0.1$. The block structure is given by $L_t = 1$ and $L_d = 10$.

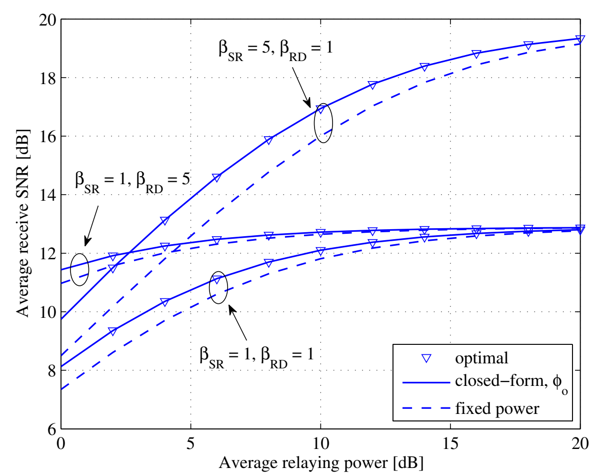

Fig. 4. The average receive SNR ρ_{ave} versus average relaying power $\overline{\mathcal{P}}_R$ in systems with a 4-antenna relay. The markers show the maximum ρ_{ave} achieved with the optimal ϕ found numerically. The solid lines show the ρ_{ave} achieved using the proposed closed-form ϕ_o in (27). The dashed lines show the ρ_{ave} achieved with fixed power relaying, *i.e.*, $\mathcal{P}_{Rt} = \mathcal{P}_{Rd}$. The transmit power at S is set to $\mathcal{P}_{St} = \mathcal{P}_{Sd} = 10$ dB. The estimation error of the individual channels has variance of $\sigma_{e1}^2 = \sigma_{e2}^2 = 0.1$. The block structure is given by $L_t = 1$ and $L_d = 50$.

power relaying. More interestingly, the amount of improvement depends on the transmission block structure, *i.e.*, the lengths of pilot and data symbols. For a fixed pilot length, the improvement from optimizing relaying energy allocation becomes more significant as the data length increases. This can be seen by comparing Fig. 3 for systems with $L_t = 1$, $L_d = 10$ and Fig. 4 for systems with $L_t = 1$, $L_d = 50$. For achieving the same level of average receive SNR, one saves around

0.7 dB of the relaying power by optimizing the relaying energy allocation instead of fixed power relaying when $L_d = 10$. This power saving boosts up to around 1.6 dB when $L_d = 50$. This result suggests that optimizing the relaying energy allocation may be important for slow fading channel that has a relatively long coherence time to allow a large number of data symbols in one transmission block.

VI. Conclusion

In this paper, we studied the relaying energy allocation between training and data transmission for a AF relay. The closed-form energy allocation derived from the high SNR analysis was shown to achieve near-optimal performance over a wide range of relaying power budget. More importantly, the derived energy allocation solution only depends on the block transmission structure but not on any other system parameters, which makes it attractive for practical implementation.

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