

Error Probability Analysis of Non-Orthogonal Multiple Access with Channel Estimation Errors

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Abstract—Non-orthogonal multiple access (NOMA) is very promising for future wireless systems thanks to its spectral efficiency. In NOMA schemes, the effect of imperfect successive interference canceler (SIC) has dominant effect on the error performances. In addition to this imperfect SIC effect, the error performance will get worse with the channel estimation errors just as in all wireless communications systems. However, all literature has been devoted to analyze error performance of NOMA systems with the perfect channel state information (CSI) at the receivers which is very strict/unreasonable assumption. In this paper, we analyze error performance of NOMA systems with imperfect SIC and channel estimation errors, much more practical scenario. We derive exact bit error probabilities (BEPs) in closed-forms. All theoretical analysis is validated via computer simulations. Then, we discuss optimum power allocation for user fairness in terms of error performance of users and propose a novel power allocation scheme which achieves maximum user fairness.

Index Terms—NOMA, performance analysis, imperfect SIC, channel estimation errors, power allocation

I. INTRODUCTION

Non-Orthogonal Multiple Access (NOMA) technique is one of the most strong candidates for future wireless systems, especially for Massive Machine Type Communication (MMTC) since it provides high spectral efficiency and allows ultra dense network [1]. In NOMA schemes, users share same resource block with different power allocation coefficients. Thus, the users can be served at the same time and massive connections can be possible in a cell. Due to sharing same resource blocks, the users suffer from inter-user-interference. Nevertheless, this interference can be eliminated by successive interference canceler (SIC) at the receivers.

The usage of NOMA in future wireless systems has been proposed in [2], and its superiority to Orthogonal multiple access (OMA) schemes has been proved in terms of achievable/sum rate. Then, NOMA has been analyzed for outage probability and its performance gain in also outage performance has been presented [3]. Thanks to these performance advantages, NOMA has attracted tremendous attention from both academia and industry [4]. Moreover, since NOMA can easily be implemented in physical layer, a great deal

of studies investigate NOMA integration with other physical layer techniques [5] such as cooperative communication [6], [7], multiple-input-multiple-output (MIMO) [8], [9], index modulation [10], [11], visible light communication [12] and reconfigurable intelligent surfaces [13].

On the other hand, the main drawback of the NOMA is the error performance due to inter-user-interference. Thus, a remarkable number of studies investigate NOMA in terms of error performance and bit/symbol/block error probabilities are derived for NOMA in various schemes according to fading channels, number of users and number of transmit/receive antenna [14]–[20]. Although, these studies consider imperfect SIC to characterize a practical scenario, all of them assume that perfect channel state information (CSI) is available at the receivers. However, considering channel estimation techniques, this assumption is too strict to be reasonable, hence it should be relaxed. To this end, in this paper, we analyze error performance of NOMA schemes when the imperfect SIC and channel estimation errors both exist. We derive exact bit error probabilities (BEPs) in closed-forms. To the best of the authors' knowledge, this is the first study which investigates error performance of NOMA schemes with imperfect SIC and CSI errors.

The rest of paper is organized as follows. In section II, the system and channel models are introduced and the detections at the users are given. In section III, the BEP expressions are derived for users. Then, the validation of the analytical analysis via computer simulations is presented in Section IV. Moreover, the optimum power allocation for user fairness is discussed in Section V. Finally, section V discusses the results and concludes the paper.

II. SYSTEM MODEL

We consider a two-user¹ downlink NOMA scheme where a base station (BS) and two mobile users (i.e., UE_i , $i = 1, 2$) are located. One of the users is close to (in terms of Euclidean distance) BS, thus it is called near user (UE_1) and the other is far user (UE_2). The channel coefficient between

¹Although more than two users can be implemented in NOMA, it is limited by two since increasing the number of users causes more inter-user-interference and users will have worse error performance. Thus, it is considered to be two users in also 3GPP standards [21].

each node follows² $CN(0, \sigma_i^2)$, $i = 1, 2$ where $\sigma_1^2 \geq \sigma_2^2$. All nodes are equipped with single antenna. The BS implements a superposition coding for the symbols of the users and transmits it to the users simultaneously. Thus, the received signals by the users are given

$$y_i = \sqrt{P_s} \left(\sqrt{\alpha} x_1 + \sqrt{(1-\alpha)} x_2 \right) h_i + n_i, \quad i = 1, 2, \quad (1)$$

where P_s is the transmit power of the BS. x_i is the base-band modulated symbol³ of the UE_i . h_i is the flat fading channel coefficient from BS to UE_i . n_i is the additive Gaussian noise (AGN) at the receiver of UE_i and follows $CN(0, N_0)$. In (1), α denotes the power allocation coefficient. Since the UE_1 has better channel condition, it is assumed to be $\alpha < 0.5$.

A. Detection at the users

1) *Far User (UE_2):* The symbols of UE_2 have more power (i.e., $\sqrt{1-\alpha}$), thus UE_2 implements maximum likelihood (ML) detection by pretending x_1 symbols as noise. The ML detection at the UE_2 is given as

$$\hat{x}_2 = \underset{n}{\operatorname{argmin}} \left| y_2 - \sqrt{(1-\alpha)} P_s x_{2,n} \hat{h}_2 \right|^2, \quad (2)$$

where $x_{2,n}$ denotes the n th point within the constellation of x_2 symbols. \hat{h}_2 is the estimated channel coefficient at the UE_2 and it is given by $\hat{h}_2 = h_2 - \epsilon$ where ϵ follows $CN(0, \delta^2)$.

2) *Near User (UE_1):* Since the x_1 symbols have less power, it is not possible to detect them directly as x_2 symbols. Thus, UE_1 should implement successive interference canceler where x_2 symbols are detected firstly like (2), then the detected \hat{x}_2 symbols are subtracted from received signal and finally the x_1 symbols are detected by ML. The detection procedure at UE_1 is given as

$$\hat{x}_1 = \underset{n}{\operatorname{argmin}} \left| y_1^+ - \sqrt{\alpha} P_s x_{1,n} \hat{h}_1 \right|^2, \quad (3)$$

where

$$y_1^+ = y_1 - \sqrt{(1-\alpha)} P_s \hat{x}_2 \hat{h}_1, \quad (4)$$

and

$$\hat{x}_2 = \underset{n}{\operatorname{argmin}} \left| y_1 - \sqrt{(1-\alpha)} P_s x_{2,n} \hat{h}_1 \right|^2. \quad (5)$$

In (3)-(5), $x_{1,n}$ is the n th point within the constellation of x_1 symbols. \hat{h}_1 is the estimated channel coefficient at the UE_1 and it is defined as $\hat{h}_1 = h_1 - \epsilon$.

III. BIT ERROR PROBABILITY (BEP) ANALYSIS

Since only ML detection is implemented at UE_2 , the erroneous detection at the UE_2 depends on whether the AGN is greater than the energy of the symbol or not. However, BS implements a superposition coding for x_1 and x_2 symbols and transmits this total symbol to the users. Thus, the energies

²In the following of this paper the notation used are as follow. $CN(\mu, \sigma)$ is a complex Gaussian distribution which has independent real and imaginary random variables with the μ mean and the $\frac{\sigma}{2}$ variance. We use $|\cdot|$ for the absolute value of a scalar/vector and $P_r(A)$ denotes the probability of the event A whereas $P_r(A|B)$ is the probability of the event A under the condition that B has already occurred.

³We assume that the symbols of both users are modulated by BPSK.

of superposition-coded symbols change according to which x_1 and x_2 symbols are superimposed. Considering BPSK is used for symbols, two different energy levels can be produced with equal probabilities. In addition, the channel estimation errors should also be considered for the erroneous detection probability. Based on these discussions, the conditional BEP of UE_2 is given by

$$P_2(e|h_2) = \frac{1}{2} P \left(n_2 + \sqrt{P_s} (\sqrt{1-\alpha} + \sqrt{\alpha}) \epsilon > \sqrt{P_s} (\sqrt{1-\alpha} + \sqrt{\alpha}) h_2 \right) \\ + \frac{1}{2} P \left(n_2 + \sqrt{P_s} (\sqrt{1-\alpha} - \sqrt{\alpha}) \epsilon > \sqrt{P_s} (\sqrt{1-\alpha} - \sqrt{\alpha}) h_2 \right) \quad (6)$$

With some simplifications and algebraic manipulations, it is determined as

$$P_2(e|\gamma_2) = \sum_{k=1}^2 \frac{1}{2} Q \left(\sqrt{2\gamma'_{2,k}} \right) \quad (7)$$

where $\gamma'_{2,k} = \frac{\beta_k \rho_s \gamma_2}{\beta_k \delta^2 \rho_s + 1}$, $\beta_k = [1 + \sqrt{\alpha - \alpha^2}, 1 - \sqrt{\alpha - \alpha^2}]$, $\rho_s = P_s/N_0$ and $\gamma_2 \triangleq |h_2|^2$ are defined. The γ_2 follows exponential distributions. When the conditional BEP in (7) is averaged over instantaneous γ_2 , with the aid of [22], the average BEP (ABEP) of UE_2 is derived as

$$P_2(e) = \sum_{k=1}^2 \frac{1}{4} \left(1 - \sqrt{\frac{\beta_k \rho_s \sigma_2^2}{\beta_k \delta^2 \rho_s + 1}} \right). \quad (8)$$

On the other hand, the BEP of x_1 symbols should be considered for two cases (i.e., correct SIC and erroneous SIC). Thus, with the law of probability, the ABEP of UE_1 is given by

$$P_1(e) = P_{SIC} P_1(e|_{\text{error}}) + (1 - P_{SIC}) P_1(e|_{\text{correct}}) \quad (9)$$

where P_{SIC} is the ABEP for x_2 symbols detection at the UE_1 . $P_1(e|_{\text{error}})$ and $P_1(e|_{\text{correct}})$ denote the ABEP of x_1 symbols when the x_2 symbols are detected erroneously and correctly, respectively.

In order to obtain P_{SIC} , we repeat the steps between (6) and (8) for h_1 , then it is derived as

$$P_2(e) = \sum_{k=1}^2 \frac{1}{4} \left(1 - \sqrt{\frac{\beta_k \rho_s \sigma_1^2}{\beta_k \delta^2 \rho_s + 1}} \right). \quad (10)$$

If the SIC is implemented erroneously at the UE_1 , this means that wrongly estimated \hat{x}_2 symbols will be subtracted from the received signal y_1 as given in (4). Without loss of the generality, we can consider this wrongly-subtraction as noise for detection of x_1 symbols. Therefore, the noise term will be greater than the energy of x_1 symbols thereby the ABEP in this case is close to the worst case as

$$P_1(e|_{\text{error}}) \cong \frac{1}{2} \quad (11)$$

As the second case, if correct SIC is implemented at the UE_1 , only x_1 symbols will remain after subtraction.

$$P_1(e) = \sum_{k=1}^2 \frac{1}{8} \left(1 - \sqrt{\frac{\beta_k \rho_s \sigma_1^2}{\beta_k \delta^2 \rho_s + 1}} \right) + \left\{ 1 - \sum_{k=1}^2 \frac{1}{4} \left(1 - \sqrt{\frac{\beta_k \rho_s \sigma_1^2}{\beta_k \delta^2 \rho_s + 1}} \right) \right\} \frac{1}{2} \left(1 - \sqrt{\frac{\alpha \rho_s \sigma_1^2}{\delta^2 \rho_s + 1}} \right) \quad (15)$$

Nevertheless, due to the SIC operation, the effect of channel estimation error will be increased. Hence, the conditional BEP of x_1 symbols in case correct SIC is given as

$$P_1(e|_{h_1 \cap \text{correct}}) = P(n_1 + \sqrt{P_s} \epsilon > \sqrt{\alpha P_s} h_1) \quad (12)$$

and it is obtained as

$$P_1(e|_{\gamma_1 \cap \text{correct}}) = Q\left(\sqrt{2\gamma'_1}\right), \quad (13)$$

$\gamma'_1 = \frac{\alpha \rho_s \gamma_1}{\delta^2 \rho_s + 1}$ and $\gamma_1 \triangleq |h_1|^2$ are defined. Just as (8), with the aid of [22], the ABEP of x_1 symbols in correct SIC case is derived as

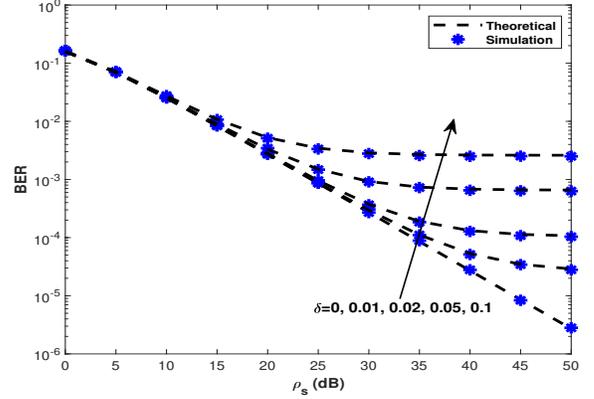
$$P_1(e|_{\text{correct}}) = \frac{1}{2} \left(1 - \sqrt{\frac{\alpha \rho_s \sigma_1^2}{\delta^2 \rho_s + 1}} \right). \quad (14)$$

Finally, substituting (10), (11) and (14) into (9), the ABEP for x_1 symbols derived as in (15) (see the top of the page).

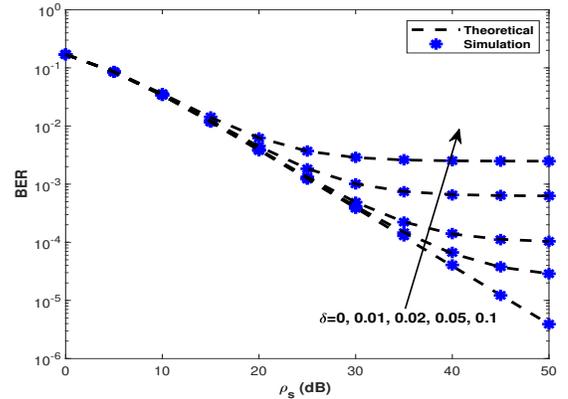
IV. NUMERICAL RESULTS

In this section, we present Monte Carlo simulations to validate theoretical analysis. In all figures, simulations are presented by markers and theoretical curves are denoted by lines. All simulation results are obtained from 10^7 channel realizations.

In Fig. 1 and Fig. 2, we present error performances of users with the change of transmit SNR for power allocation coefficients $\alpha = 0.1$ and $\alpha = 0.2$, respectively. In both figures, we assume that $\sigma_1^2 = 10\text{dB}$ and $\sigma_2^2 = 0\text{dB}$. The results are presented for five different channel estimation error effects as $\delta = 0$ (perfect CSI) and $\delta = 0.01, 0.02, 0.05, 0.1$. It is noteworthy that derived expressions match perfectly with simulations. In addition, as expected, with the increase of channel estimation errors (i.e., δ) both users' performance decrease. Especially, in high SNR region, an error floor may be observed. Besides, if we compare Fig. 1 and Fig. 2, we can easily see that increasing α provides performance gain for UE_1 whereas a performance decay occurs for UE_2 . This is also an expected results since the higher α means that the higher and lower powers are allocated to symbols of UE_1 and UE_2 , respectively. Nevertheless, the gain in the UE_1 's performance cannot be always-observed with the increase of α due to the SIC operation. In order to reveal the effect of power allocation on the performances of users, we present error performances of users with the change of α in Fig. 3. The results are provided for transmit SNR $\rho_s = 30\text{dB}$ and the channel estimation errors are assumed to be $\delta = 0$ (perfect CSI) and $\delta = 0.01, 0.02, 0.05, 0.1$. The channel conditions are assumed to be same with previous figures. One can easily see that increase of α always causes a decay in error performance of UE_2 . On the other hand, it firstly provides a gain in error



(a) UE_1



(b) UE_2

Fig. 1: Error performance of NOMA vs. transmit SNR (ρ_s) with channel estimation errors ($\delta = 0, 0.01, 0.02, 0.05, 0.1$) when $\sigma_1^2 = 10\text{dB}$, $\sigma_2^2 = 0\text{dB}$, $\alpha = 0.1$

performance of UE_1 , but too much increase in α causes a decay in performance of UE_1 . This can be explained as follow. With the increase of α , erroneous detection of UE_2 symbols at the UE_1 increases, thus these erroneously detected UE_2 symbols cause erroneous SIC operation (wrongly detected x_2 symbols are subtracted from the received signal) and the error performance of the UE_1 becomes worse. Furthermore, the effect of imperfect CSI will change on the performance of users according to the power allocation. For instance, when relatively lower power allocation is implemented (e.g., $\alpha < 0.25$), the error performance of UE_2 is affected more with the increase of channel estimation errors (i.e., δ). Nevertheless, UE_2 will have almost the same error performance when $\alpha \geq 0.25$. The same discussion is valid for UE_1 since the error performance of UE_1 is dominated by SIC operation (same with the error performance of UE_2).

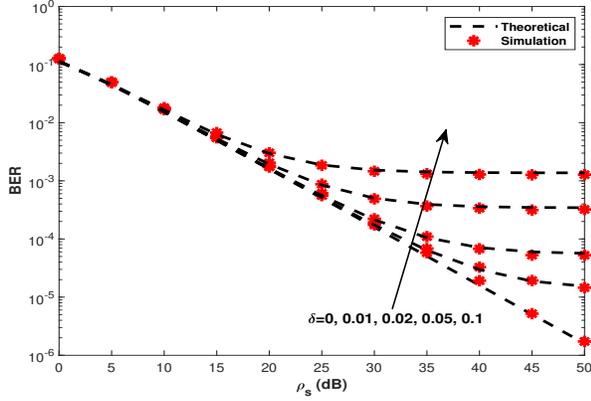
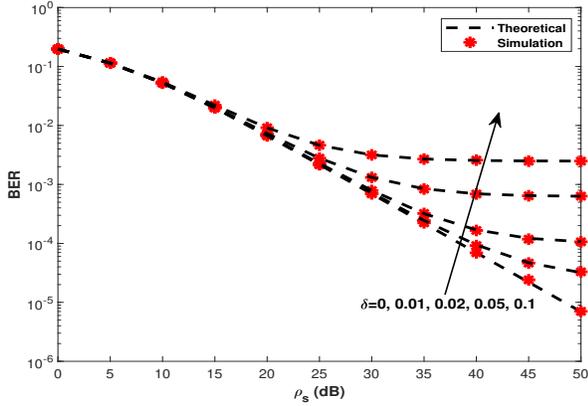
(a) UE_1 (b) UE_2

Fig. 2: Error performance of NOMA vs. transmit SNR (ρ_s) with channel estimation errors ($\delta = 0, 0.01, 0.02, 0.05, 0.1$) when $\sigma_1^2 = 10dB$, $\sigma_2^2 = 0dB$, $\alpha = 0.2$

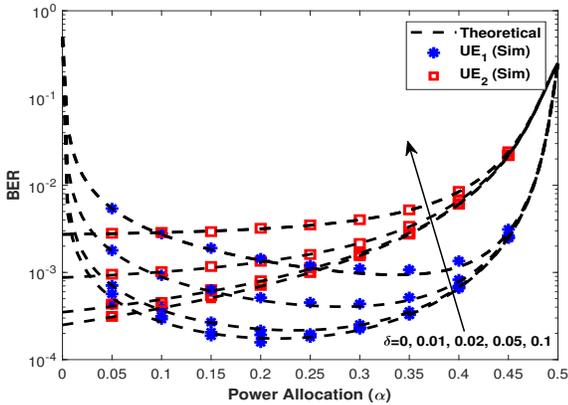


Fig. 3: Error performance of NOMA vs. power allocation (α) with channel estimation errors ($\delta = 0, 0.01, 0.02, 0.05, 0.1$) when $\sigma_1^2 = 10dB$, $\sigma_2^2 = 0dB$, $\rho_s = 30dB$

V. OPTIMUM POWER ALLOCATION

Based on extensive simulations in the previous section, we reveal that the power allocation has dominant effect on

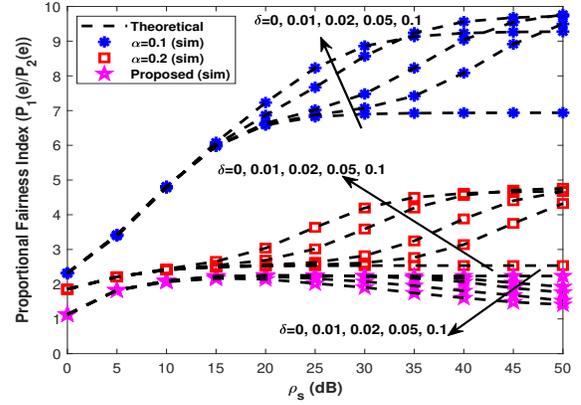


Fig. 4: Proportional fairness index ($P_1(e)/P_2(e)$) vs. transmit SNR (ρ_s) with channel estimation errors ($\delta = 0, 0.01, 0.02, 0.05, 0.1$) when $\sigma_1^2 = \sigma_2^2 = 0dB$

the error performances of users. Thus, the power allocation coefficient should be carefully chosen not to cause a user unfairness. To this end, we propose the optimum power allocation as

$$\alpha^* = \min\{\max\{P_1(e), P_2(e)\}\}. \quad (16)$$

$$\text{s.t } \alpha^* < 0.5$$

With the proposed power allocation in (16), the ABEPs of users are calculated and the power allocation is chosen as the optimum value which minimizes the maximums of the ABEPs of the users. Therefore, none of the users performs much worse than the other so that the user fairness is achieved. In (16), $\alpha^* < 0.5$ constraint should be satisfied, otherwise, the symbols of the users cannot be detected with the given SIC order. To evaluate the proposed power allocation, we present proportional fairness (PF) index⁴ (i.e., $P_1(e)/P_2(e)$) for users in Fig. 4. when users have same channel conditions (i.e., $\sigma_1^2 = \sigma_2^2 = 0dB$). As seen from Fig. 4, the proposed power allocation outperforms fixed power allocation strategies. For instance, at $20dB$ when $\delta = 0.05$, proportional fairness indexes are 8.56, 3.68 and 1.5 for $\alpha = 0.1$, $\alpha = 0.2$ and proposed power allocation, respectively. This means that UE_1 has 8.56 and 3.68 times worse performance than UE_2 for $\alpha = 0.1$ and $\alpha = 0.2$. On the other hand, the proposed power allocation provides almost same performances for users and this proves the effectiveness of that for user fairness. Moreover, the proposed power allocation scheme is more robust to channel estimation errors in terms of PF. User fairness is increased even if the channel estimation errors increase whereas it gets worse for fixed power allocations.

VI. CONCLUSION

In this paper, we investigate the error performance of NOMA schemes in the presence of channel estimation er-

⁴We hereby note that PF index of any performance metrics have the same meaning for κ and $1/\kappa$. It only defines which user has better performance. For instance, 2 and 0.5 have the same meanings in terms of PF index

rors in addition to imperfect SIC. We derive exact ABEP expressions in the closed-forms and all analyses are validated via computer simulations. Moreover, we discuss optimum power allocation considering user fairness in terms of error performances. To the best of the authors' knowledge, error analysis of any NOMA schemes with channel estimation errors is firstly conducted in this paper. Thus, the analysis in this paper could be further extended for any NOMA schemes such as cooperative-NOMA, MIMO-NOMA, etc. These are seen as future works.

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