On the genetic optimization of APSK constellations for satellite broadcasting

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Abstract—Both satellite transmissions and DVB applications over satellite present peculiar characteristics that could be taken into consideration in order to further exploit the optimality of the transmission. In this paper, starting from the state-ofthe-art, the optimization of the APSK constellation through asymmetric symbols arrangement is investigated for its use in satellite communications. In particular, the optimization problem is tackled by means of Genetic Algorithms that have already been demonstrated to work nicely with complex non-linear optimization problems like the one presented hereinafter. This work aims at studying the various parameters involved in the optimization routine in order to establish those that best fit this case, thus further enhancing the constellation.

Index Terms—Transmission, Channel coding, modulation, multiplexing, Signal processing for transmission

I. INTRODUCTION

APSK modulation with pre- and post- compensation schemes [1] is deployed in DVB over satellite standards [2] [3] for its power and spectral efficiency over nonlinear satellite channels. Nevertheless, for multimedia broadcasting applications, further improvements by means of non-uniform constellations could be obtained. As a matter of fact, multimedia streams employed in digital broadcasting are hierarchical by nature, so that bits associated with transmitted symbols present different error sensitivities. In particular, Most Significant Bits (MSB) affect the transmission more than errors on the Least Significant Bits (LSB).

Channel coding techniques with Unequal Error Protection (UEP) have been studied in [5] for QAM and in [6] for APSK, even though this introduces overhead and reduces bandwidth efficiency, which is a critical issue for satellite applications. In [7] Modulation with Unequal Power Allocation (MUPA) was proposed as a mean to improve the performance of conventional modulation schemes in case of digital wireless communication systems (e.g. DECT, Bluetooth) which do not include channel coding for some reason thus saving bandwidth. MUPA distributes the available budget power over the QAM symbols according to their sensitivity to channel errors, whereas the average transmission power per symbol

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remains unchanged. The resulting quality on received data is improved without any increase of transmission bandwidth.

The same concept was extended in [8] to the case of APSK modulations in order to achieve UEP through asymmetric layout of the constellation symbols. The approach used is similar to MUPA in the selection of the opportune radius of the constellation circles and the phase of each symbol. The best numerical solution was obtained solving the optimization problem (OP) of minimizing inter-symbol distortion by means of Genetic Algorithms (GA) [9], a numerical search technique used in many fields to solve complex problems which do not allow analytical derivation [10] [11]. This work focuses on the OP through careful selection of the parameters involved in GA in order to further improve the constellation performance with respect to [8]. Moreover, the possibility to drop various symmetric bit allocation constraints is taken into consideration as a mean to further boost the constellation optimality.

II. SYSTEM MODEL

First of all let us introduce the model used for the optimization by schematically describing the various building blocks of the communication system.

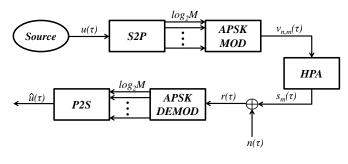


Figure 1. System model

Considering M to be the alphabet size of the constellation, each symbol will represent a stream $u(\tau)$ of $log_2 M$ bits generated by a memoryless source and put in parallel by the serialto-parallel (S2P) block. These symbols are then modulated by the APSK modulation block giving place to a complex number $v_{n,m}(\tau) = \rho_n \cdot e^{j\theta_m}$ that represents constellation symbols and where ρ_n is the radius of the n-th circle of the constellation and θ_m is the phase of the m-th symbol. As already claimed in [8] the distribution of the symbols of the constellation on the various radii is basically free. However, considering the nonlinear distorsion introduced by the HPA the best performance is obtained when 4 symbols are put in the inner circle for 16-APSK (i.e. 4 + 12) and 4 in the inner and 12 in the medium circle are put for 32-APSK (i.e. 4 + 12 + 16) as shown in Figure 2.

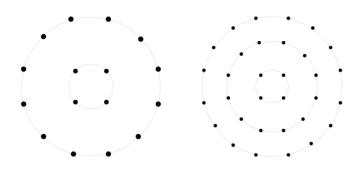


Figure 2. 16-APSK constellation (left) and 32-APSK constellation (right)

Once constellation symbols are modulated, they are amplified by the HPA prior to transmission, thus being subject to the non-linear behaviour of the amplifier, whose effects can be modeled using the Saleh model [12] resulting in the output $s_m(\tau)$. This model distinguishes two effects:

- the AM/AM non-linear effect that models amplitude distorsions on the input signal;
- the AM/PM non-linear effect that models phase distorsions on the input signal.

In this paper, optimization has been accomplished without taking into consideration the AM/PM non-linear effect due to the HPA (that is known to change the relative position of symbols) since it has been already shown in [1] that this effect can be easily compensated. Therefore only the AM/AM nonlinear distorsion is taken into account through the formula

$$A(\rho) = \frac{a\rho_n}{1+b\rho_n^2} \tag{1}$$

where a = 2.1587 and b = 1.1517 are standard values of the constants gathered from the literature and obtained by means of curve-fitting techniques.

Considering the channel to be of the Additive White Gaussian Noise (AWGN) type, transmitted symbols are affected by the addition of a random nuisance signal with zero mean and variance $N_0/2$. Therefore, the received signal at the destination is $r(\tau) = s(\tau) + n(\tau)$. This signal is passed to the demodulator that applies the Maximum Likelihood (ML) criterion in order to estimate the received symbol and it is then converted back from parallel to the serial signal $\hat{u}(\tau)$. The difference between the stream generated by the memoryless source and the one estimated at the receiver can be computed as

$$d(\tau) = [u(\tau) - \hat{u}(\tau)] \tag{2}$$

and it is called distorsion. The aim of the GA presented in the next section is to minimize this distorsion. Therefore the

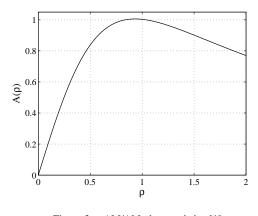


Figure 3. AM/AM characteristics [1]

minimum square error

$$MSE = E\{[d(\tau)]^2\}$$
(3)

represents our optimization criterion.

III. GENETIC ALGORITHMS OPTIMIZATION

At each iteration n, a GA gives birth to a generation G_n of potential solution vectors (also called chromosomes γ_i) that constitute the population of size p of the OP:

$$G_n = \{\gamma_1^{(n)}, \gamma_2^{(n)}, ..., \gamma_i^{(n)}, ..., \gamma_p^{(n)}\}$$
(4)

A vector of fitness scores S_n is also calculated for each generation using the objective function R:

$$S_n = \{ R(\gamma_1^{(n)}), R(\gamma_2^{(n)}), ..., R(\gamma_i^{(n)}), ..., R(\gamma_p^{(n)}) \}$$
(5)

The chromosomes with the highest fitness score are meant to be the closest to the desired solution and are thus selected for surviving and giving place to the next generation.

The next generation is created in three steps:

- 1) *Selection* of the part of population with the best fitness score that will be parents for the next generation;
- Crossover of selected parents according to a mixing criterion in order to give birth to a number of children from each couple that will constitute the next generation;
- Mutation of a percentage of the offspring in order to spread the optimum solution search and avoid local optimal solutions.

The computation is stopped when the population has converged to the same fitness value which is supposed to be the optimal solution.

In the case of APSK the chromosomes are the radii and the phase of each symbol. The optimization criterion chosen is the minimization of the MSE, i.e. the expected minimum squared error between the transmitted symbol and the received one. Therefore the function used to calculate the fitness scores is R = 1/(MSE). In [8] the importance of a constellation design that takes into account UEP has been highlighted and GA has been demonstrated to be a viable solution for solving the OP. However the choice of the best selection, crossover and mutation functions together with an appropriate number of generation iterations and population size has not been exploited yet. For this reason, this paper extends the results obtained in [8] demonstrating that a proper selection of the abovementioned parameters can further improve the optimality of the solution. In particular, this work concentrates on the best choice regarding the selection and the crossover function by comparing the results obtained through simulation in Matlab for 5 different selection functions (*stochastic uniform, remainder, uniform, roulette, tournament*) and 6 different crossover functions (*scattered, single point, two point, intermediate, heuristic, arithmetic*). For more information about the abovementioned functions, the reader can refer to the Matlab guide and a vast literature on the topic.

IV. RESULTS FOR 16-APSK

First of all, let us analyze the case of 16-APSK. The GA starts with an initial population of 80 chromosomes characterized by genes with values uniformly distributed on the constellation circles. Using these values, transmission over satellite is simulated according to the model presented in Section II. Then, the fitness function is evaluated thanks to the function R already discussed. At this point, the GA modifies the population in accordance with the fitness results using the policies defined by the specific selection and crossover functions taken into consideration. The transmission is then repeated using the new generation until convergence or the maximum number of generations (in this case set to n = 130) is reached.

CR	stochunif	remainder	uniform	roulette	tournam.
scattered	1.1045	1.2225	1.1724	1.2054	1.2287
single point	1.1758	1.1029	1.2541	1.1111	1.1577
two point	1.1336	1.1235	1.2720	1.2002	1.1470
intermediate	1.2074	1.1983	1.6869	1.1981	1.2149
heuristic	1.1695	1.1701	1.1706	1.1609	1.1514
arithmetic	1.2749	1.2968	1.8615	1.3074	1.1830

Table I TARGET VALUES FOR 16-APSK 4SIM

CR	stochunif	remainder	uniform	roulette	tournam.
scattered	1.2640	0.7629	1.0275	0.7439	0.7619
single point	1.4284	0.7259	0.9252	0.7238	0.9086
two point	0.7361	0.7527	0.7639	0.7942	0.7809
intermediate	0.7732	0.8237	2.1824	0.7921	0.7530
heuristic	0.7969	1.4852	0.8238	0.7967	0.7722
arithmetic	0.7437	0.8052	3.3424	0.8634	0.7616

Table II TARGET VALUES FOR 16-APSK 2SIM

In Tables I, II and III the results in terms of MSE at the target SNR = 10dB are shown respectively for the case of double symmetry (x and y axis), for the case with symmetry

CR	stochunif	remainder	uniform	roulette	tournam.
scattered	0.9844	0.7686	2.3634	1.1534	1.1600
single point	0.8674	0.6696	1.6173	0.8807	0.7660
two point	0.7127	1.0152	3.0634	0.7291	0.7899
intermediate	0.8709	1.2898	15.5712	0.9861	1.5404
heuristic	1.9504	1.3986	0.6794	0.7527	2.7479
arithmetic	0.9338	1.0234	13.9503	0.8291	1.1862

Table III TARGET VALUES FOR 16-APSK 0SIM

on only one axis and for the case without any symmetry constraint. The mentioned symmetry refers to the placement of the symbols with regard to the inter-symbol distorsion, as shown in Figure 4 for the case of double symmetry.

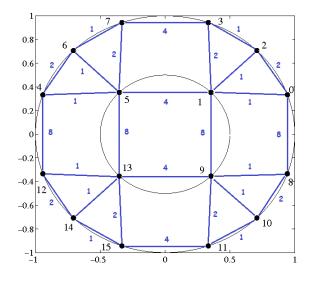


Figure 4. Example of 16-APSK constellation with double symmetry

The best three results are highlighted in bold. It can be seen that regardless of the considered symmetries, the combination of the *remainder* selection function and the *single point* crossover function always yields to the best result or to a result really close to the best one. Moreover, from a general perspective, it can be gathered from the tables that the optimization procedure benefits from the drop of the symmetry contraints. As a matter of fact, comparing the case of double symmetry and the one without symmetry, the MSE value at the target SNR is reduced by approximately 40%. However, in order to validate the results, it is important to verify whether the improvement extends for a certain SNR range or if it is only local and specific of that SNR.

For this reason, in Table IV the values for the radius and the phase of the various symbols are presented. The first column refers to the results obtained in [8]. The second, third and fourth column refer respectively to the cases of double, single and no symmetry for the case in which the *remainder* selection function and the *single point* crossover function are

Parameter	[8]	double sym.	single sym.	no sym.
ρ_0	0.6404	0.8996	0.9627	0.9593
θ_0	1.0482	1.0360	2.5650	5.0872
θ_1	0.9355	0.8867	2.3592	4.7453
θ_2	0.6374	0.5802	2.0128	4.3400
θ_3	0.4891	0.4013	1.7317	3.7447
θ_4	-	-	1.4188	3.4121
θ_5	-	-	1.2107	3.1109
θ_6	-	-	0.8849	2.7071
θ_7	-	-	0.5372	2.2326
θ_8	-	-	-	1.8925
θ_9	-	-	-	1.5490
θ_{10}	-	-	-	1.2567
θ_{11}	-	-	-	1.0438
θ_{12}	-	-	-	0.7340
θ_{13}	-	-	-	0.4687
θ_{14}	-	-	-	0.2205
θ_{15}	-	-	-	0.0699

Table IV PARAMETER VALUES FOR 16-APSK

used. The considered chromosomes for these three cases are respectively:

- $\gamma = [\rho_0, \theta_0, \theta_1, \theta_2, \theta_3]$
- $\gamma = [\rho_0, \theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7]$
- $\gamma = [\rho_0, \theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}, \theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}]$

where the different number of parameters is due to the fact that, when using symmetries, the rest of the symbols are defined by symmetry. Notice also that only one radius has been defined in the table, since we are assuming that the outer one is fixed to 1. In addition, the phases have been defined so that the subscripts of each theta correspond to the alphabet value assigned to that symbol.

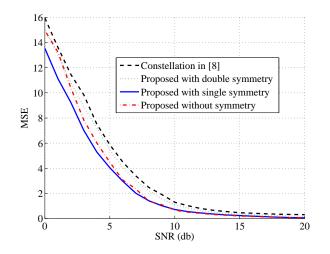


Figure 5. MSE results for the considered constellations

Figure 5 shows the results in terms of MSE as a function

of the SNR for the 4 constellations presented in Table IV. Unfortunately, in [8] it was not stated what kind of selection and crossover functions were used. However, it can be seen from the graph that a proper choice of the functions results in a better performance even when both symmetries are kept. Surprisingly, the results for a single symmetry are better than those with no symmetries, although this last case overtakes the first one from SNR = 10dB. These results demonstrate that having a better MSE at the SNR target and/or dropping all the symmetry constraints does not necessarily corresponds to an improvement of the performance. In figure 6, the constellation with single symmetry is shown.

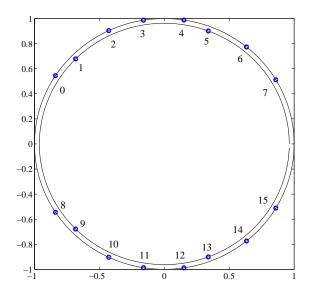


Figure 6. Optimized constellation for 16-APSK with single symmetry

V. RESULTS FOR 32-APSK

The same procedure presented in the previous section has also been applied to the 32-APSK constellation, with the necessary modifications. The first difference that can be noticed is the greater number of variables to compute. Therefore, also the convergence of the optimization is expected to be slower than in the previous case. Nevertheless, we decided to keep the population size to the value 80 and the number of generations to 130 in order to evaluate how optimal the solutions are keeping the same values as in 16-APSK. Moreover, this has been dictated by time constraints, since in the case of 32-APSK with no symmetries each combination of selection and crossover function required approximately 8 hours to be run.

Tables V, VI and VII show that, not unexpectedly, when the number of variables to compute increases the algorithm is not anymore able to converge to a solution in the number of generations set. Moreover, each combination of the selection and crossover functions converges with different paces. Another interesting result is that, when the number of symbols is changed, the best result is not obtained for the same selection

CR SEL	stochunif	remainder	uniform	roulette	tournam.
scattered	4.5572	4.6612	5.2848	4.4263	4.6581
single point	4.8225	4.9691	5.3944	4.5561	4.5573
two point	4.7172	4.7339	5.3761	4.7521	4.4136
intermediate	5.2306	5.4009	13.1101	5.5408	4.7368
heuristic	4.7713	4.5942	4.5201	5.2484	4.8416
arithmetic	5.8264	5.4936	12.0061	5.4959	4.4832

Table V TARGET VALUES FOR 32-APSK 4SIM

CR	stochunif	remainder	uniform	roulette	tournam.
scattered	8.1330	4.8600	12.7564	5.1381	4.3409
single point	9.2198	5.5814	10.9982	10.9608	4.1631
two point	4.5536	4.4141	10.2918	4.4285	4.0857
intermediate	4.6031	8.6994	11.7529	11.0164	4.9557
heuristic	4.3239	4.5903	5.1924	6.6715	4.78346
arithmetic	5.1618	5.2917	13.1309	10.7594	4.6443

Table VI TARGET VALUES FOR 32-APSK 2SIM

CR	stochunif	remainder	uniform	roulette	tournam.
scattered	6.3820	7.4839	25.7629	7.8117	5.9957
single point	8.7097	14.4059	26.9502	6.3373	6.8377
two point	15.6945	7.0951	19.4484	5.6768	5.6494
intermediate	12.9701	23.4311	72.1346	17.7114	14.7770
heuristic	12.1008	9.6265	10.5484	26.8005	9.5540
arithmetic	43.5802	36.4328	42.8421	11.6793	8.1312

Table VII TARGET VALUES FOR 32-APSK 0SIM

and crossover function. We expect the same thing to hold when some symbols are moved from one radius to another. The proof of this expectation and a deeper explanation of why this happens are left as future research on the topic.

In Table VIII the values for the parameters describing the position of the constellation symbols are given. In this case, the case without symmetry has not been included due to the lack of significance. Concerning the differences between the angle ranges of our case and the one in [8], this is simply due to the fact that we have considered $[0, \pi]$ as our optimization range while in [8] the considered range was $[\pi/2, 3\pi/2]$.

Figure 7 presents the results for the constellations described in Table VIII. Although in a smoother way, also in this case the obtained results improve those obtained in [8]. In particular, the MSE is lowered from 5.34 to 4.34, that is approximately a decrement of the 19%. Figure 8 shows the proposed constellation for 32-APSK with single symmetry.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, the use of asymmetric constellations for uncoded transmission in the satellite broadcasting scenario has been presented. Moreover, their optimization using genetic algorithms and the careful selection of the functions involved

Parameter	[8]	double sym.	single sym.
ρ_0	0.2453	0.2446	0.2487
ρ_1	0.8163	0.8285	0.8217
θ_0	3.9215	0.1664	0.5301
θ_1	3.8878	0.2998	0.7360
θ_2	3.7697	0.3009	0.7342
θ_3	3.6837	0.6293	0.8993
θ_4	3.4184	0.5831	1.1148
θ_5	3.6422	0.9550	1.2771
θ_6	3.2628	0.9219	1.4283
θ_7	3.1881	1.0028	1.5063
θ_8	2.6639	-	1.8236
θ_9	2.6409	-	1.9895
θ_{10}	2.4866	-	2.0694
θ_{11}	2.3709	-	2.1592
θ_{12}	2.2034	-	2.2590
θ_{13}	2.1479	-	2.5016
θ_{14}	2.0492	-	2.5479
θ_{15}	2.0199	-	2.5813

Table VIII PARAMETER VALUES FOR 32-APSK

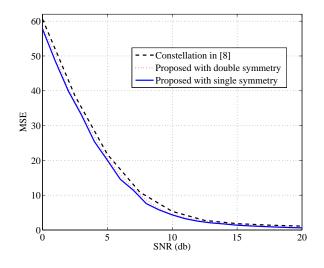


Figure 7. MSE results for the considered constellations

in the optimization routine have been discussed. Found results demonstrate that it is possible to further optimize this kind of communications by adjusting the behavior of the genetic algorithm. Moreover it has been demonstrated that dropping symmetry constraints is not always beneficial to the optimization process, especially when several variables must be computed thus slowing down the convergence to an optimal solution of the genetic algorithm. As future work, we aim at extending the results presented in this paper as well as apply the same concepts to the case of 64-APSK.

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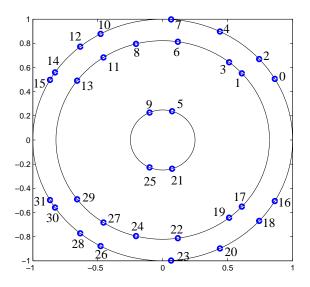


Figure 8. Optimized constellation for 16-APSK with single symmetry

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