SENSE: Semantically Enhanced Node Sequence Embedding

Swati Rallapalli

IBM T.J. Watson Research

Yorktown, NY, USA

srallapalli@us.ibm.com

Liang Ma

IBM T.J. Watson Research

Yorktown, NY, USA

maliang@us.ibm.com

Mudhakar Srivatsa

IBM T.J. Watson Research

Yorktown, NY, USA

msrivats@us.ibm.com

Ananthram Swami
Army Research Lab
Adelphi, MD, USA
ananthram.swami.civ@mail.mil

Heesung Kwon

Army Research Lab

Adelphi, MD, USA
heesung.kwon.civ@mail.mil

Graham Bent

IBM Research

Hursley, UK

gbent@uk.ibm.com

Christopher Simpkin

Cardiff University

Cardiff, UK

simpkinc1@cardiff.ac.uk

Abstract—Effectively capturing graph node sequences in the form of vector embeddings is critical to many applications. We achieve this by (i) first learning vector embeddings of single graph nodes and (ii) then composing them to compactly represent node sequences. Specifically, we propose SENSE-S (Semantically Enhanced Node Sequence Embedding - for Single nodes), a skip-gram based novel embedding mechanism, for single graph nodes that co-learns graph structure as well as their textual descriptions. We demonstrate that SENSE-S vectors increase the accuracy of multi-label classification tasks by up to 50% and link-prediction tasks by up to 78% under a variety of scenarios using real datasets. Based on SENSE-S, we next propose generic SENSE to compute composite vectors that represent a sequence of nodes, where preserving the node order is important. We prove that this approach is efficient in embedding node sequences, and our experiments on real data confirm its high accuracy in node order decoding.

I. INTRODUCTION

Accurately learning vector embeddings for a sequence of nodes in a graph is critical to many scenarios, e.g., a set of Web pages regarding one specific topic that are linked together. Such a task is challenging as: (i) the embeddings may have to capture graph structure along with any available textual descriptions of the nodes, and moreover, (ii) nodes of interest may be associated with a specific order. For instance, (i) for a set of Wikipedia pages w.r.t. a topic, there exists a recommended reading sequence; (ii) an application may consist of a set of services/functions, which must be executed in a particular order (workflow composability); (iii) in source routing [1], the sender of a packet on the Internet specifies the path that the packet takes through the network or (iv) the general representation of any path in a graph or a network, e.g., shortest path. Node sequence embedding, thus, requires us to (i) learn embeddings for each individual node of the graph and (ii) compose them together to represent their sequences. To learn the right representation of individual nodes and also their sequences, we need to understand how these nodes are correlated with each other both functionally and structurally.

A lot of work has only gone into learning single node embeddings (i.e., where node sequence length is 1), as they are

essential in feature representations for applications like multilabel classification or link prediction. For instance, algorithms in [2], [3], [4] and others try to extract features purely from the underlying graph structure; algorithms in [5], [6] and others learn vector representations of documents sharing a common vocabulary set. However, many applications would potentially benefit from representations that are able to capture both textual descriptions and the underlying graph structure simultaneously. For example, (i) classification of nodes in a network not only depends on their inter-connections (i.e., graph structure), but also nodes' intrinsic properties (i.e., their textual descriptions); (ii) for product recommendations, if the product is new, it may not have many edges since not many users have interacted with it; however, using the textual descriptions along with the graph structure allows for efficient bootstrapping of the recommendation service. For general case of sequence lengths greater than 1, despite the importance in applications like workflow composability described above, there is generally a lack of efficient solutions. Intuitively, we can concatenate or add all involved node vectors; however, such a mechanism either takes too much space or loses the sequence information; thus unable to represent node sequences properly.

We aim to learn node sequence embeddings by first addressing the single node embedding problem, as a special case of node sequence embedding, by considering both the textual descriptions and the graph structure. We seek to answer two questions: How should we combine these two objectives? What framework should we use for feature learning? Works that jointly address these two questions either investigate them under different problem settings [7], [8], under restricted learning models [9], ignore the word context within the document [10], [11], do not co-learn text and graph patterns [12] or only consider linear combinations of text and graph [13]; this is elaborated further in Section II. In contrast, we propose a generic neural-network-based model called SENSE-S (Semantically Enhanced Node Sequence Embeddings - for Single nodes) for computing vector representations of nodes

with additional semantic information in a graph. SENSE-S is built on the foundation of skip-gram models. However, SENSE-S is significantly different from classic skip-gram models in the following aspects: (i) For each word ϕ in the textual description of node v in the given graph, neighboring words of ϕ within v's textual description and neighboring nodes of v within the graph are sampled at the same time. (ii) The text and graph inputs are both reflected in the output layer in the form of probabilities of co-occurrence (in graph or text). (iii) Moreover, this joint optimization problem offers an opportunity to leverage the synergy between the graph and text inputs to ensure faster convergence. We evaluate the generated vectors on (i) [14] to show that our SENSE-S model improves multi-label classification accuracy by up to 50\% and (ii) Physics Citation dataset [15] to show that SENSE-S improves link prediction accuracy by up to 78% over the state-of-the-art.

Further, we propose SENSE for general feature representation of a *sequence* of nodes. This problem is more challenging in that (i) besides the original objectives in SENSE-S, we now face another representation goal, i.e., sequence representation while preserving the node order; (ii) it is important to represent the sequence in a compact manner; and (iii) more importantly, given a sequence vector, we need to be able to decipher which functional nodes are involved and in what order. To this end, we develop efficient schemes to combine individual vectors into complex sequence vectors that address all of the above challenges. The key technique we use here is vector cyclic shifting, and we prove that the different shifted vectors are orthogonal with high probability. This sequence embedding method is also evaluated on the Wikispeedia and Physics Citation datasets, and the accuracy of decoding a node sequence is shown to be close to 100% when the vector dimension is large.

The rest of the paper is organized as follows: in Section II we overview the most related papers, Section III details SENSE-S for computing single node embeddings that simultaneously capture text and graph structures. Section IV details SENSE for representing a sequence of nodes. Section V evaluates SENSE and finally Section VI concludes the paper.

II. RELATED WORK

We categorize the most related works as follows:

Learning vector representation from text: Vector representation of words [16] has been a long standing research topic. It has received significant attention in the recent times due to the advances in deep neural networks [17], [18], [6]. In particular, these neural-network-based schemes outperform n-gram-based techniques [19], [20] significantly as they are able to learn the similarities between words. Furthermore, paragraph2vec [5] extends the well-established word2vec [6] to learn representations of chunks of text.

Learning vector representation from graphs: Lot of research has gone into learning graph representations by translating the network/graph into a set of words or documents [2],

[21], [22], [23], [24], [4], [3]. Generic models incorporating both edge weight and direction information for graph embeddings are proposed in [25], [4] and [3]. Specifically, node2vec [3] advances the state-of-the-art in this area by designing flexible node sampling methodologies to allow feature vectors to exhibit different properties. Subgraph2vec [26] extends these schemes to learn vector representations of subgraphs. [27] proposes techniques to represent graph sequences under the assumption that each node is represented by a random binary vector.

Learning graph representation with auxiliary information: Broadly speaking, our work falls into the category of node embedding in graphs with auxiliary information. [28], [29], [30] and others address the case where nodes are associated with labels. [31] studies graph embedding when node/edge attributes are continuous. [7] investigates phrase ambiguity resolution via leveraging hyperlinks. However, all these works operate under information or network constraints. On the other hand, [8], [32] and [33] explore embedding strategies in the context of knowledge graphs, where the main goal is to maintain the entity relationships specified by semantic edges. In contrast, we consider a simpler network setting where only nodes are associated with semantic information. EP [13] and GraphSAGE [34] learn embeddings for structured graph data. However, the textual similarities are only captured by linear combinations. Planetoid [35] computes node embeddings under semi-supervised settings; metapath2vec [36] learns embeddings for heterogeneous networks (node can be author or paper); and Graph-Neural-Network-based embeddings are explored in [37] and [12]. However, these papers do not explicitly learn graph structure and text patterns simultaneously, and thus are complementary to SENSE. SNE [10], SNEA [11], TADW [38], HSCA [39], AANE [40], ANRL [41] and PLANE [9] are more related to our work. However, unlike SENSE, these do not consider the relative context of the words w.r.t. the document. Furthermore, the objective of PLANE is to maximize the likelihood that neighboring nodes have similar embeddings, which is not always the case in practice because neighboring nodes may be semantically different; more critically, it relies on strong assumptions of statistical distributions of words and edges in the network. In this regard, we propose a generic embedding scheme that jointly considers network topology as well as the nodes' semantic information.

III. SENSE-S: SENSE FOR SINGLE NODE EMBEDDINGS

To embed a general node sequence, we first consider a special case where each node sequence contains only one node. Such single node embedding is referred to as SENSE-S, which jointly learns node representations along with textual descriptions in graphs.

A. SENSE-S Objective

Let $\mathcal{G}=(V,E)$ denote a given directed or undirected graph, where V is the set of nodes and E the set of edges. Each node in V is associated with a text description. We

aim to embed each node v in V into a feature vector that captures both graphical (neighboring node inter-connections) and textual (semantic meaning of v) properties. Specifically, let ϕ denote a word in the text description of node v. Suppose we obtain a set of neighboring nodes of v in graph $\mathcal G$ via a specific node sampling strategy, e.g., biased random walk [3], and a set of neighboring words of ϕ in the text description of v by a sliding window over consecutive words [6]. We then define $N_{\mathcal G}(v)$ as a probabilistic event of observing the set of neighboring nodes of v in $\mathcal G$ (under the chosen model) and $N_T(\phi|v)$ as an event of observing the set of neighboring words of ϕ in the text description of v. Let $f: v \to \mathbb R^d$ ($v \in V$) be the embedding function that maps each node v in V into a d-dimensional vector. Our goal is to find function f that maximizes:

$$\max_{f} \sum_{v \in V} \sum_{\phi \text{ in } v} \log \Pr[N_{\mathcal{G}}(v) N_{T}(\phi|v) | \phi, f(v)]. \tag{1}$$

Since events $N_{\mathcal{G}}(v)$ and $N_{T}(\phi|v)$ are independent, (1) can be rewritten as:

$$\max_{f} \sum_{v \in V} \sum_{\phi \text{ in } v} \left(\log \Pr[N_{\mathcal{G}}(v)|\phi, f(v)] + \log \Pr[N_{T}(\phi|v)|\phi, f(v)] \right)$$

$$= \sum_{v \in V} w_{v} \log \Pr[N_{\mathcal{G}}(v)|f(v)] + \sum_{v \in V} \sum_{\phi \text{ in } v} \log \Pr[N_{T}(\phi|v)|\phi, f(v)],$$
(2)

where w_v is the number of words¹ in the description of v. Given a word in a node description, (2) jointly captures the node neighborhood in the graph and the word neighborhood in text. Below, we explain how the problem can be interpreted in a simplified way. Let

$$F_{\mathcal{G}} := \sum_{v \in V} \log \Pr[N_{\mathcal{G}}(v)|f(v)] \text{ and}$$

$$F_{T} := \sum_{v \in V} \sum_{\phi \text{ in } v} \log \Pr[N_{T}(\phi|v)|\phi, f(v)]. \tag{3}$$

 $F_{\mathcal{G}}$ is exactly the same as the optimization objective in node2vec [3] for graph embedding, and F_T is also similar to the optimization objective in paragraph2vec [5] for text embedding except that we use one word (rather than multiple words as in [5]) to predict its neighboring words within the same document, i.e., the description of node v. Therefore, (2) takes both graphical and textual features into account for node embeddings. Under this optimization objective, we next discuss our SENSE-S model and how $\Pr[N_{\mathcal{G}}(v)|f(v)]$ and $\Pr[N_{T}(\phi|v)|\phi,f(v)]$ are computed in SENSE-S.

B. SENSE-S Architecture

Due to the similarity between $F_G + F_T$ and (2), we build SENSE-S on the foundation of the skip-gram model [6] originally proposed to learn vector representation of words. In particular, Node2Vec [3] and DeepWalk [2] leverage this skip-gram model to learn vector representation of nodes in a graph by performing biased random walks on the graph and treating each walk as equivalent to a sentence, aiming to predict a neighboring *node* given the current node in a graph. On the other hand, Paragraph Vector [5] extends skip-gram models to learn embeddings of various chunks of text, e.g., sentences, paragraphs or entire documents, via sampling the neighboring words over a sliding window within the text. Motivated by the effectiveness of these models, we build two SENSE-S models, SENSE-S (add) and SENSE-S (concat), as detailed below.

1) SENSE-S (add): Let w denote the number of words (some uninformative words are skipped) in text descriptions across all nodes in the given graph \mathcal{G} , i.e., w is the size of the entire vocabulary, and n the number of nodes in \mathcal{G} . Then our model is formulated as a neural network, as shown in Figure 1. In this model, each input is a two-tuple (ϕ, v) , i.e., word ϕ is picked from the description of node v. Then (ϕ, v) is mapped to two one-hot vectors, w-dimensional word vector ϕ and n-dimensional vertex vector 2 v, where only the entries corresponding to ϕ and v are set to 1 and others are set to 0. Then as in typical fully connected neural network architectures, in the first layer, ϕ^T and \mathbf{v}^T are multiplied (implemented as look-up for efficiency) by two matrices $\mathbf{M}_{w \times d}$ and $\mathbf{M}_{n \times d}$, respectively. The resulting vectors are then added together and multiplied by another matrix $\mathbf{M}_{d\times(w+n)}$ in the second layer, i.e., a (w+n)-dimensional vector

$$\mathbf{h}^T := (\boldsymbol{\phi}^T \mathbf{M}_{w \times d} + \mathbf{v}^T \mathbf{M}_{n \times d}) \mathbf{M}_{d \times (w+n)}$$
(4)

is obtained. Finally, unlike typical skip-gram models where all entries in the output vectors of the second layer are processed by a softmax function, we decompose vector h into two subvectors, \mathbf{h}'_1 consisting of the first w entries and \mathbf{h}'_2 consisting of the rest. Then \mathbf{h}_1' and \mathbf{h}_2' are fed to separate softmax functions, yielding h_1 and h_2 , respectively (see Figure 1). The reason for this decomposition operation is that we use \mathbf{h}_1 to represent the probability vector of neighboring words of ϕ in the description of node v, and h_2 to represent the probability vector of neighboring nodes of v in the graph. Using this neural network architecture, we aim to learn the values of all entries in the matrices $\mathbf{M}_{w\times d}$, $\mathbf{M}_{n\times d}$ and $\mathbf{M}_{d\times (w+n)}$ such that the entries (i.e., probabilities) in h_1 and h_2 corresponding to neighboring words of ϕ (within the text description of node v) and neighboring nodes of v (within the given graph) are maximized; see the objective in (2). Note that as discussed before, the set of neighboring words and the set of neighboring nodes are obtained via a fixed-size sliding window as proposed in word2vec [6] and a fixed-length random walk as proposed in node2vec [3], respectively. The reason to use the random

¹Some uninformative words can be skipped as in skip-gram models [2], [3] for achieving better performance.

²One-hot *vertex vectors* are the input to SENSE, while *node vectors* are our semantically augmented node embeddings.

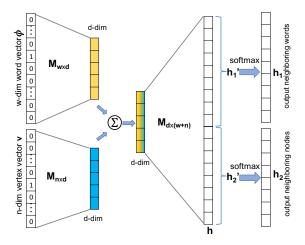


Fig. 1: SENSE-S (add) architecture (dim: dimension)

walk is that it is sufficiently flexible to incorporate different graph properties like heterophily or structural equivalence. Under this architecture, we use matrix $\mathbf{M}_{n\times d}$ as our final semantically augmented node embeddings, i.e., each row in $\mathbf{M}_{n\times d}$ corresponds to a node embedding vector.

Note that a unique property of SENSE-S is that it is a conjoined model where the textual and graphical inputs both contribute to the learning of node embeddings. Moreover, there is an *add* operation for combining these features, and thus this model is called SENSE-S (add). This is in contrast to the *concatenation* operation that is used in a different implementation of SENSE-S; see below.

2) SENSE-S (concat): SENSE-S (concat) model is illustrated in Figure 2. Clearly, SENSE-S (concat) is quite similar to SENSE-S (add), except that (i) the resulting vectors generated by the first layer are concatenated, and (ii) the matrix dimension in the second layer becomes $(2d) \times (w+n)$. In this model, the output vector after the second layer is $\mathbf{h}^T = (\phi^T \mathbf{M}_{w \times d}, \mathbf{v}^T \mathbf{M}_{n \times d}) \mathbf{M}_{(2d) \times (w+n)}$, where $(\mathbf{x}^T, \mathbf{y}^T)$ denotes the concatenation of vectors \mathbf{x} and \mathbf{y} . Then, again, matrix $\mathbf{M}_{n \times d}$ is employed as the final output for node embeddings under semantically augmented information. In Section V, we will compare SENSE (add) and SENSE (concat) against other baseline solutions to show the accuracy of our model.

IV. SENSE

In SENSE-S, our focus has been on computing semantically enhanced embeddings for individual nodes. In this section, we propose the general SENSE algorithm to represent any set of nodes following a specified order using the node vectors generated by SENSE-S, called *node sequence embedding*.

Given the original graph $\mathcal{G}=(V,E)$, let $S=v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_q$ be a node sequence constructed with $v_i \in V(i=1,2,\ldots,q)$. Note that S may contain *repeated* nodes, e.g., some functions need to be executed more than

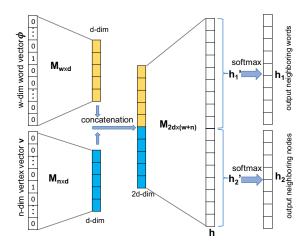


Fig. 2: SENSE-S (concat) architecture (dim: dimension)

once in one application. Intuitively, node sequence S can be represented by a $d \times q$ matrix with column i being the vector representation of node v_i . However, such a representation is costly in space. Alternatively, representing S by the vector sum $\sum_{i=1}^{q} \mathbf{v}_i$ (\mathbf{v}_i corresponds to v_i) results in missing the node order information. Hence, in this section, we seek to find a low-dimensional vector representation such that (i) node properties in the original network \mathcal{G} and (ii) the node order in S are both preserved. To this end, we propose an efficient node sequence embedding method utilizing node vectors generated for the original network \mathcal{G} . The basic idea behind our method is that (i) we first change the form of the node vectors in S, (ii) then we add these transformed vectors to represent node sequence S. The main advantage of such arithmetic operations is that all required features in a node sequence can be easily encoded and decoded in a vector of the same dimension as the node vectors.

A. Node Sequence Embedding Mechanism

We now discuss our node sequence embedding method based on node vectors generated by SENSE. In the rest of this section, all node vectors are *unit vectors* obtained by normalizing the node vectors generated by SENSE-S; this property is critical in our node sequence embedding method (see Section IV-B).

1) Node Sequence Vector Construction:: Given a node sequence $S = v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_q$ following in order from node v_1 to node v_q , let \mathbf{v}_i be the unit node vector of node v_i . We first perform positional encoding, via cyclic shift function. Specifically, given vector \mathbf{v} of dimension-d and non-negative integer m, we define $\mathbf{v}^{(m)}$ as a vector obtained via cyclically shifting elements in \mathbf{v} by $m' = (m \mod d)$ positions. Mathematically, let \mathbf{S} be a $d \times d$ binary matrix, where entries $S_{i,i+1}$ $(i=1,2,\ldots,d-1)$ and $S_{d,1}$ are 1 and

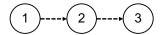


Fig. 3: Sample node sequence S consisting of a simple linear chain of nodes 1, 2 and 3.

the rest are 0. Then

$$\mathbf{v}^{(m)} := \mathbf{S}^{m} \mathbf{v}$$

$$= \begin{cases} \mathbf{v}, & \text{if } (m \mod d) = 0, \\ [v_{d-m'+1}, \dots, v_{d}, v_{1}, \dots, v_{d-m'}]^{T}, & \text{otherwise,} \end{cases}$$
(5)

where v_j is the j-th element in \mathbf{v} . Therefore, $\mathbf{v}_i^{(i-1)}$ represents a vector resulted from cyclically shifting $(i-1 \mod d)$ positions of the node vector at position i in S. Let \mathbf{S} denote the vector representation of node sequence S. Suppose $q \ll d$, Then we embed S as

$$\mathbf{S} = \sum_{i=1}^{q} \mathbf{v}_i^{(i-1)}.\tag{6}$$

Note that for repeated nodes in S, they are cyclically shifted by different positions depending on the specific order in the node sequence. In S, by imposing the restriction $q \ll d$, we ensure that wraparounds do not occur while shifting the vector, because it may lead to ambiguity of node positions within a node sequence. Simple as this embedding approach may seem, we show that it exhibits the following advantages. First, the dimension of node sequence vectors remains the same as that of the original node vectors. Second, given a node sequence vector S, we are able to infer which nodes are involved in S and their exact positions in S as explained below.

2) Node Sequence Vector Decoding:: The method for determining which nodes are included (and in which order) in a given node sequence vector is referred to as node sequence vector decoding. The basic idea in node sequence vector decoding is rooted in Corollary 5 (Section IV-B), which implies that using cyclic shifting, we essentially enable a preferable property that $\mathbf{v}_i^{(i-1)}$ and $\mathbf{v}_j^{(j-1)}$ with $i \neq j$ are almost orthogonal, i.e., $\mathbf{v}_i^{(i-1)} \cdot \mathbf{v}_j^{(j-1)} \approx 0$, even if \mathbf{v}_i and \mathbf{v}_j are related in the original graph (\mathbf{v}_i and \mathbf{v}_j may even correspond to the same node). By this property, we make the following claim, assuming that all node vectors are unit vectors.

Claim 1. Given a node sequence vector \mathbf{S} , node v, whose node vector is \mathbf{v} , is at the k-th position of this node sequence if the inner product $\mathbf{S} \cdot \mathbf{v}^{(k-1)} \approx 1$.

Claim 1 provides an efficient way to decode a given node sequence. In particular, to determine whether (and where) a node resides in a node sequence, it only takes quadratic complexity $O(d^2)$, where d is typically small.

3) Example: Suppose we have a node sequence S as shown in Figure 3. By SENSE-S and our encoding method, we construct its node sequence vector as

$$\mathbf{S} = \mathbf{v}_1^{(0)} + \mathbf{v}_2^{(1)} + \mathbf{v}_3^{(2)}.\tag{7}$$

where \mathbf{v}_i denotes the node vector of node *i*. Then each node can compute the inner product of its cyclic shifted vector with \mathbf{S} . If the result is approximately 1, then its position in this node sequence is uniquely determined. For instance,

$$\mathbf{S} \cdot \mathbf{v}_{2}^{(1)} = \mathbf{v}_{1}^{(0)} \cdot \mathbf{v}_{2}^{(1)} + \mathbf{v}_{2}^{(1)} \cdot \mathbf{v}_{2}^{(1)} + \mathbf{v}_{3}^{(2)} \cdot \mathbf{v}_{2}^{(1)} \approx 0 + 1 + 0 = 1;$$
 (8)

see Corollary 5. Thus, given the encoded node sequence S, we know node 2 is at the second position.

B. Theoretical Foundation

We now present our theoretical results to support Claim 1 for node sequence vector decoding. We first prove that two independent random vectors of dimension N are orthogonal with a high probability. Next, we prove that the inner product of a vector and another random unit vector cyclically shifted by m position is also close to 0 with a high probability, i.e., such vectors are orthogonal as well.

Theorem 2. Let \mathbf{x} and \mathbf{y} be N-dimensional unit vectors with i.i.d. (independent and identically distributed) random vector elements, then $\mathbb{E}[\mathbf{x} \cdot \mathbf{y}] = 0$ and $\operatorname{Var}[\mathbf{x} \cdot \mathbf{y}] = 1/N$.

Proof. Since both \mathbf{x} and \mathbf{y} are unit vectors, we have $\mathbf{x} \cdot \mathbf{y} = ||\mathbf{x}||_2||\mathbf{y}||_2\cos\theta = \cos\theta$, where θ is the angle between \mathbf{x} and \mathbf{y} . Since both \mathbf{x} and \mathbf{y} are independent and uniformly distributed across the sphere surface, θ is also uniformly distributed, and thus $\mathbb{E}[\mathbf{x} \cdot \mathbf{y}] = 0$.

As $\mathbf{x} \cdot \mathbf{y}$ is purely determined by the angle θ between \mathbf{x} and \mathbf{y} , without loss of generality, we select $\mathbf{y} = [1,0,\dots,0]^T$ and only consider $\mathbf{x} = [x_1,x_2,\dots,x_N]$ to be a random unit vector. Then, $\mathbf{x} \cdot \mathbf{y} = x_1$. Therefore, $\mathrm{Var}[\mathbf{x} \cdot \mathbf{y}] = \mathbb{E}[x_1^2] - \mathbb{E}^2[x_1] = \mathbb{E}[x_1^2]$. Since all entries in \mathbf{x} are identically distributed, we have $\mathbb{E}[\mathbf{x} \cdot \mathbf{x}] = \mathbb{E}[\sum_{i=1}^N x_i^2] = \sum_{i=1}^N \mathbb{E}[x_i^2] = N \cdot \mathbb{E}[x_1^2] = 1$. Therefore, $\mathrm{Var}[\mathbf{x} \cdot \mathbf{y}] = \mathbb{E}[x_1^2] = 1/N$.

Based on Theorem 2, we have the following corollary.

Corollary 3. Let \mathbf{x} and \mathbf{y} be N-dimensional unit vectors with i.i.d. random vector elements, then event $\mathbf{x} \cdot \mathbf{y} = 0$ happens almost surely, i.e., $\Pr[\mathbf{x} \cdot \mathbf{y} = 0] \approx 1$, when N is large.

Proof. According to Theorem 2,
$$\mathbb{E}[\mathbf{x} \cdot \mathbf{y}] = 0$$
 and $\lim_{N \to \infty} \operatorname{Var}[\mathbf{x} \cdot \mathbf{y}] = 0$, thus completing the proof.

Corollary 3 suggests that if two nodes are embedded into two independent vectors, then it is highly likely that these two nodes are not related in any form, i.e., their inner product is close to 0. However, in a given graph, two arbitrary nodes may not be completely unrelated due to their graphical or semantic similarities, and our goal in this paper is to capture such similarities in the embedded vectors. Fortunately, even if two nodes are (loosely or strongly) related, i.e., the inner product of their embedded vectors is non-zero, we can proactively transform these vectors so that the transformed vectors are unrelated with high probability. In particular, leveraging the concept of cyclic shifting of vectors, we have the following theorem.

Theorem 4. Let \mathbf{x} and \mathbf{y} be N-dimensional unit vectors with i.i.d. vector elements. If $\mathbf{x} \cdot \mathbf{y} = c$, then $\mathbb{E}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = 0$ and $\operatorname{Var}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = (1 - c^2)/(N - 1)$, where c ($|c| \leq 1$) is a constant and $m \neq kN$ ($m, k \in \mathbb{Z}^+$).

Proof. Based on the proof of Theorem 2, we know that $\mathbf{x} \cdot \mathbf{y}$ is only determined by the angel θ between them. Therefore, again, without loss of generality, let $\mathbf{y} = [1,0,\dots,0]^T$ and $\mathbf{x} = [x_1,x_2,\dots,x_N]$. Then $\mathbf{x} \cdot \mathbf{y} = x_1 = c$ and $\mathbf{x} \cdot \mathbf{y}^{(m)} = x_\gamma$, where $\gamma = 1 + (m \mod N)$. Then $\mathbb{E}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = \mathbb{E}[x_\gamma | \mathbf{x} \cdot \mathbf{y} = c]$. As $m \neq kN$ $(k \in \mathbb{Z}^+)$, we have $\mathbf{y} \neq \mathbf{y}^{(m)}$. Moreover, x_γ is uniformly distributed across the sphere surface under the condition that $x_1 = c$. Therefore, $\mathbb{E}[x_\gamma | \mathbf{x} \cdot \mathbf{y} = c] = 0$, i.e., $\mathbb{E}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = 0$. Next, $\mathrm{Var}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = 0$, i.e., $\mathbb{E}[\mathbf{x}^2 | x_1 = c] - \mathbb{E}^2[x_\gamma | x_1 = c] = \mathbb{E}[x_\gamma^2 | x_1 = c]$. Since x_2, x_3, \dots, x_N in \mathbf{x} are identically distributed, we have $\mathbb{E}[\mathbf{x} \cdot \mathbf{x}] = \mathbb{E}[\sum_{i=1}^N x_i^2 | x_1 = c] = \sum_{i=1}^N \mathbb{E}[x_i^2 | x_1 = c] = c^2 + (N-1)\mathbb{E}[x_\gamma^2 | x_1 = c] = 1$. Therefore, $\mathrm{Var}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = (1-c^2)/(N-1)$.

Based on Theorem 4, the following corollary can be proved.

Corollary 5. Let \mathbf{x} and \mathbf{y} be N-dimensional unit vectors with i.i.d. vector elements, then given $\mathbf{x} \cdot \mathbf{y} = c$ ($|c| \leq 1$), event $\mathbf{x} \cdot \mathbf{y}^{(m)} = 0$ ($m \neq kN$ and $m, k \in \mathbb{Z}^+$) happens almost surely, i.e., $\Pr[\mathbf{x} \cdot \mathbf{y}^{(m)} = 0 | \mathbf{x} \cdot \mathbf{y} = c] \approx 1$, when N is large.

Proof. According to Theorem 4, $\mathbb{E}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = 0$ and $\lim_{N \to \infty} \operatorname{Var}[\mathbf{x} \cdot \mathbf{y}^{(m)} | \mathbf{x} \cdot \mathbf{y} = c] = 0$, irrespective of the value of c, thus completing the proof.

Corollary 5 reveals an interesting fact: When two nodes a and b in a graph are related, then we know their embedded vectors $\mathbf{v}_a \cdot \mathbf{v}_b = c$ and c is non-zero. Nevertheless, if one of \mathbf{v}_a and \mathbf{v}_b , say \mathbf{v}_b , is cyclically shifted, yielding $\mathbf{v}_b^{(m)}$ with $\mathbf{v}_b \neq \mathbf{v}_b^{(m)}$, then $\mathbf{v}_a \cdot \mathbf{v}_b^{(m)} = 0$ happens with probability close to 1 when the vector dimension is sufficiently large. Furthermore, this conclusion holds, regardless of the values of c and m, as long as $\mathbf{v}_b \neq \mathbf{v}_b^{(m)}$. In other words, no matter how \mathbf{v}_a and \mathbf{v}_b are related, i.e., loosely or strongly, it is almost surely the case that the inner product of \mathbf{v}_a and $\mathbf{v}_b^{(m)}$ is 0. Motivated by this result, we develop the efficient node sequence embedding and decoding method in Section IV-A.

V. EVALUATION

This section evaluates SENSE under varying scenarios, with multiple datasets and applications.

A. Datasets

We evaluate SENSE on the following datasets that contain graph information along with textual descriptions: (1) [14]: it contains both Wikipedia plain text articles (text descriptions) and hyper links between articles (graph structure). It is a directed graph with 4,604 nodes (each is a Wikipedia article) and 119,882 hyper links. (2) Citation Network [15]: it contains both textual descriptions (title, authors and abstract of the paper) and graphical structure (citations). It is a directed graph with 27,770 nodes (papers) and 352,807 edges (citations).

B. SENSE-S Model Training

To train our SENSE-S model, we first define the loss function based on (2). Suppose the vocabulary size associated with each node $v \in V$ is similar, i.e., we assume $\forall v \in V, w_v \approx \tau$ (τ is a constant). Then with the concepts of $F_{\mathcal{G}}$ and F_T in (3), we define the loss function as

$$L := -\tau F_{\mathcal{G}} - F_{\mathcal{T}}.\tag{9}$$

We then use stochastic gradient descent to minimize our loss function. Let η be the learning rate. At each iteration, we update the model parameters by adding a fraction of $-\nabla L$, i.e., $-\eta \nabla L = \tau \eta \nabla F_{\mathcal{G}} + \eta \nabla F_{T}$. Since the per-node vocabulary size is much larger than 1, the node neighborhood sampling via random walk is much less frequent than the textual neighborhood sampling. Therefore, we want to inject more input data consisting of only the nodes' graphical neighborhood information. To do so, we adjust the model parameter update rule $-\eta \nabla L$ as

$$\beta_1 \nabla F_G + \beta_2 \nabla F_T \ (\beta_1 > \beta_2 > 0), \tag{10}$$

where β_1 and β_2 are the equivalent learning rates for graph inputs and text inputs, respectively.

In addition, the output of the SENSE-S (both add and concat) architecture is obtained by a softmax function, which generally incurs high cost in practice when computing ∇L . Therefore, to ensure computational efficiency, we use Noise Contrastive Estimation (NCE) [42] to approximate softmax as discussed in [6].

C. SENSE-S Evaluation

We first evaluate SENSE-S, which is compared against the following baseline solutions.

Leverage graph information alone: To compare with schemes that use graph information alone, we use node2vec since it has been shown to be flexible to capture different graph properties.

Leverage text information alone: To compare against schemes that use textual information alone, we use semantic vectors from paragraph2vec [5] since it outperforms other schemes such as Recursive Neural Tensor Networks ([43]) for tasks like Sentiment Analysis. As in SENSE-S, we also study two implementations of paragraph2vec, i.e., addition and concatenation operations at the hidden layer, referred to as paragraph2vec (add) and paragraph2vec (concat), respectively. Leverage text+graph information: For joint text/graph learning, we compare with the following:

- 1) **Initialize with semantic vectors:** We learn embeddings using node2vec, but rather than using random initialization, we initialize the vectors using paragraph2vec.
- 2) **Initialize with graphical vectors:** Here, we learn final embeddings using paragraph2vec, but initialize them with node2vec, i.e., just reverse of the scheme above.
- 3) **Iterative Vectorization:** The above approaches only leverage semantic or graphical vectors for initializations. Here, we try to capture both iteratively. Specifically, in one iteration, we compute node embedding via node2vec with the

Algorithm 1 Iterative Vectorization

```
1: procedure VECTORIZE(text, graph)
2: vector \leftarrow randomInit()
3: for i \leftarrow 1, k do
4: vector \leftarrow node2vec(graph, vector)
5: vector \leftarrow paragraph2vec(text, vector)
6: return vector
```

embeddings from the previous iteration as initializations; the corresponding results are then fed to paragraph2vec as initializations to further compute node embeddings, after which we go to the next iteration. We repeat this process multiple times (5 times in our experiment) to get the final embeddings. The pseudo-code is shown in Algorithm 1.

- 4) Concatenation of graphical and semantic vectors: Here, we simply concatenate the vectors obtained from paragraph2vec and node2vec and use them as our node embedding vectors.
- 1) Experimental Setup: We first learn the vector representations and then use these vector representations for two different tasks:

Multi-label classification: Wikipedia pages are classified into different categories, such as history, science, people, etc. This ground truth information is included in the Wikispeedia dataset (which is not used while learning the vectors). There are 15 different top level categories, and our multi-label classification task tries to classify a page into *one or more* of these categories based on the vectors obtained from different algorithms. We train the OneVsRestClassifier (SVM, linear kernel) from scikit-learn for this task.

Link prediction: Since no category information is available for the Citation Network, we evaluate for link prediction. In particular, 1% of existing citations are removed, after which vectors are learned on this network. We use these removed links as positive samples for link prediction. For negative samples, we randomly sample the same number of pairs which are not linked via a citation in the original network. To obtain the similarity features w.r.t. a pair of nodes, after experimenting with several alternatives, we chose the element-wise absolute difference and train SVM classifier (linear kernel) for link prediction.

Parameter settings: (i) We perform κ random walks starting from each node in the graph (κ is 10 for Wikispeedia and 3 for Citation Network, since Citation Network is larger); (ii) each walk is of length 80 as recommended by Node2Vec [3]; (iii) we use sliding window of size 5 for neighboring word sampling; (iv) the default node vector dimension is 128; (v) multi-label classification error is the misclassification rate over the test set; (vi) link prediction error is the percentage of incorrect link predictions over all pairs of papers in the test set; (vii) learning rates β_1 and β_2 in (10) are selected based on the validation set, and the error is reported on the test set.

2) Multi-label classification error: The error of multi-label classification is reported in Figure 4, where the first 500

characters of each Wikipedia page are selected as its textual description. Figures 4 (a), (b) and (c) correspond to different splits of the dataset (based on number of nodes) into training, validation and test samples. The following observations can be made. First, as expected, the results of all the schemes improve with higher ratio of training to test samples. Second, in general, schemes that leverage both textual and graphical information incur lower errors. Third, among the schemes that utilize both textual and graphical information, SENSE-S (add) and SENSE-S (concat) consistently perform the best. This is because we train the network to co-learn the textual and graphical information to ensure that both objectives converge. This is in contrast with other schemes where the two objectives, due to the loose coupling, are not guaranteed to converge. Finally, SENSE-S (add) (in Figure 4 (c)) outperforms node2vec by over 40\%, paragraph2vec (add) by over 55% and the closest baseline scheme that leverages both text and graph by 30%. This confirms the benefit of co-learning features from textual as well as graphical information under the SENSE-S architecture. Moreover, we also see similar trends using the first 1,000 characters; results are in Figure 5. Intuitively, this is because the most critical textual descriptions that differentiate them from others are mostly included in the first 500 characters; therefore, 1,000 characters do not further provide useful information for learning.

3) Link prediction error: The error of link prediction in the Citation Network is reported in Figure 6. We fix train:valid:test to 60%:20%:20% (based on number of links) and use the first 500 characters as text description. We make several interesting observations. First, schemes that use graph information alone (node2vec) substantially outperform schemes that use text descriptions alone (paragraph2vec) for link prediction task. Intuitively, this is because the neighborhood information, which is important for link prediction task, is captured effectively by the node embeddings obtained from node2vec-like techniques. Second, even in cases where the difference in accuracy using the two different sources of information is large, SENSE-S (add) and (cat) are robust, and can effectively extract useful information from text descriptions to further reduce the error of link prediction that uses graph structure alone. Finally, this is significant because it demonstrates the effectiveness of SENSE-S in a variety of scenarios, including cases where the two sources of information may not be equally valuable.

D. SENSE Evaluation

We now evaluate the accuracy of encoding/decoding by SENSE for node sequences. We evaluate on three experiments via constructing node sequences in the following different ways:

Experiment 1: the node at every position in a sequence is chosen uniformly at random from 4,604 Wikispeedia nodes. Note that as mentioned earlier, this node sequence may contain repeated nodes (which is allowed), and may or may not be a subgraph of the original graph.

Experiment 2: the node sequences are constructed by performing random walks on the Wikispeedia graph.

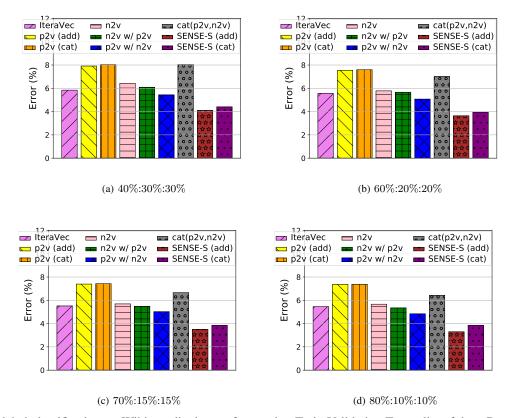


Fig. 4: Multi-label classification on Wikispeedia dataset for varying Train:Validation:Test splits of data. Document length = first 500 characters (n2v: node2vec, p2v (add)/(cat): paragraph2vec (add)/(concat), n2v w/ p2v: node2vec with paragraph2vec (add) initialization, p2v w/ n2v: paragraph2vec (add) with node2vec initialization, IteraVec: Iterative Vectorization, cat(p2v,n2v): concatenation of p2v and n2v vectors).

Experiment 3: the node sequences are constructed by performing random walks on the Physics Citation Network. Note that in both Experiments 2 and 3, adjacent nodes in the node sequences will have *related vector embeddings*. Next, for such constructed node sequences, their vector representations are computed by SENSE-S. Given these sequence vectors, we then decode the node at each position. We evaluate the decoding accuracy under different sequence lengths and vector dimensions, as reported in Figure 7.

From Figure 7 (a), we make the following observations. First, when the node sequence length is small, all vector dimensions lead to almost 100% decoding accuracy. Second, as node sequence length increases, the decoding accuracy declines sharply especially in cases where the node vector dimensions are relatively small, i.e., 128 and 256. This is because, by Theorem 4, correlations between the involved node vectors cause inevitable errors. Such error accumulates when the length of the node sequence is large. Nevertheless, with sufficiently large node vector dimension, i.e., 1024, even long sequences can be decoded perfectly, and with 512, we can decode a workflow of length 10 with over 90% accuracy. Interestingly, Figures 7 (b) and (c) also show similar trends. This is significant, as it shows that *even if node sequences are*

constructed from correlated node vectors, i.e., picked from the same graph neighborhood, the decoding still achieves high accuracy. This is because, as shown in Theorem 4, after cyclic shifting, the resulting vectors are orthogonal with high probability when the vector dimension is large (even if these vectors are originally non-orthogonal). Finally, in Figures 7 (a) and (b), the decoding algorithm needs to find the best match from among 4,592 nodes (Wikispeedia network). In contrast, in Figure 7 (c), it is much more challenging to find the match among 27,770 nodes. Yet, we are able to decode with high accuracy with theoretical guarantees.

VI. CONCLUSION

We presented SENSE that learns semantically enriched vector representations of graph node sequences. To achieve this, we first developed SENSE-S that learns single node embeddings via a multi-task learning formulation that jointly learns the co-occurrence probabilities of nodes within a graph and words within a node-associated document. We evaluated SENSE-S against state-of-the-art approaches that leverage both graph and text inputs and showed that SENSE-S improves multi-label classification accuracy in Wikispeedia dataset by up to 50% and link prediction over Physics Citation

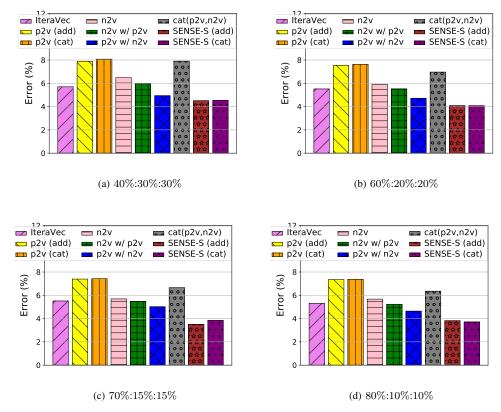


Fig. 5: Multi-label classification on Wikispeedia dataset for varying Train: Validation: Test splits of data. Document length = first 1000 characters.

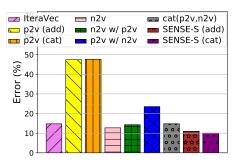


Fig. 6: Link prediction in Citation Network.

network by up to 78%. We then developed SENSE that is able to employ theoretically provable schemes for vector composition to represent node sequences using the same dimension as the individual node vectors from SENSE-S. We demonstrated that the individual nodes within the sequence can be inferred with a high accuracy (close to 100%) from such composite SENSE vectors.

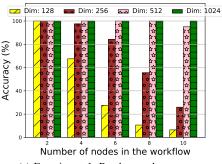
ACKNOWLEDGMENT

This research was sponsored by the U.S. Army Research Laboratory and the U.K. Ministry of Defence under Agreement Number W911NF-16-3-0001. The views and conclusions contained in this document are those of the authors and should

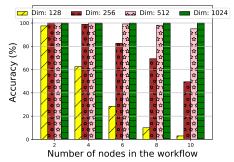
not be interpreted as representing the official policies, either expressed or implied, of the U.S. Army Research Laboratory, the U.S. Government, the U.K. Ministry of Defence or the U.K. Government. The U.S. and U.K. Governments are authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon. Author Swati Rallapalli is currently employed with Facebook, Inc.

REFERENCES

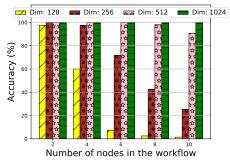
- L. Ma, T. He, K. Leung, D. Towsley, and A. Swami, "Efficient identification of additive link metrics via network tomography," in *IEEE ICDCS*, 2013.
- [2] B. Perozzi, R. Al-Rfou, and S. Skiena, "Deepwalk: Online learning of social representations," in ACM KDD, 2014.
- [3] A. Grover and J. Leskovec, "Node2vec: Scalable feature learning for networks," in ACM KDD, 2016.
- [4] J. Tang, M. Qu, M. Wang, M. Zhang, J. Yan, and Q. Mei, "Line: Large-scale information network embedding," in WWW, 2015.
- [5] Q. Le and T. Mikolov, "Distributed representations of sentences and documents," in *ICML*, 2014.
- [6] T. Mikolov, K. Chen, G. Corrado, and J. Dean, "Efficient estimation of word representations in vector space," CoRR, vol. abs/1301.3781, 2013.
- [7] Y. Cao, L. Huang, H. Ji, X. Chen, and J. Li, "Bridge text and knowledge by learning multi-prototype entity mention embedding," in ACL, 2017.
- [8] H. Xiao, M. Huang, L. Meng, and X. Zhu, "SSP: Semantic space projection for knowledge graph embedding with text descriptions," in AAAI, 2017.
- [9] T. M. V. Le and H. W. Lauw, "Probabilistic latent document network embedding," in *ICDM*, 2014.



(a) Experiment 1: Random node sequences



(b) Experiment 2: Random walk, Wikispeedia



(c) Experiment 3: Random walk, Citation Network

Fig. 7: Accuracy of decoding a node sequence vector (Dim: dimension of the vectors).

- [10] L. Liao, X. He, H. Zhang, and T. Chua, "Attributed social network embedding," CoRR, vol. abs/1705.04969, 2017.
- [11] S. Wang, C. Aggarwal, J. Tang, and H. Liu, "Attributed signed network embedding," in ACM CIKM, 2017.
- [12] Y. Li, D. Tarlow, M. Brockschmidt, and R. S. Zemel, "Gated graph sequence neural networks," in *ICLR*, 2016.
- [13] A. Garcia Duran and M. Niepert, "Learning graph representations with embedding propagation," in NIPS, 2017.
- [14] Wikispeedia, "Wikispeedia," https://snap.stanford.edu/data/wikispeedia. html, 2009.
- [15] J. Leskovec and A. Krevl, "SNAP Datasets: Stanford large network dataset collection," https://snap.stanford.edu/data/cit-HepTh.html, 2014.
- [16] H. Schütze, "Word space," in NIPS, 1993.
- [17] Y. Bengio, R. Ducharme, P. Vincent, and C. Janvin, "A neural probabilistic language model," *J. Mach. Learn. Res.*, vol. 3, pp. 1137–1155, 2003.
- [18] T. Mikolov, J. Kopecky, L. Burget, O. Glembek, and J. Cernocky, "Neural network based language models for highly inflective languages," in *IEEE ICASSP*, 2009.
- [19] S. Katz, "Estimation of probabilities from sparse data for the language model component of a speech recognizer," *IEEE Transactions on Acous*tics, Speech, and Signal Processing, vol. 35, no. 3, pp. 400–401, 1987.
- [20] F. Jelinek and R. L. Mercer, "Interpolated estimation of Markov source parameters from sparse data," in *Proceedings, Workshop on Pattern Recognition in Practice*. North Holland, 1980, pp. 381–397, 401.
- [21] T. Pimentel, A. Veloso, and N. Ziviani, "Unsupervised and scalable algorithm for learning node representations," in *ICLR*, 2017.
- [22] L. F. Ribeiro, P. H. Saverese, and D. R. Figueiredo, "Struc2vec: Learning node representations from structural identity," in ACM KDD, 2017.
- [23] D. Wang, P. Cui, and W. Zhu, "Structural deep network embedding," in ACM KDD, 2016.
- [24] L. Liu, W. K. Cheung, X. Li, and L. Liao, "Aligning users across social networks using network embedding," in *IJCAI*, 2016.
- [25] C. Zhou, Y. Liu, X. Liu, Z. Liu, and J. Gao, "Scalable graph embedding for asymmetric proximity," in AAAI, 2017.

- [26] A. Narayanan, M. Chandramohan, L. Chen, Y. Liu, and S. Saminathan, "Subgraph2vec: Learning distributed representations of rooted subgraphs from large graphs," *CoRR*, vol. abs/1606.08928, 2016.
- [27] C. Simpkin, I. Taylor, G. Bent, G. De Mel, and R. Ganti, "A scalable vector symbolic architecture approach for decentralized workflows," in *COLLA*, 2018.
- [28] S. Guo, Q. Wang, B. Wang, L. Wang, and L. Guo, "SSE: Semantically smooth embedding for knowledge graphs," *IEEE Transactions on Knowledge and Data Engineering*, vol. 29, no. 4, pp. 884–897, 2017.
- [29] R. Xie, Z. Liu, and M. Sun, "Representation learning of knowledge graphs with hierarchical types," in *IJCAI*, 2016.
- [30] X. Huang, J. Li, and X. Hu, "Label informed attributed network embedding," in ACM WSDM, 2017.
- [31] M. Niepert, M. Ahmed, and K. Kutzkov, "Learning convolutional neural networks for graphs," in *ICML*, 2016.
- [32] L. Yao, Y. Zhang, B. Wei, Z. Jin, R. Zhang, Y. Zhang, and Q. Chen, "Incorporating knowledge graph embeddings into topic modeling," in AAAI, 2017.
- [33] Z. Wang and J. Li, "Text-enhanced representation learning for knowledge graph," in *IJCAI*, 2016.
- [34] W. Hamilton, Z. Ying, and J. Leskovec, "Inductive representation learning on large graphs," in NIPS, 2017.
- [35] Z. Yang, W. W. Cohen, and R. Salakhutdinov, "Revisiting semisupervised learning with graph embeddings," in ICML, 2016.
- [36] Y. Dong, N. V. Chawla, and A. Swami, "Metapath2vec: Scalable representation learning for heterogeneous networks," in ACM KDD, 2017.
- [37] T. N. Kipf and M. Welling, "Semi-supervised classification with graph convolutional networks," in *ICLR*, 2017.
- [38] C. Yang, Z. Liu, D. Zhao, M. Sun, and E. Y. Chang, "Network representation learning with rich text information," in *Proceedings* of the 24th International Conference on Artificial Intelligence, ser. IJCAI'15. AAAI Press, 2015, pp. 2111–2117. [Online]. Available: http://dl.acm.org/citation.cfm?id=2832415.2832542

- [39] D. Zhang, J. Yin, X. Zhu, and C. Zhang, "Homophily, structure, and content augmented network representation learning," 2016 IEEE 16th International Conference on Data Mining (ICDM), pp. 609–618, 2016.
- [40] X. Huang, J. Li, and X. Hu, "Accelerated attributed network embedding," in SDM, 2017.
- [41] Z. Zhang, H. Yang, J. Bu, S. Zhou, P. Yu, J. Zhang, M. Ester, and C. Wang, "Anrl: Attributed network representation learning via deep neural networks," in *IJCAI*, 2018.
- [42] M. U. Gutmann and A. Hyvärinen, "Noise-contrastive estimation of unnormalized statistical models, with applications to natural image statistics," J. Mach. Learn. Res., vol. 13, no. 1, pp. 307–361, 2012.
- [43] R. Socher, A. Perelygin, J. Wu, J. Chuang, C. D. Manning, A. Ng, and C. Potts, "Recursive deep models for semantic compositionality over a sentiment treebank," in *EMNLP*, 2013.