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Convex Relaxation of Power Dispatch for Voltage Stability Improvement

Andreas S. Pedersen¹, Mogens Blanke^{1,2} and Hjörtur Jóhannsson³

Abstract—A method for enhancing the voltage stability of a power system is presented in this paper. The method is based on a stability-constrained optimal power flow approach, where dispatch is done such that a maximum L -index is minimised for all load busses in a transmission grid. It is shown that optimal dispatch is obtainable with enhanced margins for voltage stability using a semidefinite relaxation of the optimal power flow problem, and that this problem can be formulated as semidefinite program with a quasi-convex objective. Numerical tests are performed on the IEEE-30 bus and BPA systems. The feasibility of the method is demonstrated through demonstrating that improved voltage stability margins are obtained for both systems.

I. INTRODUCTION

The Optimal Power Flow (OPF) is an essential tool in power system operation. It is used to obtain cost-optimal operation and to maintain the security [1] and stability [2] of a power system. The OPF is a non-convex problem, and many algorithms have been proposed to solve it. Recent efforts to solve the OPF use various convex relaxations, such as the semidefinite relaxation (SDR) [3] and second-order cone relaxation [4]. To operate a power system robustly, i.e. being able to operate with acceptable stability margins, the OPF has to be complemented by means to obtain stability margins, and incorporate these into the optimisation problem. This is referred to as *stability-constrained* or *stability-enhancing* OPF. The aim of this paper is to use the semidefinite relaxation on a stability-enhancing OPF to calculate corrective actions on a power system. Emergency operation is given particular attention. In emergencies, the power flow solution obtained as remedial action need be severely constrained to avoid further overloading. When solving a severely constrained OPF, local solvers can experience trouble finding a feasible solution. When formulated using convex relaxations, the problem has guaranteed global convergence.

Voltage instability is one of the main threats to a stable operation of modern power systems. Voltage stability refers to a power systems ability to maintain system voltages such that when the load increases, load power will increase, and such that the power and voltage are controllable [5]. Various extensions to the general OPF which incorporates voltage

stability margins – stability-constrained OPF – has been proposed in the literature, e.g. [6]. Although load power margins are the standard measure of preventive stability margins [7], other measures have been used in the literature for voltage-stability-enhancing reconfiguration. Simple voltage stability indices such as the L -index [8] can be used as a quantitative measure for estimating the distance to stability limits. The L -index is a measure of distance to insolvability of the power flow equations, and the load bus with the highest L -index indicates the most vulnerable bus in the system. Therefore, it would be desirable to make a dispatch that minimize the maximal L -index on the entire system. A reason to use the L -index as a measure include that it can serve as an online voltage stability indicator.

Solving the OPF including constraints related to the L -index has been suggested in previous literature. An OPF with a maximal L -index constraint on each load bus was solved by [9] using an interior point method. An OPF solution with mixed L -index and economic objective was obtained in [10] using particle swarm optimization. A sum of squares of L was minimized by [11] using a gradient approach and [12] minimised the maximal L -index using a genetic algorithm. The optimal reactive dispatch of renewables with regard to the L -index was solved in [13] using a trust region method.

The main difficulty in solving stability-constrained OPFs stem from the non-convexity of the power flow equations. As the OPF problem can be formulated as a quadratic program in the bus voltages, an SDR can be applied, which leads to convex optimization problems that can be efficiently solved, and [3] showed that their solution was exact for several benchmark systems. Various studies into which class of networks this is true is done in [3], [14]. Voltage stability is closely associated with generator reactive-power limits and these needs to be included when calculating load margins, usually through complementarity constraints [15], [16]. By including these constraints, however, the OPF problem can no longer be formulated as a quadratic program, and the SDR can no longer be employed. An approach to deal with this obstacle is to formulate, as in [17], the constrained OPF as a mixed-integer quadratic program. In this paper we account for reactive limits with detailed models of generators to better relate the reactive-power constraints to the bus voltages. By doing this, the voltage-stability measures can still be represented as quadratic indicators in the voltages, and it will be shown that SDP relaxation can be applied through introducing this technique.

The contributions of this paper include first to show how the standard semidefinite relaxation can be applied to a voltage-stability-enhancing OPF, and then, to show how this

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is possible by inclusion of detailed models in the quadratic OPF.

The paper is organized as follows: Section 2 recalls the L stability index and sets up the problem formulation. A computational method for calculating the new dispatch is described in section 3. Two standard benchmark systems are presented in section 4, and it is shown how these are optimized using a dispatch generated by the method we suggest.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We introduce the following nomenclature: the set of busses is denoted \mathcal{N} , the set of load busses is $\mathcal{L} \subset \mathcal{N}$, $\mathcal{G} \subset \mathcal{N}$ is the set of voltage-controlled busses, and the set of transmission lines and transformers is $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$. To each bus k we associate an active and reactive power injection P_k^g, Q_k^g with $S_k^g = P_k^g + jQ_k^g$, an active and reactive power demand P_k^d, Q_k^d with $S_k^d = P_k^d + jQ_k^d$, a complex voltage V_k and a complex current I_k . We define vectors of bus voltages $V = [V_1, V_2, \dots, V_n]$ and bus currents $I = [I_1, I_2, \dots, I_n]$. The currents and voltages are related through an admittance matrix Y as $I = YV$.

The L -index was introduced in [8] as a simple measure of a system's voltage stability margin. A dimensionless number $0 \leq L_k \leq 1$ is associated to each load bus, for which 0 is no load and 1 is voltage collapse. The index is calculated as follows. The vector of bus voltages is ordered such that the first g busses are those that are voltage controlled, $V = [V_1, \dots, V_g, V_{g+1}, \dots, V_n]$ and $V_{g+1} \dots V_n$ are the load busses where $n = |\mathcal{N}|$ and $g = |\mathcal{G}|$. The relationship between bus voltages and currents can be expressed by:

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \quad (1)$$

where I_G, I_L and V_G, V_L denote the currents and voltages at generator and load busses, respectively. By rearrangement:

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F \\ K & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \quad (2)$$

where $F = -Y_{LL}^{-1}Y_{LG}$.

Using F , the L index of a bus k is given by

$$L_k = \left| 1 - \sum_{i=1}^g F_{ki} \frac{V_i}{V_k} \right| \quad (3)$$

where F_{ki} is the k, i element in F . The L -index of each bus represents the bus' proximity to instability, and $\max_{k \in \mathcal{L}} L_k$ is used as an indicator of the system's proximity to collapse.

The L -index assumes constant voltages at generator busses. When the reactive-power limits are activated, this will no longer be the case. An extension to the L -index was suggested in [18] to include these effects. This approach will be applied in the stability-constrained OPF. The network is appended with the internal node of the machines. The electrical equations for generators can be written:

$$e_d'' = v_d + r_a i_d - (x_q'' - x_l) i_q \quad (4)$$

$$e_q'' = v_q + r_a i_q + (x_d'' - x_l) i_d \quad (5)$$

Under the assumption $x_d'' \approx x_q''$, the internal voltage are calculated from behind a constant impedance $Z_k = R_a + j(X_d'' - X_l)$ as:

$$E_k = V_k + Z_k I_k \quad (6)$$

The admittance matrix is extended to include the internal nodes. Let $Y_{gg} = \text{diag } Z_k$

$$\begin{bmatrix} Y_{gg} & -Y_{gg} & 0 \\ -Y_{gg} & Y_{GG} + Y_{gg} & Y_{GL} \\ 0 & Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} E_G \\ V_G \\ V_L \end{bmatrix} = \begin{bmatrix} I_G \\ 0 \\ I_L \end{bmatrix} \quad (7)$$

By Kron reduction, the new L -index can be calculated using [18]:

$$F' = -Z'_{LL} Y'_{LG}, \quad (8)$$

where $Z'_{LL} = (Y_{LL} - Y_{LG}(Y_{GG} + Y_{gg})^{-1}Y_{GL})^{-1}$ and $Y'_{LG} = Y_{LG}(Y_{GG} + Y_{gg})^{-1}Y_{gg}$. The extended L -index for a load bus k is then calculated using

$$L'_k = \left| 1 - \sum_{i=1}^g F'_{ki} \frac{E_i}{V_k} \right| \quad (9)$$

Protective controls are present in synchronous machines in order to avoid overheating in the field windings. When the protective controls are active, they limit the machines reactive power output and instantaneously change the voltage-control capabilities of the machine (see [19] for a study of the effects on voltage stability). The reactive limits of a machine has origin in the limitations on the currents in field and armature windings [20].

For $X_d \approx X_q$ the field excitation voltage is determined by [20]:

$$|E_{f,k}| = |X_{ad} i_{fd}| = |V_k + (R_a + jX_q) I_k| \quad (10)$$

The field voltage and armature current will be constrained by $E_{f,k}^{\max}$ and I_k^{\max} .

The voltage-stability measure and the machine limitations discussed above will be included to find a stability enhanced power dispatch. The stability enhancing OPF then takes the form,

$$\min \max_{k \in \mathcal{L}} L_k \quad (11a)$$

$$\text{s.t. } I_k^* V_k = S_k^g - S_k^d, \quad \forall k \in \mathcal{N} \quad (11b)$$

$$P_k^{\min} \leq P_k \leq P_k^{\max}, \quad \forall k \in \mathcal{N} \quad (11c)$$

$$Q_k^{\min} \leq Q_k \leq Q_k^{\max}, \quad \forall k \in \mathcal{N} \quad (11d)$$

$$V_k^{\min} \leq |V_k| \leq V_k^{\max}, \quad \forall k \in \mathcal{N} \quad (11e)$$

$$|S_{l,m}| \leq S_{l,m}^{\max}, \quad \forall (l, m) \in \mathcal{E} \quad (11f)$$

$$|E_{f,k}| \leq E_{f,k}^{\max}, \quad \forall k \in \mathcal{G} \quad (11g)$$

$$|I_k| \leq I_k^{\max}, \quad \forall k \in \mathcal{G} \quad (11h)$$

The constraint (11b) is the nodal power balance, constraint (11c) the real power generation limit, constraint (11e) the bus voltage magnitude limit, constraint (11f) the transmission line flow limit, constraint (11g) the field voltage limit and constraint (11h) is the armature current limit.

The semidefinite relaxation is now done by replacing (20j)

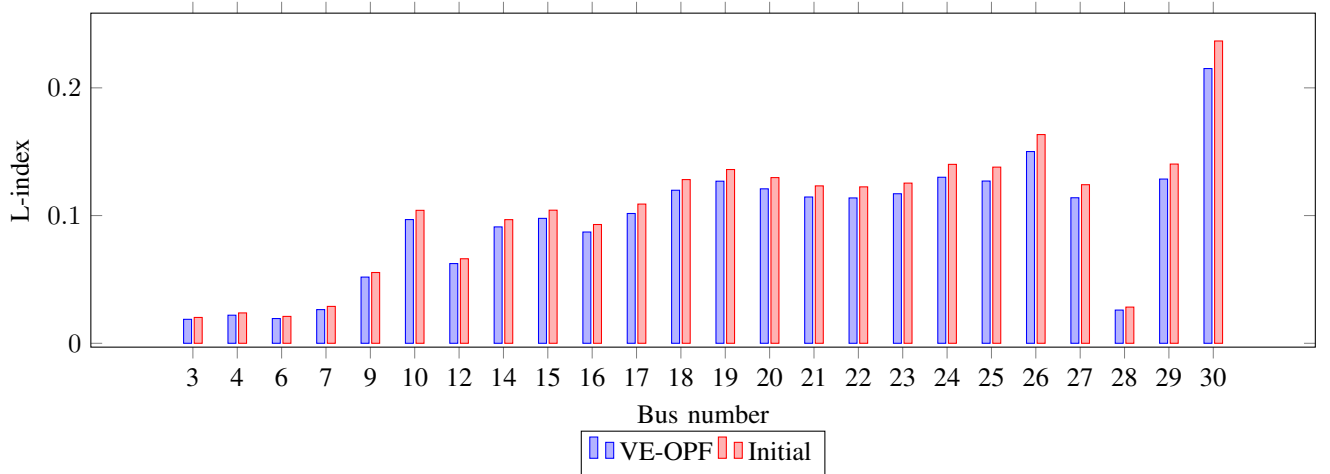


Fig. 2. L -index of all load busses on IEEE 30-bus benchmark system. A lower index indicates a better voltage stability margin.

with $W \succeq 0$. The resulting program is quasi-convex, and its solution W^* gives a lower bound on the optimal value of (11). If the solution has rank-1, the solution is exact and can be obtained from $W^* = EE^H$.

A bisection method can now be used to solve the quasi-convex optimization. This is done by finding a convex function $\theta_t(W, y_k)$ for which the objective functions t -sublevel set is the 0-sublevel set of θ_t . For a given t we solve the feasibility problem:

$$\text{find } W \quad (21a)$$

$$\text{s.t. } (20b) - (20i) \quad (21b)$$

$$W \succeq 0 \quad (21c)$$

$$\theta_t(W, y_k) \leq 0, \quad \forall k \in \mathcal{L} \quad (21d)$$

Using bisection to maximize t , the problem (21) can be solved by a series of SDPs. This problem is equivalent to solving:

$$\max t \quad (22a)$$

$$\text{s.t. } -tW_{kk} + y_k \geq 0, \quad \forall k \in \mathcal{L} \quad (22b)$$

$$(20b) - (20i) \quad (22c)$$

$$W \succeq 0 \quad (22d)$$

The optimization (22) is a main result of this paper. It shows how a semidefinite relaxation is obtained from the original non-convex voltage-enhancing OPF (11) such that the optimization (22) is solvable using standard SDP solvers.

The next section will demonstrate the voltage-stability-enhancing OPF in two case studies and show how enhanced voltage stability is obtained with the proposed method.

IV. EXAMPLE

The method is tested on two benchmark systems. The first test is done on the IEEE-30 bus system, and shows the voltage magnitude improvements using the voltage-stability enhancing OPF (VE-OPF) following a contingency. The second test is a dynamic simulation, where the method is

applied as a way of calculating a remedial action for a system in an emergency state.

A. IEEE 30-bus system

The IEEE 30-bus system (Figure 1) consists of six generator busses and 24 load busses. The standard IEEE 30-bus system [22] is modified to increase the loading on the system. This was done by increasing all loads by a factor of 1.25.

To illustrate the effects of improving the L -index, a contingency is considered where the line 27-30 is disconnected. Two different dispatches are considered for the pre-contingency system, the default dispatch and the voltage-stability enhanced OPF dispatch. The latter was found from (20) with a bisection precision of $\epsilon = 1e-4$. The optimization terminated with a rank = 1 solution W^* , which allowed recovery of the exact solution. The resulting L -index of all load busses is shown in Figure 2.

Using these dispatches, the line 27-30 disconnection contingency is applied and the resulting bus voltages are examined. The L -index and voltage magnitude of bus 30 is shown in Table I.

	Post contingency	
	Initial	VE-OPF
L_{30}	0.3310	0.2966
$ V_{30} $	0.89 pu	0.95 pu

TABLE I

RESULT OF BUS VOLTAGE MAGNITUDE UNDER CONTINGENCY FOR IEEE 30-BUS SYSTEM.

From the results in Table I it is clear that the voltage-stability enhanced dispatch has a better robustness against disturbances in the grid. For the default dispatch, the voltage on bus 30 drops to 0.89 pu, where an improvement is shown in the voltage-profile using the dispatch from (20), where voltage only drops to 0.95 pu.

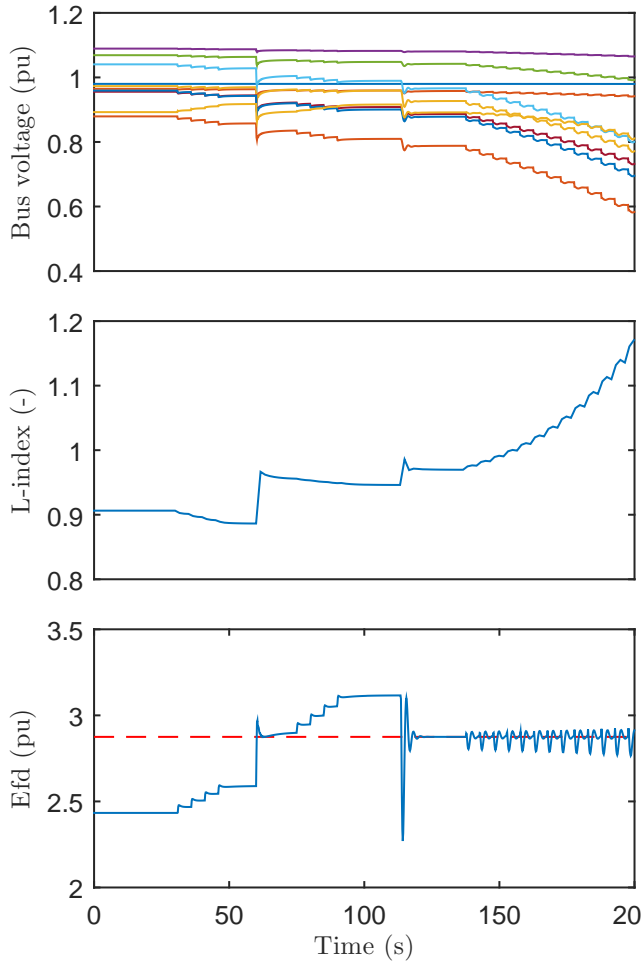


Fig. 3. Time simulation of BPA system, with a fault injected at time $t = 60$ s.

B. BPA dynamic case study

The second test case is a dynamic simulation where the proposed method is employed to calculate a remedial action for a system in an emergency state. The test system from [23] is employed to show feasibility of the method. The system (cf. Figure 5) is a 11-bus, 3 generator system, with a generating area on the left consisting of generators $G1$ and $G2$, which are connected to a local area on the right. Generator $G1$ represents an infinite bus. Generators $G2$ and $G3$ are voltage controlled. Generator $G3$ is equipped with an over-excitation limiter (OXL). The others never reach excitation limits in the test case. An On-Load Tap Changer (OLTC) is connected to bus 11 to maintain the load side voltage.

The system is operated close to its stability limits. After one of the lines between bus 6 and 7 is tripped, the OLTC at bus 11 will try to maintain the load voltage, but the operation point ends up beyond the point of maximal power deliverable and the result is a voltage collapse. A time domain simulation of the system is shown in Figure 3.

The bus voltage at bus 11 is initially too low, which the OLTC recovers. At time $t = 60$ s, one of the lines between

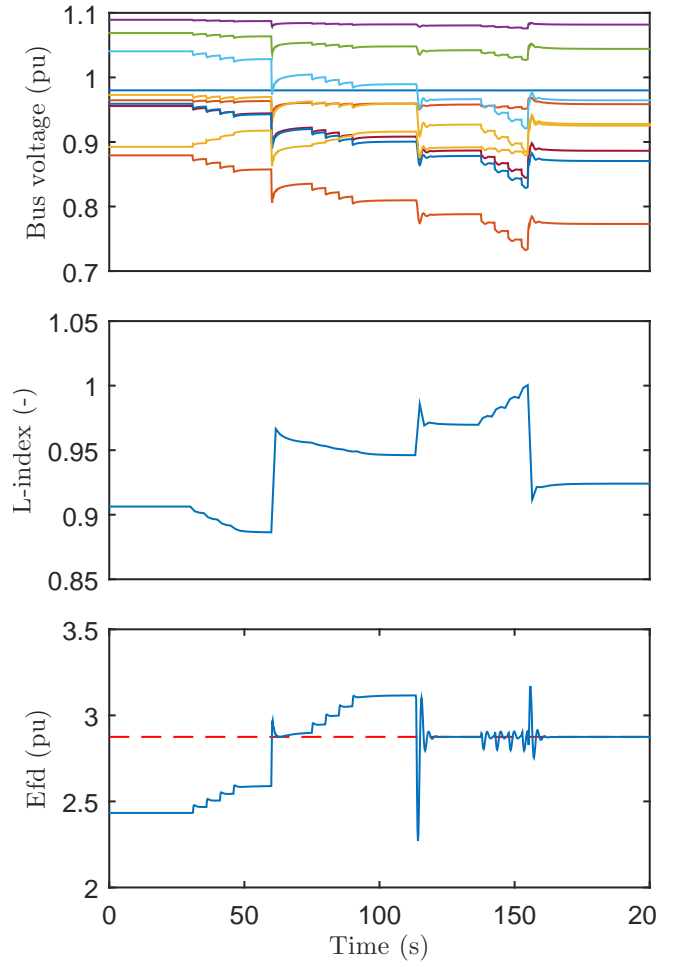


Fig. 4. Time simulation of BPA system, with a fault injected at time $t = 60$ s. The L -index is used as an emergency indicator, and when $L_{11} > 1$ at $t = 155$ s a remedial action is applied.

bus 6 and 7 is tripped. When this happens, the field voltage of generator $G3$ crosses its maximal limit. The OXL is allowed to operate at a higher field voltage for a limited period of time. Within this period, the OLTC successfully recovers the voltage at bus 11, but at time $t = 110$ s the OXL is activated and at time $t = 155$ s the L -index indicates that the system is voltage unstable. The OLTC continuously tries to maintain the load side voltage at bus 11, but a voltage collapse is the result.

Remedial action: We will now use the SDP formulation to calculate the necessary load-shedding to avoid a voltage collapse. In this case we replace the voltage-enhancing objective with an objective to minimize the load shedding at bus 11, and a constraint relating to the L -index is added, constraining $L'_k < 0.95$ for all load busses.

The optimization obtains a rank-one solution W^* to (22), such that the solution is exact. The remedial action takes place from time $t = 155$ s, by the shedding of 391 MW load on bus 11. By this remedial action, the system is able to recover stability, as seen from the L -index. The resulting response is shown in Figure 4.

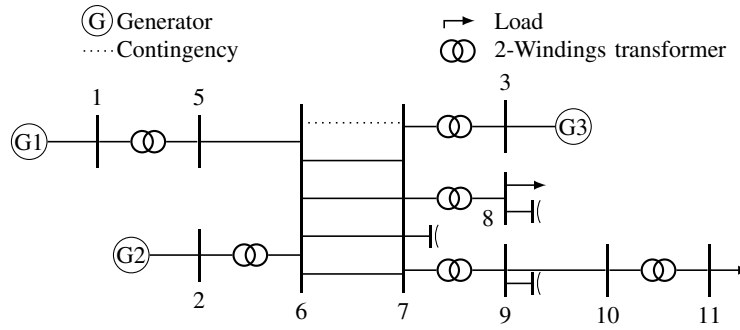


Fig. 5. BPA test system used for dynamic simulation. The dotted line indicates the considered contingency.

V. CONCLUSION

This paper presented a method for solving a stability-enhancing optimal power flow problem using a semidefinite relaxation, resulting in a quasi-convex semidefinite program. The method was tested on two benchmark systems, which showed feasibility of the method to improve the voltage-stability of a power system.

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