

How Decentral Smart Grid Control limits non-Gaussian power grid frequency fluctuations

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Abstract—Frequency fluctuations in power grids, caused by unpredictable renewable energy sources, consumer behavior and trading, need to be balanced to ensure stable grid operation. Standard smart grid solutions to mitigate large frequency excursions are based on centrally collecting data and give rise to security and privacy concerns. Furthermore, control of fluctuations is often tested by employing Gaussian perturbations. Here, we demonstrate that power grid frequency fluctuations are in general non-Gaussian, implying that large excursions are more likely than expected based on Gaussian modeling. We consider real power grid frequency measurements from Continental Europe and compare them to stochastic models and predictions based on Fokker-Planck equations. Furthermore, we review a decentral smart grid control scheme to limit these fluctuations. In particular, we derive a scaling law of how decentralized control actions reduce the magnitude of frequency fluctuations and demonstrate the power of these theoretical predictions using a test grid. Overall, we find that decentral smart grid control may reduce grid frequency excursions due to both Gaussian and non-Gaussian power fluctuations and thus offers an alternative pathway for mitigating fluctuation-induced risks.

I. INTRODUCTION

Electric power grids operate close to set reference frequencies (e.g., $f_R = 50$ Hz) to robustly ensure power distribution among generators and consumers [1]. During operation, the grid is experiencing frequency fluctuations arising from power fluctuations on the supply side, e.g. due to fluctuating feed-in from renewable energy sources [2], [3], as well as power fluctuations in consumption [4] and signals resulting from energy trading [5]. Already today, frequency fluctuations can be substantial [6], [7] which can be threatening for sensitive electronic equipment [8], [9], [10]; in particular since large shares of renewable generation, like solar generation, are placed spatially distributed on low voltage levels, making it difficult to control by central control schemes [11]. In future power grids, frequency stability is expected to become a major issue, primarily due to the replacement of synchronous machines by power electronics, which have no natural inertia [12]. Furthermore, a recent analysis showed that frequency fluctuations are anomalous, e.g. follow non-Gaussian statistics, such that large deviations

can be more likely than expected [13]. Reliably operating 100% renewable grids of the future is thus confronted with major challenges. One open question is: How to best limit the impact of power fluctuations on the grid frequency?

There exist several different approaches on how grids with a high share of renewable generation can be controlled [14]. Novel concepts for frequency control include for example the usage of storage [15], [16], controlling wind turbines directly [17] or applying demand side control [18], [19]. On the demand side, a variety of smart grid concepts [20], [21], have been proposed to balance supply-demand differences and to limit fluctuations. However, standard smart grid concepts are often centralized and based on collective supply and demand information of the generator and consumer sides in real time. Thereby, the instantaneous imbalance is computed centrally and the grid should be balanced by sending incentives, e.g. price signals, to increase or decrease supply and demand [22], [23], [24]. Such central organization comes with a number of issues that may be undesired. Most importantly, consumer (and generator) information is centrally collected, resulting in data privacy issues and making the grid vulnerable to intentional attacks, e.g. through hacking the central computer. Recent cyber attacks [25], [26] demonstrated that even large companies, like banks and logistic enterprises, are vulnerable to such hacking attempts. Moreover, we do not fully understand to date how and under which conditions central control is feasible for such large, distributed and nonlinear system as power grids; in particular, generally valid stability guarantees are missing entirely.

As an alternative to central control paradigms, decentralized control algorithms have been proposed, e.g. in [27], [28] or [29], [30], [31]. Decentral control could work by providing local feedback to the consumer or generator by evaluating local frequency deviations from the reference frequency and providing a local control signal. Thereby, it extends the basic mechanism of *primary control* [1] to decentral generators and consumers by coupling frequency and price incentives.

It remains unclear whether and how both central and decentral control actions are capable to coping with non-Gaussian fluctuations. The viability of many control methods has only been tested using historical time series [17], [19] or simple stochastic models [16]. The aforementioned decentral smart grid control has been demonstrated to induce frequency-stable operation in small and moderately sized grids with balanced demands and supplies that are static [32], [33]. Nevertheless, how stochastic perturbations of the power are influenced by such a control is not understood.

To gain first fundamental insights about these questions,

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and see whether non-Gaussian fluctuations may be limited at all by such decentral control schemes, we here study the impact of decentral smart grid control on power grid networks. We integrate external power fluctuations as external noise sources and study how these transform to frequency fluctuations when modeling the power grid as coupled (virtual) synchronous machines. We combine noise-driven grid modeling with stochastic analysis of frequency data recorded in real transmission grids that exhibit non-Gaussian frequency fluctuations. We consider both Gaussian noise processes and non-Gaussian frequency measurements to extract the power fluctuations, analyze and quantify how decentralized control may reduce frequency fluctuations and may thus contribute to improve dynamic stability of electricity grids.

II. MODELING POWER GRID FREQUENCY FLUCTUATIONS

We model power grid dynamics using a coarse-grained model to integrate real frequency measurements with mathematical methods of stochastic analysis. We aggregate several synchronous generators within one region together with the regional loads into one (virtual) synchronous machine [34]. An excess of generated power will make such a machine an effective generator, while urban regions are modeled as effective loads (consumers). Within a simple dynamical description of coupled synchronous machines using the swing equation [1], [8], each node $i \in \{1, \dots, N\}$ is modeled via its voltage phase angle $\theta_i(t)$ and angular velocity $\omega_i(t)$ as

$$\begin{aligned} \frac{d}{dt}\theta_i &= \omega_i, \\ M_i \frac{d}{dt}\omega_i &= F_i(\theta, \omega, t) + C_i(\theta, \omega, t), \end{aligned} \quad (1)$$

with inertia M_i , intrinsic, potentially noisy dynamics, $F_i(\theta, \omega, t)$ and control $C_i(\theta, \omega, t)$. Neglecting ohmic losses and assuming constant voltage amplitudes (well justified for high voltage transmission grids [8], [35]), the intrinsic dynamic is given as

$$F_i(\theta, \omega, t) = P_i^{\text{in}}(t) - \kappa_i^D \omega_i + \sum_{j=1}^N K_{ij} \sin(\theta_j - \theta_i), \quad (2)$$

where at each node we have active injected power $P_i^{\text{in}}(t)$, including fluctuations, damping κ_i^D and coupling matrix K_{ij} . For now, we analyze the impact of random perturbations in the power $P_i^{\text{in}}(t)$ in the absence of (decentral) control, $C_i \equiv 0$. Throughout this manuscript, we use a per-unit notation so that all quantities have unit 1 or powers of 1/second, see e.g. [35]

A. Gaussian noise

As a simple model for power fluctuations, we consider white Gaussian noise such that the active injected power

$$P_i^{\text{in}}(t) = P_{0,i}^{\text{in}} + \sigma_i^P \Gamma_i(t) \quad (3)$$

at node i is a sum of a constant term $P_{0,i}^{\text{in}}$ and a noise term $\Gamma_i(t)$ multiplied by an noise amplitude σ_i^P . Thereby, we

allow different noise distributions at individual nodes and only require a time-average of zero for the noise.

We assume the damping κ_i^D to be proportional to the inertia at each node [36] so that $\gamma = \kappa_i^D / M_i$ for all i . While this may not be true in all cases, this assumption is necessary to obtain analytical results. Furthermore, we assume the power to be balanced on average, i.e., $\sum_{i=1}^N P_{0,i}^{\text{in}} = 0$ and the coupling to be symmetrical $K_{ij} = K_{ji}$. The dynamics of the bulk angular velocity $\bar{\omega} = \sum_{i=1}^N \omega_i M_i / \sum_{i=1}^N M_i$ is then given by

$$\frac{d}{dt}\bar{\omega} = -\gamma\bar{\omega} + \frac{\sum_{i=1}^N \sigma_i^P \Gamma_i(t)}{\sum_{i=1}^N M_i}. \quad (4)$$

The sum of Gaussian processes is again a Gaussian process [37], [38] such that

$$\frac{\sum_{i=1}^N \sigma_i^P \Gamma_i(t)}{\sum_{i=1}^N M_i} = \bar{\sigma}^P \bar{\Gamma}, \quad (5)$$

where the aggregated noise amplitude $\bar{\sigma}^P$ is defined as

$$\bar{\sigma}^P = \frac{\sqrt{\sum_{i=1}^N (\sigma_i^P)^2}}{\sum_{i=1}^N M_i}. \quad (6)$$

To obtain the probability distribution for the angular velocity $p(\bar{\omega})$, we have to formulate and solve the Fokker-Planck equation of the dynamics (4). The resulting distribution [13], [37], [39] is (also) a Gaussian with standard deviation

$$\bar{\sigma}^\omega = \frac{\bar{\sigma}^P}{\sqrt{2\gamma}}. \quad (7)$$

A final quantity of interest is the autocorrelation $c(\Delta t)$ as a function of the time lag Δt , which for process (4) is a decaying exponential [13], [37]

$$c(\Delta t) = \exp(-\gamma\Delta t). \quad (8)$$

With these results, we now analyze frequency recordings from a real power grid, assuming that we measure the bulk frequency \bar{f} and convert it to the bulk angular velocity via $\bar{\omega} = 2\pi(\bar{f} - f_R)$. Given the statistics for $\bar{\omega}$, we estimate its standard deviation $\bar{\sigma}^\omega$ and compute the autocorrelation $c(\Delta t)$ to determine the damping to inertia ratio γ from an exponential fit.

We compare real frequency measurements with our theory and a naive Gaussian assumption in Figure 1. Panels a,b illustrate an artificial trajectory following such an Ornstein-Uhlenbeck process, as given by Eq. (4) and a real frequency time series recorded in 2015 in the Continental European Grid [40]. Comparing the statistics systematically, we find that the real frequency data exhibit a higher likelihood of large fluctuations than that predicted by the best fitting Gaussian distribution (panel c). To model these heavy tails, we thus generalize Gaussian to arbitrary stable distributions [38] to characterize the frequency deviations. Finally, panel d of Fig. 1 shows the exponential decay of the autocorrelation function predicted by Eq. (8).

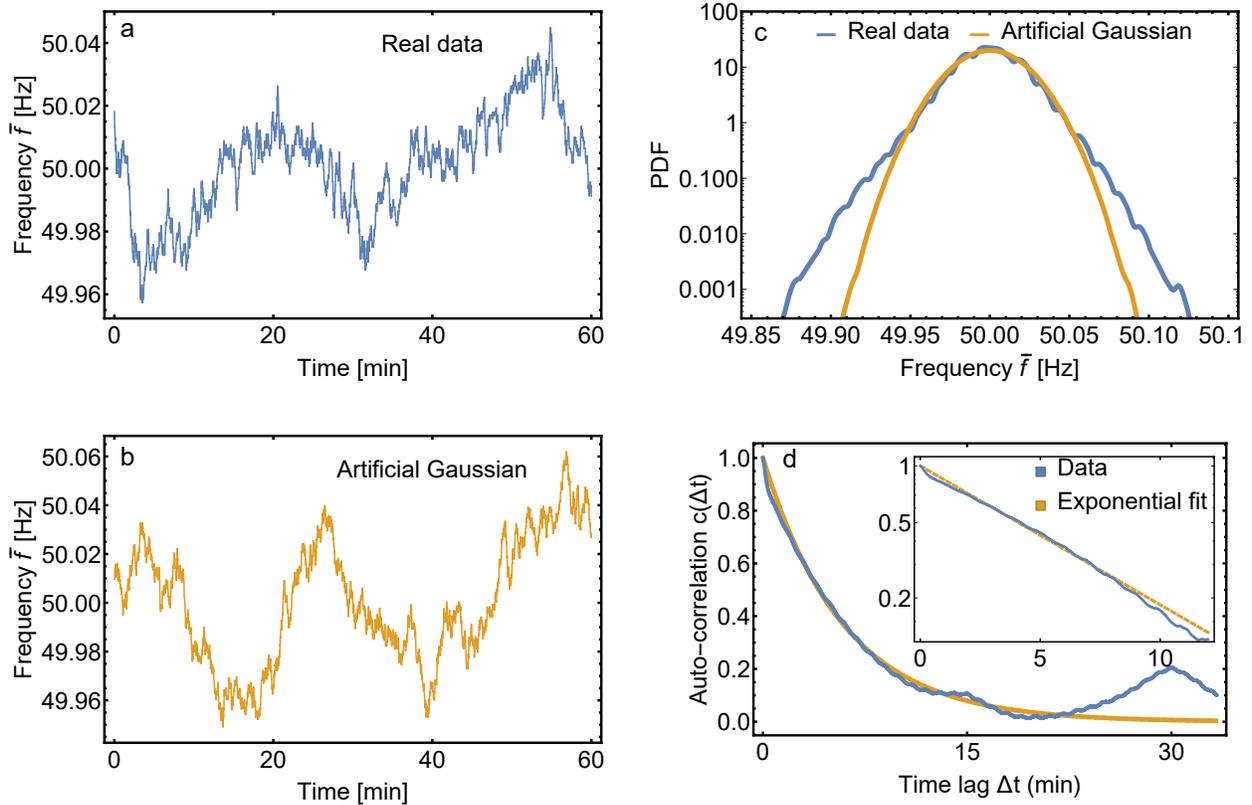


Fig. 1. Real power grid frequency fluctuations are not Gaussian and show an exponential decay of the autocorrelation. a: Sample trajectory of the real frequency trajectory. b: Sample trajectory of an artificial Gaussian frequency trajectory, assuming an underlying Ornstein-Uhlenbeck process (4). c: The probability density function (PDF) of the bulk frequency \bar{f} comparing real data with artificial Gaussian data. The inset uses a log-linear plot to highlight the exponential decay. In addition, we observe regular correlation peaks, associated with trading [13]. The real data has much heavier tails than a Gaussian distribution would predict (Kurtosis of approx. 3.8 instead of 3 for Gaussian distribution). We use data by *50Hertz* from 2015 with a 1 second resolution describing the Continental European power grid [40].

B. Extracting power fluctuations from frequency fluctuations

Given real frequency measurements, we extract the statistics of the aggregated power fluctuations following a generalized Fokker-Planck equation [41], i.e. we still use Eq. (4) but allow the noise $\Gamma_i(t)$ to follow non-Gaussian distributions [42]. Let us denote the characteristic equation, i.e., the Fourier transform of the probability density function, of the collective power noise as $S^P(k)$. Then, the following relations with respect to the characteristic function of the angular velocity $S^\omega(k)$ hold [13]:

$$S^\omega(k) = \exp \left[\frac{1}{\gamma} \int_0^k \frac{\ln(S^P(z))}{z} dz \right], \quad (9)$$

$$S^P(k) = \exp \left[\gamma k \frac{\partial}{\partial k} \ln(S^\omega(k)) \right], \quad (10)$$

similar to the Gaussian case, individual nodes might have different distributions, but we require the average of the perturbation to be zero and it has to be possible to aggregate all contributions.

Using measurements from the continental European grid [40], we first perform a maximum likelihood analysis [43] to

identify Gaussian and the more general stable distributions as generally good fits to the data. Next, we extract the approximate characteristic function of the angular velocity $S^\omega(k)$ from the distribution and apply Eq. (10) to extract the distribution of the aggregated power fluctuations $S^P(k)$. Fig. 2 displays the results when using the data directly (red dots), compared to the assumption of Gaussian noise (orange curve) and non-Gaussian stable noise (blue curve). Gaussian distributions tend to underestimate the tails of the distribution (see Fig. 1) so that we use stable distributions to complement our analysis. Stable distributions (also known as α -stable or Lévy-stable) are characterized by a stability parameter α_S , determining the heavy tails, a skewness parameter β_S , a scale parameter σ_S , which fulfills a similar role as the standard deviation in the Gaussian case, and a location parameter μ_S . In the remainder of the paper we assume that the noise follows a stable distribution, simplifying it by assuming centered and symmetric distributions, i.e. $\beta_S = \mu_S = 0$ for all nodes. Furthermore, we assume that the stability parameter α_S is identical at all nodes, thereby ensuring that the resulting bulk distribution is a stable distribution, as observed in the data. Stable distributions contain Gaussian distributions as a special case given by the stability parameter $\alpha_S = 2$.

In the plots below, we use the term stable distributions only in connection with $\alpha_S < 2$, i.e. non-Gaussian stable distributions.

To aggregate several independent stable distributions, we require that the noise at each node has identical stability parameter α_S but it may have an arbitrary scale parameter σ_S . Let $\sqrt{2}\sigma_{S,i}^P$ be the scale parameter of the stable power perturbations at node i , where the factor $\sqrt{2}$ is necessary to reproduce the Gaussian case for $\alpha_S = 2$. Then, the scale parameter of the bulk angular velocity is given as [13], [41]

$$\bar{\sigma}_S^\omega = \frac{1}{\sqrt{2}\sum_{i=1}^N M_i} \left[\frac{1}{\gamma\alpha_S} \sum_{i=1}^N (\sigma_{S,i}^P)^{\alpha_S} \right]^{1/\alpha_S}. \quad (11)$$

III. DECENTRAL CONTROL

In the previous section, we have explored how to extract the statistics of power fluctuations from power grid frequency measurement data, inferring the underlying distribution of power disturbances. These considerations assumed that there is no control C_i at a given node. We follow the proposal of Decentral Smart Grid Control, introduced in [30], [31] and mathematically modeled in [32], [33] to motivate an additional control term for the swing equation (1) as follows. All grid participants use the local grid frequency to determine an energy shortage (in case of low frequencies) or energy abundance (in case of high frequencies). Based on price incentives, customers then adapt their consumption and generation to keep the grid closer to the desired frequency, see Fig. 3 for an illustration.

Let us assume that the injected power $P_i^{\text{in}}(t)$ at node i is given as the difference of supply $\mathcal{S}_i(p_i)$ and demand $\mathcal{D}_i(p_i)$, which both depend on the price p_i

$$P_i^{\text{in}}(t) = \mathcal{S}_i(p_i) - \mathcal{D}_i(p_i). \quad (12)$$

We expect supply to increase and demand to decrease with increasing prices so that overall, $P_i^{\text{in}}(p_i)$ increases with increasing price p_i . For simplicity, we assume this dependency to be linear for prices close to the equilibrium

$$P_i^{\text{in}}(p_i) = P_{i,0}^{\text{in}} + c_i^{(1)}(p_i - p_{i,0}), \quad (13)$$

with equilibrium injected power $P_{0,i}^{\text{in}}$, equilibrium price $p_{i,0}$ and price-dependency $c_i^{(1)}$. Next, we need to determine how the price adapts with respect to the frequency. Following recent proposals [30], [31], we will assume that the price is a linear decreasing function of the frequency f_i , given in terms of the angular velocity $\omega_i = 2\pi(f_i - f_R)$ as

$$p_i(\omega_i) = p_{i,0} - c_i^{(2)}\omega_i, \quad (14)$$

with price constant $c_i^{(2)}$. Thereby, the injected power becomes a linear function of the angular velocity as

$$P_i^{\text{in}}(\omega_i) = P_{0,i}^{\text{in}} - \kappa_i^C \omega_i, \quad (15)$$

with $\kappa_i^C = c_i^{(1)}c_i^{(2)}$, i.e., it is a product of the price-dependency of the node i and the slope of the price-frequency

curve. Using the notation of Eq. (1), we formulate our control term as

$$C_i(\theta, \omega, t) = -\kappa_i^C \omega_i. \quad (16)$$

Inspecting (16), we notice that adding our decentralized control adds effective damping to the grid, similar to control applied on generator sides today [44]. The main benefit is that we generalize the notion of primary control and make it applicable to all grid participants, generators, consumers as well as other services that e.g. provide electricity storage.

IV. REDUCING FLUCTUATION RISKS WITH DECENTRAL CONTROL

Does a decentralized control C_i , as proposed above, reduce fluctuation risks? We investigate this question using a ten node test grid, see Fig. 4, considering different control settings, using Gaussian and non-Gaussian stable noise with stability parameter $\alpha_S = 1.5$. For each setting, we evolve the stochastic differential equation given by Eq. (1) and compare it to the analytical prediction of the bulk frequency distribution $p(\bar{\omega})$, based on the theory from the last section and [13]. We vary the strength of control applied via a control parameter κ^C . For our analysis restricted to Gaussian noise, we extract the standard deviation of the distribution and compare it to the predicted standard deviation in Eq. (7). Similarly, for non-Gaussian stable noise, we extract the scale parameter and compare it to the predicted one in Eq. (11). All simulations use $\sigma_S = 1/100$ at all nodes, i.e. for the Gaussian case, we use $\sigma = \sqrt{2}/100$. Heterogeneous noise does produce similar results (not shown).

a) Homogeneous control to inertia: If we assume the grid to have a homogeneous damping to inertia ratio $\kappa_i^D/M_i = \gamma = 0.1s^{-1}$, we may use equations (7) and (11) for Gaussian and non-Gaussian noise respectively. Furthermore, if the control κ_i^C at each node is also proportional to the inertia $\kappa_i^C = 10\kappa^C \kappa_i^D$, we have a precise prediction of how the standard deviation and scale parameter depend on the control. Fig. 5 shows the great agreement of theory and simulations. Namely, increasing control κ^C decreases the scale parameter (or standard deviation) and thereby reduces fluctuation risks.

b) Heterogeneous control to inertia: Next, let us drop the assumption that the damping κ_i^D is proportional to the inertia M_i . Instead, we determine damping values κ_i^D so that averaged over all nodes the total ratio is the same as before $\sum_{i=1}^N \kappa_i^D / \sum_{i=1}^N M_i = 0.1s^{-1}$. The control at each node is still proportional to the damping with $\kappa_i^C = 10\kappa^C \kappa_i^D$. Inspecting Fig. 6 reveals that increasing the control κ^C still decreases the observed width of the distribution, i.e. standard deviation σ and scale parameter σ_S decrease. However, the simulations do no longer perfectly align with the theoretical predictions. Instead, the simulations tend to display a wider distribution than expected based on a homogeneous damping to inertia ratio. These deviations between simulations and theory appear to be smaller for non-Gaussian noise. Overall, while increasing control reduces fluctuations, heterogeneous damping and inertia values lead to larger fluctuations than homogeneous values.

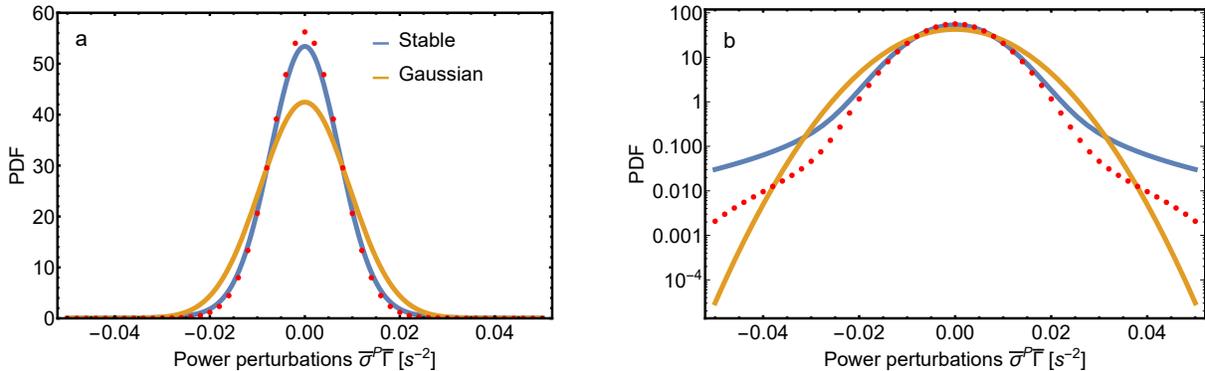


Fig. 2. Using frequency measurements, we extract approximate power fluctuations. We plot the probability density function (PDF) of the estimated power fluctuations, based on frequency measurements, using Eq. (10) and assuming noise following the best fitting Gaussian or non-Gaussian stable distribution (solid curves). The stable noise distribution is a more likely description for our data (red dots), only overestimating the tails slightly. a: Linear scale for the PDF. b: Log-scale for the PDF. We use data by *50Hertz* from 2015 with a 1 second resolution describing the Continental European power grid [40].

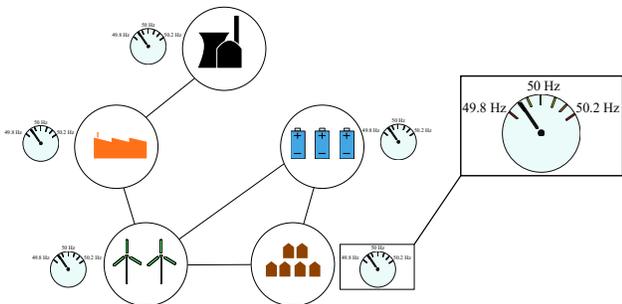


Fig. 3. Decentral Smart Grid Control uses local frequency measurements at all nodes, regardless of whether they are consumers, generators or storage facilities to stabilize the grid. Driven by price incentives, customers should increase generation (reduce consumption) in case of low frequencies and decrease generation (increase consumption) in case of high frequencies.

c) Controlling generators only: Often, control is assumed to be a task to be fulfilled primarily by the generators and not by the consumers [1], [4]. Here, we implement this control paradigm using identical machines with $M_i \equiv M = 1$ for all i and homogeneous damping $\kappa_i^D = \kappa^D = 0.1s^{-1}$. The control is $\kappa_i^C = 10\kappa^C \kappa_i^D$ for generators, i.e. for nodes with $P_i^{\text{in}} > 0$, and zero otherwise. To match the simulation, we include only $\kappa^C/2$ in our predictions, since only half of the network is controlled, and observe a good match between theory and simulation in Fig. 7. Applying frequency-dependent control only at the generator nodes already decreases the overall frequency fluctuations in the network. Interestingly, theory and simulation agree very well in this case, although the theory was derived assuming homogeneous parameters.

V. DISCUSSION

Overall, we have shown that non-Gaussian effects are present in real power grid frequency statistics, with non-Gaussian stable distributions as good fits (Fig. 1). Furthermore, we have made some earlier results [13] more explicit, for example when extracting the distribution of power fluctu-

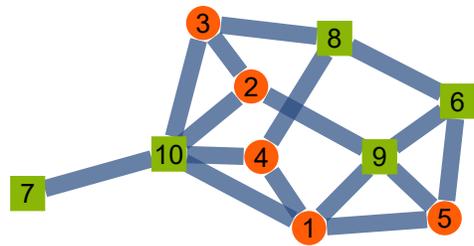


Fig. 4. Ten node test grid used for the simulations. Red circles (indices 1–5) represent consumers with $P^{\text{in}}(\text{Con}) = -1s^{-2}$ and green squares (indices 6–10) represent generators with $P^{\text{in}}(\text{Gen}) = +1s^{-2}$. If not stated otherwise, the inertia values are $M_i = (1.1, 1.7, 7, 8.7, 3.2, 9.8, 0.7, 5.8, 0.2, 0.9)$ which was obtained by randomly drawing an inertia value in the interval $M_i \in [0.1, 10]$ for each node. The coupling matrix is $K_{ij} = 0$ if two nodes are not connected and $K_{ij} = 4s^{-2}$ otherwise.

ations from pure frequency measurements (Fig. 2). To reduce the effect of these power fluctuations on the grid frequency, a decentralized control paradigm could be used that relies on local grid frequency measurements and price incentives to effectively provide additional droop control at all nodes (Fig. 3). Such a control intrinsically avoids privacy issues or vulnerability concerns as no communication infrastructure is necessary. Using both simulations and stochastic analysis, we have predicted the effectiveness of such a decentralized control, extending previous results on deterministic stability [32], [33] to stochastic stability. In particular, we have derived a scaling of the general scale parameter in equation (11). Investigating a ten node test grid, we found that our stochastic analysis matches the simulations results for the standard deviation (or scale parameter in the case of non-Gaussian stable noise) very well (Fig. 5). Increasing decentral control reduces fluctuation risks by decreasing standard deviation (or scale parameter) of the resulting frequency distribution. Even for heterogeneous damping to inertia ratios, we observe a reduction of fluctuations. However, fluctuation risks are reduced less efficiently in the presence of heterogeneous ratios when compared to the predictions and the precise

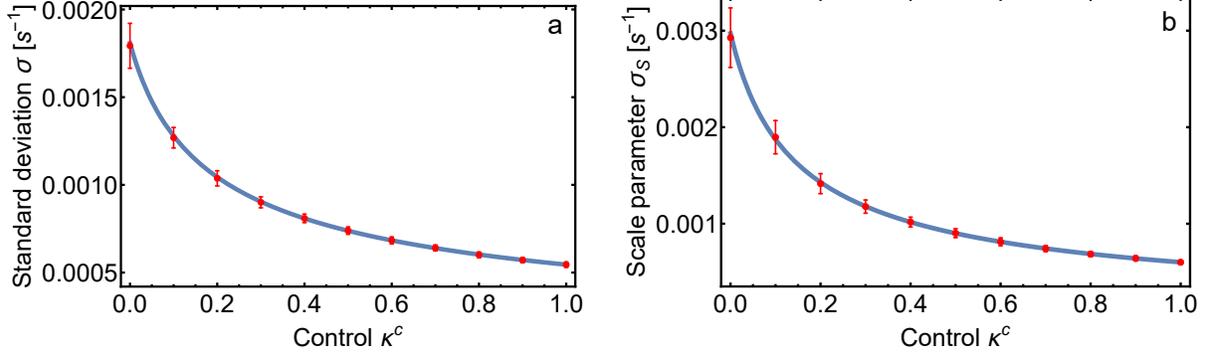


Fig. 5. Fluctuations are reduced by control as predicted by the theory. a: We plot the predicted standard deviation σ , based on (7) versus simulation results. b: We plot the predicted scale parameter σ_S , based on (11), versus simulation results. Simulations used the ten node network, see Fig. 4, with both damping κ_i^D and control κ_i^C proportional to the inertia M_i with $\kappa_i^D/M_i = 0.1s^{-1}$ and $\kappa_i^C = 10\kappa^C \kappa_i^D$. The dots give the mean value based on 100 independent experiments of independent random realizations of the noise trajectory. For each trajectory, we evaluate 1000 data points. The error bars show the standard deviation between these 100 runs.

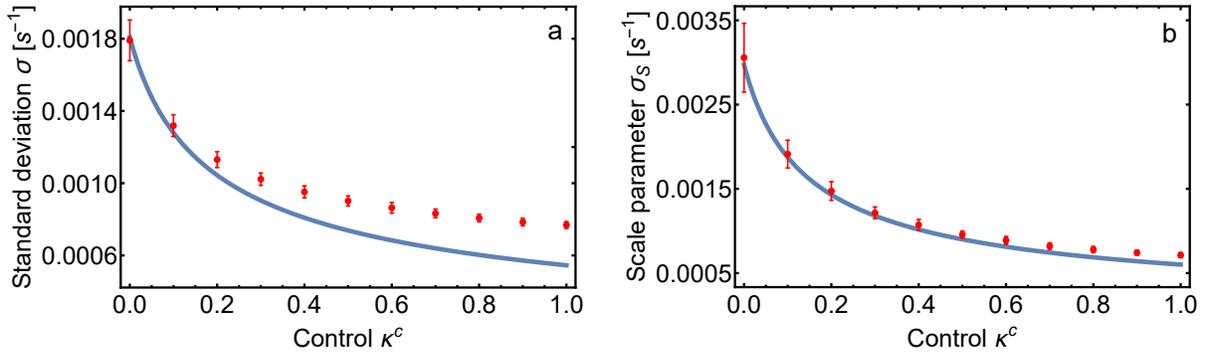


Fig. 6. Control also reduces fluctuations when using heterogeneous damping to inertia ratios. a: We plot the predicted standard deviation σ , based on (7) versus simulation results. b: We plot the predicted scale parameter σ_S , based on (11), versus simulation results. Simulations used the ten node network, see Fig. 4, with the average damping being proportional to the average inertia $\sum_{i=1}^N \kappa_i^D / \sum_{i=1}^N M_i = 0.1s^{-1}$ and control κ_i^C proportional to the individual damping $\kappa_i^C = 10\kappa^C \kappa_i^D$. The prediction overestimates the effect of control. We use the mean damping and mean inertia for the analytical prediction (blue curves). The damping realization for this plot is $\kappa_i^D = (0.306, 0.494, 0.158, 0.188, 0.573, 0.089, 0.592, 0.849, 0.425, 0.236)s^{-1}$. The dots give the mean value based on 100 independent experiments of independent random realizations of the noise trajectory. For each trajectory, we evaluate 1000 data points. The error bars show the standard deviation between these 100 runs.

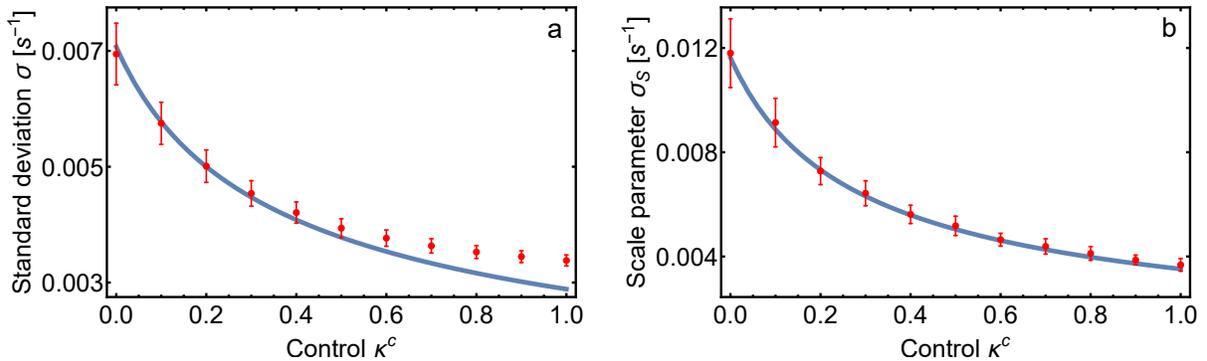


Fig. 7. Control also reduces fluctuations when only controlling generators. a: We plot the predicted standard deviation σ , based on (7) versus simulation results. b: We plot the predicted scale parameter σ_S , based on (11), versus simulation results. Simulations used the ten node network, see Fig. 4, with identical inertia $M_i = M = 1$ and damping $\kappa_i^D = \kappa^D = 0.1s^{-1}$ for all nodes. The control is proportional to the inertia but only for generators, i.e., for nodes with $P_i^{in} > 0$, the control is $\kappa_i^C = 10\kappa^C \kappa_i^D$ and zero otherwise. The prediction overestimates the effect of control slightly. We use half the value of κ^C for our analytical prediction (blue curves). The dots give the mean value based on 100 independent experiments of independent random realizations of the noise trajectory. For each trajectory, we evaluate 1000 data points. The error bars show the standard deviation between these 100 runs.

scaling of the fluctuations is only approximately described by our theory (Fig. 6). Similarly, controlling only parts of the network, e.g. because consumers do not participate in demand control schemes, still reduces fluctuation risks, approximately as predicted by the theory (Fig 7).

Our analysis used several necessary simplifying assumptions. Most crucially, we assumed the inertia to be proportional to the damping at each node. This assumption is crucial since thereby the stochastic equation of motion Eq. (4) becomes a linear one-dimensional equation. Otherwise, we would end with a high-dimensional, nonlinear Fokker-Planck equation for the bulk frequency $p(\bar{\omega})$. Since Fokker-Planck equations are partial differential equations, solving such a complex equation analytically becomes impossible [37]. Instead, we made the above-mentioned simplifications to also include the non-Gaussian effects observable in real power grid dynamics.

In conclusion, decentral smart grid control may be capable of reducing frequency fluctuation in power grids which exhibit Gaussian or more general non-Gaussian fluctuations of the grid frequency. We formulated an approximate stochastic theory to predict the effectiveness of control for a family of noise distributions and supported these predictions with simulations for Gaussian and stable noise. Based on all performed simulations (not all shown), the precise network topology and distribution of power seem negligible, as long as lines are not heavily loaded (the coupling K_{ij} is large compared to the power P_i^{in}). The main benefit of the decentralized control scheme is its inclusion of all grid participants, thereby distributing the control burden throughout the network.

Future research may focus on open theoretical and practical questions. First, it remains unclear how individual node frequency statistics are affected by Gaussian and non-Gaussian noise, respectively. Second, there is no explanation yet for why our approximation seems to more closely resemble the simulation data for non-Gaussian noise than it does for Gaussian noise (see Fig. 5 b). Third, control actions may not always be instantaneous but instead could follow a delayed signal, compare for instance [32], [33], [45]. How such a delay affects the effectiveness of decentral control of fluctuations remains an open question as well.

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