Iterative Controller Tuning Using Bode's Integrals *

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Abstract

An iterative controller tuning based on a frequency criterion is proposed. The frequency criterion is defined as the weighted sum of squared errors between the desired and measured gain margin, phase margin and crossover frequency. The method benefits from specific feedback relay tests to determine the gain margin, the phase margin and the crossover frequency of the closed-loop system.The Bode's integrals are used to approximate the derivatives of amplitude and phase of the plant model with respect to the frequency without any model of the plant. This additional information is employed to estimate the gradient and the Hessian of the frequency criterion in the iterative controller tuning method. Simulation examples and experimental results illustrate the effectiveness and the simplicity of the proposed method to design and tuning the controllers.

Keywords: Iterative method, relay feedback test, phase margin, gain margin, auto-tuning

1 Introduction

Recently, a great attention has been given to the data-driven controller design methods without or with little use of models. These methods use the experimental closed-loop data and do not suffer from unmodeled dynamics like the model-based approaches. They can generally be applied to the slowly time-varying systems as well. The Iterative controller tuning methods are usually based on the minimization of a time domain criterion. The main drawback is the effect of the measurement noise and the disturbances. The real-time experiments are commonly time consuming and expensive. On the other hand, the robustness specifications cannot be explicitly included into the criterion. In this paper, a frequency criterion is proposed which will be minimized iteratively. The frequency criterion is defined as the weighted sum of squared errors between the desired and measured gain margin, phase margin and crossover frequency. Thus the robustness of the closed -loop system is directly taken into account in the criterion. The method benefits from specific feedback relay tests to determine the gain margin, the phase margin and the crossover frequency of the closed-loop system. In each experiments only the amplitude and the phase of the loop transfer function are measured. These experiments are shorter than the conventional experiments for controller tuning and are less sensitive to the measurement noise and disturbances.

The main contribution of this paper is the use of Bode's integrals for the estimation of the gradient and the Hessian of the frequency criterion. The Bode's integrals [1] show the relation between the phase and the amplitude of minimum phase stable systems. It was shown in [5] how these integrals can be used to approximate the derivatives of the amplitude and the phase of a system with respect to frequency at a given frequency. It is interesting to notice that the approximation is made only with the knowledge of the amplitude and the phase of the system at the given frequency and the system static gain. In this paper the results will be extended to the estimation of the derivatives of the amplitude and phase of the system with respect to the controller parameters. As a result, with a simple relay test, not only the phase margin and the crossover frequency are identified (criterion evaluation) but also the gradient and the Hessian of

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the frequency criterion are approximated with no parametric model of the plant. The simulations and the experimental results show the fast convergence of the proposed iterative method.

An iterative procedure for gain and phase margin adjustment for a PI controller was presented in [2]. The approach uses similar relay tests, but the gradient of the criterion is not calculated and an ad hoc algorithm is used instead. As a result the algorithm converges much slower than the proposed method.

The paper is organized as follows: In Section 2 several relay feedback tests will be presented for measuring the gain margin, phase margin and the crossover frequency and an adaptive algorithm is used to improve the precision of the measurements. Section 3 shows how the Bode's integrals can be used for the estimation of the derivatives of amplitude and phase with respect to the frequency. The iterative tuning method for desired phase margin and crossover frequency is presented in Section 4 and is extended to adjust the gain margin in Section 5. A simulation example is given in Section 6 and the experimental results for a three-tank system are presented in Section 7. Finally, Section 8 gives some concluding remarks.

2 Relay feedback test

The relay method is a well-known technique to identify useful points on the Nyquist curve by generating an appropriate oscillation with a relay feedback. This section recalls some schemes to measure the critical frequency (the frequency at which the phase of the loop is $-\pi$), gain margin, phase margin and crossover frequency.

In the standard relay method the controller is replaced by an on-off relay in closed-loop. Fig. 1 depicts the standard relay test where $G(s)$ represents the transfer function of the process. For many systems

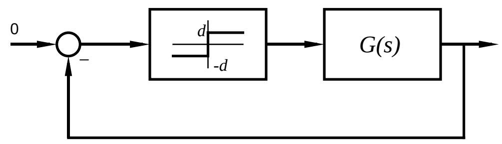

Figure 1: Standard relay method

there will be an oscillation where the control signal is a square wave and the process output is close to a sinusoid. Since the process attenuates higher harmonics, it is usually sufficient to consider only the first harmonic component of the relay output. Then the relay with the output amplitude d can be described by a simple gain $N(a) = \frac{4d}{\pi a}$, where a is the amplitude of the signal at the relay input. This gain is called the describing function. The condition for obtaining a limit cycle can be given simply by $N(a)G(j\omega) = -1$. It means that an oscillation may occur if there is an intersection between the two curves $G(j\omega)$ and $\frac{-1}{N(a)}$ in the Nyquist plot. Since the function $\frac{-1}{N(a)}$ is real, an oscillation may occur if the Nyquist curve intersects the negative real axis. The amplitude and the frequency of the limit cycle obtained give, respectively, the point where the Nyquist curve intersects the negative real axis and its corresponding frequency.

2.1 Gain Margin Measurement

Relay feedback can also be applied to closed-loop systems including a linear controller. Fig. 2 shows an experiment where the output of the relay is the reference signal for the closed-loop system with an existing controller $K(s)$ and the output of the closed-loop system is fed back to the relay input.

The condition for obtaining a limit cycle is given by

$$\frac{K(j\omega)G(j\omega)}{1 + K(j\omega)G(j\omega)} = -\frac{\pi a}{4d}$$

which is equivalent to

$$K(j\omega)G(j\omega) = -\frac{\pi a}{4d + \pi a} \in (-1, 0)$$

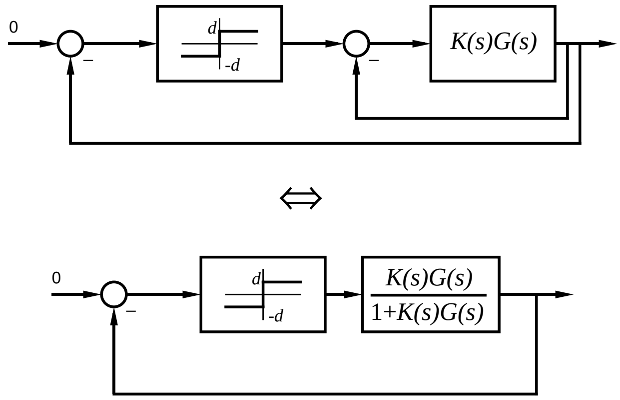

Figure 2: Relay experiment for gain margin measurement

The identified point, obtained from the amplitude of the limit cycle, is on the intersection of the open-loop Nyquist curve with the segment $(-1, 0)$. The gain margin is defined as the inverse of the amplitude of the identified point.

2.2 Phase margin measurement

The experiment proposed in [9] and shown in Fig. 3 generates a limit cycle at the crossover frequency of the open-loop transfer function $K(j\omega)G(j\omega)$. The phase margin can be approximated by the amplitude and frequency of the generated limit cycle [6].

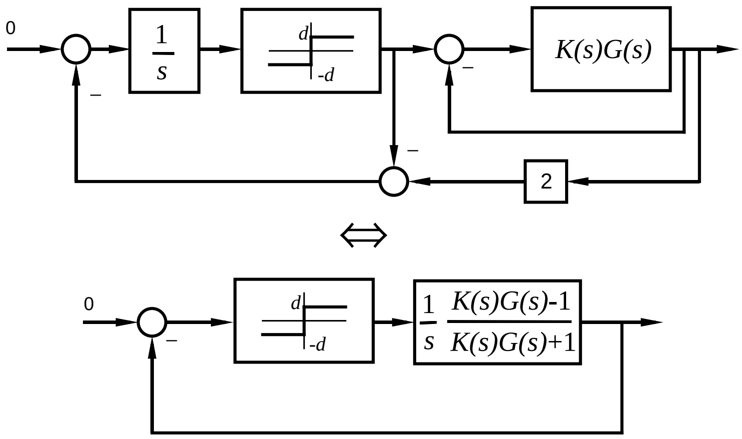

Figure 3: Relay experiment for phase margin measurement

Consider Figure 3. The condition for obtaining a limit cycle is given by the following equation :

$$\frac{1}{j\omega}\frac{K(j\omega)G(j\omega)-1}{K(j\omega)G(j\omega)+1} = -\frac{\pi a}{4d} \tag{1}$$

which is equivalent to

$$K(j\omega)G(j\omega) = \frac{1-j\frac{\omega\pi a}{4d}}{1+j\frac{\omega\pi a}{4d}} \tag{2}$$

The term on the right hand side of Eq. (2) has a modulus equal to 1. This means that the identified point is on the intersection of the open-loop transfer function with the unit circle. If $K(j\omega)G(j\omega)$ intersects the unit circle several times, the phase margin is defined by the closest point to -1. This point corresponds to the limit cycle with the largest amplitude a which is stable.

Let ω_c be the measured limit cycle frequency (equal to the crossover frequency) and a_c the relay input amplitude. Then the phase of the identified point can be computed as follows:

$$\angle K(j\omega_c)G(j\omega_c) = -2 \ \arctan(\frac{\pi a_c \omega_c}{4d})$$

The phase margin Φ_m is then given by

$$\Phi_m = \pi - 2 \ \arctan(\frac{\pi a_c \omega_c}{4d})$$

2.3 Improving the approximation

The proposed methods for measuring the gain and phase margins by the relay method have the drawback that the method of the describing function must be used for the identification of the frequencies and the points on the Nyquist curve of interest. This approach is approximative. The assumption that the process attenuates higher harmonics is not always guaranteed especially for the phase margin measurement scheme, where the relative degree of the transfer function $\frac{1}{s}\frac{K(s)G(s)-1}{K(s)G(s)+1}$ is 1. Experiments have shown relative errors between the measurements and the real values of the crossover frequencies and phase margins of about 30%. To improve the precision of the approximations, an adaptive identification method of the critical gain proposed in [8] can be used. In this method the relay is replaced by a saturation nonlinearity and a time varying gain k. Thus the describing function of the nonlinear part is the interval $(-\infty, -1/k]$ on the negative real axis. As $k > 0$ approaches infinity, the system behaves like a standard relay feedback so the system oscillates. As k is decreased gradually from this value, the point $-1/k$ of the describing function moves toward the critical point, i.e. the intersection of $G(j\omega)$ with the negative real axis. At the same time the output of the saturation block $u(t)$ becomes smaller and smaller and approaches a sinusoidal function. If k is smaller than the critical gain K_u there will be no intersection between $G(j\omega)$ and the Nyquist curve of the describing function of the saturation block, and the oscillations will be damped in a finite time. Thus K_u may be identified by tuning k using the scheme of Fig. 4. The absolute value of the error between the input and output of the saturation block indicates whether the output of the block is saturated or not. Thus the scheme uses the following rule: if the difference $|e(t)-u(t)|$ is nonzero, decrease k, otherwise increase k. So the next equation can be proposed for the time varying gain.

$$\frac{dk}{dt} = -\delta \left| e(t) - u(t) \right| + \varepsilon$$

where $\delta > 0$ and ε is a small positive number. The feedback system with this tuning rule is illustrated in Figs 4, where $|.|$ stands for absolute value. The convergence of the identification procedure is guaranteed theoretically [8]. For sufficiently small ε, $k(t)$ converges to a steady state in the vicinity of K_u, and the frequency of the oscillation corresponds to the critical frequency.

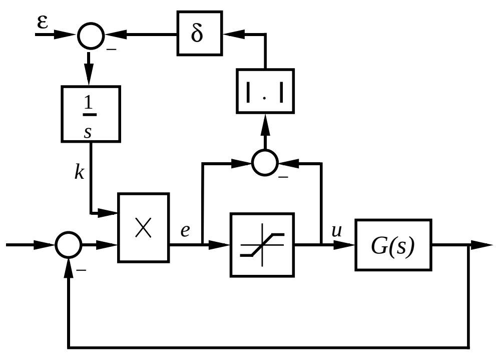

Figure 4: Adaptive measurement of the critical gain

This method can also be applied to closed-loop systems in order to determine the gain and phase margins as well as the critical and crossover frequencies of the open-loop transfer function $K(s)G(s)$ by replacing the relay in Fig. 2 and 3 with the saturation and the time varying gain.

3 Bode's integrals

The relations between the phase and the amplitude of a stable minimum-phase system have been investigated for the first time by Bode [1]. The results are based on Cauchy's residue theorem and have been extensively used in network analysis. Two integrals are presented in this section. The first one, which is well known in the control engineering field, shows the relation between the phase of the system at each frequency as a function of the derivative of its amplitude. But the second integral has been used in the control design for the first time in [5]. The integral shows how the amplitude of the system at each frequency is related to the derivative of the phase and the static gain of the system.

3.1 Derivative of amplitude

Bode has shown in [1] that for a stable minimum-phase transfer function $G(j\omega)$, the phase of the system at ω_0 is given by:

$$\angle G(j\omega_0) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{d\ln|G(j\omega)|}{d\nu} \ln\coth\frac{|\nu|}{2} d\nu \tag{3}$$

where $\nu = \ln\frac{\omega}{\omega_0}$. Since $\ln\coth\frac{|\nu|}{2}$ decreases rapidly as ω deviates from ω_0, the integral depends mostly on $\frac{d\ln|G(j\omega)|}{d\nu}$ (the slope of the Bode plot) near ω_0. Therefore, assuming that the slope of the Bode plot is almost constant in the neighborhood of ω_0, $\angle G(j\omega_0)$ can be approximated by:

$$\angle G(j\omega_0) \approx \frac{1}{\pi}\left.\frac{d\ln|G(j\omega)|}{d\nu}\right|_{\omega_0}\int_{-\infty}^{+\infty} \ln\coth\frac{|\nu|}{2} d\nu = \frac{\pi}{2}\left.\frac{d\ln|G(j\omega)|}{d\nu}\right|_{\omega_0} \tag{4}$$

This property is often used in loop shaping where the slope of the amplitude Bode plot at crossover frequency is limited to -20dB/decade in order to obtain approximately a phase margin of 90°. Here the measured phase of the system at ω_0 is used to determine approximately the slope of the amplitude Bode plot (s_a):

$$s_a(\omega_0) = \left.\frac{d\ln|G(j\omega)|}{d\nu}\right|_{\omega_0} = \omega_0\left.\frac{d\ln|G(j\omega)|}{d\omega}\right|_{\omega_0} \approx \frac{2}{\pi}\angle G(j\omega_0) \tag{5}$$

3.2 Derivative of phase

The second Bode's integral shows that the amplitude of a stable minimum-phase system can be determined uniquely from its phase and its static gain. More precisely, the logarithm of the system amplitude at ω_0 is given by [1]:

$$\ln|G(j\omega_0)| = \ln|K_g| - \frac{\omega_0}{\pi}\int_{-\infty}^{+\infty} \frac{d(\angle G(j\omega)/\omega)}{d\nu} \ln\coth\frac{|\nu|}{2} d\nu \tag{6}$$

where K_g is the static gain of the plant. In the same way, assuming that $\angle G(j\omega)/\omega$ is linear (in a logarithmic scale) in the neighborhood of ω_0, one has:

$$\ln|G(j\omega_0)| \approx \ln|K_g| - \frac{\omega_0}{\pi}\left.\frac{d(\angle G(j\omega)/\omega)}{d\nu}\right|_{\omega_0}\frac{\pi^2}{2} \tag{7}$$

$$\ln|G(j\omega_0)| \approx \ln|K_g| - \frac{\pi\omega_0^2}{2}\left[\frac{1}{\omega_0}\left.\frac{d\angle G(j\omega)}{d\omega}\right|_{\omega_0} - \frac{\angle G(j\omega_0)}{\omega_0^2}\right] \tag{8}$$

which gives the slope of the Bode phase plot at ω_0:

$$s_p(\omega_0) = \left.\frac{d\angle G(j\omega)}{d\nu}\right|_{\omega_0} = \omega_0\left.\frac{d\angle G(j\omega)}{d\omega}\right|_{\omega_0} \approx \angle G(j\omega_0) + \frac{2}{\pi}[\ln|K_g| - \ln|G(j\omega_0)|] \tag{9}$$

Note that for the systems containing an integrator, the static gain cannot be computed. For such systems, the static gain of the system without the integrator should be estimated and used in the above formula (note that the phase of the integrator is constant and its derivative is zero).

4 Iterative procedure for phase margin adjustment

First of all, a performance criterion in the frequency domain is defined as follows:

$$J(\rho) = \frac{1}{2}[\lambda_1(\omega_c - \omega_d)^2 + \lambda_2(\Phi_m - \Phi_d)^2] \tag{10}$$

where ρ is the vector of the controller parameters, λ_1 and λ_2 are weighting factors, ω_c and ω_d are respectively the measured and desired crossover frequencies, and Φ_m and Φ_d are the measured and desired phase margins. Then the controller parameters minimizing the criterion are obtained by the iterative Gauss-Newton formula:

$$\rho_{i+1} = \rho_i - \gamma_i R^{-1} J'(\rho_i) \tag{11}$$

where i is the iteration number, γ_i is a positive scalar representing the step size, R is a square positive definite matrix of dimension n_ρ and $J'(\rho)$ is the gradient of the criterion with respect to ρ. This algorithm converges to the vector of parameters minimizing the criterion, provided that the step size is properly chosen. The matrix R can be chosen equal to the identity matrix or to the Hessian of the criterion to obtain faster convergence.

The gradient of the criterion is given by:

$$J'(\rho) = \lambda_1(\omega_c - \omega_d)\frac{\partial \omega_c}{\partial \rho} + \lambda_2(\Phi_m - \Phi_d)\Phi'_m \tag{12}$$

where Φ'_m is the derivative of the phase margin with respect to ρ computed through the chain rule (note that Φ_m is a function of ρ and ω_c):

$$\Phi'_m = \frac{\partial \Phi_m}{\partial \rho} + \left.\frac{\partial \Phi_m}{\partial \omega}\right|_{\omega_c} \frac{\partial \omega_c}{\partial \rho} \tag{13}$$

Now replacing Φ_m in the above equation by $\angle L(j\omega_c) + \pi$ gives (where $L(j\omega) = K(j\omega)G(j\omega)$):

$$\Phi'_m = \frac{\partial \angle L(j\omega_c)}{\partial \rho} + \left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_c} \frac{\partial \omega_c}{\partial \rho} \tag{14}$$

The first term in the above equation is equal to $\partial \angle K(j\omega_c)/\partial \rho$ which is completely known at each iteration. Furthermore one has:

$$\left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_c} = \left.\frac{\partial \angle K(j\omega)}{\partial \omega}\right|_{\omega_c} + \left.\frac{\partial \angle G(j\omega)}{\partial \omega}\right|_{\omega_c} = \left.\frac{\partial \angle K(j\omega)}{\partial \omega}\right|_{\omega_c} + \frac{s_p(\omega_c)}{\omega_c} \tag{15}$$

Again, the first term is completely known and $s_p(\omega_c)$ can be approximated using Eq. (9). Now, it only remains to compute $\partial \omega_c/\partial \rho$. For this purpose, we use the fact that the loop gain at ω_c in each iteration is equal to 1, therefore its derivative (or derivative of its logarithm) with respect to ρ will be zero. Then one has:

$$\frac{\partial \ln |L(j\omega_c)|}{\partial \rho} + \left.\frac{\partial \ln |L(j\omega)|}{\partial \omega}\right|_{\omega_c} \frac{\partial \omega_c}{\partial \rho} = 0 \tag{16}$$

but from the Bode's integral (see Eq. (5)) one has:

$$\left.\frac{\partial \ln |L(j\omega)|}{\partial \omega}\right|_{\omega_c} \approx \frac{2\angle L(j\omega_c)}{\pi \omega_c} = \frac{2(\Phi_m - \pi)}{\pi \omega_c} \tag{17}$$

Thus $\partial\omega_c/\partial\rho$ can be approximated as follows (note that $\partial \ln|L(j\omega)|/\partial\rho = \partial \ln|K(j\omega)|/\partial\rho$):

$$\frac{\partial\omega_c}{\partial\rho} \approx -\frac{\pi\omega_c}{2(\Phi_m - \pi)} \frac{\partial \ln|K(j\omega_c)|}{\partial\rho} \tag{18}$$

Notice that the gradient of the criterion is computed using only the measured crossover frequency ω_c and the measured amplitude and phase of the plant. In the same way, the Hessian of the criterion can be approximated without any additional information as follows:

$$H = J''(\rho) = \lambda_1 \frac{\partial\omega_c}{\partial\rho}(\frac{\partial\omega_c}{\partial\rho})^T + \lambda_2 \Phi'_m(\Phi'_m)^T + \lambda_1(\omega_c - \omega_d)\frac{\partial^2\omega_c}{\partial\rho^2} + \lambda_2(\Phi_m - \Phi_d)\Phi''_m \tag{19}$$

The last two terms can be neglected because they are small especially in the neighborhood of the final solution. In addition, this simplification makes the Hessian always positive which fixes the numerical problems normally encountered in the iterative Newton algorithm. The use of the Hessian matrix in the iterative formula (Eq. 11) instead of R significantly improves the convergence speed.

$$R = H \approx \lambda_1 \frac{\partial\omega_c}{\partial\rho}(\frac{\partial\omega_c}{\partial\rho})^T + \lambda_2 \Phi'_m(\Phi'_m)^T \tag{20}$$

5 Iterative procedure for phase and gain margins adjustment

In the previous section, the controller is designed using only the information on one frequency point of the plant. Although this approach is very fast and simple and works well for the majority of the industrial processes, it may not work appropriately for some complex high-order systems. This means that the specifications may be satisfied just locally at the crossover frequency and it is possible that the behavior of the system changes drastically elsewhere, leading for example to a very small gain margin. For such systems, it may be preferable to tune the phase and gain margins at the same time. However, this necessitates two relay experiments in each iteration: one relay test to measure the gain margin and the other for the phase margin. A frequency criterion can be defined as follows:

$$J(\rho) = \frac{1}{2}[\lambda_1(\omega_c - \omega_d)^2 + \lambda_2(\Phi_m - \Phi_d)^2 + \lambda_3(K_u - K_d)^2] \tag{21}$$

where $K_u = |L(j\omega_u)|$ is the amplitude of the loop transfer function where it intersects the negative real axis ($\angle L(j\omega_u) = -\pi$). ω_u is the critical loop frequency, K_d is the inverse of the desired gain margin and λ_1, λ_2 and λ_3 are weighting factors. The gradient of the criterion is given by:

$$J'(\rho) = \lambda_1(\omega_c - \omega_d)\frac{\partial\omega_c}{\partial\rho} + \lambda_2(\Phi_m - \Phi_d)\Phi'_m + \lambda_3(K_u - K_d)K'_u \tag{22}$$

where K'_u is the derivative of the critical loop gain with respect to ρ computed as follows:

$$K'_u = \frac{\partial|L(j\omega_u)|}{\partial\rho} + \left.\frac{\partial|L(j\omega)|}{\partial\omega}\right|_{\omega_u} \frac{\partial\omega_u}{\partial\rho} \tag{23}$$

The first term is equal to $|G(j\omega_u)| \cdot \partial|K(j\omega_u)|/\partial\rho$ which can be easily computed at each iteration. The second term can be approximated using the Bode's integrals in a similar way as is done for the phase margin. First the derivative of the amplitude of the loop gain is computed using Eq. (5) as follows:

$$\left.\frac{\partial|L(j\omega)|}{\partial\omega}\right|_{\omega_u} = |L(j\omega_u)| \left.\frac{\partial \ln|L(j\omega)|}{\partial\omega}\right|_{\omega_u} \approx K_u \frac{2\angle L(j\omega_u)}{\pi\omega_u} = -\frac{2K_u}{\omega_u} \tag{24}$$

Then $\partial\omega_u/\partial\rho$ is computed using the fact that in each iteration $\angle L(j\omega_u) = -\pi$ and consequently its derivative with respect to ρ will be zero, which gives:

$$\frac{\partial\angle L(j\omega_u)}{\partial\rho} + \left.\frac{\partial\angle L(j\omega)}{\partial\omega}\right|_{\omega_u} \frac{\partial\omega_u}{\partial\rho} = 0 \tag{25}$$

The first term in the above equation is equal to $\partial \angle K(j\omega_u)/\partial \rho$ which can be computed knowing the controller at each iteration. Furthermore one has:

$$\left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_u} = \left.\frac{\partial \angle K(j\omega)}{\partial \omega}\right|_{\omega_u} + \left.\frac{\partial \angle G(j\omega)}{\partial \omega}\right|_{\omega_u} \tag{26}$$

Again, knowing the controller at each iteration, the first term in the right hand side can be computed while the second term is approximated using the Bode's integral of Eq. (9). Therefore $\partial \omega_u/\partial \rho$ is given by:

$$\frac{\partial \omega_u}{\partial \rho} = -\left(\left.\frac{\partial \angle K(j\omega)}{\partial \omega}\right|_{\omega_u} + \frac{s_p(\omega_u)}{\omega_u}\right)^{-1} \frac{\partial \angle K(j\omega_u)}{\partial \rho} \tag{27}$$

In a similar way an approximation of the Hessian can also be computed as follows:

$$R = H \approx \lambda_1 \frac{\partial \omega_c}{\partial \rho}(\frac{\partial \omega_c}{\partial \rho})^T + \lambda_2 \Phi_m'(\Phi_m')^T + \lambda_3 K_u'(K_u')^T \tag{28}$$

6 Simulation example

Now let us consider a complex plant model including a time delay as follows:

$$G(s) = \frac{e^{-0.3s}}{(s^2+2s+3)^3(s+3)} \tag{29}$$

This model was proposed in [7] and represents a difficult problem to be solved using a PID controller. It should be noted that neither the traditional Ziegler-Nichols method nor a more advanced method based on a first-order plant model [3] can give a stabilizing controller for this plant. In the following, it is shown that a PID controller which confers appropriate specifications on the closed-loop system can be tuned with the iterative phase and gain margins adjustment method. This method requires two experiments per iteration and is more convenient for high-order complex systems. However, one should note that the experimental time for the gain margin measurement can be chosen to be smaller than that of the phase margin, because the critical frequency is appreciably larger than the crossover frequency.

The specifications are set for this example at $\omega_d = 0.2$ rad/s for the crossover frequency, and $\Phi_d = 70°$ and $K_d = \frac{1}{3}$ for the phase and for the inverse of the gain margin, respectively. The initial controller is designed using the Kappa-Tau tuning rule proposed in [4] which gives the following stabilizing controller:

$$K(s) = 4.5\,(1 + \frac{1}{0.41s} + 0.033s) \tag{30}$$

The performance criterion of Eq. (21) is used and the weighting factors are chosen such that almost the same weighting be obtained for the three parts of the criterion:

$$\lambda_1 = \frac{1}{\omega_d^2} \qquad \lambda_2 = \frac{1}{\Phi_d^2} \qquad \lambda_3 = \frac{1}{K_d^2}$$

Two closed-loop experiments are performed to measure a crossover frequency of 0.139 rad/s, a phase margin of 78.5° and a gain margin of 4.39, which give a performance criterion of 0.104. With these values, the gradient and Hessian are calculated, and a new controller is obtained. After one other iteration the following controller is obtained:

$$K(s) = 4.93\,(1 + \frac{1}{0.316s} + 0.125s) \tag{31}$$

with a performance criterion of 0.0017. This controller gives 0.1997 rad/s for the crossover frequency, 66.0° for the phase margin and 2.97 for the gain margin. It should be noted that no significant improvement can be achieved with further iterations. A comparison of the closed-loop performance between the initial

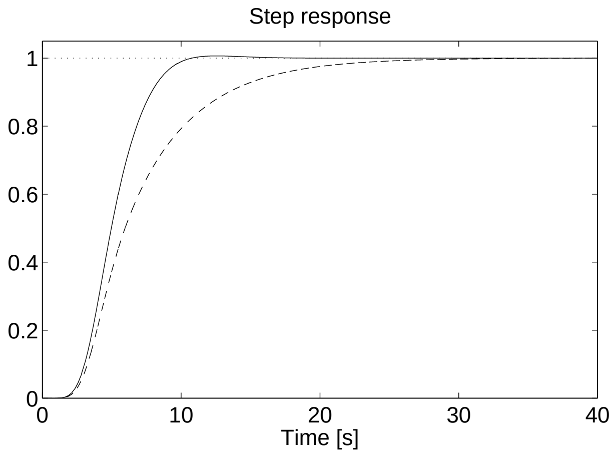

Figure 5: Step response of the closed-loop system (dashed line: Kappa-Tau, solid line: proposed)

controller and the final one in Fig. 5 shows that a much smaller settling time is achieved with an overshoot of only 0.7 %.

It is worth mentioning that even if the estimation errors of s_a might be large due to the presence of an unknown pure time delay in the plant model, the criterion converges rapidly enough. The reason is that in the Gauss-Newton algorithm the quadratic criterion will converge to its minimum value even with an approximative gradient.

7 Real-time experiment

In this section, experimental results obtained with a three-tank system (Fig. 6) are presented. The process consists of three cylinders T1, T2 and T3 which are interconnected in series by two connecting pipes. Two pumps, driven by DC motors, supply the tanks T1 and T2 with the liquid (water) collected in a reservoir. The water level in each tank is measured by a piezo-resistive pressure transducer. The first tank is equipped with a manually adjustable valve which lets the water outflow at rate Q_{out} to the reservoir. The input u of the process is an electrical signal which controls the flow rate Q_1 into the first tank. The output y of the process is the voltage of the piezo-resistive pressure transducer of T2, which is proportional to the level L with a negative gain. The second pump acts as an output disturbance to the system. The control objective is to maintain the level of T2 at a desired value. Due to nonlinear relations between the input flow rate Q_1, the output flow rate Q_{out} and the tanks' levels, the transfer function of the considered system is nonlinear.

Consider, that the system operates at a set point corresponding to $Q_1 = \frac{Q_{1\,\max}}{2}$, where $Q_{1\,\max}$ is the maximum flow rate of the pump. For this operating point, an initial controller is designed using the Kappa-Tau tuning rule proposed in [4] based on the step response of the system. The resulting PID controller is:

$$K_{\kappa\tau}(s) = 29.3\,(1 + \frac{1}{20.84s} + 4.72s) \tag{32}$$

It should be noted that the Ziegler-Nichols method gives an unstable controller for this operating point. After measuring a static gain of about 1, a closed-loop experiment as proposed in Section 2.3 identifies for this configuration a phase margin of 64° and a crossover frequency of 0.097 rad/s. In order to improve the stability of the closed-loop system and reduce the control effort, new specifications with an increased phase margin Φ_d and a reduced crossover frequency ω_d compared with the actual ones, are defined: $\Phi_d = 80°$, $\omega_d = 0.08$ rad/s. The ratio between the integral time and the derivative time is chosen to be constant ($T_d = 0.25T_i$). After choosing the weighting factors $\lambda_1 = \frac{1}{\omega_d^2}$ and $\lambda_2 = \frac{1}{\Phi_d^2}$ to obtain almost the same weightings for the crossover frequency and phase margin, the gradient and the Hessian can be calculated and a new controller is obtained after the first iteration:

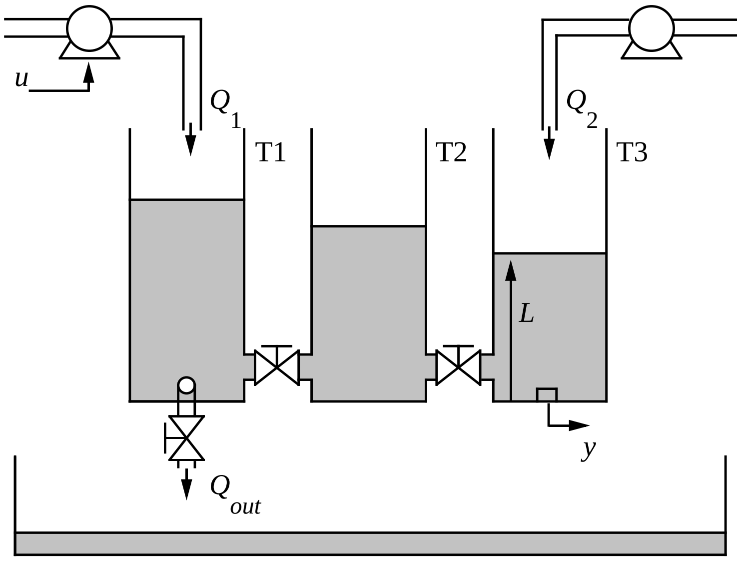

Figure 6: Schematic diagram of the three-tank system

$$K(s) = 20.4\,(1 + \frac{1}{31.5s} + 7.88s) \tag{33}$$

The closed-loop test applied to the system with the controller in Eq.(33) gives the following estimates:

$$\Phi_m = 86.2° \qquad \omega_m = 0.085\text{rad/s}$$

As the estimated values are close to the desired ones, there is no need for further iterations. A comparison between the time response of the closed-loop system with the Kappa-Tau controller and the proposed one is shown in Fig. 7. The step response is normalized and the output disturbance concerns a constant flow rate Q_2 for the pump 2 applied at $t = 160s$. It can be seen that the proposed controller gives a much better performance for the closed-loop system in terms of disturbance rejection, overshoot and settling time.

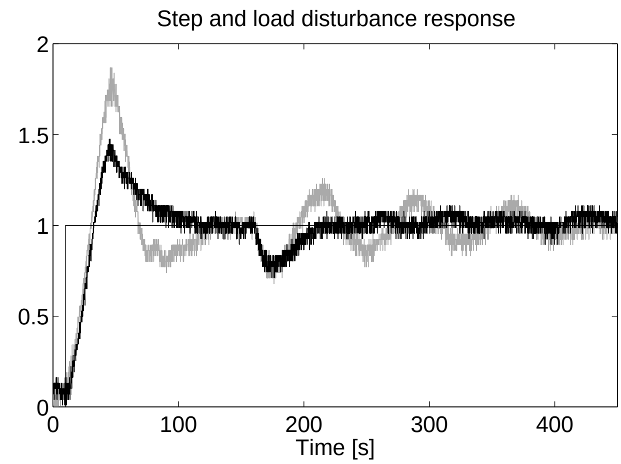

Figure 7: Normalized step and disturbance response of the closed-loop system (grey line: Kappa-Tau, black line: proposed)

Now, consider that the operating point of the system is changed and, due to the system nonlinearity, the closed-loop performance deteriorates. Consequently, the same controller at the new operating point does not meet the specification (a phase margin $\Phi_m = 55°$ and a crossover frequency $\omega_m = 0.066$ rad/s are obtained) and the controller should be retuned. After one iteration, using the proposed tuning method, the following controller is obtained:

$$K(s) = 23\,(1 + \frac{1}{42.3s} + 10.6s) \tag{34}$$

The closed-loop system with this controller gives the following performances which are very close to the desired ones:

$$\Phi_m = 73.7° \qquad \omega_m = 0.0791\text{rad/s}$$

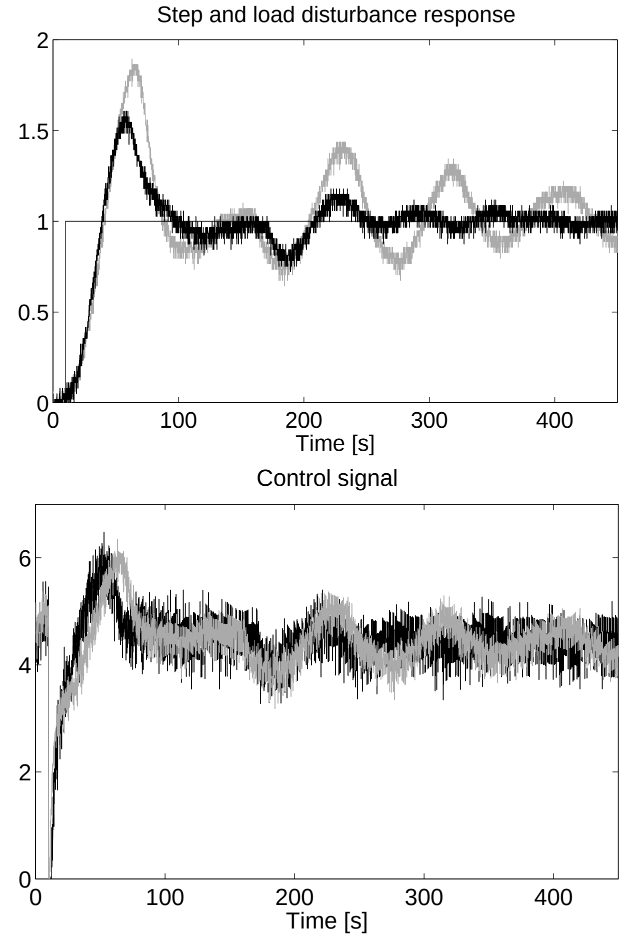

Figure 8: Normalized step and disturbance response of the closed-loop system (grey line: controller (33), black line: controller (34))

A comparison of the normalized step response and disturbance response of the closed-loop system with the two controllers is shown in Fig. 8. The effectiveness and rapidity of the proposed algorithm is shown in Fig. 9 for an auto-tuning experiment. During the first 500 seconds the initial controller is implemented on the real system and measurements are performed. Then the controller is updated and a new test is carried out for measuring the phase margin as well as the crossover frequency.

As shown in this section, the proposed iterative tuning algorithm is fast enough to be used for the auto-tuning of real processes, and is particularly appropriate for readjusting the controller parameters of systems whose operating point changes slowly.

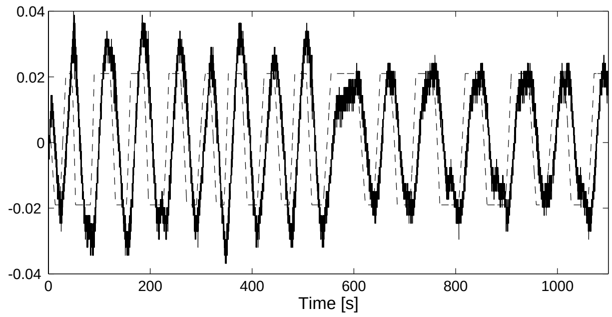

Figure 9: Auto-tuning experiment (dashed: generated reference signal of the closed-loop system, solid: output of the closed-loop system)

8 Conclusions

An iterative approach for tuning the controller parameters with specifications on gain margin, phase margin and crossover frequency was proposed. This approachtakes advantage of the Bode's integrals to estimate the gradient and the Hessian of a frequency criterion and therefore requires no parametric model of the plant. The proposed iterative approach converges in a few iterations to the minimum of the frequency criterion and can be used for auto-tuning of industrial plants. Simulation and experimental results show the effectiveness of the proposed method to design and tuning of the PID controllers. However the proposed method is not restricted to the PID controllers and can be applied to other type of controllers.

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