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A new physics

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Abstract

This session considers the application of mathematics from control theory to several persistent mysteries at the foundations of physics where interconnected, multiscale systems issues arise. In addition to the ubiquity of power laws in natural and man-made systems, these include a new view of turbulence in highly sheared flows that results from design for drag minimization, the origin of macroscopic dissipation and thermodynamic irreversibility in microscopically reversible dynamics, the universality of quantum gates for quantum computing, decoherence minimization in quantum systems, and entanglement witnessing. The latter ones are problems at the heart of several important tasks such as quantum computing, teleportation and quantum key distribution. Much of the original motivation for a new science of complexity came from the hope that methods of theoretical physics could contribute to a theory of complex engineering and biological networks and systems. This collection of work shows that apparently exactly the opposite is true. The role that robust control methods play in this research will be the central theme of this paper, around which the other issues will be woven. The aim is not to provide a control-friendly rederivation of known results in physics, but rather to illustrate through representative examples, how exciting new results and important insight, as assessed by physicists themselves, can be obtained through the mathematics and methods that the control community has developed.Since this work is largely being published in the scientific literature, the controls community may be largely unaware of these developments.

1 Overview of the session

The next two papers deal with problems arising in the context of fluid mechanics and statistical mechanics for which classical physics fails to provide a simple explanation: (i) the transition from laminar to turbulent flow in highly sheared fluids and (ii) the question of how microscopic conservative, time-reversible dynamic laws give rise to apparently irreversibly laws at the macroscopic level. As shown in the paper by Bobba et al, a compelling explanation of many aspects of turbulent shear flows can be obtained by analyzing the disturbance amplification properties of the linearized Navier-Stokes equations. The theoretical tools (input-output operators, induced norms, etc.) are at the heart of modern robust control theory, and provide another stunning example of the many advantages of a system-theoretic viewpoint. This paper then extends that analysis to a global, nonlinear treatment of the dominant structures found from the linearized 3d analysis.

The paper, by Sznaier, Doherty and Barahona, argues that apparently irreversible dynamics can (and perhaps should) be understood as an artifact of incomplete observations over a finite horizon. These results are illustrated with a simple example consisting of an harmonic oscillator in a thermal bath, where it is shown that, by adopting a model reduction approach one can furnish new bounds on the number of bath oscillators required to capture the behavior of the system over a finite horizon. Surprisingly, this number is far lower than what conventional wisdom in physics dictates, opening up the door for substantially more efficient tools for the analysis and design of systems consisting of a collection of complex interconnected Hamiltonian dynamics.

The remaining papers of the session are concerned with the use of systems motivated tools to address several problems arising in the context of Quantum Mechanics. A remarkable feature here is that the use of these tools allows not only for solving some hitherto open problems, but also opens the door for new practical applications of Quantum Mechanics with far reaching technological implications.

A common theme on the papers by D'Alessandro and Khaneja is the exploration of the connections between Geometric Control Theory and several "controllability type" problems in Quantum Mechanics, i.e., the ability to drive the state of a quantum system from a given initial condition to a final desired state. This problem is of special relevance to Quantum Computing and Nuclear Magnetic Resonance applications.

The paper by D'Alessandro surveys different aspects of

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optimal and geometric control notions in QM, emphasizing the role of Lie algebra rank conditions for controllability. The rich $SU(n)$ group structure in quantum dynamics enables surprising reductions in the computational burden to check the controllability conditions, when compared to standard nonlinear systems. The potential of this approach is illustrated by the fact that it not only allows for proving that almost every pair of quantum logic gates is universal, but also assures the existence of an upper bound on the number of stages required to perform any unitary operation.

The approach by Khaneja is concerned with achieving the transition between given quantum states in a time-optimal fashion, while minimizing decoherence effects, a practical concern in Quantum Computing and NMR spectroscopy applications. As shown there, this problem can be recast as as one of finding sub-Riemannian geodesics–geodesics where the tangent to the path is constrained to a given set of admissible directions–in a given space. Moreover, these geodesics can be explicitly characterized. These results are illustrated with a practical example arising in the context of NMR spectroscopy, concerning the time optimal sequence of pulses required to transfer coherence in a three coupled spin $\frac{1}{2}$ nuclei.

Finally, the last paper of the session, by Parrilo, Doherty and Spedalieri, addresses the so-called *entanglement* question: to determine whether or not the state of a given multipartite quantum system can be written as a mixture of tensor products of the states of its subsystems. The entanglement problem is not only of crucial theoretical importance–it is at the heart of Bell's inequalities– but is also the key to many applications impossible in a classical setup, such as teleportation and quantum computation. As shown in the paper, entanglement can be verified by considering a hierarchy of convex conditions that can be checked via semi-definite programming, providing a complete generalization of previously available results.

2 Historical perspective

Erwin Schrodinger's 1994 essay *What is Life?* is often regarded as the quintessential reductionist manifesto and as a prophecy about the ultimate triumph of molecular biology. What is often less highlighted is that Schrodinger's central theme was that biology demanded a "new physics." It is interesting to re-examine Schrodinger's largely ignored central thesis in the light of recent developments in both "post-genomic biology" and control theory, and particularly with respect to the persistent mysteries that remain at the heart of fluid, statistical, and quantum mechanics. The remainder of this paper emphasizes the connection between control theory and these persistent mysteries in physics and the challenges facing research in complex biological and technological networks.

A central problem in "post-genomic biology" is reverse and forward engineering the dynamics and control of intracellular networks of genes and proteins. These interact through intercellular chemical and mechanical signaling to direct both development and regulate an organism's response to its environment. Similar challenges exist in understanding the more global regulatory strategies that maintain organism and ecosystem homeostasis. These complex networks can overwhelm intuition and informal models and thus modeling and simulation methods are playing an increasingly central role, inspired by computer-aided engineering and scientific computation. One important lesson can and must be drawn from the history of these areas: brute-force modeling and computation has no chance of succeeding for systems with the type of complexity of a biological cell, let alone an organism or ecosystem. Thus the success of post-genomic biology will be contingent upon the development of new algorithms and methodologies guided by rigorous mathematical theory.

A corresponding challenge in "better, cheaper, faster" engineering is to create robust, reliable systems using more virtual and less physical prototyping, and greater component reuse. Hard problems include multiscale integration of electrical, mechanical, and chemical subsystems. Furthermore, the verification of complex engineering systems with embedded software closely parallels the robustness analysis of complex biological developmental and regulatory pathways controlled by "embedded" computational networks of genes and proteins.

Two great abstractions of 20th century engineering are that control, communications, and computing could be developed 1) largely separately from each other, and 2) independently of the details of physical substrates. This horizontal and vertical isolation of systems held both in practical applications and in academic research. It facilitated massively parallel, wildly successful, explosive growth in both mathematical theory and technology, but left many fundamental problems unresolved and a poor foundation for future systems of systems in which these elements must be integrated. While the search for "unified theories" both of systems and of multiscale physics has been an appealing intellectual challenge for decades, it has only recently become both an urgent technological challenge, and a tangibly reachable research objective. New research, hinted at in this session, offers not only a theoretical research direction of unprecedented promise, but also one that has already proven promising and already useful in a wide variety of practical applications, including biological regulatory networks in signal transduction, metabolism, and gene regulation, shear flow turbulence, networking protocols, global optimization, forest ecology, and financial market volatility.

3 HOT

One unifying theme in this work has been the concept of Highly Optimized Tolerance (HOT) ([2]-[7]). There are now a number of papers on HOT in the physics literature, so this paper will only briefly review the main motivation, and the central role that robustness plays. HOT systems arise when deliberate robust design aims for a specific level of tolerance to uncertainty. The resulting "robust, yet fragile" features of HOT systems are high performance and high throughput, but potentially serious sensitivities to design flaws and unanticipated or rare events, particularly those that can cause catastrophic, cascading failures. The literature on HOT ([2]-[7]) has focused on contrasting it with the orthodox views of complexity loosely organized around such rubrics as Complex Adaptive Systems (CAS), New Science of Complexity (NSOC), Chaoplexity, and more specifically Self-Organized Criticality (SOC) and Edge-of-Chaos (EOC). They emphasize so-called "emergent phenomena," including power law distributions, self-similarity, and fractals, and describe systems of interest as adaptive, self-organizing, far-from-equilibrium, nonlinear, heterogeneous, and so on. While there are many differences, from the perspective of this paper *they are quite minor*, and thus we will lump these various approaches all together under the acronym CCC (for Chaos, Criticality, and Complexity).

The foundations of the CCC approach are that 1) emergent complexity occurs between states of order and disorder characterized by phase transitions and bifurcations in otherwise largely generic interconnections of components, and 2) computer simulations of such generic interconnections with even very simple models of components can reveal the essential nature of this emergent complexity. These claims are not just different but exactly opposite from HOT. CCC researchers are inspired by phase transitions and critical phenomena, fractals, self-similarity, pattern formation, and self-organization in statistical physics, and bifurcations and deterministic chaos from dynamical systems. Motivating examples vary from equilibrium statistical mechanics of interacting spins on a lattice to the spontaneous formation of spatial patterns in systems far from equilibrium, such as Raleigh-Benard convection cells and certain driven chemical reactions. Favorite model systems include percolation lattices, cellular automata, random boolean networks, and various networks of interacting agents.

What the HOT perspective shares with CCC is that there are universal and important features of complex systems that transcend the details of specific domains. It is in what those universal features are and what mathematical theory and methods are most relevant that we come to not only different, but essentially exactly opposite, conclusions. CCC has its origins in the physics of simple systems that can produce apparently complex phenomena, coupled with the technology of computer simulation, also of simple systems. We are inspired by the Internet and other engineering networks as well as biological networks, and claim that these are radically different in their nature than random networks of simple components. HOT's origins are in the mathematics of control, communications, and computing systems, but translated into the language and models of CCC. CCC focuses on what happens when a few parameters are adjusted in an otherwise random configuration, whereas we focus on systems in which design or evolution has effectively resulted in the fine tuning of *entire protocols*, which in turn confer the extreme robustness that we observe.

It is becoming increasingly clear that robustness and complexity in biology, ecology, technology, and social systems are so intertwined that they must be treated in a unified way. Engineering theories of controls, communications and computing have matured in recent decades, facilitating the creation of systems of bewildering complexity, but relying on technology and mathematics only understand by experts, and fragmented into narrow technical disciplines. Interestingly, all involve theories of robustness, including feedback control of uncertain systems, error-correcting codes, software verification, and high-confidence algorithms. Advanced technologies have also been used for automated, high throughput assays that add to the molecular biologist's evidence that biological networks are at least as complex as those in engineering, and perhaps much more so. In all cases it is now clear that complexity is driven by robustness, and it is unlikely that creation of a "science of complexity" is possible without an emphasis on robustness.

Through design or evolution, complex systems in engineering and biology develop highly structured, elaborate internal configurations, with layers of feedback and signaling. This makes them robust to the uncertainties in their environment and components for which such complexity was selected, but also makes the resulting system potentially vulnerable to rare or unanticipated perturbations. Such fragility can lead to large cascading failures from tiny initiating events. Perturbation of one gene or a single line of software code, or the introduction of a novel parasite, an exotic specie, or trace amounts of a toxin, rarely causes significant system-wide impact, yet occasionally can cascade into complete system failure. This "robust, yet fragile" character is overwhelmingly the most important feature of a complexity "phenotype," and is not an accident. The corresponding complexity "genotype" of highly structured, elaborate, nongeneric, heterogeneous, far-from-equilibrium, internal configurations, with layers of feedback regulation, communication, signaling, and protocols, is largely driven by the need to create barriers, in state space, to cascading failure events. Thus, genotype and phenotype can co-evolve with their environment to create a spiral of increasing complexity, more finely tuned for expanding robustness, but with more extreme fragilities.

It is widely recognized that in complex engineering systems there is often a severe tradeoff between nominal performance in an ideal environment with perfect components, and the more practical need for robust performance in an uncertainty environment with real, hence uncertain, components. Indeed, the complexity of most systems is driven far more by robustness considerations than by nominal performance. While there are obviously fundamental differences between biology and engineering, the design and evolution processes and the resulting system-level characteristics may be much less different than often realized. High performance robust systems must have certain highly structured features involving signaling and feedback. This is largely independent of the design process, whether it be deliberate or random mutation and natural selection. Highly complex engineering systems are very new, far from optimal, and heavily constrained by both historical and nontechnical considerations. Biological "design" involves pure trial and error, but at least the "primitive" biosphere of microorganisms has had billions of years of evolution and appears to be highly optimized and extraordinarily robust. As we better understand the role of complexity and robustness, the more they appear to use the same system-level regulatory strategies as engineering systems

Only a few hundred genes are required for minimal life in an idealized laboratory environment, but free-living bacteria often have many thousands of genes, most of which are not lethal knockouts in laboratory conditions. This is because most genes code for sensors, actuators, and the complex regulatory networks that control them, and thus confer to the cell robustness to variations rather than the mere basic functionality required for survival in ideal circumstances. Our central claim is that the essence of this robustness, and hence of biological as well as engineering complexity, is the elaboration of highly structured mechanisms that create barriers to cascading failure events.

The extreme "robust, yet fragile" character of complex systems severely complicates the challenge of connecting phenomena on widely different time and space scales, and in particular, exactly those phenomena most critical to understanding and preventing large cascading events. A consequence is that the "typical" behavior of complex systems is often quite simple, so that a naive view leads to simple models and (wrong) explanations of most phenomena. Much of the original motivation for CCC came from the hope that methods of theoretical physics, developed to understand the simplest, most basic aspects of nature, could contribute to a theory of complex engineering and biological networks and systems. An irony of this session and the work that it represents is that perhaps exactly the opposite is true. In particularly, the mathematics of control theory promises to contribute substantially to the foundations of theoretical physics, where complex, multiscale phenomena is involved.

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