Proceedings of the 42nd IEEE
Conference on Decision and Control
Maui, Hawaii USA, December 2003

Robust Adaptive Tracking Control for Time-varying Nonlinear Systems with Higher Order Relative Degree

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Abstract

This paper deals with a controller design problem for time-varying nonlinear systems with nonparametric uncertainties. A robust adaptive tracking control for uncertain time-varying nonlinear systems with higher order relative degree will be proposed based on the high-gain adaptive output feedback and backstepping strategies. The proposed method is useful in the case where only the output signal is available.

1 INTRODUCTION

A nonlinear system is said to be OFEP (output feedback exponentially passive) [1] if there exists an output feedback such that the resulting closed-loop system is exponentially passive. The sufficient conditions for the nonlinear system to be OFEP are that (1) the system be globally exponential minimum-phase, (2) the system has relative degree of 1 and (3) the nonlinearities of the system satisfy the Lipschitz condition. Under these conditions, there exists a static output feedback such that the resulting closed-loop system is exponentially passive[1]. It has also been shown that one can stabilize uncertain nonlinear systems with OFEP property by a high-gain feedback based adaptive control with simple structure [2], [4]. Since the control methods utilize only the output signal in order to design the controller and have strong robustness with respect to bounded disturbances in spite of its simple structure, these methods are considered powerful control tools for uncertain nonlinear systems. Unfortunately the OFEP conditions give very severe restrictions to practical applications of the above-mentioned adaptive schemes because most practical systems do not satisfy the OFEP condition.

With this problem in mind, some alleviation methods to the OFEP condition have been proposed [3], [4], [6]. The method by [3] and [4] alleviated the OFEP condition by introducing a parallel feedforward compensator (PFC) in parallel to the controlled system. Although this method can solve the restriction for relative degree, since the controller is designed for an augmented controlled system with PFC, the bias error from the PFC output may remain. The method by [6] is a robust control scheme for non-OFEP systems with nonlinear uncertainties but the method was for systems with relative degree of 1.

In this paper, we will propose a robust adaptive tracking control, which is based on high-gain feedback based adaptive control, for a class of uncertain time-varying nonlinear systems with higher order relative degree. We extend the robust adaptive control method by [6] to uncertain time-varying nonlinear systems with higher order relative degree by utilizing a *backstepping* strategy. It is shown that if the upper limit of uncertain nonlinearities can be evaluated by a function of the output signal then one can design an adaptive controller by using only the output signal without a state observer.

2 PROBLEM STATEMENT

We consider the following nth order time-varying nonlinear system with relative degree of n.

$$\begin{aligned} \dot{x}_i &= f_i(\boldsymbol{x},t) + g_i(t)x_{i+1} \ (1 \le i \le n-1) \\ \dot{x}_n &= f_n(\boldsymbol{x},t) + g_n(t)u \\ y &= x_1 \end{aligned} \quad (1)$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^T \in R^n$ is the state variable, u and $y \in R$ are the input and output, respectively. $f_i(\boldsymbol{x},t)$ are uncertain nonlinearities and $g_i(t), (1 \le i \le n)$ are unknown time-varying functions. We assume that the uncertainties satisfy the following assumptions.

Assumptions: (A-1) Uncertain nonlinear functions $f_i(\boldsymbol{x},t)$ can be evaluated for all $x \in R^n$ and $t \in R^+$ by

$$|f_i(\boldsymbol{x},t)| \le d_{1i}|\psi_i(y)| + d_{0i} \ (1 \le i \le n) \quad (2)$$

with unknown positive constants d_{1i}, d_{0i} and a known smooth function $\psi_i(y)$ which has the following property for any variable y_1 and y_2:

$$|\psi_i(y_1+y_2)| \le |\psi_{1i}(y_1,y_2)||y_1| + |\psi_{2i}(y_2)| \quad (3)$$

with a known smooth function ψ_{1i} and a function ψ_{2i} which is bounded for all bounded y_2.
(A-2) Unknown functions $g_i(t)$ are smooth and bounded with bounded derivatives for any $t \ge 0$, and are positive functions such that $g_i(t) \ge g_{0i} > 0$ with positive constants g_{0i}.

The control objective is to achieve the goal:

$$\lim_{t\to\infty} |y(t) - y^*(t)| \le \delta \tag{4}$$

under the assumptions (A-1) and (A-2) for a given δ and a command signal $y^*(t)$ such that $|y^*(t)| \le \beta_0$, $|\dot{y}^*(t)| \le \beta_1$ with positive constants β_0 and β_1.

3 ADAPTIVE CONTROLLER DESIGN

3.1 Virtual System

In order to design a robust adaptive controller based on the high-gain feedback strategy, we must first consider the virtual controlled system by introducing a virtual control input filter.

Consider the $(n-1)$th order virtual filter:

$$\begin{aligned} \dot{u}_{f_i} &= -\lambda_i u_{f_i} + u_{f_{i+1}} \ (1 \le i \le n-2) \\ \dot{u}_{f_{n-1}} &= -\lambda_{n-1} u_{f_{n-1}} + u \\ \lambda_i &> 0, \ (1 \le i \le n-1) \end{aligned} \tag{5}$$

The virtual system, which is obtained by considering u_{f1} given from a virtual filter as the control input, can be represented by the following form with appropriate variable transformation using filtered signals u_{fi}.

$$\begin{aligned} \dot{y}(t) &= a(y,\boldsymbol{\eta},t) + g_{1,n}(t)u_{f_1} + f_1(y,\boldsymbol{\eta},t) \\ \dot{\boldsymbol{\eta}}(t) &= \boldsymbol{q}(y,\boldsymbol{\eta},t) + \boldsymbol{F}(y,\boldsymbol{\eta},t) \end{aligned} \tag{6}$$

where $y = x_1, \boldsymbol{\eta}^T = [\eta_2, \cdots, \eta_n]$ and η_i are given as follows:

For $n = 2$; $\quad \eta_2 = \bar{g}_2 x_2 - \chi_{2,1}\bar{g}_{1,2}x_1 - u_{f_1}$ (7)

For $n \ge 3$ $(3 \le k \le n-1)$;

$$\begin{aligned} \eta_2 &= \bar{g}_{2,n}x_2 - \chi_{2,2}\bar{g}_{1,n}x_1 - u_{f_1} \\ \eta_k &= \bar{g}_{k,n}x_k - \sum_{l=3}^{k} \chi_{k,l-1}\bar{g}_{k-l+2,n}x_{k-l+2} \\ &\quad -\chi_{k,k}\bar{g}_{1,n}x_1 - u_{f_{k-1}} \\ \eta_n &= \bar{g}_n x_n - \sum_{i=1}^{n-2} \chi_{n,i}\bar{g}_{n-i,n}x_{n-i} \\ &\quad -\chi_{n,n-1}\bar{g}_{1,n}x_1 - u_{f_{n-1}} \end{aligned} \tag{8}$$

with

$$\begin{aligned} \chi_{1,1} &= 0, \ \chi_{i,1} = \lambda_{i-1} + g_{i,n}\dot{\bar{g}}_{i,n} \ (2 \le i \le n) \\ \chi_{n,n} &= -\lambda_{n-1}\chi_{n,n-1}\bar{g}_{1,n} - \dot{\chi}_{n,n-1}\bar{g}_{1,n} - \chi_{n,n-1}\dot{\bar{g}}_{1,n} \\ \chi_{i,2} &= \sum_{k=2}^{n} \chi_{k,1} - \sum_{k=2}^{i} \chi_{k-1,1} \ (2 \le i \le n-1, \ n \ge 3) \\ \chi_{i,i+1} &= -\lambda_{i-1}\chi_{i,i}\bar{g}_{1,n} + \chi_{i+1,n-1}\bar{g}_{1,n} \\ &\quad -\dot{\chi}_{i,i}\bar{g}_{1,n} - \chi_{i,i}\dot{\bar{g}}_{1,n} \ (2 \le i \le n-1, \ n \ge 3) \\ \chi_{n,i} &= -\lambda_{n-1}\chi_{n,i-1} - \dot{\chi}_{n,i-1} \\ &\quad -\chi_{n,i-1}g_{n+1-i,n}\dot{\bar{g}}_{n+1-i,n} \\ &\quad (2 \le i \le n-1, \ n \ge 3) \\ \chi_{k,i} &= -\lambda_{k-1}\chi_{k,i-1} + \chi_{k+1,n+i-k-2} \\ &\quad -\dot{\chi}_{k,i-1} - \chi_{k,i-1}g_{n+1-i,n}\dot{\bar{g}}_{n+1-i,n} \\ &\quad (3 \le k \le n-1, \ 3 \le i \le n-1, \ n \ge 4) \end{aligned} \tag{9}$$

where

$$g_{m,n}(t) := \prod_{i=m}^{n} g_i(t) \ , \ \bar{g}_{m,n}(t) := \frac{1}{g_{m,n}(t)} \tag{10}$$

and $g_n = g_{n,n}, \bar{g}_n = \bar{g}_{n,n}$. Further, a, $\boldsymbol{q}$ and $\boldsymbol{F}$ are given by the following form:

For $n = 2$

$$a(y,\boldsymbol{\eta},t) = \chi_{2,1}y + g_{1,2}\eta_2 \tag{11}$$

$$q(y,\eta_2,t) = \chi_{2,2}y - \lambda_1\eta_2 \tag{12}$$

$$F(y,\eta_2,t) = \bar{g}_2 f_2 - \chi_{2,1}\bar{g}_{1,2}f_1 \tag{13}$$

For $n \ge 3$

$$a(y,\boldsymbol{\eta},t) = \chi_{2,2}y + g_{1,n}\eta_2 \tag{14}$$

$$\boldsymbol{q}(y,\boldsymbol{\eta},t) = \begin{pmatrix} \chi_{2,3} \\ \chi_{3,4} \\ \vdots \\ \chi_{l,l+1} \\ \vdots \\ \chi_{n-1,n} \\ \chi_{n,n} \end{pmatrix} y + T\boldsymbol{\eta} \tag{15}$$

$$\boldsymbol{F}(y,\boldsymbol{\eta},t) = \begin{pmatrix} \bar{g}_{2,n}f_2 - \chi_{2,2}\bar{g}_{1,n}f_1 \\ \vdots \\ \bar{g}_{l,n}f_l - \sum_{k=0}^{l-2} \chi_{l,l-k}\bar{g}_{k+1,n}f_{k+1} \\ \vdots \\ \bar{g}_{n-1,n}f_{n-1} - \sum_{k=0}^{n-3} \chi_{n-1,n-k-1}\bar{g}_{k+1,n}f_{k+1} \\ \bar{g}_n f_n - \sum_{k=1}^{n-1} \chi_{n,k}\bar{g}_{n-k,n}f_{n-k} \end{pmatrix} \tag{16}$$

$$T = \begin{pmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & -\lambda_{i-1} & \ddots & 0 \\ \vdots & & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & 0 & -\lambda_{n-1} \end{pmatrix} \tag{17}$$

For the obtained virtual system, it is easy to confirm that $\boldsymbol{q}(y,\boldsymbol{\eta},t)$ and $a(y,\boldsymbol{\eta},t)$ are bounded with respect to time t and Lipschitz with respect to $y, \boldsymbol{\eta}$ so that there exist positive constants L_1 and L_2 such that

$$||\boldsymbol{q}(y_1,\boldsymbol{\eta}_1) - \boldsymbol{q}(y_2,\boldsymbol{\eta}_2)|| \le L_1(|y_1 - y_2| + ||\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2||) \tag{18}$$

$$|a(y_1,\boldsymbol{\eta}_1) - a(y_2,\boldsymbol{\eta}_2)| \le L_2(|y_1 - y_2| + ||\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2||) \tag{19}$$

Further, the uncertain vector function $\boldsymbol{F}(y,\boldsymbol{\eta},t)$ can be evaluated from assumption (A-1) by

$$||\boldsymbol{F}(y,\boldsymbol{\eta},t)|| \le \sum_{i=1}^{M_2} p_i|\phi_i(y)| + p_0 \tag{20}$$

with unknown positive constants p_i and p_0 and a known smooth function $\phi_i(y)$ which has the following property for any variable y_1 and y_2:

$$|\phi_i(y_1+y_2)| \le |\phi_{1i}(y_1,y_2)||y_1| + |\phi_{2i}(y_2)| \quad (21)$$

with a known smooth function ϕ_{1i} and a function ϕ_{2i} which is bounded for all bounded y_2.

The obtained virtual controlled system (6) has the relative degree of 1 and the zero-dynamics given by (12) or (15), which is obtained by a nominal system with f_1 and $\boldsymbol{F}$ as disturbances, is exponentially stable so that from the converse theorem of Lyapunov [5], there exists a positive definite function $W(\boldsymbol{\eta})$ and positive constants κ_1 to κ_4 such that

$$\begin{aligned} \frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} \boldsymbol{q}(0,\boldsymbol{\eta}) \ &\le\ -\kappa_1 ||\boldsymbol{\eta}(t)||^2,\ ||\frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}}|| \le \kappa_2 ||\boldsymbol{\eta}(t)|| \\ \kappa_4 ||\boldsymbol{\eta}(t)||^2 &\le ||W(\boldsymbol{\eta})|| \le \kappa_3 ||\boldsymbol{\eta}(t)||^2 \end{aligned} \quad (22)$$

3.2 Controller Design

We first design a virtual input for the virtual system (6) through a robust adaptive output feedback control. The actual control input will be designed through the backstepping procedure as follows:

Step 1: Defining $\nu(t) = y(t) - y^*(t)$ as the tracking error, the virtual controlled system (6) can be rewritten as the following error system

$$\begin{aligned} \dot{\nu}(t) &= a(\nu + y^*, \boldsymbol{\eta}, t) + g_{1,n} u_{f_1} + f_1(\nu + y^*, \boldsymbol{\eta}, t) - \dot{y}^* \\ \dot{\boldsymbol{\eta}}(t) &= \boldsymbol{q}(\nu + y^*, \boldsymbol{\eta}, t) + \boldsymbol{F}(\nu + y^*, \boldsymbol{\eta}, t) \end{aligned} \quad (23)$$

We design a virtual control input α_1 for the input u_{f1} in this error system (23) by

$$\alpha_1(t) = -[k(t)\nu(t) + u_R(t)] \quad (24)$$

$$k(t) = k_I(t) + k_P(t) \quad (25)$$

$$\dot{k}_I(t) = \gamma_I \nu(t)^2 - \sigma_I k_I(t),\ k_I(0) \ge 0 \quad (26)$$

$$k_P(t) = \sum_{i=1}^{M_2} \gamma_{pi} \phi_{1i}(\nu, y^*)^4 \nu(t)^2 \quad (27)$$

$$u_R(t) = \gamma_R \psi_1(y)^2 \nu(t) \quad (28)$$

where γ_I, γ_{pi}, $\gamma_R$and σ_I are arbitrary positive constants. Here consider a positive definite function $V_0(\nu, \boldsymbol{\eta}, k)$:

$$V_0 = \mu_0 W(\boldsymbol{\eta}) + \frac{1}{2}\mu_1 \nu(t)^2 + \mu_1 \frac{g_m}{2\gamma_I}[k_I(t) - k^*]^2 \quad (29)$$

where μ_0 and μ_1 are any positive constants and k^* is an ideal feedback gain to be determined later. g_m is a positive constant such that $0 < g_m \le g_{1,n}$.

The time derivative of $V_0(\nu, \boldsymbol{\eta}, k)$ along the trajectories (23) and (26) yields that

$$\begin{aligned} \frac{dV_0}{dt} \ =\ & \mu_0 \frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} [\boldsymbol{q}(\nu + y^*, \boldsymbol{\eta}, t) + \boldsymbol{F}(\nu + y^*, \boldsymbol{\eta}, t)] \\ & + \mu_1 \nu [a(\nu + y^*, \boldsymbol{\eta}, t) + f_1(\nu + y^*, \boldsymbol{\eta}, t) - \dot{y}^*] \\ & + \mu_1 \nu [-g_{1,n}\{k\nu + u_R(t, \nu + y^*)\}] \\ & + \mu_1 g_{1,n} \nu (u_{f_1} - \alpha_1) \\ & + \mu_1 \frac{g_m}{\gamma_I}[k_I - k^*][\gamma_I \nu^2 - \sigma_I k_I] \end{aligned} \quad (30)$$

It follows from assumptions (A-1), (A-2) and (18) to (22) that we can evaluate the time derivative of V_0 by

$$\begin{aligned} \frac{dV_0}{dt} \le\ & -\mu_0\kappa_1||\boldsymbol{\eta}||^2 + \mu_0\kappa_2||\boldsymbol{\eta}||L_1(|\nu| + \beta_0) \\ & + \mu_0\kappa_2||\boldsymbol{\eta}|| \left[\sum_{i=1}^{M_2} p_i\{|\phi_{1i}(\nu, y^*)||\nu| + |\phi_{2i}(y^*)|\} \right] \\ & + \mu_0\kappa_2||\boldsymbol{\eta}||p_0 + \mu_1 L_2(|\nu| + |y^*| + ||\boldsymbol{\eta}||)|\nu| \\ & + \mu_1|\nu|d_{11}|\psi_1(\nu + y^*)| + \mu_1 d_{01}|\nu| + \mu_1\beta_1|\nu| \\ & - \mu_1 g_{1,n} k\nu^2 - \mu_1 g_m k^* \nu^2 + \mu_1 g_m k_I \nu^2 \\ & - \mu_1 g_{1,n}\gamma_R\psi_1(\nu + y^*)^2\nu^2 - \mu_1\frac{g_m}{\gamma_I}\sigma_I[k_I - k^*]k^* \\ & + \mu_1 g_{1,n}\nu(u_{f_1} - \alpha_1) - \mu_1\frac{g_m}{\gamma_I}\sigma_I[k_I - k^*]^2 \end{aligned} \quad (31)$$

Further since we have

$$-\mu_1 g_{1,n} k\nu^2 + \mu_1 g_m k_I \nu^2 \le -\mu_1 g_m k_P \nu^2 \quad (32)$$

from the fact that $k(t) \ge 0$ obtained from (26) and (27), and since we also have

$$\begin{aligned} & -\mu_1 g_{1,n}\gamma_R\psi_1(\nu + y^*)^2\nu^2 \\ & \le\ -\mu_1 g_m \gamma_R \psi_1(\nu + y^*)^2\nu^2, \end{aligned} \quad (33)$$

we obtain from (31) that

$$\begin{aligned} \frac{dV_0}{dt} \le\ & -\left[\mu_0\kappa_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2}\rho_{3i} \right] ||\boldsymbol{\eta}||^2 \\ & - [\mu_1 g_m k^* - k_0]|\nu|^2 + \mu_1 g_{1,n}\nu(u_{f_1} - \alpha_1) \\ & + \frac{\mu_1^2}{4\rho_4}(L_2\beta_0 + d_{01} + \beta_1)^2 + \frac{1}{4\rho_5}(\frac{\mu_1 g_m \sigma_I}{\gamma_I})^2 k^{*2} \\ & - \mu_1 g_m \frac{\sigma_I}{\gamma_I}(1 - \frac{\rho_5\gamma_I}{\mu_1 g_m \sigma_I})[k_I - k^*]^2 + \frac{\mu_1}{4 g_m \gamma_R} d_{11}^2 \\ & + \frac{(\mu_0\kappa_2)^2}{4\rho_2}\left[L_1\beta_0 + \sum_{i=1}^{M_2} p_i|\phi_{2i}(y^*)| + p_0 \right]^2 \\ & + \sum_{i=1}^{M_2} \frac{1}{4\mu_1 g_m \gamma_{pi}} \left[\frac{(\mu_0\kappa_2 p_i)^2}{4\rho_{3i}} \right]^2 \end{aligned} \quad (34)$$

with any positive constants ρ_1 to ρ_5 and

$$k_0 = \mu_1 L_2 + \rho_4 + \frac{1}{4\rho_1}(\mu_0\kappa_2 L_1 + \mu_1 L_2)^2 \quad (35)$$

Finally the time derivative of V_0 can be evaluated by

$$\begin{aligned} \dot{V}_0 \le\ & -\left[\mu_0\kappa_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2}\rho_{3i} \right] ||\boldsymbol{\eta}||^2 \\ & - [\mu_1 g_m k^* - k_0]|\nu|^2 - \mu_1 g_m \frac{\sigma_I}{\gamma_I}(1 - \rho_5')[k_I - k^*]^2 \\ & + \mu_1 g_{1,n}\nu\omega_1 + R_0 \end{aligned} \quad (36)$$

where $\omega_1 = u_{f1} - \alpha_1$, $\rho_5' = \frac{\rho_5 \gamma_I}{\mu_1 g_m \sigma_I}$ and

$$R_0 = \frac{\mu_1}{4 g_m \gamma_R} d_{11}^2 + \frac{\mu_1^2}{4\rho_4}(L_2\beta_0 + d_{01} + \beta_1)^2 + \frac{(\mu_0\kappa_2)^2}{4\rho_2}\left[L_1\beta_0 + \sum_{i=1}^{M_2} g_i \phi_{2iM} + g_0\right]^2 + \frac{\mu_1 g_m \sigma_I}{4\rho_5' \gamma_I} k^{*2} + \sum_{i=1}^{M_2} \frac{1}{4\mu_1 g_m \gamma_{pi}}\left[\frac{(\mu_0\kappa_2 p_i)^2}{4\rho_{3i}}\right]^2.$$

ϕ_{2iM} in R_0 is a positive constant such that $|\phi_{2i}(y^*)| \le \phi_{2iM}$. Since y^* is bounded, such a constant exists from the assumption that $\phi_{2i}(y_2)$ is bounded for all bounded y_2.

Step 2: In *step 2*, we consider an error system, ω_1-system, between u_{f_1} and α_1. ω_1-system is given from (5) by

$$\dot{\omega}_1 = -\lambda_1 u_{f_1} + u_{f_2} - \dot{\alpha}_1 \tag{37}$$

where

$$\begin{aligned}\dot{\alpha}_1 &= \frac{\partial \alpha_1}{\partial y}\dot{y} + \frac{\partial \alpha_1}{\partial y^*}\dot{y}^* + \frac{\partial \alpha_1}{\partial k_I}\dot{k}_I \\ &= \frac{\partial \alpha_1}{\partial y}[\chi_{2,2} y + g_{1,n}\eta_2 + f_1 + g_{1,n} u_{f_1}] \\ &\quad + \frac{\partial \alpha_1}{\partial y^*}\dot{y}^* + \frac{\partial \alpha_1}{\partial k_I}[\gamma_I \nu^2 - \sigma_I k_I]\end{aligned} \tag{38}$$

For this ω_1-system, we design the virtual input α_2 for u_{f_2} as follows:

$$\begin{aligned}\alpha_2 &= -c_1\omega_1 + \lambda_1 u_{f_1} - \epsilon_{10}\nu^2\omega_1 - \epsilon_{1,1}(\frac{\partial \alpha_1}{\partial y})^2 y^2 \omega_1 \\ &\quad -\epsilon_{1,2}(\frac{\partial \alpha_1}{\partial y})^2\omega_1 - \epsilon_{1,3}(\frac{\partial \alpha_1}{\partial y})^2 {u_{f_1}}^2 \omega_1 \\ &\quad -\epsilon_{1,4}(\frac{\partial \alpha_1}{\partial y})^2 \psi_1^2 \omega_1 - \epsilon_{1,5}(\frac{\partial \alpha_1}{\partial y})^2\omega_1 \\ &\quad -\epsilon_{1,6}(\frac{\partial \alpha_1}{\partial y^*})^2\omega_1 + \frac{\partial \alpha_1}{\partial k_I}(\gamma_I\nu^2 - \sigma_I k_I)\end{aligned} \tag{39}$$

where c_1, ϵ_{10} and $\epsilon_{1,1}$ to $\epsilon_{1,6}$ are any positive constants.

Consider the following positive definite function V_1 for the obtained ω_1-system.

$$V_1 = \frac{1}{2}\omega_1^2 + V_0 \tag{40}$$

The time derivative of V_1 is obtained by

$$\begin{aligned}\dot{V}_1 &= \omega_1(\dot{u}_{f_1} - \dot{\alpha}_1) + \dot{V}_0 \\ &= \omega_1\Big[-\lambda_1 u_{f_1} + \omega_2 + \alpha_2 - \frac{\partial \alpha_1}{\partial y}\big(\chi_{2,2} y + g_{1,n}\eta_2 \\ &\quad + f_1 + g_{1,n}u_{f_1}\big) - \frac{\partial \alpha_1}{\partial y^*}\dot{y}^* - \frac{\partial \alpha_1}{\partial k_I}[\gamma_I\nu^2 - \sigma_I k_I]\Big] \\ &\quad + \dot{V}_0\end{aligned} \tag{41}$$

where $\omega_2 = u_{f_2} - \alpha_2$.

Since $g_{1,n}$ and $\chi_{2,2}$ are bounded from assumption (A-2), we can evaluate the time derivative of V_1 by applying α_2 given in (39) as follows:

$$\begin{aligned}\dot{V}_1 &\le -c_1\omega_1^2 + \omega_1\omega_2 + \frac{g_M^2}{4\epsilon_{1,2}}|\eta_2|^2 + R_1 \\ &\quad - \left[\mu_0\kappa_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2}\rho_{3i}\right]||\eta||^2 \\ &\quad - [\mu_1 g_m k^* - k_0]|\nu|^2 \\ &\quad - \mu_1 g_m \frac{\sigma_I}{\gamma_I}(1 - \rho_5')[k_I - k^*]^2 + R_0\end{aligned} \tag{42}$$

with positive constants g_M and χ_M such as $g_{1,n} \le g_M$ and $\chi_{2,2} \le \chi_M$ and

$$R_1 = \frac{(\mu_1 g_M)^2}{4\epsilon_{10}} + \frac{\chi_M^2}{4\epsilon_{1,1}} + \frac{g_M^2}{4\epsilon_{1,3}} + \frac{d_{11}^2}{4\epsilon_{1,4}} + \frac{d_{01}^2}{4\epsilon_{1,5}} + \frac{\beta_1^2}{4\epsilon_{1,6}}$$

Step i ($3 \le i \le n-1$)**:** In *step i*, we consider the error system, ω_{i-1}-system, between $u_{f_{i-1}}$ and α_{i-1}, as is in *step 2*, where $\omega_i = u_{f_i} - \alpha_i$. ω_{i-1}-system is given from (5) by

$$\dot{\omega}_{i-1} = -\lambda_{i-1}u_{f_{i-1}} + u_{f_i} - \dot{\alpha}_{i-1} \tag{43}$$

where

$$\dot{\alpha}_{i-1} = \frac{\partial \alpha_{i-1}}{\partial y}\dot{y} + \frac{\partial \alpha_{i-1}}{\partial y^*}\dot{y}^* + \frac{\partial \alpha_{i-1}}{\partial k_I}\dot{k}_I + \sum_{k=1}^{i-2}\frac{\partial \alpha_{i-1}}{\partial u_{f_k}}\dot{u}_{f_k} \tag{44}$$

For this ω_{i-1}-system, the virtual input α_i for u_{f_i} is designed by

$$\begin{aligned}\alpha_i &= -c_{i-1}\omega_{i-1} - \omega_{i-2} \\ &\quad + \lambda_{i-1}u_{f_{i-1}} - \epsilon_{i-1,1}(\frac{\partial \alpha_{i-1}}{\partial y})^2 y^2 \omega_{i-1} \\ &\quad -\epsilon_{i-1,2}(\frac{\partial \alpha_{i-1}}{\partial y})^2\omega_{i-1} - \epsilon_{i-1,3}(\frac{\partial \alpha_{i-1}}{\partial y})^2 u_{f_1}^2\omega_{i-1} \\ &\quad -\epsilon_{i-1,4}(\frac{\partial \alpha_{i-1}}{\partial y})^2\psi_1^2\omega_{i-1} - \epsilon_{i-1,5}(\frac{\partial \alpha_{i-1}}{\partial y})^2\omega_{i-1} \\ &\quad -\epsilon_{i-1,6}(\frac{\partial \alpha_{i-1}}{\partial y^*})^2\omega_{i-1} + \frac{\partial \alpha_{i-1}}{\partial k_I}(\gamma_I\nu^2 - \sigma_I k_I) \\ &\quad + \sum_{k=1}^{i-2}\frac{\partial \alpha_{i-1}}{\partial u_{f_k}}(-\lambda_k u_{f_k} + u_{f_{k+1}})\end{aligned} \tag{45}$$

where c_{i-1} and $\epsilon_{i-1,1}$ to $\epsilon_{i-1,6}$ are any positive constants. Here we consider the following positive definite function V_{i-1}:

$$V_{i-1} = \frac{1}{2}\omega_{i-1}^2 + V_{i-2} \tag{46}$$

The time derivative of V_{i-1} is then given by

$$\begin{aligned}\dot{V}_{i-1} &= \omega_{i-1}\Big[-\lambda_{i-1}u_{f_{i-1}} + \omega_i + \alpha_i - \frac{\partial \alpha_{i-1}}{\partial y}\big(\chi_{2,2}y + g_{1,n}\eta_2 \\ &\quad + f_1 + g_{1,n}u_{f_1}\big) - \frac{\partial \alpha_{i-1}}{\partial y^*}\dot{y}^* - \frac{\partial \alpha_{i-1}}{\partial k_I}[\gamma_I\nu^2 - \sigma_I k_I] \\ &\quad - \sum_{k=1}^{i-2}\frac{\partial \alpha_{i-1}}{\partial u_{f_k}}(-\lambda_k u_{f_k} + u_{f_{k+1}})\Big] + \dot{V}_{i-2}\end{aligned} \tag{47}$$

It follows from the structure of α_i given in (45) that $\dot{V}_{i-1}$ can be evaluated as

$$\dot{V}_{i-1} \leq -\sum_{k=1}^{i-1} c_k\omega_k^2 + \omega_{i-1}\omega_i + \sum_{k=1}^{i-1}\frac{g_M^2}{4\epsilon_{k,2}}|\eta_2|^2 + \sum_{k=1}^{i-1} R_k$$
$$-\left[\mu_0\kappa_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2}\rho_{3i}\right]||\eta||^2$$
$$-\left[\mu_1 g_m k^* - k_0\right]|\nu|^2$$
$$-\mu_1 g_m \frac{\sigma_I}{\gamma_I}(1-\rho_5')[k_I - k^*]^2 + R_0 \quad (48)$$

where

$$R_k = \frac{\chi_M^2}{4\epsilon_{k,1}} + \frac{g_M^2}{4\epsilon_{k,3}} + \frac{d_{11}^2}{4\epsilon_{k,4}} + \frac{d_{01}^2}{4\epsilon_{k,5}} + \frac{\beta_1^2}{4\epsilon_{k,6}} \quad (49)$$

Step n: This is the final step. In *step n*, we design the actual control input as follows:

$$u = \alpha_n \quad (50)$$

using α_i given in (39) or (45).

In this final step, we finally consider the following positive definite function V:

$$V = V_{n-1} = \frac{1}{2}\omega_{n-1}^2 + V_{n-2} \quad (51)$$

The time derivative of V can be evaluated by

$$\dot{V} \leq -\sum_{k=1}^{n-1} c_k\omega_k^2 - \left[\mu_1 g_m k^* - k_0\right]|\nu|^2$$
$$-\left[\mu_0\kappa_1 - \rho_1 - \rho_2 - \sum_{n=1}^{M_2}\rho_{3i} - \sum_{k=1}^{n-1}\frac{g_M^2}{4\epsilon_{k,2}}\right]||\eta||^2$$
$$-\mu_1 g_m \frac{\sigma_I}{\gamma_I}(1-\rho_5')[k_I - k^*]^2 + R_T \quad (52)$$

using the same manner as in previous steps. Where $R_T = \sum_{k=0}^{n-1} R_k$.

Setting $\rho_1 = \rho_2 = \frac{\mu_0\kappa_1}{10}$, $\rho_{3i} = \frac{\mu_0\kappa_1}{10M_2}$ and $\rho_5' = \frac{1}{2}$ and considering μ_0 such as $\mu_0 \geq \frac{5(n-1)g_M^2}{4\epsilon_{k,2}\kappa_1}$, we have

$$\dot{V} \leq -\sum_{k=1}^{n-1} c_k\omega_k^2 - \frac{\kappa_1}{2\kappa_3}\mu_0 W(\eta) - \left[\mu_1 g_m k^* - k_0\right]|\nu|^2$$
$$-\mu_1 g_m \frac{\sigma_I}{2\gamma_I}[k_I - k^*]^2 + R_T \quad (53)$$

from the fact that $||\eta||^2 \geq \frac{1}{\kappa_3}W(\eta)$. Further by considering the ideal feedback gain k^* such that

$$k^* \geq \frac{1}{\mu_1 g_m}\left[\frac{\kappa_1\mu_1}{4\kappa_3} + k_0\right] \quad (54)$$

it follows that

$$\dot{V} \leq -\sum_{k=1}^{n-1} c_k\omega_k^2 - \frac{\kappa_1}{2\kappa_3}\left[\mu_0 W(\eta) + \frac{1}{2}\mu_1|\nu|^2\right]$$
$$-\frac{1}{2}\sigma_I\mu_1\frac{g_m}{\gamma_I}[k_I - k^*]^2 + R_T$$

$$\leq -\sum_{k=1}^{n-1} c_k\omega_k^2 - c_m V_0 + R_T \quad (55)$$

where $c_m = \min\left[\frac{\kappa_1}{2\kappa_3}, \sigma_I\right]$.

Finally, the time derivative of V can be evaluated by

$$\dot{V} \leq -\alpha_v V + R_T \quad (56)$$

where $\alpha_v = \min[2c_1, \cdots, 2c_{n-1}, c_m]$.

It is apparent from (56) that all the signals in the closed-loop system with the controller (50) are bounded. We also obtain from (56) that

$$\lim_{t\to\infty} V \leq R_T/\alpha_v \quad (57)$$

From the fact that $|\nu|^2 \leq 2V/\mu_1$, it follows that

$$\lim_{t\to\infty} |\nu|^2 \leq 2R_T/\alpha_v\mu_1 \quad (58)$$

Thus the control objective (4) is achieved for δ such that $\delta^2 \geq 2R_T/\alpha_v\mu_1$ and it is also easy to confirm that an appropriate choice of μ_0, μ_1 and ρ_4 and design parameters γ_I, γ_{pi}, γ_R, ϵ_{10} and $\epsilon_{i,1}$ to $\epsilon_{i,6}(1 \leq i \leq n-1)$ ensure the control objective for any positive constant δ. As a conclusion we have the following theorem.

Theorem: Under assumptions (A-1) and (A-2) on the controlled system (1), all the signals in the resulting closed-loop system with controller (50) designed according to each step with adaptive adjusting law (25) to (27) and the robust control term (28) are bounded and there exist appropriate design parameters γ_I, γ_{pi}, γ_R, ϵ_{10} and $\epsilon_{i,1}$ to $\epsilon_{i,6}(1 \leq i \leq n-1)$ such that the tracking error $\nu(t)$ converges to any given bound $|\nu(t)| \leq \delta$ as $t \to \infty$.

4 NUMERICAL SIMULATION

Here the effectiveness of the proposed method will be confirmed through a numerical simulation for a model of a DC motor. The model considered here is given as follows:

$$\dot{x}_1 = -\frac{F(x_1)}{J} + \frac{K_t}{J}x_2 - \frac{T(t)}{J}$$
$$\dot{x}_2 = -\frac{K_b}{L(t)}x_1 - \frac{R(t)}{L(t)}x_2 + \frac{1}{L(t)}u$$
$$y = x_1 \quad (59)$$

x_1 is the rotational velocity of the motor and x_2 is the armature current. $R(t)$ and $L(t)$ are the resistance of armature winding and the self-inductance, respectively, which are assumed to be time-varying related to temperature in the motor. K_b, K_t, $T(t)$ and J are the back-emf parameter, the torque motor parameter, the load torque and the rotor inertia, respectively. $F(x_1)$ is the friction given in the following form [7].

$$F(x_1) = F_0(x_1) + \theta x_1 \quad (60)$$
$$F_0(x_1) = F_c sgn(x_1) + (F_s - F_c)\exp\{-(\frac{x_1}{v_s})^2\}sgn(x_1).$$

Table.1 Motor and friction model parameters.

Symbol	Values	units
J	0.2	kgm^2
K_t	0.306	Nm/A
K_b	3.15	Vs/rad
θ	0.4	Ns/m
v_s	0.001	rad/s
F_c	1	Nm
F_s	1.5	Nm

The parameters of the DC motor and the friction model are shown in Table.1. In this simulation we assumed that $R(t)$ and $L(t)$ are varied as shown in Fig.1 and $T(t)$ is given as shown in Fig.2.

In order to apply the proposed method, we consider the following nonsingular transformation:

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= \frac{JR(t)}{K_t L(t)} x_1 + x_2 \end{aligned} \tag{61}$$

so that the system (59) can be represented by

$$\begin{aligned} \dot{z}_1 &= f_1(z_1, t) + g_1 z_2 \\ \dot{z}_2 &= f_2(z_1, t) + g_2(t) u \\ y &= z_1 \end{aligned} \tag{62}$$

where

$$\begin{aligned} f_1(z_1,t) &= -\frac{1}{J}\left[F_0(z_1) + \theta z_1 + \frac{JR(t)}{L(t)} z_1\right] - \frac{T(t)}{J} \\ f_2(z_1,t) &= -\frac{R(t)}{K_t L(t)} F_0(z_1) - \frac{T(t)R(t)}{K_t L(t)} \\ &\quad + \left[\frac{J}{k_t}\frac{d}{dt}\left(\frac{R(t)}{L(t)}\right) - \frac{\theta R(t)}{K_t L(t)} - \frac{K_b}{L(t)}\right] z_1 \\ g_1 &= \frac{K_t}{J},\ g_2(t) = \frac{1}{L(t)}. \end{aligned} \tag{63}$$

Since the transformed system (62) satisfies the assumptions (A-1) and (A-2) with known functions as $\psi_1(y) = \psi_2(y) = y$, we can design the controller for the DC motor according to the proposed procedure.

Figs 3 and 4 show the simulation results. A good control result is obtained in spite of the controlled system having unknown nonlinearities and time-varying unknown coefficients in control input term.

5 CONCLUSIONS

In this paper, we proposed a robust adaptive tracking control, which is based on high-gain feedback based adaptive control, for a class of uncertain time-varying nonlinear systems with higher order relative degree. The effectiveness of the proposed method was confirmed through a numerical simulation for a model of a DC motor.

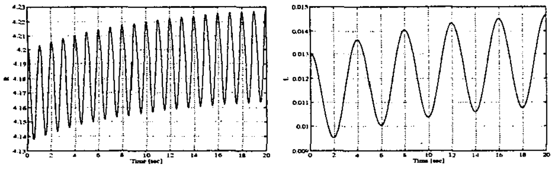

Figure 1: Resistance: $R(t)$ and inductance: $L(t)$

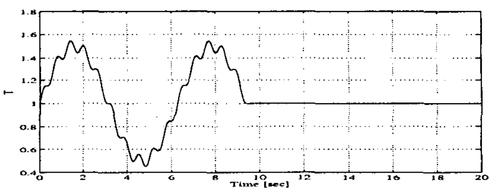

Figure 2: Load torque : $T(t)$

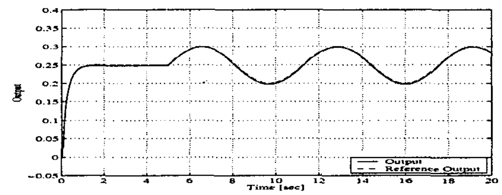

Figure 3: Output and reference signals

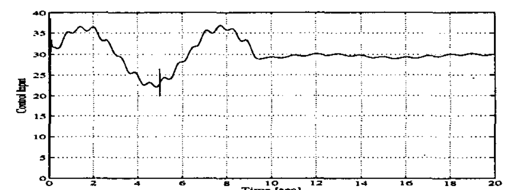

Figure 4: Control input

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