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# On Limit Cycles in Event-Based Control Systems

Anton Cervin and Karl Johan Åström

**Abstract**—Event-based control is a promising alternative to time-triggered control, especially for systems with limited computation and communication capacities. In the paper, the architecture of a general structure for event-based control is presented. The resulting system has many interesting properties. For instance, a constant load disturbance will typically make the process output oscillate according to a stable limit cycle. Necessary conditions for the limit cycle are given and its local stability is analyzed. Finally, a simple way to achieve integral action based on times between events is proposed.

## I. INTRODUCTION

Periodic, time-triggered control is the dominating paradigm in computer-controlled systems. In spite of its success, there are some disadvantages. First, time-triggered control does not utilize the resources in an optimum way. Sensor values and control actions are communicated every period regardless of whether anything significant has occurred in the system. By contrast, in an event-based control system, signals are calculated and communicated “on demand.” Analysis of first-order stochastic systems [3], [12] indicates that event-based control may require only a fraction of the computation and communication bandwidth compared to periodic control to achieve the same performance. The spare capacity could be used for other applications, or the developer could opt for a cheaper implementation platform.

Periodic control systems also have problems coping with implementation effects such as multiple sampling rates, transmission jitter, and unsynchronized computers. Further, there exist some applications where event-based sampling is inherent in the physics. Examples include wheel encoders and accelerometers that deliver pulse trains rather than continuous measurement signals. Queueing systems are another application where the signals are updated in an event-based rather than continuous fashion.

It should be noted that event-based control as a technology is not new. Mostly, however, it has been applied in an ad-hoc way. This can be attributed to the lack of a comprehensive theory, which in turn can be explained by the mathematical difficulties involved. Only recently have some analytical results on event-based state estimation and control started to appear [3], [9], [10], [11], [4], [5].

Event-based control systems, being hybrid and nonlinear systems, can exhibit very interesting behavior. The relation to nonlinear systems analysis is discussed in [2]. In this paper we focus on limit cycles generated by constant load disturbances.

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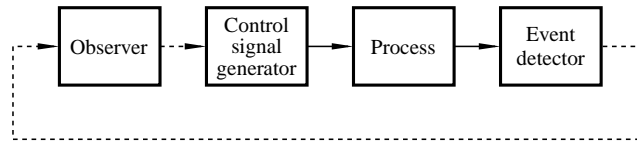


Fig. 1. A general event-based controller structure. Solid lines represent continuous signal transmission while dashed lines represent event-based signal transmission.

The rest of the paper is organized as follows. In Section II, a general structure for event-based control is presented and its various components are discussed. Section III gives the system model. In Section IV, limit cycles in a double integrator process are studied. The general case is treated in Section V, and integral action is discussed in Section VI. Finally, the conclusions are given in Section VII.

## II. A GENERAL STRUCTURE

A block diagram of a system with event-based control is shown in Fig. 1 [2]. The system consists of the process, an event detector, an observer, and a control signal generator. The event detector generates a signal when an event occurs; for instance when the output passes certain levels. The levels are typically tuned according to a trade-off between the number of events per time unit and the control performance. Different events may be generated for the up- and down-crossings. The observer updates the estimates when an event occurs and passes information to the control signal generator, which generates the input signal to the process. The observer and the control signal generator run in open loop between the events. Note however that the absence of events is also information that can be used by the observer.

The control strategy is a combination of feedback and feedforward. Feedback actions occur only at the events. The actuator is driven by the control signal generator in open loop between the events. Design of the control signal generator is therefore a central issue. It is interesting to compare with a conventional sampled-data system, where the events are generated by a clock and the behavior of the system is primarily determined by the control law. Such a system can also be represented by Fig. 1 with a block representing the control law inserted between the sampler and the control signal generator and a clock to generate the events. For a conventional sampled system the behavior of the closed-loop system is essentially determined by the control algorithm, but in an event-based controller the behavior is instead determined by the control signal generator. It therefore makes sense to use a special name, even if the control signal generator can be regarded as a generalized hold [6], [13].

### III. SYSTEM MODEL

We assume that the process is given by

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t) + B_w w(t) \\ y(t) &= Cx(t), \end{aligned} \quad (1)$$

where  $x \in \mathbb{R}^n$  is the state,  $u \in \mathbb{R}$  is the control signal,  $w \in \mathbb{R}$  is a disturbance and  $y \in \mathbb{R}$  is the output. The control signal is limited to  $|u| \leq 1$ .

Events are generated whenever the magnitude of the output passes the threshold  $d$ . We disregard the observer and assume that the full system state can be obtained whenever an event occurs. Optimal event-based observers are studied in [8].

Design of the control signal generator is crucial in event-based control systems. If a simple threshold detector is used, it is important to ensure that disturbances cannot make the output drift away from the detection band. We therefore apply the full control signal

$$u(t) = -\text{sgn}(y(t)), \quad (2)$$

when we are outside the detection band. Upon entering the detection band, we want the state to asymptotically reach zero. One option would be to use optimal control. A simpler alternative is to use a control signal generator in the form

$$\begin{aligned} \frac{dz(t)}{dt} &= Az(t) + Bu(t) \\ u(t) &= -\text{sat}(Lz(t)), \end{aligned} \quad (3)$$

where  $z \in \mathbb{R}^n$  is the generator state, and where  $L$  is chosen to give  $A - BL$  the desired eigenvalues. The generator is initialized to  $z = x$  when the detection band is entered. If there are no disturbances or model errors, the process state will follow the generator state without error.

### IV. DOUBLE INTEGRATOR

We will first examine a special case where an analytical solution can be obtained. Consider the double integrator process

$$\begin{aligned} \frac{dx(t)}{dt} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{aligned}$$

The event detection threshold is set to  $d = 1$ . The control signal generator is given by (3) with the gain vector  $L = \begin{bmatrix} 1 & 2 \end{bmatrix}$ . This choice of  $L$  gives a critically damped response and a control signal limited to  $|u(t)| < 1$  if  $x_1 = 1$  and  $-1 < x_2 < 0$  at the event.

#### A. System Response

To get a flavor for how the event-based control system behaves, in Fig. 2 we show the system response when the disturbance  $w(t)$  is a white noise process with intensity 0.01. Note the shape of the control pulses. The deviation is basically kept within the limits even though the events are quite sparse.

Next we simulate the system assuming a constant load disturbance  $w(t) = w_0 = 0.05$ , see Fig. 3. The disturbance

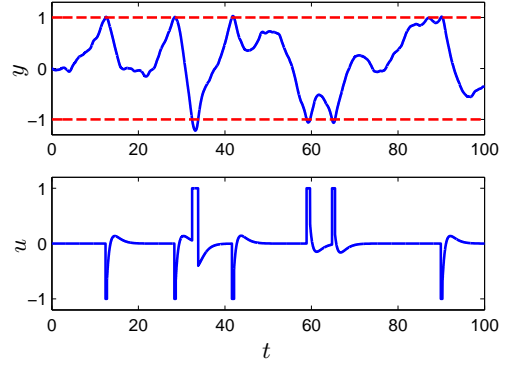


Fig. 2. Simulation of the double integrator process with event-based control and random disturbance.

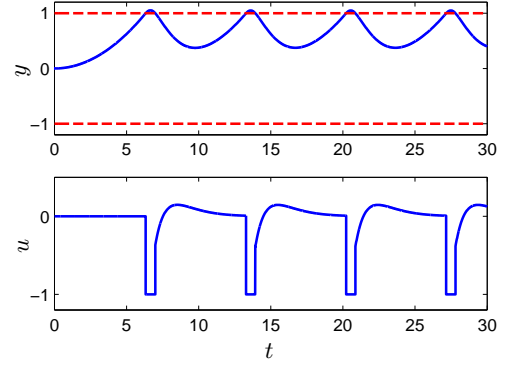


Fig. 3. Simulation of the double integrator process with event-based control and constant disturbance  $w_0 = 0.05$ .

causes the output to drift towards the upper threshold, where a limit cycle is quickly established. Increasing the load disturbance to  $w_0 = 0.5$  in Fig. 4, the resulting oscillation has a higher frequency while the limit cycle is established at a slower rate.

It is clear that the times between events contain information about the magnitude of the disturbance. It is hence interesting to analyze the relationship between  $w_0$  and the properties of the limit cycle. In the next step, this information could be used to design a simple disturbance observer and a compensator.

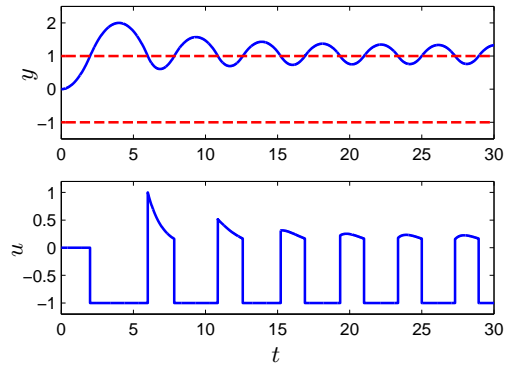


Fig. 4. Simulation of the double integrator process with event-based control and constant disturbance  $w_0 = 0.5$ .

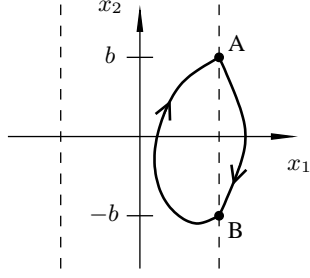


Fig. 5. Possible limit cycle for the double integrator. The dashed lines are the detection threshold  $|x_1| = 1$ .

### B. Necessary Conditions for Limit Cycle

We will investigate the possibility of a limit cycle starting at the point A in Fig. 5. (Due to symmetry, we will only consider limit cycles around the upper detection threshold in this paper.) Let  $x(0) = x^A = [1 \ b]^T$ , and assume that  $w(t) = w_0$ ,  $0 < w_0 < 1$ . Outside the detection band, the closed-loop system is given by

$$\frac{dx_1(t)}{dt} = x_2(t), \quad \frac{dx_2(t)}{dt} = w_0 - 1.$$

Integrating the equations gives

$$\begin{aligned} x_1(t) &= 1 + bt + (w_0 - 1)t^2/2 \\ x_2(t) &= b + (w_0 - 1)t. \end{aligned}$$

Point B will thus be reached after a time  $t_1 = 2b/(1 - w_0)$ , with  $x_2(t_1) = -b$ . Next consider the transition from B to A. Let  $x(0) = x^B = [1 \ -b]^T$ . Assume that the control signal is never saturated, meaning that  $|1 - 2b| \leq 1$ . The state then evolves as

$$x(t) = \Phi(t)x^B + \Gamma(t)w_0,$$

where

$$\Phi(t) = e^{(A-BL)t} = \begin{bmatrix} 1+t & t \\ -t & 1-t \end{bmatrix} e^{-t}$$

accounts for the influence of the control generator, and

$$\Gamma(t) = \int_0^t e^{As} B ds = \begin{bmatrix} t^2/2 \\ t \end{bmatrix}$$

accounts for the influence of the disturbance. The point A will be reached after a time  $t_2$  when

$$x(t_2) = \Phi(t_2)x^B + \Gamma(t_2)w_0 = x^A.$$

We hence have the system of equations

$$\begin{bmatrix} 1 \\ b \end{bmatrix} = \begin{bmatrix} 1+t_2 & t_2 \\ -t_2 & 1-t_2 \end{bmatrix} e^{-t_2} \begin{bmatrix} 1 \\ -b \end{bmatrix} + \begin{bmatrix} t_2^2/2 \\ t_2 \end{bmatrix} w_0. \quad (4)$$

Here it is not possible to obtain a closed-form expression for  $t_2$ . However, the equations can be solved explicitly for  $w_0$  and  $b$  if  $t_2$  is given, hence

$$\begin{aligned} w_0 &= \frac{2(1 - 2t_2e^{-t_2} - e^{-2t_2})}{t_2^2(1 - (1+t_2)e^{-t_2})} \\ b &= \frac{2(1 - (1+t_2 + \frac{t_2^2}{2})e^{-t_2})}{t_2(1 - (1+t_2)e^{-t_2})}. \end{aligned}$$

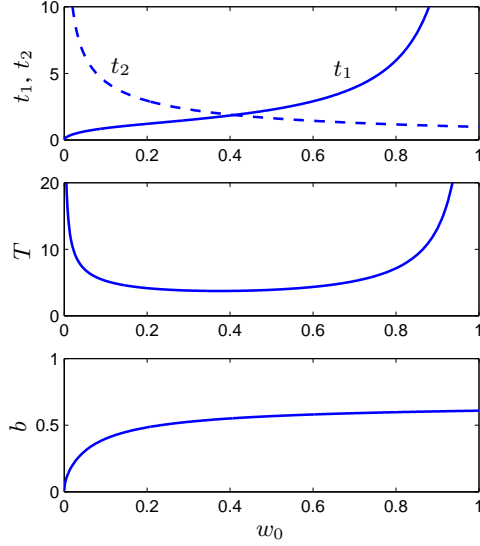


Fig. 6. Parameters of the limit cycle as functions of the magnitude of the disturbance for the double integrator. Note that the limit cycle period  $T$  is not a monotonic function of  $w_0$ .

Being monotonic, these functions can easily be inverted numerically to obtain  $b$  and  $t_2$  as functions of  $w_0$ . Finally, the limit cycle period is given by

$$T = t_1 + t_2 = \frac{2b}{1 - w_0} + t_2.$$

The limit cycle parameters  $t_1$ ,  $t_2$ ,  $T$  and  $b$  as functions of  $w_0$  are shown in Fig. 6. We see that  $0 < b < 1$  for all valid values of  $w_0$ . Hence the assumption  $|1 - 2b| \leq 1$  is not violated. It is interesting to note that  $T$  is not a monotonic function of  $w_0$ . For large disturbances, the trajectory spends more and more time outside the detection band. Obviously, for  $w_0 \geq 1$ , the process cannot be stabilized and the output will diverge.

### C. Stability of Limit Cycle

We will here investigate the local stability of the limit cycle by computing the Jacobian of the Poincaré map according to the method given in [1]. Global stability can be analyzed using quadratic surface Lyapunov functions [7].

Consider a solution with a disturbed initial state,  $x(0) = x^A + \delta x$ , where  $\delta x$  is such that  $Cx(0) = 1$ . According to the analysis above, the detection threshold will be reached after some time  $\tau = t_1 + \delta t_1$  with  $x(\tau) = x^B - \delta x$ . The transition from B to A is then given by

$$x(\tau + t) = \Phi(t)(x^B - \delta x) + \Gamma(t)w_0.$$

The threshold will be reached after some time  $t_2 + \delta t_2$ :

$$x(\tau + t_2 + \delta t_2) = \Phi(t_2 + \delta t_2)(x^B - \delta x) + \Gamma(t_2 + \delta t_2)w_0.$$

Series expansions of  $\Phi$  and  $\Gamma$  give

$$\begin{aligned} x(\tau + t_2 + \delta t_2) &= (I + (A - BL)\delta t_2)\Phi(t_2)(x^B - \delta x) \\ &\quad + (I + A\delta t_2)\Gamma(t_2)w_0 + Bw_0\delta t_2 + \mathcal{O}(\delta^2) \\ &= x^A - \Phi(t_2)\delta x + v\delta t_2 + \mathcal{O}(\delta^2), \end{aligned}$$

where

$$v = Ax^A - BL\Phi(t_2)x^B + Bw_0 = \begin{bmatrix} b \\ w_0 + ((1-b)t_2 + 2b)e^{-t_2} \end{bmatrix}.$$

Ignoring  $\mathcal{O}(\delta^2)$  terms and multiplying both sides by  $C$  gives

$$1 = 1 - C\Phi(t_2)\delta x + Cv\delta t_2$$

Hence

$$\delta t_2 = \frac{C\Phi(t_2)}{Cv} \delta x,$$

and finally

$$x(\tau + t_2 + \delta t_2) = x^A + \left( \frac{vC}{Cv} - I \right) \Phi(t_2)\delta x.$$

The Jacobian of the Poincaré map of the limit cycle is hence given by

$$W = \left( \frac{vC}{Cv} - I \right) \Phi(t_2), \quad (5)$$

and it follows that the limit cycle is locally stable if and only if  $W$  has all eigenvalues inside the unit circle. Note that one eigenvalue of  $W$  is always zero. The non-zero eigenvalue is in our case equal to

$$\lambda = \frac{((w_0 + b)t_2 - b + ((1-b)t_2 + 2b - 1)te^{-t_2})e^{-t_2}}{b}.$$

A plot of  $\lambda(w_0)$  is shown in Fig. 7. The limit cycle is locally stable for all admissible values of  $w_0$ . It is seen that the convergence is very fast for  $w_0 = 0.05$  and moderately fast for  $w_0 = 0.5$ . This agrees with the behavior displayed in Figs. 3 and 4.

## V. THE GENERAL CASE

We will now investigate the general case when the system (1) is controlled by an event-based controller with the detection threshold  $|y| = d$  and the control signal generator (3). The disturbance is given by  $w(t) = w_0$ ,  $0 < w_0 < 1$ . We assume that coordinates are chosen such that  $y = x_1$ , i.e.,  $C = [1 \ 0 \ \dots \ 0]$ .

To determine a possible limit cycle we consider a trajectory starting at the point A on the upper detection limit, see Fig. 5. The starting point is

$$x^A = \begin{bmatrix} d \\ b \end{bmatrix}, \quad b \in \mathbb{R}^{n-1}.$$

The analysis is complicated by the possibility of control signal saturation inside the detection band. We will only consider two possible cases, see Fig. 8. In case (a), the control does not saturate inside the detection band. In case (b), the control signal is saturated at  $u = 1$  for some time  $t_{\text{sat}}$  upon entering the detection band.

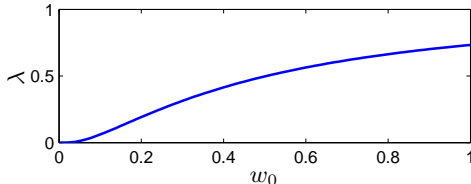


Fig. 7. Convergence rate (i.e., the non-zero eigenvalue of the Jacobian of the Poincaré map) towards the limit cycle for the double integrator process.

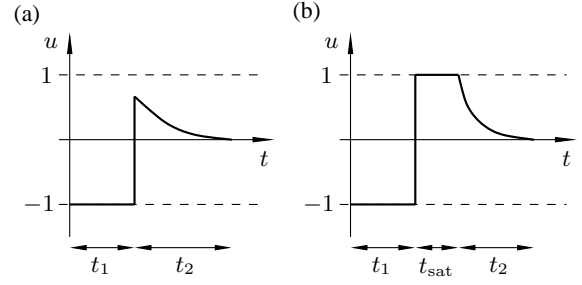


Fig. 8. Two cases of control signal saturation in the limit cycle: (a) The control is only saturated outside the detection band. (b) The control is saturated at  $u = 1$  for some time  $t_{\text{sat}}$  upon entering the detection band.

### A. Necessary Conditions for Limit Cycle

First consider the transition from A to B. Let  $x(0) = x^A$ . In this region we have  $u = -1$ , and the solution is given by

$$x(t) = \Phi_1(t)x^A - \Gamma(t) + \Gamma_w(t)w_0,$$

where

$$\begin{aligned} \Phi_1(t) &= e^{At} \\ [\Gamma(t) \ \Gamma_w(t)] &= \int_0^t e^{As} ds [B \ B_w]. \end{aligned}$$

The condition for reaching B at time  $t_1$  is given by

$$Cx^B = Cx(t_1) = d.$$

Next consider the transition from B to A. Let  $x(0) = x^B$ .

(a) *No saturation.* In the case of no saturation, the trajectory from B to A is given by

$$x(t) = \Phi_2(t)x^B + \Gamma_w(t)w_0,$$

where  $\Phi_2(t) = e^{(A-BL)t}$ . The condition for reaching A at time  $t_2$  is given by

$$x(t_2) = \Phi_2(t_2)x^B + \Gamma_w(t_2)w_0 = x^A.$$

Summarizing we obtain the following system of equations:

$$\begin{aligned} Cx^B &= d \\ x^A &= \Phi_2(t_2)x^B + \Gamma_w(t_2)w_0, \end{aligned} \quad (6)$$

where

$$x^B = \Phi_1(t_1)x^A - \Gamma(t_1) + \Gamma_w(t_1)w_0.$$

Since  $d$  and  $w_0$  are known we have  $n + 1$  equations to determine the  $n + 1$  unknowns  $t_1$ ,  $t_2$ , and  $b$ .

(b) *Saturation.* In the case the control signal saturation inside the detection band, the equations must be extended. During saturation, the control generator state evolves according to

$$z(t) = \Phi_1(t)z(0) + \Gamma(t),$$

where  $z(0) = x^B$ . Let  $t_{\text{sat}}$  be the time during which the control is saturated, and let  $z^C$  be the state of the signal generator when the saturation is released. We then have the condition  $u(t_{\text{sat}}) = -Lz^C = 1$ . After the saturation, we exploit the fact that the effects of the control signal generator and the disturbance can be superimposed to get

$$x(t_{\text{sat}} + t) = \Phi_2(t)z^C + \Gamma_w(t_{\text{sat}} + t).$$

Summarizing we obtain the system of equations

$$\begin{aligned} Cx^B &= d \\ -Lz^C &= 1 \\ x^A &= \Phi_2(t_2)z^C + \Gamma_w(t_{\text{sat}} + t_2), \end{aligned} \quad (7)$$

where

$$\begin{aligned} x^B &= \Phi_1(t_1)x^A - \Gamma(t_1) + \Gamma_w(t_1)w_0 \\ z^C &= \Phi_1(t_{\text{sat}})x^B + \Gamma(t_{\text{sat}}). \end{aligned}$$

We thus have  $n + 2$  equations to determine the  $n + 2$  unknowns  $t_1$ ,  $t_{\text{sat}}$ ,  $t_2$ , and  $b$ .

### B. Stability of Limit Cycle

First consider the transition from A to B. Let  $x(0) = x^A + \delta x$ . Carrying out the analysis as in the previous section, after some calculations we obtain

$$x(t_1 + \delta t_1) = x^B + \left( I - \frac{v^B C}{C v^B} \right) \Phi_1(t_1) \delta x,$$

where  $v^B = Ax^B - B + B_w w_0$ .

Next consider the transition from B to A. Let  $x(0) = x^B + \delta x$ .

(a) *No saturation.* In the case of no saturation, we obtain

$$x(t_2 + \delta t_2) = x^A + \left( I - \frac{v^A C}{C v^A} \right) \Phi_2(t_2) \delta x,$$

where  $v^A = Ax^A - BL\Phi_2(t_2)x^B + B_w w_0$ . Local stability of the limit cycle is hence ensured if the eigenvalues of the matrix

$$W = \left( I - \frac{v^A C}{C v^A} \right) \Phi_2(t_2) \left( I - \frac{v^B C}{C v^B} \right) \Phi_1(t_1) \quad (8)$$

are inside the unit circle.

(b) *Saturation.* In the case of saturation, the transition from B to A must be divided into two parts. First considering the state of the control signal generator, we obtain

$$z(t_{\text{sat}} + \delta t_{\text{sat}}) = z^C + \left( I - \frac{v^C L}{L v^C} \right) \Phi_1(t_{\text{sat}}) \delta x,$$

where  $v^C = Az^C + B$ .

Next, considering the complete transition from B to A, after some calculations we get

$$x(t_{\text{sat}} + \delta t_{\text{sat}} + t_2 + \delta t_2) = x^A + \left( I - \frac{v^A C}{C v^A} \right) \Psi \delta x,$$

where

$$\begin{aligned} \Psi &= \Phi_2(t_2) \left( I - \frac{v^C L}{L v^C} \right) \Phi_1(t_{\text{sat}}) - \frac{v^w L}{L v^C} \Phi_1(t_{\text{sat}}) \\ v^A &= Ax^A - BL\Phi_2(t_2)z^C + B_w w_0 \\ v^w &= (A\Gamma_w(t_{\text{sat}} + t_2) + B_w)w_0. \end{aligned}$$

Local stability of the limit cycle is hence ensured if the eigenvalues of

$$W = \left( I - \frac{v^A C}{C v^A} \right) \Psi \left( I - \frac{v^B C}{C v^B} \right) \Phi_1(t_1) \quad (9)$$

are inside the unit circle.

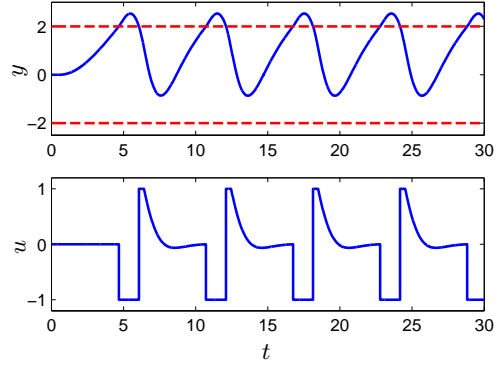


Fig. 9. Simulation of the third-order system with constant disturbance  $w_0 = 0.1$

### C. Example

We apply the analysis above to the third-order, non-minimum-phase process

$$G(s) = \frac{2(4-s)}{s(s+1)^2},$$

with the state space representation

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0].$$

The detection threshold is set to  $d = 2$ , and the control generator gain  $L$  is chosen such that the eigenvalues of  $A - BL$  are placed in  $1.5e^{ik\pi/4}$ ,  $k = 3, 4, 5$ .

A simulation of the control system assuming a constant disturbance  $w_0 = 0.1$  is shown in Fig. 9. It can be noted that the control signal saturates after the down-events. Further, the convergence towards the limit cycle is very fast.

Next, we analyze the properties of the limit cycle for different  $w_0$ . For each value of  $w_0$ , we solve the system of equations (6) using nonlinear optimization. If the solution does not fulfill the condition  $-Lx^B < 1$  we proceed to solve (7). As a measure of the convergence rate, we compute the spectral radius  $\sigma(W)$  of either (8) or (9) for each value of  $w_0$ . The solutions are reported in Fig. 10. The control signal saturates inside the detection band for disturbances in the range  $0.04 < w_0 < 0.63$ . The convergence is very fast throughout and virtually immediate for some values of  $w_0$ . For  $w_0 = 0.1$  for instance we have  $\rho(W) = 0.003$ .

## VI. INTEGRAL ACTION

Integral action is an essential feature in many feedback control systems, since it allows robust regulation of constant load disturbances and error-free tracking of constant reference values. In event-based control systems, a constant disturbance may cause a large number of extra events. These extra events potentially diminish the benefits of the event-based approach. Hence, it would be useful to introduce some form of integral action also in these systems.

One way to achieve integral action is to introduce a disturbance observer. If the convergence towards the limit

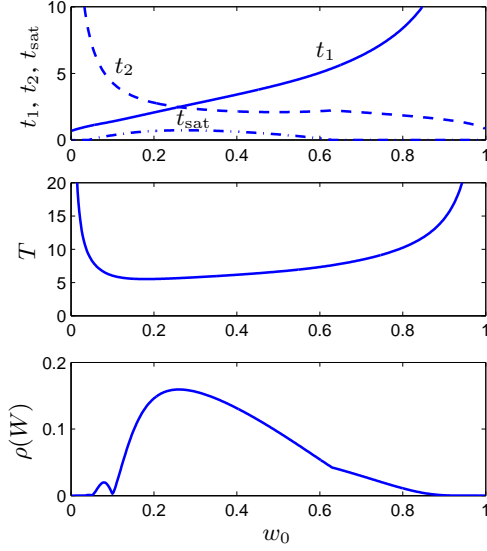


Fig. 10. Properties of the limit cycle as functions of the magnitude of the disturbance for the third-order system.

cycle is fast, the time between events will contain accurate information about the magnitude of the disturbance.

Again consider the double integrator in Section IV. In stationarity, the times between events are given by the monotonic functions  $t_1 = f_1(w_0)$  and  $t_2 = f_2(w_0)$  shown in Fig. 6. After consecutive events on the same side of the detection band, we can use either or both of the inverse functions  $w_0 = f_1^{-1}(t_1)$  and  $w_0 = f_2^{-1}(t_2)$  to estimate  $w_0$ . We explore that approach here and let the disturbance observer be given by

$$\hat{w} := \hat{w} + k \frac{f_1^{-1}(t_1) + f_2^{-1}(t_2)}{2}.$$

The estimate is updated whenever there have been two consecutive down-events on the same side. The gain  $k$ ,  $0 < k \leq 1$ , is chosen as a trade-off between the convergence rate and the sensitivity to noise. The disturbance estimate is subtracted from the control signal before it is applied to the process.

Fig. 11 shows a simulation where the process is disturbed both by a constant acceleration  $w_0 = 0.5$  and by white noise with intensity 0.01. The observer gain was chosen as  $k = 0.1$ . The plot shows that the disturbance estimate converges to the correct value. Once the estimate has converged, much fewer events are generated.

## VII. CONCLUSION

Event-based control systems are nonlinear and hybrid systems and may hence exhibit very rich behaviors. In this paper we first presented a general structure of an event-based control system. We then focused on limit cycles induced by constant disturbances. The local stability of as well as the convergence rate towards the limit cycle were studied. The analysis becomes complicated when there are control signal saturations inside the detection band. In the examples, it was seen that the convergence was often very fast. This

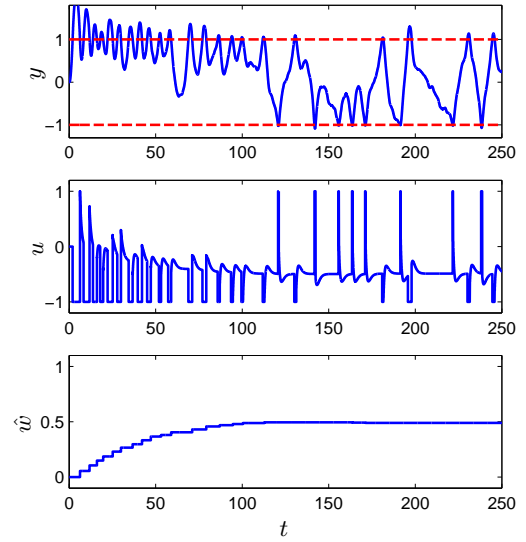


Fig. 11. Simulation of the double integrator with event-based control and integral action. The process is disturbed both by a constant acceleration  $w_0 = 0.5$  and by white noise. Once the disturbance estimate  $\hat{w}$  has converged, much fewer events are generated.

inspired the development of a simple disturbance observer, which only used the times between events as its inputs.

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