Robust power control for CDMA cellular communication networks using quantitative feedback theory

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Abstract— **In this work, a new transmission power control algorithm based on the use of Quantitative Feedback Theory (QFT) is proposed for CDMA wireless cellular networks. The QFT based loop-shaping framework that is considered fully compensates the effect of link round-trip time delay within the network. The design supports predefined levels of performance robustness in the presence of channel uncertainty and signal interference. A novel stability boundary is introduced based on the use of the Jury Array that informs the necessary trade-off between disturbance attenuation and system stability. Extensive simulation results are provided that illustrate the effectiveness of the proposed methodology.**

Index Terms— **Power control, Wireless cellular network, Quantitative feedback theory (QFT).**

I. Introduction

With the rapid growth of wireless networks, there is an increasing demand for Quality of Service (QoS). In code-division-multiple-access (CDMA) system many users share the same channel. In such a case, the power receiver from users close to the base station is much higher than that received from users further away. Therefore, a user close to the base will constantly create a lot of interference for users far from the base station, making their reception impossible. This near-far effect can be solved by applying an appropriate power control (PC) algorithms so that all users are received by the base station with the same average power. As wireless radio channels are typically affected by exogenous, uncertain factors that have an adverse impact on system performance such as path loss, shadowing and fading effects as well as noise and time delay; PC is clearly a challenging objective.

In this work, a new Active Power Control (APC) algorithm is presented based on quantitative feedback theory. The basis for the work relies on the following assumptions that: (i) it is feasible to adjust the transmission power in an appropriate fashion in each base-mobile, (ii) the received signal quality at a receiver can be reliably measured using a signal-to-interference-noise ratio (SINR) type metric. It is also noted that the focus of the is on uplink system (communication from mobile users to the base station).

Consider a wireless cellular system consisting of N user and M fixed base stations. The SINR for i-th user in the desired cell is given by, [17]:

$$\gamma_i(k) = \frac{g_{ii}(k)p_i(k)}{\sum_{j=1,\ j\neq i}^{N} g_{ii}(k)p_j(k) + \sigma_i^2} = \frac{g_{ii}(k)p_i(k)}{I_i(k)} \qquad (1)$$

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where g_{ij} represents the channel gain between i-th base staion and j-th subscriber. p_j also denotes the power that $j-$th user consumes for packet transmission. σ_i^2 is the power of the white noise at the base station i. I_i represents both inter and intracell interferences. SINR represented in (1), contains sufficient information on channel gain and interference so that an APC algorithm based on SINR feedback control can address the existing issues.

In practice, an APC algorithm should have the properties of simplicity, the ability to adjust the power levels of each transmitted signal using only local measurements, in a reasonable time, and to maintain a desired signal-to-interference ratio denoted as γ_t. An outer loop adjusts γ_t in terms of Frame Error Rate (FER) [15], but since the update rate for target SIR is generally much slower than the update rate for the corresponding power signal, it is considered as fixed for the purposes of this work as per [4], [8], [10]–[12], [14]. Since the channel knowledge is generally unpredictable and imperfect, any APC algorithm should contain a certain level of performance robustness against any or all of the aforementioned exogenous uncertain factors.

II. literature review

A brief literature review in relation to the limitations of existing APC techniques is now provided. Simple fixed-step (decision feedback) power control schemes [1] were initially considered during the early work on distributed power control (DPC) for wireless systems and has thus been typically employed in practical CDMA systems where power control is considered. However, the large time delay considerably increases the variance of the system response. In [4], [14], several adaptive APC algorithms based minimum variance (MV) and generalized MV control have been proposed. However, robustness cannot be fully guaranteed using either MV and GMV and furthermore accurate information relating to the delay of the process is required for both approaches. In addition, the stability of MV or GMV algorithms is still an open issue that has been highlighted in the literature [14] and [4]. It is noted that on-line algorithms to estimate the appropriate network parameters so that the necessary tuning of the control parameters is completed may be time-consuming and can also lead to additional computational complexity. Moreover, convergence problem should be taken into account in such adaptive techniques which estimate or predict a linear model of the plant [4], [14] or channel uncertainty [10].

In [11], a robust PC design based on Smith predictor concepts and H_∞ theory has been proposed. However it is well-known that when significant uncertainty exists either within the time-delay or the rational part of the system model, SPC may be very sensitive to the mismatch between the inner-loop model and actual system model [6]. In [12], an alternative robust PC design has been proposed using linear matrix inequality (LMI) concept and and H_∞ theory. The effect of link delay is compensated through the state feedback control. Both [11] and [12] result in fixed-order controller, however larger uncertainty on the round-trip delay leads to higher order system and then controller. Moreover, since the worst-case influence of unwanted factors is minimized, there is an inherent conservatism which can lead to high order controller.

In this paper, an alternative robust APC algorithm is presented based on the use of QFT. In the proposed approach the effect of link time delay is fully addressed using the Nichols chart. Exact information in relation to channel uncertainty and interference are not explicitly required by a QFT design. The convergence problem is fully addressed in a QFT-based design approach as the desired tracking specifications are determined using settling time based constraints. In addition, a novel stability analysis based on the use of the Jury Array is presented. This has particular benefits for the problem at hand. While it is to be expected that the larger the network round-trip time delay, the smaller a stability region becomes it is argued that the Jury test presented here is particularly informative regarding the management of the trade-off between stability and attenuation of time-varying link uncertainties.

The rest of paper is organized as follows. In section III the closed-loop model for APC is presented and control objectives within that paradigm are then introduced. The Jury stability test is presented in section IV. Section V considers the design methodology and in particular discusses the necessary QFT loop-shaping design framework. Several computational scenarios are then presented to evaluate the effectiveness of the proposed technique. These results are illustrated in section VI.

III. System model and control objectives

A. System model

Throughout this paper, the decibel value of a variable x, is denoted by $\bar{x} = 10log_{10}x$. In logarithmic scale, the SINR expression (1) becomes:

$$\bar{\gamma}_i(k) = \bar{p}_i(k) + \bar{g}_{ii}(k) - \bar{I}_i(k) \tag{2}$$

Suppose that the mobile power is updated according to a distributive bilinear control law given by:

$$\bar{p}_i(k) = \bar{p}_i(k-1) + u_i(k-d), \tag{3}$$

where d represents sum of downlink delay and uplink delay and u_i denotes APC law decision for the i-th user. Equations (2) and (3) result in a simplified linearized closed-loop APC system that has been widely adopted by the literature, illustrated in Fig. 1. Note that an integrator inherently has been considered in the plant dynamic which modifies tracking performance [11]. Moreover, the effects of channel gain, interference from other connections and then noise are treated as output disturbance $\bar{D}_i(k)$.

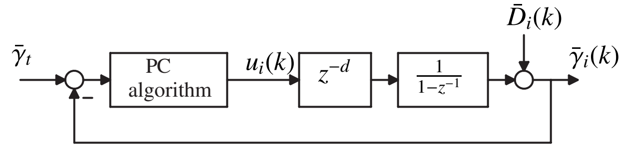

Fig. 1. Simplified power control feedback systems.

A typical value of d varies between 2 (for WCDMA systems) and 4 (for IS-95 systems), while $\bar{D}_i(k)$ is determined by the levels of thermal noise and channel gains. The receiver noise is, as usual, assumed to be a white Gaussian process with zero mean and variance σ_i^2. g_{ij}'s are determined as follows, [7]:

$$g_{ij}(k) = l_{ij}(k)s_{ij}(k)f_{ij}(k). \tag{4}$$

where, l_{ij}, s_{ij} and f_{ij} represent path loss, shadowing, and fading between i-th base station and j-th user, respectively. These exogenous, uncertain factors distort received signal strength at the receiver and thus cause variations in the received SINR according to the following equations:

$$l_{ij}(k) = \frac{A_p}{r_{ij}^{d_p}(k)}, \tag{5}$$

$$\bar{s}_{ij}(k+1) = a\bar{s}_{ij}(k) + n_{sj}(k),\ a_j = e^{-\delta_{ij}/X_c}, \tag{6}$$

$$f_{ij}(k) = X^2. \tag{7}$$

In equation (5), A_p is a constant depending on the antenna properties, transmission wavelength, and the environment (rural, suburban, urban). A_p is arbitrarily selected equal to 1 in this work. r_{ij} is the distance between the transmitter and receiver at k-th sampling time and d_p is the path loss exponent with typical values ranging from 2 (free space propagation) to 5 (dense urban areas). In (6), δ_{ij} is the distance between two consecutive samples. For users moving with velocity v_j, the shadowing decorrelation in time t is obtained by substituting $\delta_{ij} = v_j T_s$, where T_s and k represent the sampling period and k-th iteration, respectively. $n_{sj}(k)$ is a white Gaussian noise process with zero mean and variance $\sigma_{ns}^2 = (1 - a_j^2)\sigma_s^2$. The term σ_s denotes the log-standard deviation of $\bar{s}_{ij}(k)$, which depends on the environment. The decorrelation distance X_c in this model is the distance at which the signal autocorrelation equals $1/e$ of its maximum value and is on the order of the size of the blocking objects or clusters of these objects. For outdoor systems X_c typically ranges from 50 to 100, and finally X is a random variable with Rayleigh distribution in (7).

B. Control objectives

The main objective of the proposed APC system is to obtain an appropriate APC law so that every mobile operates with a SINR around the target value γ_t.

Outage probability, P_o is a supplementary performance metric that is considered here to benchmark the performance of a power control scheme [4], [10]–[12]. It is defined as the probability of failing to achieve minimum required SINR, γ_{min}. It is assumed that the mobile user will be disconnected if $\bar{\gamma} < \bar{\gamma}_{min}$. P_o is computed as the ratio of the number of times that $\bar{\gamma} < \bar{\gamma}_{min}$ to the total number of iterations.

IV. Stability Analysis based Jury array

Since the proposed APC structure is a type-I system [1] and an integrator has inherently been considered in the model dynamic, it can be concluded that a proportional controller would achieve the desired tracking requirements. In this section the stability region of the proportional controller, $C(z) = K$, is investigated for a network of nodes wherein the proposed APC scheme has been adopted to counter the effect of round-trip time delay. The region that is determined, describes the boundary values of the controller gain that will stabilize the system. In this sense, the characterization of a stability region gives an insight that highlights how appropriate a QFT loop-shaping process for this problem. Two control theory concepts are used in the following procedure.

1) A discrete-time closed-loop system $H(z) = \frac{B(z)}{A(z)}$ is stable if and only if all roots of the characteristic polynomial $A(z)$ lie in the circle with radius equal to 1 in the complex plane [13].
2) Suppose that the plant transfer function $G(z)$ belongs to a set $\mathcal{G}$ that covers uncertainty. The closed-loop system is robustly stable if and only if all plants over the set $\mathcal{G}$ are stable [5].

Theorem: Consider a power control system of the wireless network shown in Fig. 1 with proportional controller $C(z) = K$. Moreover assume that the round-trip time delay varies within $2 \leq d \leq 4$ that is compatible with IS-95 systems. Then selection of K within $0 < K < 0.445$ stabilizes the closed-loop system for all plants over the uncertainty region.

Proof: It is simple to show that the characteristic polynomial of the closed-loop system Fig. 1, is as follows:

$$A(z) = z^d - z^{d-1} + K \tag{8}$$

where d denotes the wireless link delay and based on the assumption that $2 \leq d \leq 4$. A Jury stability test can be separately applied to determine the stabilizing region for a feedback compensator taken over a discrete set of plants $\mathcal{G} = \left\{\frac{z^{-2}}{1-z^{-1}}, \frac{z^{-3}}{1-z^{-1}}, \frac{z^{-4}}{1-z^{-1}}\right\}$, corresponding to that two to four-step delay within the wireless link. The obtained stability regions are summarized in Table I. The intersection of the obtained boundaries, shows that the stabilizing region for the design is $0 \leq K \leq 0.445$. Details of the necessary computations are provided in Appendix I.□

Remark: The results of Table I clearly illustrate the limiting effects of time delay. The larger the time delay, the smaller the stability region becomes. Thus, bounding the controller gain limits the engineer's freedom to reduce the sensitivity function for a link uncertainty. This suggests that the necessary trade off between the disturbance attenuation and system stability can be easily managed through the use of QFT design techniques [9]. □

TABLE I
Stabilizing region of the controller K.

Time daly	stability region
$d = 2$	$0 \leq K \leq 1$
$d = 3$	$0 \leq K \leq 0.618$
$d = 4$	$0 \leq K \leq 0.445$

[1] Defined as the number of pure integrators in a system [13].

V. Robust power tracking control based quantitative feedback theory

The QFT loop-shaping paradigm introduced by Horowitz in [9] is essentially a frequency domain technique using standard feedback architectures, as illustrated in Fig. 2, that achieves client-specified levels of desired performance over a region of plant uncertainty and unknown disturbances. The methodology requires that some desired constraints be generated in terms of closed-loop frequency response, which in turn lead to design bounds in the loop function on the Nichols chart. The feedback compensator, $C(z)$ is designed using an iterative shaping of the loop gain function so that the design bounds are satisfied.

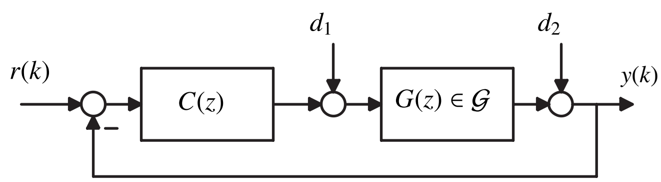

Fig. 2. QFT control structure based loop-shaping framework.

The design procedures are presented in the following.

A. Description of system data and desired specifications

1) Templates and nominal plant: A discrete set of plants should be selected over the uncertainty region that totally covers the variable dynamics of the system. For the proposed APC structure, plant uncertainty has been considered on the link delay d which varies within $2 \leq d \leq 4$, that is $\mathcal{G} = \left\{\frac{z^{-2}}{1-z^{-1}}, \frac{z^{-3}}{1-z^{-1}}, \frac{z^{-4}}{1-z^{-1}}\right\}$.

The term *template* denotes the range of variations on the plant frequency response at a particular design frequency. Since the worst case (biggest template) of frequency response corresponds quite naturally to the maximum wireless link delay, the plant model corresponds to the largest link delay and is selected as the nominal plant. It is noted that this same nominal plant must be used throughout the design procedure [2].

2) Sampling frequency: Based on the IS-95 standard, the sampling period for SINR computation is $T_s = 1/9600$ (s). However to reduce collisions and to prevent poor link utilization, the APC closed-loop system and then $u_i(k)$ are periodically updated every $T_p = 12T_s(s)$.

3) Desired specifications: A set of desired specifications are subsequently introduced in terms of the magnitude of the frequency response of the closed-loop system so as to achieve a satisfactory level of robust stability *and* performance. This section outlines a method for obtaining these desired specifications so that system SINR tracks the target value and the influences of link uncertainties are minimized. First of all, robust stability, a necessary fundamental specification for any uncertain feedback system, must be guaranteed *a priori*. In QFT design, the notion of robust stability is usually incorporated into gain and phase margins through the use of the following constraint:

$$\left|\frac{CG}{1+CG}(z)\right|_{z=e^{j\omega T_S}} \leq \mu, \qquad (9)$$
$$for\ all\ G \in \{\mathcal{G}\}, \omega \in [0, \pi/T_p].$$

This criterion corresponds to lower bounds on the gain margin of $K_M = 1 + 1/\mu$ and the phase margin angle of $\phi_M = 180° - \cos^{-1}(0.5/\mu^2 - 1)$, [2]. Throughout the design, we adopt $\mu = 1.5$ that guarantees the phase and gain margin equal to 50° and 1.44, respectively.

The following design constraint is accordingly used to ensure the tracking performance,

$$|T_L(j\omega)| \leq \left|F\frac{CG}{1+CG}(z)\right|_{z=e^{j\omega T_s}} \leq |T_U(j\omega)|, \qquad (10)$$
$$for\ all\ G \in \{\mathcal{G}\}, \omega \in [0, \omega_h].$$

where ω_h denotes desired performance bandwidth. Equation (10) implies the system SINR should be placed in a predefined region which are specified by upper and lower bounds $T_U(z)$ and $T_L(z)$, respectively. $T_U(z)$ and $T_L(z)$ are typically defined based on the nature of the system, by using time-domain concepts such as settling time and overshoot, [2]. Supposing that *i)* the SINR should be required to settle around the target value of $10T_p \leq t_{ss} \leq 20T_p$, and *ii)* the critically damped response, $\xi = 1$, is selected to reduce outage probability at the outset of communication, the following transfer functions can be selected so as to achieve the desired tracking bounds:

$$T_U(z) = \frac{0.08967z + 0.06862}{z^2 - 1.291z + 0.4493} \qquad (11)$$

$$T_L(z) = \frac{0.01752z + 0.01534}{z^2 - 1.637z + 0.67030} \qquad (12)$$

Finally, the third design constraint relates to attenuating the effects of link uncertainties which have been treated as a disturbance. To do this, it is sufficient to over-bound the transfer function from $\bar{D}(k)$ to $\bar{y}(k)$ with an appropriate disturbance rejection ratio as follows:

$$\left|\frac{1}{1+CG}(z)\right| \leq |W_D(z)|, \qquad (13)$$
$$z = e^{j\omega T_p}\ for\ all\ G \in \{\mathcal{G}\}, \omega \in [0, \omega_h].$$

where W_D represents weighting function of the disturbance rejection. There must always be a trade off between stability and disturbance attenuation that has to be taken into consideration in the selection of W_D. It is argued that the combination of Jury stability test as well as the use of a graphical design environment inherent to the QFT design procedure gives a novel insight into the selection of an appropriate W_D that results in a minimised over design and conservatism. Sweeping W_D from 0.1 to 1 demonstrates that $W_D = 1$ is the best choice of disturbance attenuation ratio to achieve a feasible controller that best satisfies the Jury stability test.

4) Design frequencies: A set of frequencies must be selected within the performance bandwidth, ω_h, to compute the related design bounds. Equation (14) illustrates how ω_h can be computed based on damping ratio and settling time for the tracking specifications [18].

$$\omega_h = \frac{4}{T_s\xi}\sqrt{1 - 2\xi^2 + \sqrt{4\xi^4 - 4\xi^2 + 2}}. \qquad (14)$$

Based on assumptions outlined previously, a damping ratio $\xi = 1$ and $t_{ss} = 20T_p(s)$ is adopted that result in $\omega_h = 100(rad/s)$. Since a tracking region $T_L(z) \leq \frac{CG}{1+CG}(z) \leq T_U(z)$ is required, it is noted that there is an approximation inherent to the computation of ω_h. So, the set of $\omega_1 = \{1, 10, 100\}\,(rad/s)$ is selected as the design frequencies within the performance bandwidth. For robust stability at higher frequencies, equation (9) is computed within the Nyquist frequency range $[0, \pi/T_p]$ for the set of $\omega_2 = \{1, 10, 100, 1000, 10000, 20000\}\,(rad/s)$.

B. Computation of QFT bounds

Using the MATLAB QFT-Toolbox [2], related design bounds corresponding to the proposed requirements are generated at each design frequency. The intersection of the bounds at each design frequency, is the final bound taken for the design of the feedback compensator, $C(z)$. Fig. 3 shows the obtained QFT design bounds. $C(z)$ is designed by adding appropriate poles and zeros to the loop function such that the nominal loop function frequency response satisfies the worst case design specification for the bounds at each frequency. For robustness, the nominal loop function must be shaped such that the frequency response lies above the design bounds at each design frequency and does not enter the U-contours described in Fig. 3. Moreover the critical point $(-180°, 0dB)$ must also be avoided. Fig. 3 illustrates that the feedback compensator $C(z) = 0.1$ satisfies all required specifications.

VI. Simulation results

In this section the performance of the proposed uplink APC algorithm of a CDMA system is evaluated. Based on an IS-95 system, operating frequency and bandwidth of each channel are assumed to be is 900 MHz and 1.23 MHz, respectively. Data rate R is set at $9.6(kb/s)$ and the processing gain is thus taken to be $21(dB)$. For ease of illustration we consider a single hexagonal cell CDMA system with approximate radius of $100(m)$, where the base station has been located at the center of the cell. No new point of principle arises in an extension to a completely arbitrary

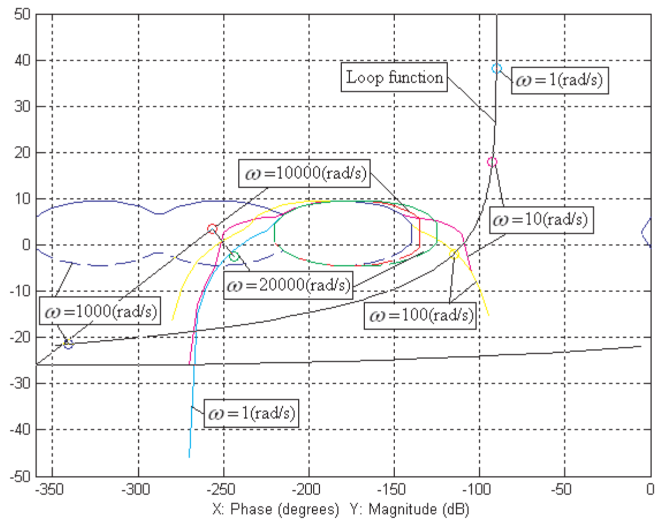

Fig. 3. Design bounds on Nichols Chart, Loop function has been considered for the system model with link delay of $d = 4$ and has been shaped with a proportional controller $C(z) = 0.1$.

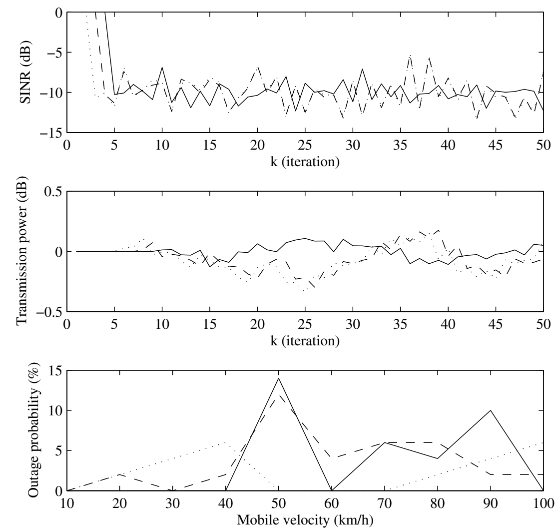

Fig. 4. The effect of proportional controller against link time-delay, $d = 4$ 'solid', $d = 3$ 'dashed', $d = 2$ 'dotted'.

configuration. Mobile users are randomly placed in the cell using a Gaussian distribution. The number of users will be assumed to be $N = 10$ unless otherwise stated. Each user can be stationary, moving toward or away from the base station. This is selected randomly at the beginning of the simulation. Non-stationary users are assumed to have a velocity of $v = 20(km/h)$ when outage probability is computed. Based on this type of motion, path loss is computed for each user at each iteration according to (5). The APC algorithm is applied to all users at each iteration. The minimum allowable SINR is given by $\gamma_{min} = (E_b/I_o)(W/R)^{-1}$ where E_b denotes the energy dissipated per information bit and I_o is the total interference and noise power spectral density. W and R represents the system bandwidth and data rate respectively. To achieve an acceptable level of communication quality a BER less than 10^{-3} is necessary, leading to an E_b/I_o of $7(dB)$ which results in $\bar{\gamma}_{min} = -14(dB)$. For simplicity, the target SINR $\bar{\gamma}_t$ is set as $-10(dB)$. Standard deviation of noise is selected as a random value between 0.5 and 1. Log-standard deviation of shadowing is set on $\sigma_s = 4.3(dB)$, and finally the Rayleigh distribution parameter is set at 0.5.

A. QFT controller design in the presence of time delay

In this section, the robustness of QFT-based controller in the presence of link time-delay is investigated. The simulation has been run for a designed proportional controller while link delay is varying between $d = 2$ and $d = 4$. The results depicted on Fig 4, shows that the SINR has settled around the target value with an acceptable transmission power and outage probability. All of these specifications have been achieved and the effect of link time delay has been fully addressed using the Nichols chart, during the design procedure.

B. Benchmark Comparison of QFT controllers versus other existing methods

In this section, the performance of QFT-based APC is compared with the fixed-step size [1] and adaptive step size [16] APC algorithms that are most popular within the communication engineering community at the present time. Tuning process of [1] and [16] are briefly presented in [12].

Figures 5 compares the system response for all proposed algorithms. It is clear that QFT-based APC results in a system with less power consumption that increases battery life. In contrast to QFT-based and fixed-step size techniques, power consumption is much higher in an adaptive step-size approach thus leading to an unnecessarily high user (or link) cost. This is due to the fact that power constraints are not explicitly considered during the adaptation procedure. In addition, Fig. 5 illustrates that the outage probability can be reduced using a QFT-based approach thus implying effective attenuation of interference and link uncertainties.

VII. Conclusion

A novel design methodology has been presented for power control within wireless cellular networks. It has been shown through the use of quantitative feedback theory based loop-shaping techniques, that an easy-to-implement and quick power control law can be determined that is robust to a wide range of signal and system uncertainty. In addition, a Jury test has been employed to determine a stability region for a feedback compensator that is very informative regarding the management of the trade-off between various conflicting system performance constraints. The effectiveness of the proposed controller designed based QFT loop-shaping approach has been validated through the use of several operating scenarios.

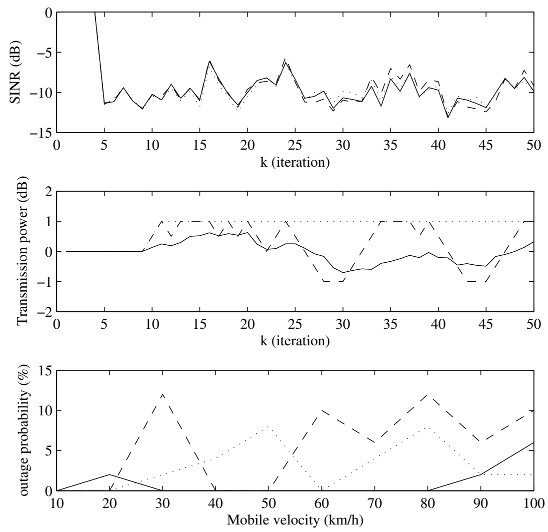

Fig. 5. Comparison of QFT-based Power Control 'solid', with existing fixed and adaptive power controllers 'dashed' and 'dotted' for $d = 4$, respectively.

Appendix I
Proof of the theorem
Jury test for APC system of wireless cellular networks

As mentioned earlier, the characteristic polynomial of the closed-loop system is determined to be:

$$A(z) = z^d - z^{d-1} + K$$

Case 1: for $d = 2$, the following constraints must simultaneously be satisfied according to the Jury test:

1) $|a_2| < |a_0| \rightarrow |K| < 1$
2) $A(1) > 0 \rightarrow K > 0$
3) $(-1)^2 A(-1) > 0 \rightarrow K > -2$

Intersection of the obtained boundaries results in $0 < K < 1$ as the stability region of the closed-loop system with $d = 2$.

Case 2: For $d = 3$, the following constraints must simultaneously be satisfied according to the Jury test:

1) $|a_3| < |a_0| \rightarrow |K| < 1$
2) $A(1) > 0 \rightarrow K > 0$
3) $(-1)^3 A(-1) > 0 \rightarrow K < 1$
4) $|b_2| > |b_0| \rightarrow K^2 + K - 1 < 0$

Intersection of the obtained boundaries results in $0 < K < 0.618$ as the stability region of the closed-loop system with $d = 3$.

Case 3: For $d = 4$, the following constraints must accordingly be satisfied simultaneously:

1) $|a_4| < |a_0| \rightarrow |K| < 1$
2) $A(1) > 0 \rightarrow K > 0$
3) $(-1)^4 A(-1) > 0 \rightarrow K > -2$
4) $|b_3| > |b_0| \rightarrow 1 > 0$ (always true)
   $|c_2| > |c_0| \rightarrow K^4 - 3K^2 - K + 1 > 0$

Intersection of the obtained boundaries results in $0 < K < 0.445$ as the stability region for the closed-loop system for the case where $d = 4$.

Table I summarises the resultant boundaries.

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