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Effects of Small Time-Delays on Dynamic Output Feedback Control of Offshore Steel Jacket Structures Subject to Wave-induced Forces

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Abstract— **This paper is to investigate the effect of a small time-delay on dynamic output feedback control of an offshore steel jacket structure subject to a nonlinear wave-induced self-excited hydrodynamic force. Firstly, a conventional dynamic output feedback controller is designed to reduce the internal oscillations of the offshore structure. It is found that the obtained controller is of a large gain in the sense of Euclidean norm, which demands a large control force. Secondly, a small time-delay is introduced intentionally to design a new dynamic output feedback controller such that (i) the controller is of a small gain in the sense of Euclidean norm; and (ii) the internal oscillations of the offshore structure can be dramatically reduced. It is shown through simulation results that purposefully introducing time-delays can be used to improve control performance.**

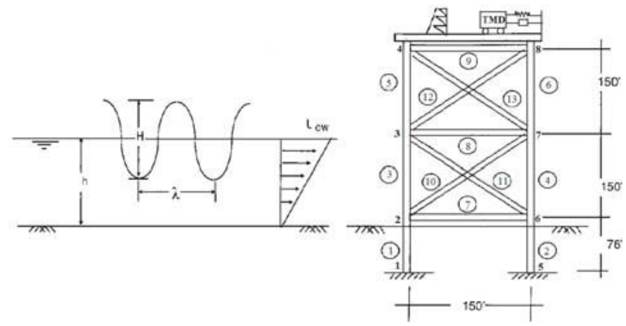

Fig. 1. Steel jacket structure with TMD [14]

I. INTRODUCTION

Modern offshore structures are mainly used in oil and gas extraction and are usually built in a water depth of more than 1000 feet. Equipped with a helicopter pad, drilling derrick, cranes, offices and accommodations, generally speaking, these structures are very large, sophisticated and flexible. Since offshore structures are located in a hostile environment, they are typically subject to severe loads due to water currents, waves and wind. In addition, their flexibilities further induce self-excited nonlinear hydrodynamic forces and make themselves large deformations due to nonlinear responses. Therefore, continuing research on the safety of these structures has been conducted in the past decade. One can refer to [4], [3] and references therein.

In order to ensure the safety of an offshore structure, some efforts have been made in the recent years. An easy approach is to increase the stiff of the structure so as to shift the natural frequencies away from the resonant range of frequencies. This requires a large number of construction materials, which leads to a huge cost. Alternatively, passive and active control methods have been proposed to reduce the internal oscillations by regulating the motion of the structure, see, for example, [9], [3]. By placing a tuned mass damper (TMD) onto the top of the structure, both a passive TMD and an active TMD mechanism to regulate the motion of the structure were discussed in [3]. An active TMD scheme, which aims to design a suitable controller to drive the hydraulic servo, can more effectively reduce the internal oscillations than a passive TMD method. However, for an active TMD mechanism, the controller is usually difficult to be designed mainly due to the self-excited nonlinear hydrodynamic force induced by the flexibility of the structure. For this reason, more attention has been paid to the active TMD control of an offshore structure, and some control methods have been proposed in the recent literature. For instance, in [14], multi-loop feedback control method was introduced: an inner loop is used to stabilize the linear part of the platform dynamical model while an outer loop aims to cope with the nonlinearities to maintain the stability of the whole structure. In [15], two different control schemes, namely nonlinear control and robust linear state feedback control, have been reported. In [11], [10], feedforward and feedback control was studied. The above mentioned methods are implemented by using the system's states. To the best of our knowledge, in the open published literature there is little research on dynamic output feedback control of an offshore structure.

In this paper, we will consider dynamic output feedback control of an offshore steel jacket structure subject to a nonlinear wave-induced force. A conventional dynamic output feedback controller will be first designed by solving a set of LMIs. Then we will investigate whether purposefully introduced time-delays in dynamic output feedback controllers can be used to improve the control performance.

Throughout this paper, for simplicity, the symmetric term in a symmetric matrix is denoted by *, e.g., $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$

II. DYNAMIC MODEL OF AN OFFSHORE STRUCTURE

Consider a simple offshore steel jacket platform in Figure 1 [14]. This structure consists of cylindrical steel tube

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$$A=\begin{bmatrix}0&1&0&0&0&0\\-\omega_1^2-K_T\phi_1^2&-2\xi_1\omega_1-C_T\phi_1^2&-K_T\phi_1\phi_2&-C_T\phi_1\phi_2&\phi_1K_T&\phi_1C_T\\0&0&0&1&0&0\\-K_T\phi_1\phi_2&-C_T\phi_1\phi_2&-\omega_2^2-K_T\phi_2^2&-2\xi_2\omega_2-C_T\phi_2^2&\phi_2K_T&\phi_2C_T\\0&0&0&0&0&1\\\omega_T^2\phi_1&2\xi_T\omega_T\phi_1&\omega_T^2\phi_2&2\xi_T\omega_T\phi_2&-\omega_T^2&-2\xi_T\omega_T\end{bmatrix},\ B=\begin{bmatrix}0\\-\phi_1\\0\\-\phi_2\\0\\\frac{1}{m_T}\end{bmatrix},\ F=\begin{bmatrix}0&0\\1&0\\0&0\\0&1\\0&0\\0&0\end{bmatrix}$$

members, and an active TMD mechanism is mounted on the top. One end of the TMD is connected to joint 8 and the other to a hydraulic servo mechanism. The motion of the damper is influenced by the motion of the structure and the hydraulic servo. Since this structure is often exposed to nonlinear hydrodynamic forces, the induced self-excited load by the nonlinear forces usually make the structure internally vibrate. An active TMD control is to design a controller to drive the hydraulic servo so as to reduce the vibration by the motion of the damper. Since the first two modes of vibration are the most dominant, for simplicity, we just take these two modes into account.

The motion equations of the first two modes of vibration with the coupled TMD can be expressed as [15]

$$\begin{cases}\ddot{z}_1 = -2\xi_1\omega_1\dot{z}_1-\omega_1^2z_1-\phi_1K_T(\phi_1z_1+\phi_2z_2)+\phi_1K_T\bar{y}\\ \qquad -\phi_1C_T(\phi_1\dot{z}_1+\phi_2\dot{z}_2)+\phi_1C_T\dot{y}-\phi_1u+f_{t1}+f_{t2},\\ \ddot{z}_2 = -2\xi_2\omega_2\dot{z}_2-\omega_2^2z_2-\phi_2K_T(\phi_1z_1+\phi_2z_2)+\phi_2K_T\bar{y}\\ \qquad -\phi_2C_T(\phi_1\dot{z}_1+\phi_2\dot{z}_2)+\phi_2C_T\dot{\bar{y}}-\phi_2u+f_{t3}+f_{t4},\\ \ddot{\bar{y}} = -2\xi_T\omega_T\dot{\bar{y}}+2\xi_T\omega_T(\phi_1\dot{z}_1+\phi_2\dot{z}_2)-\omega_T^2\bar{y}\\ \qquad +\omega_T^2(\phi_1z_1+\phi_2z_2)+\dfrac{1}{m_T}u.\end{cases} \tag{1}$$

where z_1 and z_2 denote the generalized coordinates of vibrational modes 1 and 2, respectively; $\bar{y}$ is the horizontal displacement of the TMD; ξ_1 and ξ_2 are the damping ratio in the first two modes of vibration, respectively; ω_1 and ω_2 are the natural frequencies of the first two modes of vibration, respectively; ϕ_1 and ϕ_2 are the contributions of first two mode shapes (for the steel jacket platform, $\phi_1=-0.003463$ and $\phi_2=0.003463$); ξ_T is the damping ratio of the TMD. We denote the damping, the mass and the stiffness of the TMD by C_T, m_T and K_T, respectively; $\omega_T=\sqrt{K_T/m_T}$ is called the natural frequency of the TMD. u is the control action of the system; and f_{t1}, f_{t2}, f_{t3}, f_{t4} are the nonlinear self-excited hydrodynamic force terms.

Let $x_1=z_1,\ x_2=\dot{z}_1,\ x_3=z_2,\ x_4=\dot{z}_2,\ x_5=\bar{y},\ x_6=\dot{\bar{y}}$ and $x:=[x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$. Rewrite (1) as

$$\dot{x}(t)=Ax(t)+Bu(t)+Fg(x,t) \tag{2}$$

where $A,\ B$ and F are listed on the top of this page; and

$$g(x,t)=\begin{bmatrix}f_{t1}+f_{t2}\\f_{t3}+f_{t4}\end{bmatrix}$$

As pointed out in [14], the nonlinear function $g(x,t)$ is uniformly bounded and can be assumed to satisfy the following cone-bounding constraint

$$\|g(x,t)\|_2\le\mu\|x\|_2 \tag{3}$$

where μ is a positive number. In fact, the nonlinear function $g(x,t)$ is usually regarded as a sinusoidal disturbance, one can refer to [12], [13].

In Figure 1, suppose $H=40ft, h=250ft, \lambda=600ft, U_{ow}=0.4ft/sec$ and the other data of the structure can be referred to [3] or [15]. The natural frequencies of the first two modes of vibration are assumed to be $\omega_1=1.818$ and $\omega_2=10.8717$, respectively. By employing the known data, the coefficient matrices, A and B of (2) can be calculated, which are given in (4), on the top of the next page.

Suppose the wave frequency to be $1.8\ rps$, the nonlinear wave forces can be computed as Appendix A in [15]. When no control is applied to the offshore structure, the responses of the first, second and third floors of the structure subject to nonlinear wave-induced forces are plotted in Figure 2 [15], respectively, from which it is clearly seen that, the uncontrolled responses oscillate with amplitudes of peak to peak about $2.2627ft, 2.4518ft$ and $2.5379ft$, respectively. In what follows, we propose dynamic output feedback schemes to reduce the internal oscillations to guarantee the safety of the structure.

III. Dynamic Output Feedback Control

Let y denote the output vector of the system

$$y(t)=Cx(t) \tag{5}$$

where C is a constant real matrix of appropriate dimensions. The corresponding state space model of the offshore structure can be expressed as

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)+Fg(x,t)\\ y(t)=Cx(t)\\ x(t)=0,\quad t=0.\end{cases} \tag{6}$$

We seek a dynamic output feedback controller of the form

$$\begin{cases}\dot{x}_c(t)=A_Kx_c(t)+B_Ky(t)\\ u(t)=C_Kx_c(t)+D_Ky(t)\end{cases} \tag{7}$$

where $x_c\in\mathbb{R}^6$ and A_K, B_K, C_K and D_K are real matrices of appropriate dimensions to be determined, such that the resulting closed loop system is asymptotically stable.

Introduce an augmented vector

$$\zeta(t):=[x^T(t)\ \ x_c^T(t)]^T$$

then the resulting closed-loop system by (6) and (7) is given by

$$\dot{\zeta}(t)=(A_0+HKL)\zeta(t)+E^TFg(x,t) \tag{8}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -3.3235 & -0.0212 & 0.0184 & 0.0030 & -5.3449 & -0.8819 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0.0184 & 0.0030 & -118.1385 & -0.1117 & 5.3468 & 0.8822 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -0.0114 & -0.0019 & 0.0114 & 0.0019 & -3.3051 & -0.5454 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.003445 \\ 0 \\ -0.00344628 \\ 0 \\ 0.00213 \end{bmatrix} \tag{4}$$

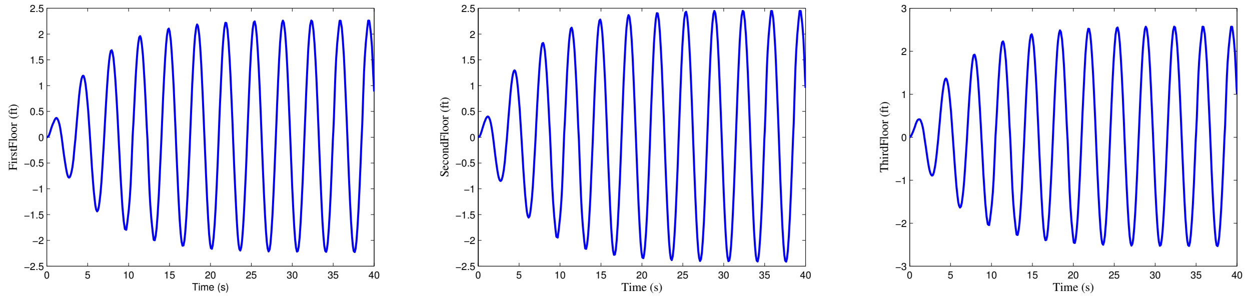

Fig. 2. Responses of the first, second and third floors with no control

where $E = [I\ 0]$, $A_0 = \text{diag}\{A, 0\}$, $H = \text{diag}\{B, I\}$, $L = \text{diag}\{C, I\}$ and

$$K := \begin{bmatrix} D_K & C_K \\ B_K & A_K \end{bmatrix} \tag{9}$$

For system (8), we have the following result.

Proposition 1: Let W_1 and W_2 be the orthogonal complements of B and C^T, respectively. A dynamic output feedback controller of form (7) is solvable for system (6) if there exist 6×6 real matrices $X > 0, Y > 0$ such that

$$\begin{bmatrix} W_1^T(AX+XA^T+FF^T)W_1 & \mu W_1^T X \\ * & -I \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} W_2^T(YA+A^TY+\mu^2 I)W_2 & W_2^T YF \\ * & -I \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} X & I \\ * & Y \end{bmatrix} > 0 \tag{12}$$

Moreover, if (10)-(12) are feasible on matrix variables X, Y, then the controller parameters K defined in (9) can be obtained by the following LMI

$$P(A_0 + HKL) + (A_0 + HKL)^T P + \mu^2 E^T E + PE^T FF^T EP < 0 \tag{13}$$

where

$$P := \begin{bmatrix} Y & I \\ N^T & 0 \end{bmatrix} \begin{bmatrix} I & X \\ 0 & M^T \end{bmatrix}^{-1}, \quad MN^T = I - XY \tag{14}$$

Proof: See the full version of this paper [16]. □

Now, based on Proposition 1, we design a dynamic output feedback controller for system (2) with (4). Let

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{15}$$

Applying Proposition 1, for $\mu = 1$, the obtained dynamic output feedback controller, which is denoted by K_1, is given in (7) with $K_1 := \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$ being shown on the top of next page. Clearly, the controller K_1 is of a large gain in the sense of Euclidean norm, namely, $\|K_1\|_2 = 4.1308 \times 10^8$, which demands a large control force. It is worth pointing out that this feature remains even for small value of μ. For example, taking $\mu = 0.01$ and applying proposition 1 yields a dynamic output feedback controller K_2, which is also of a large gain with $\|K_2\|_2 = 5.6472 \times 10^8$. Figures 3 depicts the responses of the first, second, third floors of the offshore structure under the control of K_2. From these figures, we can find that the oscillation magnitudes of peak to peak have been reduced from $2.2627ft, 2.4518ft, 2.5379ft$ to $0.5433ft, 0.5967ft$ and $0.6388ft$, respectively.

IV. Dynamic Output Feedback Control by Introducing Delayed Measurements

In practical feedback control systems, small time-delays in the control action are inevitable because of involved dynamics of actuators and sensors. As is seen from the previous section, the obtained dynamic output feedback controller is of a large gain in the sense of Euclidean norm. In this section, we are interested in investigating the effect of a small time-delay on dynamic output feedback control of the offshore structure. More specifically, we introduce a small time-delay $h > 0$ when we measure the output of the system, i.e.

$$y(t) = Cx(t-h) \tag{16}$$

The corresponding state space model of the offshore structure is stated as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Fg(x,t) \\ y(t) = Cx(t-h) \\ x(t) = 0, \quad t \in [-h, 0]. \end{cases} \tag{17}$$

We now design a dynamic output feedback controller of form (7) based on the system (17). The resulting closed-loop

$$K_1 = 10^8 \times \left[\begin{array}{rrrrrr|rrr} -0.0058 & -0.0083 & 0.0023 & -0.0011 & -0.0064 & -0.0000 & -0.0035 & -0.0015 & -0.0107 \\ -0.2918 & -0.4229 & 0.1174 & -0.0554 & -0.3246 & -0.0007 & -0.1780 & -0.0769 & -0.5410 \\ 1.2906 & 1.2299 & -0.2677 & 0.1075 & 0.9412 & 0.0028 & 0.4031 & -0.5884 & 1.5677 \\ 1.4708 & -0.5941 & 0.4779 & -0.3062 & -0.4671 & 0.0020 & -0.7372 & -3.5625 & -0.7830 \\ -1.1264 & -1.3226 & 0.4061 & -0.1785 & -1.2684 & -0.0012 & -0.1052 & 0.2773 & -2.1279 \\ -1.3491 & -1.2400 & -0.2681 & 0.1182 & 0.8629 & -0.0126 & -3.2107 & -0.2128 & 1.5399 \\ \hline -2.9911 & 1.3116 & 0.0897 & -0.0887 & 0.2414 & 0.0026 & 1.0962 & -1.7721 & 0.3633 \end{array}\right]$$

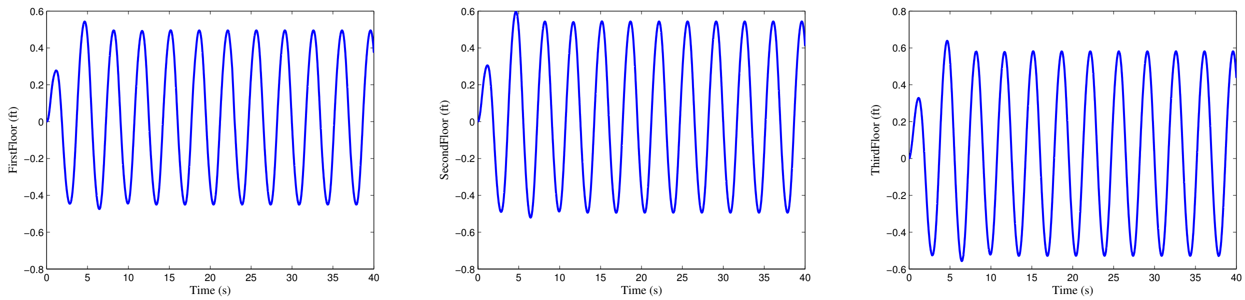

Fig. 3. Responses of the first, second and third floor under the control of K_2

system by (17) and (7) is

$$\dot{\zeta}(t) = (A_0 + HKL_1)\zeta(t) + HKL_2E\zeta(t-h) + E^TFg(x,t) \tag{18}$$

where A_0, H, K are the same as those in (8) and

$$L_1 := \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \quad L_2 := \begin{bmatrix} C \\ 0 \end{bmatrix}$$

We now state and establish the following stability criterion.

Proposition 2: For given positive scalars μ and h, the system (18) is asymptotically stable if there exist real matrices $P > 0$, $Q > 0$ and $R > 0$ of appropriate dimensions such that matrix inequality (20), on the top of next page, holds.

Proof: See the full version of this paper [16]. □

Proposition 2 provides a delay-dependent stability criterion for the nonlinear system (18). However, this condition cannot be used to design the controller parameters directly due to some nonlinear terms, such as $PHKL_1$ and $PHKL_2$ etc. in matrix inequality (20). In what follows, we propose a controller design method based on Proposition 2. Similar to the proof of Proposition 1, a sufficient condition of the existence of a desired controller can be obtained, which is stated in the following proposition.

Proposition 3: Let W_1 and $[W_2^T \;\; W_3^T]^T$ be the orthogonal complements of C^T and $[B^T \;\; B^T]^T$, respectively. Matrix inequality (20) is feasible on matrix variable K if and only if there exist real 6×6 matrices $X > 0$, $Y > 0$, $Q > 0$ and $R > 0$ such that (12) and

$$\begin{bmatrix} \Xi_{11} & RW_1 & YF & hA^TR \\ * & W_1^T(-Q-R)W_1 & 0 & 0 \\ * & * & -I & hF^TR \\ * & * & * & -R \end{bmatrix} < 0 \tag{21}$$

$$\Omega := \begin{bmatrix} \Omega_{11} & W_2^TXR \\ * & -Q-R \end{bmatrix} < 0 \tag{22}$$

where

$$\begin{aligned} \Omega_{11} := {} & W_2^T[AX + XA^T + X(Q - R + \mu^2 I)X]W_2 \\ & + (W_2 + W_3)^T FF^T (W_2 + W_3) + W_2^T XA^T W_3 \\ & + W_3^T AXW_2 - h^{-2} W_3^T R^{-1} W_3 \\ \Xi_{11} := {} & YA + A^TY + Q - R + \mu^2 I. \end{aligned}$$

□

One can see that matrix inequality (22) is still nonlinear on matrix variables, which is a non-convex feasible problem. Now, we will convert this non-convex feasible problem into a nonlinear minimization problem subject to a set of LMIs. Define $J := \mathrm{diag}\{I, \;\; X\}$, then pre- and post-multiplying both sides of Ω in (22) by J^T and its transpose, respectively, yield

$$J^T\Omega J = \begin{bmatrix} \Omega_{11} & W_2^TXRX \\ * & -XQX - XRX \end{bmatrix} < 0 \tag{23}$$

Introducing two new matrix variables $S > 0$ and $Z > 0$ such that

$$XRX \geq S, \quad XQX \geq Z$$

which are equivalent to, respectively

$$\begin{bmatrix} R & X^{-1} \\ X^{-1} & S^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} Q & X^{-1} \\ X^{-1} & Z^{-1} \end{bmatrix} \geq 0.$$

Notice that

$$\begin{bmatrix} -W_2^TXRXW_2 & W_2^TXRX \\ * & -XRX \end{bmatrix} \leq \begin{bmatrix} -W_2^TSW_2 & W_2^TS \\ * & -S \end{bmatrix}$$

One can see that matrix inequality (23) is implied by the following

$$\begin{bmatrix} \tilde{\Omega}_{11} & W_2^TS \\ * & -Z-S \end{bmatrix} < 0 \tag{24}$$

where

$$\tilde{\Omega}_{11} := W_2^T[AX + XA^T + X(Q + \mu^2 I)X - S]W_2$$

$$\Theta := \begin{bmatrix} \Theta_{11} & PHKL_2 + E^T R & PE^T F & h(A_0 + HKL_1)^T E^T R \\ * & -Q - R & 0 & hHKL_2^T E^T R \\ * & * & -I & hF^T R \\ * & * & * & -R \end{bmatrix} < 0 \tag{20}$$

$$\Theta_{11} := P(A_0 + HKL_1) + (A_0 + HKL_1)^T P + E^T(Q - R + \mu^2 I)E. \tag{21}$$

$$+ (W_2 + W_3)^T FF^T (W_2 + W_3) + W_2^T XA^T W_3 + W_3^T AXW_2 - h^{-2} W_3^T R^{-1} W_3$$

By using Schur complement, (24) is equivalent to

$$\begin{bmatrix} \breve{\Omega}_{11} & W_2^T S & W_2^T X & \mu W_2^T X \\ * & -Z - S & 0 & 0 \\ * & * & -Q^{-1} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{25}$$

where

$$\breve{\Omega}_{11} := W_2^T [AX + XA^T - S] W_2 - h^{-2} W_3^T R^{-1} W_3 + (W_2 + W_3)^T FF^T (W_2 + W_3) + W_2^T XA^T W_3 + W_3^T AXW_2$$

Therefore, setting $\bar{R} = R^{-1}, \bar{X} = X^{-1}, \bar{S} = S^{-1}, \bar{Z} = Z^{-1}, \bar{Q} = Q^{-1}$, and similar to the proof of Proposition 2, we can derive a new sufficient condition for the existence of the dynamic output feedback controller, which is formulated in the following result.

Proposition 4: Let W_1 and $[W_2^T \;\; W_3^T]^T$ be the orthogonal complements of C^T and $[B^T \;\; \bar{B}^T]^T$, respectively. For given scalars $\mu > 0$ and $h > 0$, the dynamic output feedback control problem for system (17) is solvable if there exist 6×6 real matrices $X > 0, Y > 0, Q > 0, R > 0, \bar{R} > 0, \bar{X} > 0, S > 0, \bar{S} > 0, Z > 0, \bar{Z} > 0$ and $\bar{Q} > 0$ such that (12), (21) and

$$\begin{bmatrix} \Upsilon & W_2^T S & (W_2^T + W_3^T)F & W_2^T X & \mu W_2^T X \\ * & -Z - S & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\bar{Q} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} R & \bar{X} \\ \bar{X} & \bar{S} \end{bmatrix} \geq 0, \quad \begin{bmatrix} Q & \bar{X} \\ \bar{X} & \bar{Z} \end{bmatrix} \geq 0 \tag{27}$$

$$R\bar{R} = I, \; X\bar{X} = I, \; S\bar{S} = I, \; Z\bar{Z} = I, \; Q\bar{Q} = I \tag{28}$$

where

$$\Upsilon := W_2^T [AX + XA^T - S] W_2 + W_2^T XA^T W_3 + W_3^T AXW_2 - h^{-2} W_3^T \bar{R} W_3.$$

Proposition 4 is based on a set of LMIs subject to equality constraints. By employing the cone complementary method proposed in [5], it can be converted into a nonlinear minimization problem (NMP) subject to LMIs, which is stated in the following.

Nonlinear Minimization Problem (NMP)

$$\text{Minimize} \quad \mathrm{Tr}(\bar{X}X + \bar{R}R + \bar{Q}Q + \bar{Z}Z + \bar{S}S) \tag{29}$$

Subject to (12), (21), (26), (27) and

$$\begin{bmatrix} R & I \\ I & \bar{R} \end{bmatrix} \geq 0, \begin{bmatrix} X & I \\ I & \bar{X} \end{bmatrix} \geq 0, \begin{bmatrix} Q & I \\ I & \bar{Q} \end{bmatrix} \geq 0, \tag{30}$$

$$\begin{bmatrix} Z & I \\ I & \bar{Z} \end{bmatrix} \geq 0, \begin{bmatrix} S & I \\ I & \bar{S} \end{bmatrix} \geq 0. \tag{31}$$

The following iterative algorithm can be used to solve the above NMP.

Algorithm 1 Solve the NMP (29).

Step 1 Find a feasible set $(X^0, Y^0, Q^0, R^0, S^0, Z^0, \bar{X}^0, \bar{Q}^0, \bar{R}^0, \bar{S}^0, \bar{Z}^0)$ satisfying 12), (21), (26), (27) and (30), (31). Set $l = 0$.

Step 2 Solve the following LMI problem for the matrix variables $(X, Y, Q, R, S, Z, \bar{X}, \bar{Q}, \bar{R}, \bar{S}, \bar{Z})$:

$$\text{Minimize} \quad \mathrm{Tr}\begin{pmatrix} \bar{X}^l X + X^l \bar{X} + \bar{R}^l R + R^l \bar{R} + \bar{Q}^l Q \\ + Q^l \bar{Q} + \bar{Z}^l Z + Z^l \bar{Z} + \bar{S}^l S + S^l \bar{S} \end{pmatrix}$$
$$\text{Subject to} \quad (12), (21), (26), (27) \text{ and } (30), (31)$$

Set $X^{l+1} = X, Q^{l+1} = Q, R^{l+1} = R, Z^{l+1} = Z, S^{l+1} = S, \bar{X}^{l+1} = \bar{X}, \bar{Q}^{l+1} = \bar{Q}, \bar{R}^{l+1} = \bar{R}, \bar{Z}^{l+1} = \bar{Z}, \bar{S}^{l+1} = \bar{S}$.

Step 3 If matrix inequality (22) and

$$\left| \mathrm{Tr}\begin{pmatrix} \bar{X}^l X + X^l \bar{X} + \bar{R}^l R + R^l \bar{R} + \bar{Q}^l Q \\ + Q^l \bar{Q} + \bar{Z}^l Z + Z^l \bar{Z} + \bar{S}^l S + S^l \bar{S} \end{pmatrix} - 10n \right| < \varepsilon \tag{32}$$

where ε is a prescribed sufficiently small positive number, are satisfied, then set $l = l + 1$ and go to Step 2. If one of the conditions (22) and (32) is not satisfied within a specified number of iterations, then exit.

Finally, if the nonlinear minimization problem (29) is feasible on the matrix variables R, Q etc., then the desired dynamic output feedback controller of form (7) can be obtained by solving the LMI (20) on the matrix variable K in (9) with the known R, Q and P of form (14).

Now, we are in position to design a dynamic output feedback controller for system (2) with (4) based on Proposition 3 incorporating with Algorithm 1. Setting $h = 0.02$, for $\mu = 1$, the dynamic output feedback controller, which is denoted by K_3, can be derived, and the controller parameters $K_3 := \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$ are given on the top of this page.

It is worth noting that the controller gain of K_3 with $\|K_3\|_2 = 6.6224 \times 10^4$ is much smaller than those of K_1 with $\|K_1\|_2 = 4.1308 \times 10^8$ and K_2 with $\|K_1\|_2 =$

$$K_3 = \left[\begin{array}{cccccc|ccc} -19.821 & 1.3551 & 4.6275 & -5.0287 & -0.0824 & -0.3689 & 0.0042 & -0.0028 & 0.0008 \\ -1.9217 & -1.3374 & -1.2510 & 1.3649 & 0.0221 & 0.0997 & 0.0103 & 0.0013 & 0.0024 \\ 44.687 & -2.4412 & -5.3613 & 2.3260 & -0.5188 & -2.5452 & 0.6109 & -118.7021 & -3.5789 \\ 53.866 & -1.1599 & -2.9063 & -3.0366 & -0.5630 & -2.9301 & 1.2531 & -119.9537 & -3.4823 \\ 52.437 & 9.1799 & 4.4460 & 1.8615 & -0.7438 & -2.9704 & 19.6555 & -80.3220 & 2.1280 \\ 574.38 & 129.27 & 116.50 & -97.968 & -0.4820 & -38.056 & 234.6118 & -1331.8 & 10.8054 \\ \hline -62745 & 2519 & 14215 & -15450 & -253 & -1133 & -0.0016 & 0.3327 & 0.0059 \end{array}\right]$$

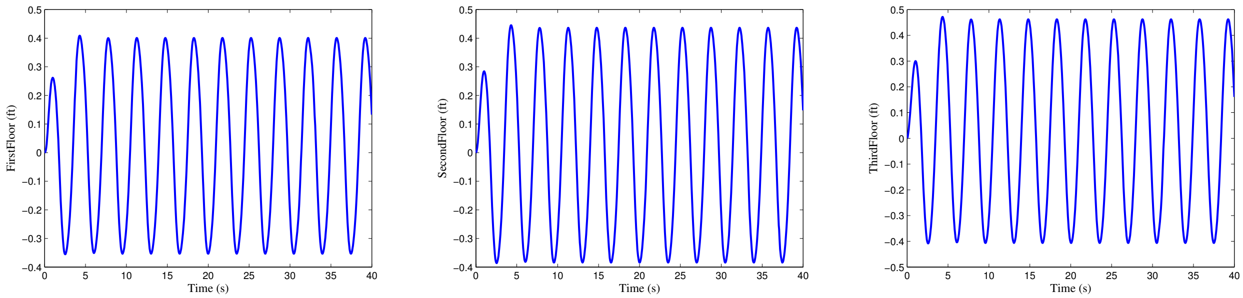

Fig. 4. Responses of the first, second and third floors under the control of K_3

5.6472×10^8. So it is more easily implemented than K_1 and K_2 for the offshore structure. In addition, we plot the response curves of the first, second and third floors of the offshore structure via the controller of K_3, which are shown in Figure 4. Clearly, the oscillation amplitudes of peak to peak are apparently reduced from the uncontrolled values of $2.2627ft$, $2.4518ft$, $2.5739ft$ to $0.4088ft$, $0.4451ft$, $0.4716ft$, respectively. Therefore, by appropriately introducing a small time-delay into the output channel, the designed controller is of a small gain in the sense of Euclidean norm, which demands a small control force, and the internal oscillation of the offshore structure can be dramatically reduced.

Remark 1: For an offshore steel jacket structure, the system parameters are usually subject to uncertainty. In this case, the proposed method can easily be extended to design a robust dynamic output feedback controller by intentionally introducing a small time-delay to make the uncertain offshore steel jacket structure work in a safe environment. Due to page limitation, it is omitted.

V. Conclusions

We have investigated the effect of a small time-delay on dynamic output feedback control of an offshore structure subject to a nonlinear wave-induced force. We have found that appropriately introducing a small time-delay into the output channel, the controller is of a small gain in the sense of Euclidean norm, which demands a small control force, and the internal oscillations of the offshore structure can be dramatically reduced. Simulation results have confirmed our finding.

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