Set-membership identification of block-structured nonlinear feedback systems

V. Cerone, D. Piga, D. Regruto
Dipartimento di Automatica e Informatica
Politecnico di Torino (Italy)

e-mail: vito.cerone@polito.it, dario.piga@polito.it, diego.regruto@polito.it

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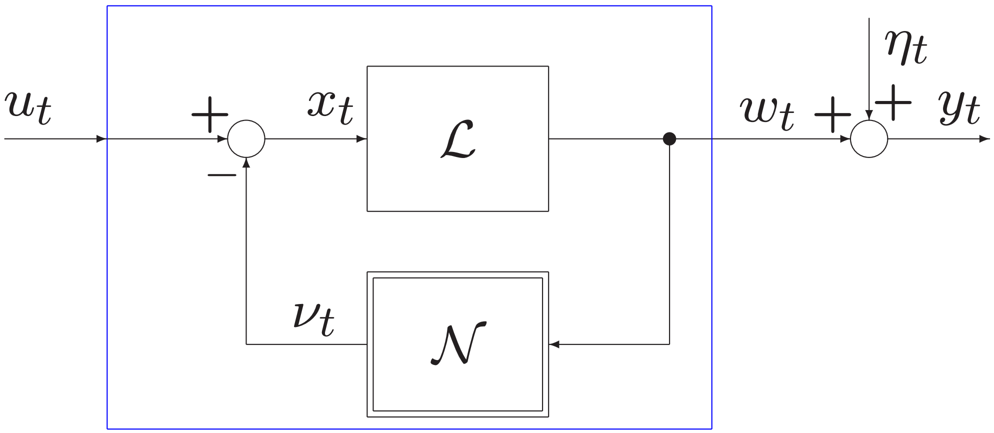
Nonlinear feedback systems



$\mathcal{N}$: nonlinear static block

$\mathcal{L}$: linear dynamic subsystem

x_t, ν_y: not measurable inner signals

u_t: known input signal

y_t: noise-corrupted measurement of w_t

$$\nu_t = \mathcal{N}(w_t) = \sum_{k=1}^{n} \gamma_k w_t^k$$ with n: polynomial degree

$$w_t = \frac{B(q^{-1})}{A(q^{-1})} x_t \quad \text{with} \quad \begin{array}{rcl} A(q^{-1}) & = & 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \\ B(q^{-1}) & = & b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb} \\ q^{-1} w_t & = & w_{t-1} \end{array}$$

Problem formulation

- **Aim:** compute bounds on the parameters $\gamma^\top = [\gamma_1, \; \gamma_2 \; \dots \; \gamma_n]$ and $\theta^\top = [a_1 \dots a_{na} \; b_0 \dots b_{nb}]$
- **Prior assumption on the system:**
  - BIBO stability
  - na and nb are known
  - n is finite and known
  - the steady-state gain of the linear subsystem is not zero
  - a rough upper bound on the settling time of the system is known
- **Prior assumption on the measurement uncertainty:**
  - η_t is UBB: $\mid \eta_t \mid \leq \Delta\eta_t$
  - $\Delta\eta_t$ is known

Proposed solution

ree-stage procedure:

- First stage: computation of bounds on the nonlinear block parameters γ.
- Second stage: computation of bounds on the inner (unmeasurable) signal x_t.
- Third stage: computation of bounds on the linear block parameters θ.

Proposed solution: first stage

Bounds on the parameters γ of the nonlinear block:

- Stimulate the system with square-wave of M different amplitude and get *steady-state* measurements
- The feasible parameters set $\mathcal{D}_\gamma$ of the nonlinear block is described as:

$$\mathcal{D}_\gamma = \{\gamma \in R^n : (\overline{y}_s - \overline{\eta}_s) + \sum_{k=1}^{n} \gamma_k (\overline{y}_s - \overline{\eta}_s)^k = \overline{u}_s,$$
$$|\, \overline{\eta}_s \,| \leq \Delta \overline{\eta}_s; \quad s = 1, \ldots, M\},$$

- $\mathcal{D}_\gamma$ is the set of all parameters γ consistent with the M given measurements, the error bounds and the assumed model structure
- Bounds on parameter γ_k:

$$\gamma_k^{min} = \min_{\gamma \in \mathcal{D}_\gamma} \gamma_k \qquad \gamma_k^{max} = \max_{\gamma \in \mathcal{D}_\gamma} \gamma_k$$

Proposed solution: first stage

Computation of γ_k^{min} and γ_k^{max}:

$$\gamma_k^{min} = \min_{(\gamma,\overline{\eta})\in\mathcal{D}_{\gamma\overline{\eta}}} \gamma_k \qquad \gamma_k^{max} = \max_{(\gamma,\overline{\eta})\in\mathcal{D}_{\gamma\overline{\eta}}} \gamma_k$$

ere:

$$= [\overline{\eta}_1 \; \overline{\eta}_2 \ldots \overline{\eta}_M]^T \, ,$$

$$_{\overline{\eta}} = \{(\gamma,\overline{\eta})\in\mathbb{R}^n \times \mathbb{R}^M : (\overline{y}_s - \overline{\eta}_s) + \sum_{k=1}^{n} \gamma_k (\overline{y}_s - \overline{\eta}_s)^k = \overline{u}_s,$$
$$| \, \overline{\eta}_s \, | \leq \Delta\overline{\eta}_s; \;\; s = 1,\ldots,M\}$$

$\mathcal{D}_{\gamma\overline{\eta}}$ is a semialbegraic set over $\mathbb{R}^{n+M}$

The above problems are semialgebraic (nonconvex) optimization problems

Proposed solution: first stage

Standard nonlinear optimization tools can not be exploited to compute γ_k^{min} and γ_k^{max}
ce they can trap in local minima

$$\Downarrow$$

The true value of γ_k could not lie in $\left[\gamma_k^{min},\ \gamma_k^{max}\right]$

Relax original identification problems to convex optimization problems

$$\Downarrow$$

Bounds on each parameter γ_k can be obtained

Convex relaxation

MI relaxation for semialgebraic optimization problems:

- SOS decomposition

P. Parrillo, "Semidefinite programming relaxations for semialgebraic problems", *Mathematical Programming 2003*

- Theory of moments

J. B. Lasserre, "Global optimization with polynomials and the problem of moments", *SIAM J. on Opt. 2001*

-relaxed bounds $\gamma_k^{min^\delta}$ and $\gamma_k^{max^\delta}$ computed solving the following SDP problems:

$$\gamma_k^{min^\delta} = \min_{x \in \mathcal{D}_x^\delta} f(x) \quad \gamma_k^{max^\delta} = \max_{x \in \mathcal{D}_x^\delta} f(x)$$

here:

LMI decision variables

$x)$: linear function

: Convex set described by LMI constraints

Tightness and convergence

operty 1 — δ-relaxed bounds become tighter as δ increases:

$$\begin{aligned} \gamma_k^{min^{\delta}} \leq \ & \gamma_k^{min^{\delta+1}} \leq \gamma_k^{min} \\ \gamma_k^{max^{\delta}} \geq \ & \gamma_k^{max^{\delta+1}} \geq \gamma_k^{max} \end{aligned}$$

operty 2 — δ-relaxed bounds converge to the true bounds as $\delta \to \infty$:

$$\begin{aligned} \lim_{\delta\to\infty} \gamma_k^{min^{\delta}} &= \gamma_k^{min} \\ \lim_{\delta\to\infty} \gamma_k^{max^{\delta}} &= \gamma_k^{max} \end{aligned}$$

Computational complexity of the LMI relaxation

n practice, due to an high computational complexity, LMI relaxation techniques can be ploited only for a small set of measurements

$$\Downarrow$$

A reduction of the complexity of SDP relaxed problems is necessary

Reduced complexity of the relaxed problems

$$\mathcal{D}_{\gamma\overline{\eta}} = \{(\gamma,\overline{\eta}) \in \mathbb{R}^n \times \mathbb{R}^M : (\overline{y}_s - \overline{\eta}_s) + \sum_{k=1}^{n} \gamma_k (\overline{y}_s - \overline{\eta}_s)^k = \overline{u}_s,$$
$$|\, \overline{\eta}_s \,| \leq \Delta\overline{\eta}_s; \quad s = 1, \ldots, M\}$$

operty 3 The variables $\overline{\eta}_s$ defining $\mathcal{D}_{\gamma\overline{\eta}}$ are not correlated with each other

ea In constructing moment matrix defining $\mathcal{D}_x^{\delta}$ do not consider the correlation between
riables $\overline{\eta}_s$

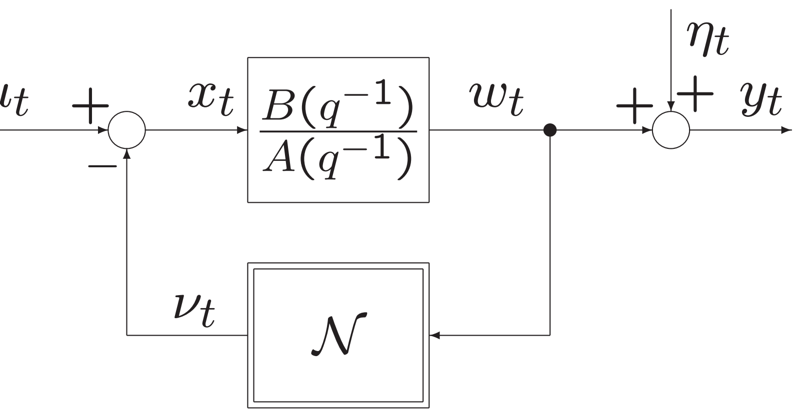
Reduced complexity of the relaxed problems

Value of M greater than 400 can be exploited in the identification (for $\delta \leq 4$)

operty 4 — Convergence to tight bounds is preserved

Proposed solution: second stage

Bounds on the inner signal x_t:



$$
\begin{aligned}
x_t^{min} &= u_t - \nu_t^{max} \\
x_t^{max} &= u_t - \nu_t^{min} \\
\nu_t^{min} &= \min_{(\gamma,\overline{\eta})\in\mathcal{D}_{\gamma\overline{\eta}},|\eta_t|\leq\Delta\eta_t} \sum_{k=1}^{n} \gamma_k (y_t-\eta_t)^k \\
\nu_t^{max} &= \max_{(\gamma,\overline{\eta})\in\mathcal{D}_{\gamma\overline{\eta}},|\eta_t|\leq\Delta\eta_t} \sum_{k=1}^{n} \gamma_k (y_t-\eta_t)^k
\end{aligned}
$$

- Stimulate the system with a persistently exciting input signal u_t
- Bounds on ν_t can be computed by means of LMI relaxation
- Structure of the problem can be exploited to reduce the computation complexity

Proposed solution: third stage

Bounds on the linear block parameters θ:

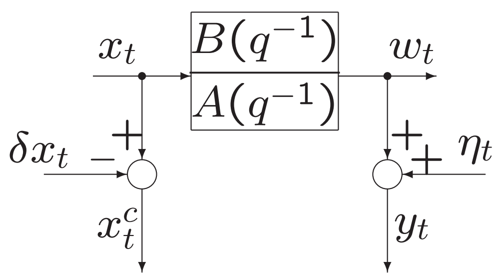
nner signal x_t described in terms of its central value x_t^c and its perturbation δx_t:

$$x_t = x_t^c + \delta x_t$$

h:

$$x_t \,|\leq \Delta x_t, \quad x_t^c = \frac{x_t^{min} + x_t^{max}}{2}, \quad \Delta x_t = \frac{x_t^{max} - x_t^{min}}{2}$$

dentification of a linear model with noisy output sequence $\{y_t\}$ and uncertain input se-
ence $\{x_t\}$



Errors-in-variables (EIV) problem with bounded errors

Proposed solution: bounds on θ

ploiting previous results on EIV problems with bounded errors

Cerone, **"Feasible parameter set of linear models with bounded errors in all variables"**, *Automatica 1993*)

$$\Downarrow$$

Bounds on θ_j are computed by means of linear programming

Example

rameters of the simulated system

$(w_t) = -1.5w_t + 1.2w_t^2 + 0.9w_t^3$

$q^{-1}) = 1 - 1.5193q^{-1} + 0.5326q^{-2}$

$q^{-1}) = 0.1549q^{-1} - 0.1416q^{-2}$

easurements output errors

$_s \mid \leq \Delta\overline{\eta}_s, \ \ \{\overline{\eta}_s\}$ random variables belonging to $[-\Delta\overline{\eta}_s, +\Delta\overline{\eta}_s]$

$_t \mid \leq \Delta\eta_t, \ \ \{\eta_t\}$ random variables belonging to $[-\Delta\eta_t, +\Delta\eta_t]$

ring the simulated experiment the SNR is about $25db$.

nlinear block parameters: central estimates and parameters bounds ($M = 50$, $\delta = 3$)

True Value	γ_k^{min}	γ_k^{c}	γ_k^{max}	$\Delta\gamma_k$
-1.5000	-1.5369	-1.4890	1.4410	0.0480
1.2000	1.1931	1.2072	1.2213	0.0141
0.9000	0.8898	0.9020	0.9141	0.0121

$$\Delta\gamma_k = \frac{\gamma_k^{max} - \gamma_k^{min}}{2}$$

Linear block parameters: central estimates and parameters bounds

N	True Value	θ_j^{min}	θ_j^{c}	θ_j^{max}	$\Delta\theta_j$
100	-1.5193	-2.0326	-1.6422	-1.2518	0.3904
	0.5326	0.3046	0.6364	0.9681	0.3318
	0.1549	0.1424	0.1579	0.1734	0.0155
	-0.1416	-0.2201	-0.1232	-0.0264	0.0969
300	-1.5193	-1.8569	-1.5633	-1.2697	0.2936
	0.5326	0.3265	0.5761	0.8256	0.2496
	0.1549	0.1452	0.1555	0.1659	0.0104
	-0.1416	-0.1951	-0.1348	-0.0746	0.0602

$$\Delta\theta_j = \frac{\theta_j^{max} - \theta_j^{min}}{2}$$

Conclusion

- Three stage procedure to evaluate parameters bounds of a nonlinear feedback system
- Bounds on the nonlinear block parameters have been evaluated by means of LMI relaxation techniques
- The particular structure of the identification problems allows the reduction of the complexity of the LMI relaxation
- Convergence to tight bounds is guaranteed
- Bounds on the parameters of the linear block has been computed through the evaluation of bounds on the unmeasurable inner signal x_t