# Optimal (static output feedback) design through Direct Search methods 

Emile Simon UCL Belgium

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## Aim of this work:

Putting forward a class of optimization methods largely overlooked in systems and control: the direct search methods.

They can however be very adequate and powerful for many complex problems (possibly including yours).

Direct search optimization
$\square$ Other problem, without gradient

## Direct search optimization methods

- What are these methods?
- Why are they overlooked in systems and control?
- Why should they be used?


## What are these methods?

- The aim is to $\min (/ m a x)$ imize an objective function $f(x): \mathbb{R}^{n} \rightarrow \mathbb{R}$, possibly under constraints, starting from one (or several) feasible initial solution(s).
- Only use function evaluations to decide how to explore $\mathbb{R}^{n}$,
- need no gradient or Hessian information.
- In this talk, direct search $=$ derivative-free optimization
- See e.g. webpages of Luis Vicente and Charles Audet.

Why are they overlooked in systems and control?: A history

- 1957-1961: The birth (Box, Davidon, Hooke, Jeeves)
- 1961-1971: The golden age (1965 Nelder-Mead)
- Very efficient in practice.
- 1972-1989: The downfall (after survey of W. Swann 1972)
- No proofs of convergence and can sometimes be slow.
- 1990-...: The resurrection = Proofs of convergence
- Torczon and Dennis: MDS, proof on smooth $f(x)$ : 1997
- Audet and Dennis: MADS, proof on non-smooth $f(x): 2003$
- Vicente and Custodio: proof on discontinuous $f(x): 2010$

Last item arrived too late for systems and control:
From the early nineties, our field got saturated by convex optimization and LMIs, solvable very efficiently but...

## Why should they be used? $(1 / 2)$

- Current problems of interest have non-convex feasible sets.
- The best that can be done is to propose methods with guaranteed convergence to locally optimal solutions.
- There exist (new) direct search methods guaranteed to converge even on most non-smooth or discontinuous $f(x)$.


## Why should they be used? (2/2)

When (clean) gradients of $f(x)$ are not (easily) available.
The main advantages are:

- Recent strong convergence guarantees
- Exponential performance of computers
- Simulations are more routine and accurate
- Ease of use and implementation


## Optimization methods: A comparison

A basic comparison of optimization methods:

| Class of methods | Computational time | Adequate problems |
| :--- | :---: | :---: |
| Convex/LMI | Very efficient $^{1}$ | Many specific <br> (sub)problems |
| Gradient-based | Efficient | Clean derivatives <br> must be available |
| Derivative-free | May be slow | 'Any' $f(x): \mathbb{R}^{n} \rightarrow \mathbb{R}$ <br> $(n<25-100)$ |

${ }^{1}$ May need to introduce a lot of additional variables to convexify

Direct search optimization

Problems in the paper (SOF)

Other problem, without gradient

## The problems considered in the paper (1/2)

Static Output Feedback optimization, $m \times p$ MIMO LTI systems:

1) Stabilization of the closed-loop:

Find $K \in \mathbb{R}^{m \times p}$ s.t. $\sigma(A+B K C)<0$ (continuous time)
2 and 3) Min. the $\mathcal{H}_{2}$ or $\mathcal{H}_{\infty}$ norm of the performance channel:
$\min _{K}\left\|T_{w z}(K, s)\right\|_{2}$ or $\min _{K}\left\|T_{w z}(K, s)\right\|_{\infty}$, s.t. $\sigma(A+B K C)<0$

With unstable models from COMPlib: a library of actual and academic models, currently often used for benchmarking.

## The problems considered in the paper (2/2)

Non-convex: ZMIts (and iterative LMI schemes rarely converge)
The objectives admit however gradients or Clarke subgradients, which should then be used for optimization purposes.

This is implemented in the methods HIFOO and hinfstruct.
Paper's benchmarks: comparison of the objectives values reached and computational times required, by a DS method and by HIFOO.

Motivation: to verify convergence and assess performance of DS. Then other $f(x)$ without gradients can be considered.

## Note on the direct search method used

Currently, the best two derivative-free methods are certainly MADS and SID-PSM, having strong theoretical convergence guarantees.

In practice, a very efficient method easily accessible and usable is the Nelder-Mead algorithm restarted at the last solution until no improvement is obtained (to a given accuracy).

This generates a set of dense (enough) exploring directions in the search space $\mathbb{R}^{n}$ (but without formal theoretical guarantee).

Packs a serious punch in practice, sufficing to make the point.

## Summary of 13000 tests

The direct search method converges as expected, and is reasonably fast (meant for off-line design), despite not using gradients:

- 1) Stabilization success: DS 92.3\%, HIFOO 90.6\%. Ratio computational time DS on HIFOO $\cong 0.74$
- 2) Performance channel $\mathcal{H}_{2}$ norm minimization: Similar: 60\% / Better for DS: 14\% / Better for HIFOO: 26\% Ratio computational time DS on HIFOO $\cong 10$
- 3) Performance channel $\mathcal{H}_{\infty}$ norm minimization: Similar: 40\% / Better for DS: 23\% / Better for HIFOO: 37\% Ratio computational time DS on HIFOO $\cong 1.3$

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## Time-response shaping problem

Objective considered: to optimize explicitly the features of a time response $z(x, t)$, with the controller parameters $x \in \mathbb{R}^{n}$.

Interesting objective: $f(x)=t_{r}+\lambda$ max $_{\text {dev }}$, where:
$t_{r} \in \mathbb{R}^{+}$is the rise time needed by the response $z(x, t)$ to reach a desired settling region (above $z_{\max }(t)$ and under $z_{\text {min }}(t)$ ),
$\max _{\text {dev }} \in \mathbb{R}^{+}$is the maximum deviation of the response $z(x, t)$ outside of the desired region (for $t>0$ above, and $t>t_{r}$ under)
and $\lambda \in \mathbb{R}^{+}$is the scalar trade-off parameter.

## Difficulty and solution

The objective $m_{\text {ax }}^{\text {dev }}$ is non-smooth but locally Lipschitz:

- Subgradients can be used for minimization to locally optimal solutions (see Bompart, Apkarian and Noll 2008).
- (New) DS methods also converge on $m_{\text {ax }}$ (but slower).

The objective $t_{r}$ is however more difficult:

- It is actually discontinuous and no gradient analysis exists.
- (New) DS have guarantees of convergence for such objectives!


## Two results

Time-response $z(x, t)$ shaping examples:
Optimization of PID parameters $\left(x=\left[K_{p}, K_{i}, K_{d}\right]\right)$, by $\min _{x} f(x)$.
Choice of time envelope: $z_{\min }(t)=0.98$ and $z_{\max }(t)=1.02$
Two animated figures present two optimizations.

## Results animation (1/2)

Initial solution: Ziegler-Nichols parameters.


## Results animation (2/2)

Initial solution: random parameters, not stabilizing.


## Take-home messages

- There definitely exists alternatives to LMIs. Which method is adequate for the considered problem?
- Direct search methods now have strong theoretical convergence guarantees, and are acceptable in practice (efficient computers, methods and simulators).
- Should be much more considered for many open problems of optimization in systems and control.

Thank you for your attention!

Questions?

## Emile.Simon@uclouvain.be

