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Economic COP Optimization of a Heat Pump with Hierarchical Model Predictive Control

Fatemeh Tahersima, Jakob Stoustrup, Henrik Rasmussen and Soroush A. Meybodi

Abstract—A low-temperature heating system is studied in this paper. It consists of hydronic under-floor heating pipes and an air/ground source heat pump. The heat pump in such a setup is conventionally controlled only by feed-forwarding the ambient temperature. Having shown >10% cut-down on electricity bills by involving feedback control in a previous study, this paper has continued the same line of argument and has investigated effects of *a priori* knowledge on weather forecast and electricity price profile to alleviate the total electricity cost subject to constraints on resident's thermal comfort. A two level hierarchical control structure is chosen for this purpose. While local PI controllers at the bottom level maintain individual temperature set-points of the rooms, a model predictive controller at the top level minimizes water supply temperature, and hence maximizes the heat pump's coefficient of performance. At the same time, it determines the actual temperature set-points of the rooms by deviating from the user-defined set-points within a thermal tolerance zone. Simulations results confirm significant cut-down on electricity bills without sacrificing resident thermal comfort. The proposed control strategy is a leap forward towards balanced load control in Smart Grids where individual heat pumps in detached houses contribute to preserve load balance through intelligent electricity pricing policies.

I. INTRODUCTION

Low-temperature heating systems with renewable energy sources have become more popular due to growing public attention to the environmental issues. Hydronic under-floor heating system is an example of such systems which offers a profitable heating solution in suburban areas by utilizing a ground/air-source heat pump. A heat pump acts like a refrigerator and transfers heat from a colder medium, e.g. ambient air, shallow ground or water to the building which is at a higher temperature. An electrically driven heat pump can generate 3-4 kWh of heat from 1 kWh of electricity for driving the heat pump's compressor. A geothermal heat pump system is shown in Fig. 1. There are typically two hydronic and one refrigerant circuits interconnected through two heat exchangers. These are: 1) the underground buried brine-filled – mixture of water and anti-freeze – pipes with a small circulating pump; 2) the refrigerant-filled circuit, equipped with an expansion valve and driven by a compressor which is called heat pump; and 3) the indoor under-surface grid of pipes with another small circulating pump which distributes heat to the concrete floor of the building.

The underground temperature is fairly constant during several days and slowly varies with an annual pattern. This

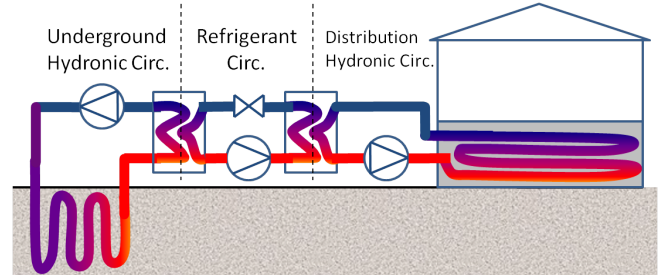


Fig. 1. An under-floor heating system with a geothermal heat pump

slow dynamic is due to the huge capacity of the ground and is an advantage to the air-source heat pump with the brine pipes exposed to the ambient air. The higher temperature at the evaporator side of the refrigerant circuit potentially increases the heat pump's Coefficient of Performance (COP) in the cold season. It is also an advantage in the warm season when heat pump works in reverse to cool down the building; because underground temperature is cooler than the ambient air in summer. Therefore, we specifically focus on geothermal heat pumps and assume a constant brine temperature.

Most commercial control solutions for heat pumps are based on feed-forwarding the ambient temperature. The forward temperature of water in the distribution hydronic circuit is adjusted based on *a priori* known adjustment curves. This method is further explained in one of our recent works [1]. In that paper, we investigated feedback control of heat pumps based on specific heat demands of individual houses. Effects of calculating the minimum heat demand of a building that handles all system constraints systematically were studied using a model predictive controller (MPC). It turned out that approximately 13% saving can be achieved in electricity consumption compared to pure feed-forward control.

Feedback control for a similar heating/cooling system is investigated in several other references too. Reference [2] conducts a comparison study among proportional (P), P-integral (PI), PI-derivative (PID) and relay controllers with a fixed control strategy. The approach is to lock the floor heating valve at fully-open position and control the forward temperature based on feedback from the room temperature. This method is practically efficient for a single-temperature zone. Multiple-temperature zones with different heat demands in a residential/office building can not be controlled by this control scheme. Reference [3] presents an MPC controller for both cooling and heating purposes. It focuses on a distributed model predictive control (DMPC)

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where different zones are controlled by semi-separated MPCs that only communicate their temperature setpoints with the adjacent zones. In another approach [4], the main simplifying assumption is to choose a constant COP for the heat pump. However, the amount of electricity saving by controlling a heat pumps' varying COP is considerable and should not be neglected at all.

This paper presents an integrated framework for COP and cost optimization of the specified hydronic heating system. We optimized COP by minimizing the supply temperature and shifting power consumption according to variations of the ambient temperature. The principal idea for this optimization method is developed in our previous work [1]. Optimization of electricity price is also feasible by load shifting, [5]. This is maintained by incorporating the concrete floor as a heat reservoir to store heat. By deferring daily power consumption from price-peak times to off-peak periods, residents can cut down electricity bills. According to [6], approximately half of the economic potential for saving in annual electricity bills, can be achieved by postponing power consumption in each day for a couple of hours.

The rest of this paper is structured as follows. As the first detailed step, our control strategy for the heating system of a specific apartment is given in Section II. Section III presents problem formulation by describing the plant model and introducing an optimization problem. The optimization problem is tackled by the control strategy and the results are presented in Section IV. Section V concludes the paper by offering a discussion on results and a road map to future works.

II. CONTROL SYSTEM STRUCTURE

Our case study is a 54 m² apartment which consists of three separate heat zones, i.e. rooms, shown in Fig. 2.

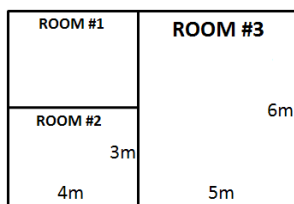


Fig. 2. Sketch of the apartment with three separate heat zones

Each room in Fig. 2 has a separate grid of under-surface floor heating pipes. As a whole, they form the hydronic distribution circuit of the apartment. The flow of heating water in each room is controlled by a valve. Valve openings are adjustable and are controlled by local PI controllers such that room-specific temperature setpoints are followed in presence of exogenous disturbances.

The circulation pump in the distribution circuit is controlled to regulate the differential pressure across all three parallel branches of the rooms' pipe grids. Thus, the flow through each valve is assumed to be only dependent on its opening percentage.

As of the refrigerant circuit, the expansion valve has a built-in mechanical feedback mechanism to marginally prevent flow of condensed refrigerant into the compressor, i.e. the heat pump. The heat pump could be continuously controlled. In [1], we have employed a Model Predictive Controller (MPC) to reduce heat pump's power consumption as much as possible. This goal was achieved by minimizing the *forward temperature*. Forward temperature is the temperature of water at inlet of the distribution piping grid; and it should be high enough in order to facilitate room temperature control by local PI controllers without driving any of the room valves into the fully-open saturated status, otherwise no actuation capacity is left for compensating exogenous disturbances that may hit the system at any time.

In the aforementioned paper, however, we did not consider the influence of a priori known disturbances like the ambient temperature and electricity price. Knowledge about the ambient temperature in advance could help to improve thermal comfort and result in a higher daily COP. A higher COP means less electricity consumption and a cut down in energy costs. The control strategy which lead us toward this objective is deferring heat load from nighttime to daytime. We can store heat in the concrete floor during day when the demanded forward temperature is lower than in night, or in the other words, COP is higher. The buffered heat can then be used in night time when COP is normally higher.

A priori knowledge about price of electricity could be provided by the grid utility 24 hours in advance. Two types of electricity tariffs are considered: daily and hourly prices. Deferring the load can be well accommodated by daily price variations, however hourly variations do not influence the pattern of daily load shifting. The concrete floor, as a low-pass filter, can be cooperated to diminish the influence of slow disturbances. A similar control strategy is envisioned for this purpose like the one employed in [5]. The difference with the latter study, however, is that the grid utility provides users with the electricity price profile instead of a power setpoint for the heat pump. Besides, we proposed a MPC in order to systematically reduce daily energy prices of the heating, by including future disturbances in the optimization process.

MPC can systematically incorporate forecast data of weather and price along with other system constraints in the optimization procedure. Besides constraint handling, MPC gives systematic feedforward design based on future demands [7]. Therefore, we designed a MPC in the top level of control hierarchy to orchestrate functioning of local controller units at the lower level.

The closed loop hierarchical control system is shown in Fig. 3. There are as many internal loops as the number of rooms and an outer loop with a multiplexer and a MPC. In the inner loops, each PI regulates a specific room's temperature to the setpoint value received from the MPC. In the outer loop, the room with highest heat demand is selected. The MPC controller then minimizes the supply temperature based on the dynamics of that room.

In the real case, a PI's signal to a thermal wax actuator

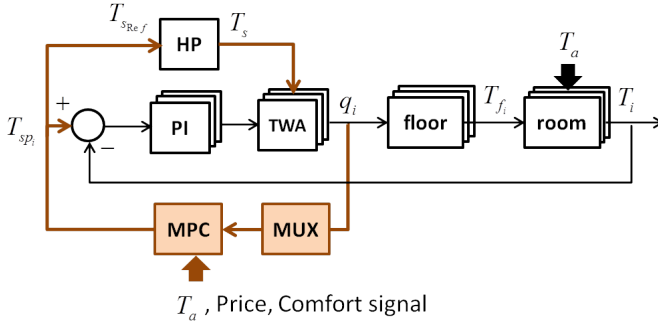


Fig. 3. Block diagram of the closed loop system consisting of three individual rooms with under-floor heating pipes. Temperature of the rooms are controlled by local PI controllers which give signal to the Thermal Wax Actuators (TWA) to adjust the flow rate. The MPC controller specifies temperature setpoints of supply, T_{sRef} , and each room, T_{sp} , based on the actual heat demand of individual rooms.

(TWA) is a two-level Pulse Width Modulation (PWM) signal which is translated into a continuous signal in simulations. Thus, the corresponding control signal determines the valves opening percentage.

III. PROBLEM FORMULATION AND METHOD

A. Plant Model

This section gives an introduction to the model of the plant and control model that is used in simulations. A description of all symbols, subscripts and parameter values are given later in table I. The chosen values for all parameters are in accordance with experimental data. Some experiments have been conducted on a low-energy building in Copenhagen for the purpose of model verification and testing designated control solutions.

The state space equations which govern a single room's dynamics are derived based on the analogy between thermal systems and electrical circuits [8]. The energy balance equations based on three main thermal masses: air, concrete floor, and water are as follows:

$$\begin{aligned} C_i \dot{T}_i &= B_a(T_a - T_i) + B_{ij}(T_j - T_i) + B_{if}(T_{fi} - T_i) \\ C_{fi} \dot{T}_{fi} &= B_{if}(T_i - T_{fi}) + B_{fw}(T_{wi} - T_{fi}) \\ C_{wi} \dot{T}_{wi} &= B_{fw}(T_{fi} - T_{wi}) + c_w q_i(T_s - T_{fi}) \end{aligned} \quad (1)$$

in which i and j are indices of two adjacent rooms, $i, j \in \{1, 2, 3\}$. B represents the equivalent convection/conduction heat transfer coefficient between two connected nodes. For instance, B_{fw} is the conduction heat transfer coefficient between the concrete layer and floor heating pipes that are at temperature T_w . The third equation which models heat flow to the concrete floor through a network of pipes is the simplified version of a more accurate simulation model which is presented in [5].

The local PI controller for the i^{th} room in state space form is:

$$\begin{aligned} \dot{\xi} &= \frac{K_p}{T_{int}}(T_{spi} - T_i) \\ q_i &= K_p(T_{spi} - T_i) + \xi \end{aligned} \quad (2)$$

with ξ as the auxiliary state. The parameters of the PI controller are chosen based on the plant step response around the desired operating point which is $q = 90\%q_{max}$. The choice of the operating point is originated from the fact that water supply temperature should be high enough not to drive floor heating valves to the fully open position.

The heat pump dynamics is much faster than the fastest dynamic in the building. Therefore, we consider it as a static gain. Relation between the transferred heat from the condenser to water in the distribution circuit, Q_c , and the heat pump's electrical work, W_c , is given by:

$$W_c = \frac{Q_c}{\eta_{cop}} \quad (3)$$

with η_{COP} representing the coefficient of performance. This term depends on the temperature difference between the evaporator i.e. brine water temperature, and the condenser i.e. floor heating supply temperature. COP as a manufacturer parameter is usually documented in the heat pump data sheet. We have used a COP curve, see Fig. 4 based on the statistical data given in [9]. The aforementioned models comprise the plant's simulation model.

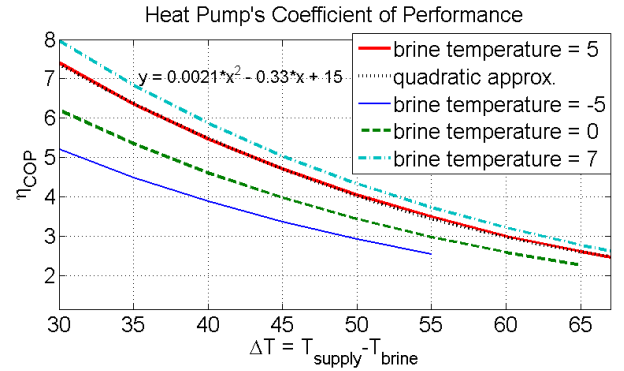


Fig. 4. Statistical data showing the relation between $T_s - T_{brine}$ and the heat pump's COP. x represents the temperature lift, $T_s - T_{brine}$.

Assuming a geothermal heat pump with deeply buried pipes in brine side, the brine temperature is assumed to be constant during heating season. Presuming $T_{brine} = 5^\circ\text{C}$, $\eta_{cop}(T_s)$ is formulated by interpolation in the following:

$$\eta_{cop} = 0.0021T_s^2 - 0.35T_s + 16.7 \quad (4)$$

The prediction model for MPC controller can be formulated as a linear time invariant system in spite of a bilinear term in the last row of 1. In the vicinity of the desired operating point which is $q = 0.9q_{max}$, the bilinear term can be linearized. Hence the internal model of the MPC controller can be written in a state space form as:

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_d d \\ y &= Cx + D_u u + D_d d \end{aligned} \quad (5)$$

with $x = [T_i, T_{fi}, T_{wi}, \xi]^T$, $u = [T_s, T_{spi}]^T$, $y = [T_i, q_i]^T$, and $d = T_a$. Matrices A , B , C and D are derived based on (1) and (2). Room temperatures are measured and the flow rate is estimated by knowing the valve opening degree

and the differential pressure across the valve. The other state variables, ξ , T_{w_i} and T_{f_i} are estimated using a Kalman state observer. The above model is discretized using a sampling time, t_s which is chosen based on the fastest dynamic of the system.

B. The Optimization Problem

The main objective is to minimize power consumption and the corresponding energy price. Power consumption, as mentioned earlier in 3, is:

$$W_c = \frac{c_w q(T_s - T_f)}{-aT_s^2 + bT_s + c} \quad (6)$$

with a , b and c defined in (4). W_c is positively correlated with supply temperature T_s for a constant transferred heat to the building. In the above equation, lessening T_s does not change the numerator because the mass flow rate will be increased in return. Denominator will increase as T_s decreases (the quadratic approximation function is negative definite until $T_s < 83.5$) which consequently leads to reduction of W_c . Therefore, the optimization problem with discretized model (5) is formulated as:

$$\begin{aligned} \min_{T_s, T_{sp_i}} & \sum_{k=1}^N c_s(k)T_s(k) + |T_i(k) - T_{cmf_i}(k)| \\ \text{s.t. } & x(k+1) = Ax(k) + B_u u(k) + B_d d(k) \\ & y(k) = Cx(k) + D_d d(k) \\ & 0 \leq q_i(k) \leq 0.9q_{max} \\ & T_{s,min} \leq T_s(k) \leq T_{s,max} \\ & -TT \leq T_{sp_i}(k) - T_{cmf_i}(k) \leq TT \end{aligned} \quad (7)$$

The prediction model is selected according to the dynamics of the room with the highest heat demand. N is the prediction horizon. In the cost functional, the weight $c_s(k)$ represents electricity price and $T_{cmf_i}(k)$ stands for user-defined temperature setpoint, both of them at time instant k . T_{sp_i} is the manipulated variable that must be bounded within comfort levels defined by the user. TT stands for *Thermal Tolerance*. We also considered constraints on the manipulated variables rate of change which is not indicated in the above formulation. Supply temperature variations rate is limited to 1°C and the setpoint temperature modification rate is limited by 0.1°C , both per sample time t_s .

IV. SIMULATION RESULTS

We have selected discretization sampling rate of the system equal to the MPC sample time, $t_s = 6\text{min}$ which is chosen based on the operation time of the TWAs, i.e. less than 5min .

A. Weather Forecast Data

This section investigates the improvement achieved for COP optimization by exploiting weather forecast data. Recorded weather data was provided by the Danish Meteorological Institute (DMI) for 12 days from January 20 to 31, 2012. In the simulations where weather forecast is involved, we assumed that a perfect forecast was available 6 hours

in advance. The coefficient $c_s(k)$ and the Thermal Tolerance level (TT) in (7) are zero indicating that price of energy does not influence the optimization. Also, T_{sp_i} is not a control input in this simulation scenario, but it is equivalent to the user specified comfort temperature, T_{cmf_i} .

The simulation results for the three-room apartment is shown in Fig. 5. However, only the room with the highest heat demand at each time instant affects the results. In other words, the graph is associated with only one room.

Both comfort and energy costs are improved compared to the case without weather forecast data. In order to quantify comfort improvement, the variance of error in both cases are compared using (8). ΔT is the evaluation time horizon over which the variance is integrated.

$$\sigma = \int_{\Delta T} \frac{|T_i(t) - T_{cmf_i}(t)|}{\Delta T} dt \quad (8)$$

It turned out that in case of employing weather forecast, the variance of error was approximately 0.018, while it was around 0.04 when no forecast data was available. Thus, the comfort level is improved by almost 55%.

In order to evaluate the effect of weather forecast on the average COP values, we calculated the average COP over 10 days using (4). The average COP with and without weather forecast data is 7.24 and 7.25, respectively. The COP is degraded around 0.17% compared to the situation without weather forecast involvement. This does not convey any meaningful outcome in regard to power savings. In the contrary, it confirms that despite having a significant positive influence on thermal comfort, weather forecast have a minor negative impact on the total energy consumption cost. The effect of weather forecast was diminishing fluctuations in the water temperature, therefore the average water temperature in both simulation scenarios is quite the same which means weather forecast does not change or improve COP, nor the energy consumption cost.

B. Price Profile

To satisfy monetary interests of end users, another mechanism is devised in this section to directly affect electricity consumption based on the instantaneous price of electrical power. In this method a list of provisional price values for the coming 24 hours is communicated through the power grid by the power utility provider. Such a price profile is designed in a way to encourage less consumption during peak hours by assigning a higher price. However, the task of the MPC controller at the end user is not to reduce the overall consumption which adversely affects user comfort. Instead, its job is to force the heat pump to consume energy when it is cheap and deprive it of energy consumption when the price is high.

To fulfill its job, the MPC modifies the setpoint of each zone according to the energy price in order to shift the heat demand from peak hours to off-peak periods, based on (7). Fig. 6 illustrates how it becomes possible to decrease the consumption cost with the same average water temperature and not sacrificing thermal comfort of residents. It shows

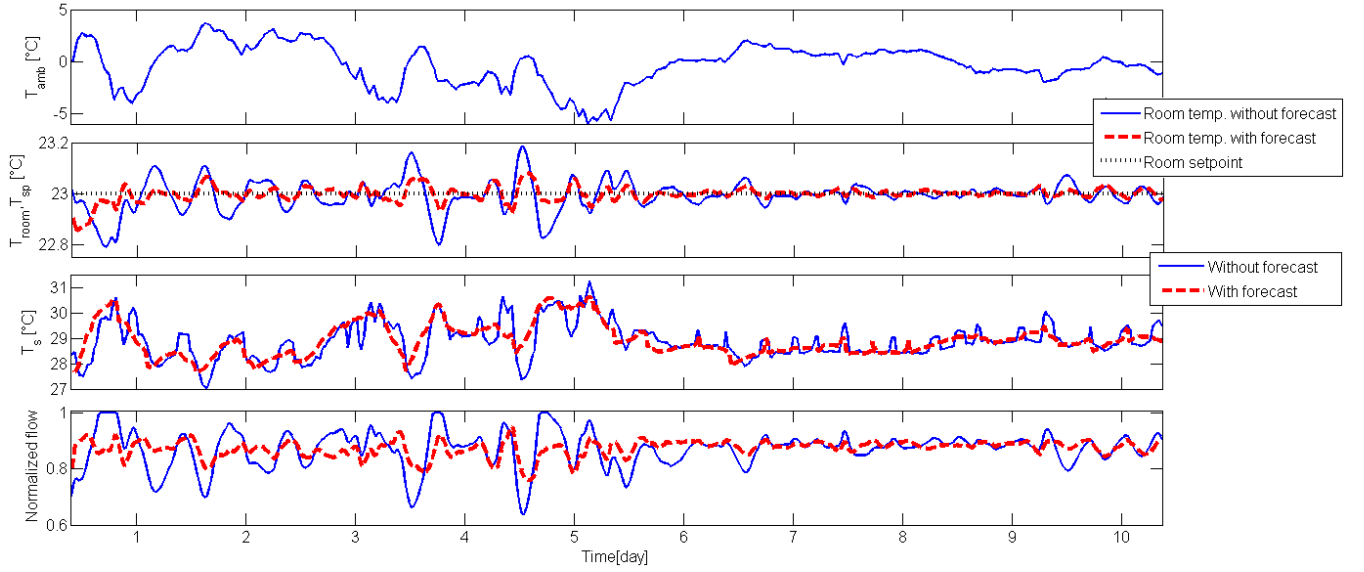


Fig. 5. Simulation results with and without accurate weather forecast data

that the average water temperature is even increased 2.2% in average compared to the scenario when energy is minimized not the energy cost. COP is also increased 1.2% which is due to the increased average water temperature. However, the cost of electricity consumption is reduced by 10% in average which is subject to the Elspot price variations shown in Fig.6. Higher fluctuations of the electricity price would lead to much more cost benefits.

Increase of the average water temperature and by this mean reduction of COP is due to the fact that load shifting for the purpose of cost minimization might not be in the same direction as the energy efficiency. More clearly, the two objectives could be in contradiction depending on the periodic signal of price. From the energy perspective, it is more efficient to shift the load from night to daytime when COP is usually higher. However, electricity price is normally higher in daytime due to peak load. Therefore it is more economic to consume in night time than during the day. This contradiction has led to a deficit in the system energy efficiency, but to a lower energy cost which is the final target of the optimization problem.

Starting in the steady state, when the price goes down, the actual temperature setpoint in the building increases. Therefore, the valves tend to become fully open. The local PI controllers interpret this situation as saturation and impaired regulation. However, in reality the building is intentionally getting warmer than what the user had desired in order to store energy for the next peak period. On the other hand, when the price goes up, the actual temperature setpoint in the building decreases. This will result in tightening of the valves on floor heating pipes and preventing expensive power consumption. Deviating from the user-defined setpoint is of course already permitted and approved by the user through adjustment of the thermal tolerance level.

It should also be noted that the constraint on flow may not

be replaced with an additional term in the objective function in (7). The reason is that the free move of floor heating valves in a permissible interval is essential if the combination of local PI controllers and the MPC controllers should be able to function properly. It is not consistent design if the top level MPC directly regulates both the setpoint and the control signal of PI controllers. At least, one should be free and we have chosen to let PI controllers have complete control on their actuators. This is a consistent hierarchical design.

In summary, it can be stated that the main contribution of the paper is to formulate objectives and constraints in the optimization problem in (7) such that a consistent hierarchical structure is created.

V. CONCLUSION

In this paper we studied the effects of: 1) weather forecast data, 2) electricity price profile, and 3) the indirectly found heat demand, on control of a heat pump. This was done by employing a two level control system structure. The lower level consisted of local PI controllers which were used to regulate temperature setpoints of individual heat zones in a building. The size of the control signal in each of the heat zones was interpreted as an indication of the heat demand in that zone and was taken into account as the basis for selection of the zone with the highest heat demand. Afterwards, the weather forecast data and electricity price profile were involved in an optimization problem by the top level MPC controller. An interesting result was that a priori knowledge on weather conditions proved to have negligible effects on saving money despite its significant role in improving user's comfort and improving temperature regulation capability of the control system. On the contrary, a priori knowledge on electricity price profile turned out to have a vast potential for providing monetary savings in electricity bills. At the end, it is the user who adjusts his desired thermal tolerance, and

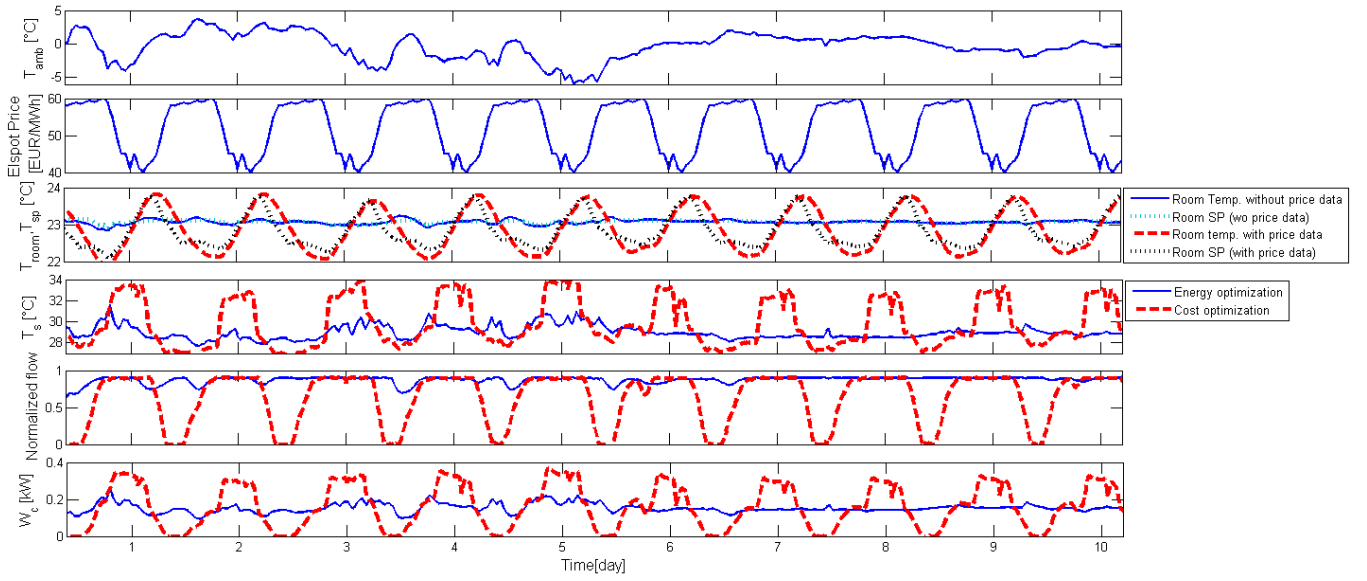


Fig. 6. Simulation results with and without price profile data

hence determines the constraints that must be satisfied by the control system. It is a deal between end users and the power utility company. Should the company send out inexpensive bills, it requires to affect control of users' heat pumps via their pricing policies.

REFERENCES

- [1] F. Tahersima, J. Stoustrup, and H. Rasmussen, "Optimal power consumption in a central heating system with geothermal heat pump," in *IFAC World Congress*, Milano, Italy, August 2011.
- [2] Z. Yang, G. Pedersen, L. Larsen, and H. Thybo, "Modeling and control of indoor climate using a heat pump based floor heating system," in *IECON 2007 - 33rd Annual Conference of the IEEE Industrial Electronics Society*, 2007, pp. 2985–2990.
- [3] V. Chandan, S. Mishra, and A. G. Alleyne, "Predictive control of complex hydronic systems," in *American Control Conference*, Baltimore, MD, USA, June 2010.
- [4] R. Halvgaard, N. K. Poulsen, H. Madsen, and J. B. Jrgensen, "Economic model predictive control for building climate control in smart grid," in *IEEE PES Conference on Innovative Smart Grid Technologies*, Washington, USA, June 2012.
- [5] F. Tahersima, J. Stoustrup, S. A. Meybodi, and H. Rasmussen, "Contribution of domestic heating system to smart grid control," in *Conference on Decision and Control*, Orlando, FL, USA, December 2011.
- [6] S. K. Jensen, "Potentials and opportunities for flexible electricity consumption with special focus on individual heat pumps," EnergiNet, Tonne Kjaersvej 65, Fredericia, Denmark, Tech. Rep., 31 January 2011.
- [7] J. A. Rossiter, *Model-Based Predictive Control*. Raton, Florida: CRC Press LLC, 2003.
- [8] G. Hudson and C. P. Underwood, "A simple building modeling procedure for matlab/simulink," *Building Simulation*, vol. 99, no. 2, pp. 777–783, 1999.
- [9] C. Heerup, "Effektivitet og temperatur sammenhng," Denmark Technological Institute, Energi og Kilima, Gregersensvej, Taastrup, Denmark, Tech. Rep., 2011.

TABLE I
SYMBOLS AND SUBSCRIPTS

Nomenclature	
B	heat transfer coefficient between two nodes in an electric circuit ($kJ/s^{\circ}K$)
B_{if}	heat transfer coefficient between a room air and the layer of concrete floor($kJ/s^{\circ}K$)
B_{ij}	heat transfer coefficient between two adjacent rooms ($kJ/s^{\circ}K$)
B_{fw}	heat transfer coefficient between influent water and concrete floor ($kJ/s^{\circ}K$)
C	thermal capacitance ($J/kg^{\circ}C$)
c_s	electric power price
K_p	proportional gain
Q_c	transferred power to the house
Q	heat (W)
q	water flow rate in floor heating (kg/sec)
q_{max}	maximum water flow rate in floor heating pipes (kg/sec)
T	temperature ($^{\circ}C$)
T_{cmf}	comfort temperature set by the user ($^{\circ}C$)
T_{int}	integration time
T_s	supply temperature (also called forward temperature) ($^{\circ}C$)
$T_{s,min}$	minimum supply temperature ($^{\circ}C$)
$T_{s,max}$	maximum supply temperature ($^{\circ}C$)
TT	Thermal Tolerance ($^{\circ}C$)
W_c	consumed power by compressor
ξ	auxiliary state
η_{cop}	coefficient of performance
Subscripts	
a	ambient
cmf	comfort
f	floor (with i index corresponding to the i^{th} room concrete floor)
f_i	floor of the i^{th} room
i, j	room number
k	time instant (s)
s	supply temperature
sp	setpoint
w	water