Detection of Biasing Attacks on Distributed Estimation Networks

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***Abstract*— The paper addresses the problem of detecting attacks on distributed estimator networks that aim to intentionally bias process estimates produced by the network. It provides a sufficient condition, in terms of the feasibility of certain linear matrix inequalities, which guarantees distributed input attack detection using an H_∞ approach.**

I. Introduction

With recent rapid developments in the area of networked control and estimation, the security of networked systems against input attacks and faults becomes increasingly important. The mainstream of the results in the literature focus on centralized attack and fault detection, however some recent work has been done on distributed attack and fault detection due to the fact that not all measurements might be available at each node of the network; see [2], [9], [10], [13], [5], [3] and the references therein.

This paper considers the problem of detection of attacks on consensus-based distributed estimation networks. The topic of distributed estimation has gained considerable attention in the literature, in a bid to reduce communication bottlenecks and improve reliability and fidelity of centralized state observers. Filter cooperation and consensus ideas have proved to be instrumental in the design of distributed state observers [7], [15], [16]. At the same time, consensus-based systems are particularly vulnerable to intentional attacks since the compromised agents can interfere with the functions of the entire network in a significant way [8]. Uncertainty and noise represent another challenge from the attack detection viewpoint — state observers are typically required in applications where uncertainty and noise make accessing the system state difficult; this may allow the attackers to remain undetected by injecting signals compatible with the noise statistics [9]. This motivates an increased interest in the literature in detection of rogue behaviours of state observers.

In this paper, we consider a general framework of distributed state estimation considered, for example, in [15], [16], [18] and assume that some of the nodes of the network are compromised. Mathematically, this situation is modelled by allowing the compromised observers to be driven by certain attack/fault inputs. The purpose of the attack under consideration is to force the compromised node to produce biased state estimates and then exploit the consensus mechanism within the network to propagate those estimates across the network. Conventional false data injections into measurements can also be included in the model as a routine extension of our results.

From the viewpoint of fault detection/input estimation, the system subject to attack is distributed itself. This is similar to [13], but is different from [5], [3] which were focused on detecting faults applied to the observed plant. We use an H_∞ fault detection approach which allows for a broad range of uncertainty in the sensors and the plant model, as well as a quite broad range of attack inputs. Furthermore, to detect the attack/fault, the proposed attack observers use the same plant measurements and the state estimate information communicated from the neighbours as the state observers themselves. The key idea is to use this information, without additional communication overheads, to determine which of the node observers' behaviour differs from what this information predicts.

Our idea of governing the detectors by neighbours' state estimates to track the attack input is similar to [12], where integral action controllers governed by diffusive couplings were used for averaging constant disturbances. More precisely, in [12] distributed integral action controllers were used for averaging constant disturbances to enable all agents in the system to synchronize to a common reference system governed by the averaged constant disturbance. In contrast, here we are interested in tracking individual attack inputs, rather than tracking an averaged attack vector. Technically this required us to introduce additional dynamics into the fault detectors. Also unlike [12], the H_∞ formulation adopted here does not restrict the attack inputs to be constants.

The paper is organised as follows. In Section II, a background on distributed consensus based estimation is presented. Also, the idea of distributed attack estimation with H_∞ consensus is explained and the attack detection problem is formulated in that section. The main result is given in Section III, where a sufficient condition in terms of coupled linear matrix inequalities is expressed. Concluding remarks are given in Section IV.

Notation: $\mathbf{R}^n$ denotes the real Euclidean n-dimensional vector space, with the norm $\|x\| = (x'x)^{1/2}$; here the symbol $'$ denotes the transpose of a matrix or a vector. The symbol I_n denotes the $n \times n$ identity matrix, and $0_{m\times n}$

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denotes the zero matrix of size $m \times n$. We will occasionally use I and 0 for notational convenience if no confusion is expected. For real symmetric $n \times n$ matrices X and Y, $Y > X$ (respectively, $Y \geq X$) means the matrix $Y - X$ is positive definite (respectively, positive semidefinite). The notation $L_2[0,\infty)$ refers to the Lebesgue space of $\mathbf{R}^n$-valued vector-functions $z(.)$, defined on the time interval $[0,\infty)$, with the norm $\|z\|_2 \triangleq \left(\int_0^\infty \|z(t)\|^2 dt\right)^{1/2}$ and the inner product $\int_0^\infty z_1'(t) z_2(t) dt$.

II. Formulation of the distributed attack detection problem

A. Network topology

Consider a filter network with N nodes and a directed graph topology $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ where $\mathbf{V}$ and $\mathbf{E}$ are the set of vertices and the set of edges (i.e, the subset of the set $\mathbf{V} \times \mathbf{V}$), respectively. Without loss of generality, we let $\mathbf{V} = \{1, 2, \ldots, N\}$. The graph $\mathbf{G}$ is assumed to be directed, reflecting the fact that while node i receives the information from node j, this relation may not be reciprocal. The notation (j,i) will denote the edge of the graph originating at node j and ending at node i. It is assumed that the nodes of the graph $\mathbf{G}$ have no self-loops, i.e., $(i,i) \notin \mathbf{E}$.

For each $i \in \mathbf{V}$, let $\mathbf{V}_i = \{j : (j,i) \in \mathbf{E}\}$ be the set of nodes supplying information to node i. The cardinality of $\mathbf{V}_i$, known as the in-degree of node i, is denoted p_i; i.e., p_i is equal to the number of incoming edges for node i. Also, q_i will denote the number of outgoing edges for node i, known as the out-degree of node i. Let $\mathbf{A} = [\mathbf{a}_{ij}]$ be the adjacency matrix of the digraph $\mathbf{G}$, i.e., $\mathbf{a}_{ij} = 1$ if $(j,i) \in \mathbf{E}$, otherwise $\mathbf{a}_{ij} = 0$. Then, $p_i = \sum_{j=1}^N \mathbf{a}_{ij} = \sum_{j \in \mathbf{V}_i} \mathbf{a}_{ij}$, $q_i = \sum_{j=1}^N \mathbf{a}_{ji}$.

B. Background: distributed consensus-based H_∞ estimation

A typical distributed consensus-based H_∞ estimation problem considers a plant described by the equation

$$\dot{x} = Ax + B_2 \xi(t), \quad x(0) = x_0, \quad x \in \mathbf{R}^n, \tag{1}$$

governed by an disturbance input $\xi \in \mathbf{R}^m$. A network of filters connected according to the graph $\mathbf{G}$ takes measurements of the plant with the purpose to produce an estimate of x. It is assumed that each filter takes measurements

$$y_i = C_{2i} x + D_{2i} \xi + \bar{D}_{2i} \xi_i, \tag{2}$$

where $\xi_i(t) \in \mathbf{R}^{m_i}$ represents the measurement disturbance at the local sensing node i, and processes them locally using an information communicated by its neighbours j, $j \in \mathbf{V}_i$. Depending on the nature of the disturbances ξ, ξ_i, the processing can be done using Kalman [7] or H_∞ [15], [16], [18] filters, both using innovations in the measurements and the neighbours' information for feedback. To be concrete, from now on we build the presentation around the distributed H_∞ consensus filter introduced in [16], [18], although the approach to bias attack detection proposed in this paper is general enough to allow extensions to other types of filters in an obvious manner.

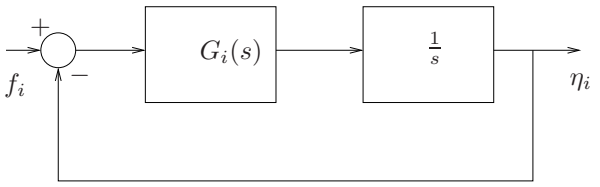

Fig. 1. An auxiliary 'input tracking' model.

According to [16], suppose the disturbances ξ, ξ_i belong to $L_2[0,\infty)$; this assumption suffices to guarantee that equation (1) has an L_2-integrable solution on any finite time interval $[0,T]$, even when the matrix A is unstable. Then using the Luenberger type observer, each filter produces an estimate $\hat{x}_i$ of the state x

$$\dot{\hat{x}}_i = A\hat{x}_i + L_i(y_i(t) - C_{2i}\hat{x}_i) + K_i \sum_{j \in \mathbf{V}_i} (\hat{x}_j - \hat{x}_i), \quad \hat{x}_i(0) = 0, \tag{3}$$

where the matrices L_i, K_i are the parameters of the filter. The observer structure indicates that each node takes advantage of being interconnected with other nodes in that each filter uses its neighbours estimates $\hat{x}_j$, $j \in \mathbf{V}_i$. The problem in [16] was to determine estimator gains L_i and K_i in (3) to ensure the filter internal stability and acceptable H_∞ attenuation of the effect which disturbances have on the consensus performance of the filter.

C. The bias attack model

The particular problem of interest in this paper is to consider the situation where one or several nodes of the network of observers described in the previous sections are subject to bias attack. While a commonly considered situation is when the attacker interferes with the measurements and/or communications between the nodes, here in contrast, we consider the situation where the attacker mounts an attack on the observer dynamics. That is, we consider the situation where in lieu of (3), some of the nodes generate their estimates according to

$$\dot{\hat{x}}_i = A\hat{x}_i + L_i(y_i(t) - C_{2i}\hat{x}_i) + K_i \sum_{j \in \mathbf{V}_i} (\hat{x}_j - \hat{x}_i) + f_i, \quad \hat{x}_i(0) = 0, \tag{4}$$

where f_i is the attack input. From now on, our focus is exclusively on the network of observers (4).

To present the class of admissible attack signals under consideration in this paper, consider an auxiliary 'input tracking' model shown in Fig. 1, with a stable square $n \times n$ transfer function $G_i(s)$, with invertible $G_i(0)$. Since $G_i(s)$ is square, then input f_i and output η_i of the system in Fig. 1 are of dimension n.

Assumption 1: Given a stable square $n \times n$ transfer function $G_i(s)$, the class of admissible bias inputs under consideration consists of all signals $f_i(t)$, $t \geq 0$, such that

$$\int_0^\infty \|f_i - \eta_i\|^2 dt < \infty. \tag{5}$$

Consider the tracking error of the system shown in Fig. 1 $\nu_i = \eta_i - f_i$. Under Assumption 1, ν_i is a finite energy signal. Denoting the Laplace transforms of f_i and ν_i as $f_i(s)$ and $\nu_i(s)$ respectively, and noting that

$$\nu_i(s) = -(I_n + \frac{1}{s}G_i(s))^{-1} f_i(s),$$

condition (5) is equivalent to

$$\int_{-j\infty}^{+j\infty} \|(I + \frac{1}{s}G_i(s))^{-1} f_i(s)\|^2 ds < \infty. \tag{6}$$

Note also that the invertibility of $G_i(0)$ guarantees that $\lim_{t\to\infty} \|f_i(t) - \eta_i(t)\| = 0$ for inputs f_i that have a finite limit at ∞.

In practice, of course the transfer function $G_i(s)$ must be selected by the designer based on the anticipated behaviour of the attack inputs $f_i(t)$. It remains unknown to the attacker. For example, to capture a class of bias injection attack inputs consisting of a steady-state component and an exponentially decaying transient component generated by a low pass filter [14] it suffices to choose $G_i(s) = \frac{1}{s+2\epsilon_i} I_n$, where I_n is the $n \times n$ identity matrix, and $\epsilon_i > 0$ is a constant. It must be noted that even with this choice of $G_i(s)$, the designer does not need to know the asymptotic steady-state value or the shape of the transient, as all such signals f_i satisfy condition (6). Furthermore, such signals have the property that $\lim_{t\to\infty} f_i(t)$ exists and therefore we can ensure that $\|f_i(t) - \eta_i(t)\| \to 0$ as $t \to \infty$. More generally, signals representing a combination of constants and L_2-integrable inputs satisfy (6). In addition to bias attack policies f_i described above, L_2-integrable inputs f_i are included which represent attack inputs with limited energy resource [14].

It can be readily shown that the state-space model for the system in Fig 1 can be written as

$$\begin{aligned} \dot{\omega}_i &= \Omega_i \omega_i + \Gamma_i \nu_i, \\ \eta_i &= [I\ 0]\omega_i, \qquad \omega_i(0) = 0, \end{aligned} \tag{7}$$

where $\nu_i = \eta_i - f_i$ is an L_2-integrable input, according to Assumption 1. In particular, in the special case $G_i(s) = \frac{1}{s+2\epsilon_i} I_n$, we have $\omega_i \in \mathbf{R}^{2n}$, and

$$\Omega_i = \begin{bmatrix} 0 & I \\ 0 & -2\epsilon_i I \end{bmatrix}, \quad \Gamma_i = \begin{bmatrix} 0 \\ -I \end{bmatrix}. \tag{8}$$

D. The proposed attack detector

The objective of the paper is to design a (distributed) attack detection system which is capable of tracking attack inputs satisfying Assumption 1. To this end, we consider the following outputs which summarize the information about the network available at node i, and can be used by the attack detector

$$\begin{aligned} \zeta_i &= y_i - C_{2i}\hat{x}_i \\ &= C_{2i}(x - \hat{x}_i) + D_{2i}\xi + \bar{D}_{2i}\xi_i, \qquad (9) \\ \bar{\zeta}_i &= \sum_{j\in\mathbf{V}_i} (\hat{x}_j - \hat{x}_i). \qquad (10) \end{aligned}$$

The idea behind introducing these outputs is as follows. If node i is under attack, then its predicted sensor measurement $C_{2i}\hat{x}_i$ is expected to be biased, compared to the actual measurement y_i. This must lead to a significant difference between these two signals, i.e., we must expect a large energy in ζ_i. Likewise, the observer under attack is expected to cause the system to deviate from the state of consensus, causing the state of the observer i, $\hat{x}_i$ to deviate from the average estimate produced at the neighbouring nodes. Thus, the disagreement variable $\bar{\zeta}_i$ at node i is expected to differ from similar variables produced by the rest of the network. This motivates using these outputs for detecting the attack.

Let $e_i = x - \hat{x}_i$ be the local estimation error at node i. Using (1) and (4), it is straightforward to verify that the local filter errors satisfy the following equation:

$$\begin{aligned} \dot{e}_i &= (A - L_i C_{2i})e_i + K_i \sum_{j\in\mathbf{V}_i} (e_j - e_i) \\ &\quad + (B_2 - L_i D_{2i})\xi - L_i \bar{D}_{2i}\xi_i - f_i, \quad e_i(0) = x_0. \end{aligned} \tag{11}$$

The outputs (9), (10) can be rewritten in terms of the estimation errors as

$$\zeta_i = C_{2i}e_i + D_{2i}\xi + \bar{D}_{2i}\xi_i, \tag{12}$$

$$\bar{\zeta}_i = -\sum_{j\in\mathbf{V}_i} (e_j - e_i). \tag{13}$$

Hence, we can consider the collection of systems (11) as a large-scale plant governed by the vector of attack inputs $f = [f_1', \ldots, f_N']'$, and equipped with the outputs (12), (13). It is worth stressing that these outputs can be readily generated at the observer i, computing them only requires the local measurements y_i, the local estimate $\hat{x}_i$ computed by the observer at node i and the neighbours estimates $\hat{x}_j$, $j \in \mathbf{V}_i$, available to that observer. Therefore the outputs (12), (13) are available for tracking the attack inputs. To achieve this, consider the system combining the estimation error dynamics (11) and the auxiliary input tracking model (7):

$$\begin{aligned} \dot{e}_i &= (A - L_i C_{2i})e_i + K_i \sum_{j\in\mathbf{V}_i} (e_j - e_i) - [I\ 0]\omega_i \\ &\quad + (B_2 - L_i D_{2i})\xi - L_i \bar{D}_{2i}\xi_i + \nu_i, \quad e_i(0) = x_0, \\ \dot{\omega}_i &= \Omega_i \omega_i + \Gamma_i \nu_i \quad \omega_i(0) = 0. \end{aligned} \tag{14}$$

The system (14) equipped with the outputs (12), (13) is an uncertain system governed by L_2-integrable inputs ξ, ξ_i and ν_i. Each such system is interconnected with its neighbours via inputs e_j, and the collection of all such systems represents a large-scale system. We propose the following distributed H_∞ observer for this large-scale system which utilizes the outputs (12), (13) to obtain estimates of e_i and ω_i while attenuating the disturbances ξ, ξ_i and ν_i,

$i = 1, \ldots, N$:

$$\begin{aligned}
\dot{\hat{e}}_i &= (A - L_i C_{2i})\hat{e}_i + K_i \sum_{j \in \mathbf{V}_i} (\hat{e}_j - \hat{e}_i) - [I\ 0]\hat{\omega}_i \\
&\quad + F_i(\zeta_i - C_{2i}\hat{e}_i) + H_i \left(\bar{\zeta}_i + \sum_{j \in \mathbf{V}_i} (\hat{e}_j - \hat{e}_i) \right), \\
\dot{\hat{\omega}}_i &= \Omega_i \hat{\omega}_i + F_i^\eta(\zeta_i - C_{2i}\hat{e}_i) + H_i^\eta \left(\bar{\zeta}_i + \sum_{j \in \mathbf{V}_i} (\hat{e}_j - \hat{e}_i) \right), \\
&\quad \hat{e}_i(0) = 0, \quad \hat{\omega}_i(0) = 0. \qquad (15)
\end{aligned}$$

The coefficients F_i, H_i, F_i^η, H_i^η are to be found in such a way that $\hat{\eta}_i : \hat{\eta}_i = [I\ 0]\hat{\omega}_i$ tracks the output η_i of the auxiliary system (7). Then, since according to the definition of the auxiliary signal η_i, this signal represents f_i asymptotically, we propose using $\hat{\eta}_i$ as a residual variable indicating whether the attack is taking place.

To formalize the above idea, introduce the error vectors $z_i = e_i - \hat{e}_i$, $\delta_i = \omega_i - \hat{\omega}_i$. Using the extended system model (14) and the corresponding observer (15), the evolution of these error vectors is governed by the following equations

$$\begin{aligned}
\dot{z}_i &= (A - L_i C_{2i})z_i + K_i \sum_{j \in \mathbf{V}_i} (z_j - z_i) - [I\ 0]\delta_i \\
&\quad - F_i C_{2i} z_i + H_i \sum_{j \in \mathbf{V}_i} (z_j - z_i) \\
&\quad + (B_2 - L_i D_{2i})\xi - L_i \bar{D}_{2i}\xi_i + \nu_i \\
&\quad - F_i D_{2i}\xi - F_i \bar{D}_{2i}\xi_i, \quad z_i(0) = x_0, \\
\dot{\delta}_i &= \Omega_i \delta_i - F_i^\eta C_{2i} z_i + H_i^\eta \sum_{j \in \mathbf{V}_i} (z_j - z_i) \\
&\quad - F_i^\eta D_{2i}\xi - F_i^\eta \bar{D}_{2i}\xi_i + \Gamma_i \nu_i, \quad \delta_i(0) = 0. \qquad (16)
\end{aligned}$$

Note that we can introduce new variables $\tilde{L}_i = L_i + F_i$, $\tilde{K}_i = K_i + H_i$, and re-write (16) as

$$\begin{aligned}
\dot{z}_i &= (A - \tilde{L}_i C_{2i})z_i + \tilde{K}_i \sum_{j \in \mathbf{V}_i} (z_j - z_i) - [I\ 0]\delta_i \\
&\quad + (B_2 - \tilde{L}_i D_{2i})\xi - \tilde{L}_i \bar{D}_{2i}\xi_i + \nu_i, \quad z_i(0) = x_0, \\
\dot{\delta}_i &= \Omega_i \delta_i - F_i^\eta C_{2i} z_i + H_i^\eta \sum_{j \in \mathbf{V}_i} (z_j - z_i) \\
&\quad - F_i^\eta D_{2i}\xi - F_i^\eta \bar{D}_{2i}\xi_i + \Gamma_i \nu_i, \quad \delta_i(0) = 0. \qquad (17)
\end{aligned}$$

Problem 1 (The H_∞ detector design problem): The distributed attack detection problem under consideration in this paper is to determine $\tilde{L}_i$, $\tilde{K}_i$, F_i^η, H_i^η such that the following conditions hold:

(i) The large-scale system (17) is internally stable. Equivalently, the disturbance and attack-free large-scale system

$$\begin{aligned}
\dot{z}_i &= (A - \tilde{L}_i C_{2i})z_i + \tilde{K}_i \sum_{j \in \mathbf{V}_i} (z_j - z_i) - [I\ 0]\delta_i, \\
\dot{\delta}_i &= \Omega_i \delta_i - F_i^\eta C_{2i} z_i + H_i^\eta \sum_{j \in \mathbf{V}_i} (z_j - z_i), \qquad (18) \\
&\quad z_i(0) = x_0, \quad \delta_i(0) = 0,
\end{aligned}$$

must be asymptotically stable.

(ii) In the presence of disturbances and attack signals, all from the class of L_2-integrable signals, the system (17) achieves a guaranteed level of H_∞ filtering performance:

$$\sup_{x_0, \mathbf{w} \neq 0} \frac{\int_0^\infty \sum_{i=1}^N (\delta_i' Q_i \delta_i + z_i' \bar{Q}_i z_i) dt}{\|x_0\|_P^2 + \sum_{i=1}^N \|\mathbf{w}_i\|_2^2} \leq \gamma^2, \quad (19)$$

where $Q_i = Q_i' > 0$, $\bar{Q}_i = \bar{Q}_i' \geq 0$ are given matrices, $\|x_0\|_P^2 = x_0' P x_0$, $P = P' > 0$ is a fixed matrix to be determined later, $\mathbf{w}_i \triangleq [\xi', \xi_i', \nu_i']'$, $\mathbf{w} \triangleq [\mathbf{w}_1', \ldots, \mathbf{w}_N']'$, and $\gamma > 0$ is a constant.

It follows from (19) that each attack detector variable $\hat{\omega}_i$ provides an H_∞ estimate of ω_i. We now show that provided Assumption 1 holds, the output $\hat{\eta}_i = [I\ 0]\hat{\omega}_i$ of the observer (15) converges to f_i, and hence it can be used as a residual indicator of attack.

Lemma 1: Suppose Assumption 1 holds and the observer network (4) is such that the disturbance and attack-free large-scale system (18) is asymptotically stable, and also (19) holds with $\bar{Q}_i > 0$. Then $\|\hat{\eta}_i - f_i\| \to 0$ as $t \to \infty$ for all f_i that have a finite limit at ∞.

Note that (19) with $\bar{Q}_i > 0$ requires the observer to ensure disturbance attenuation with respect to both δ_i and z_i, even though only the variable δ_i captures the tracking error of interest. When $\bar{Q}_i = 0$ and condition (19) reduces to a weaker condition we can guarantee that $\hat{\eta}_i$ converges to f_i in L_2 sense, even when f_i does not have a finite limit at ∞.

Lemma 2: Suppose Assumption 1 holds and the observer network (4) is such that the disturbance and attack-free large-scale system (18) is asymptotically stable, and also condition (19) holds with $\bar{Q}_i = 0$,

$$\sup_{x_0, \mathbf{w} \neq 0} \frac{\int_0^\infty \sum_{i=1}^N \delta_i' Q_i \delta_i dt}{\|x_0\|_P^2 + \sum_{i=1}^N \|\mathbf{w}_i\|_2^2} \leq \gamma^2. \qquad (20)$$

Then $\sum_{i=1}^N \int_0^\infty \|\hat{\eta}_i - f_i\|^2 dt < \infty$.

It is worth noting that the system (15) is governed by the outputs of the observer network (4); therefore it can be implemented to monitor the health of the network. We explain in the next section how to design $\tilde{L}_i$, $\tilde{K}_i$, F_i^η, and H_i^η such that the above conditions hold.

III. Attack detector design

Problem 1 belongs to the class of distributed stabilization by output injection problems. References [4], [16], [18] developed a vector dissipativity approach to solve this class of problems which will be applied here as well. For each node i, consider a candidate storage function $V_i(z_i, \delta_i) = [z_i'\ \ \delta_i'] X_i [z_i'\ \ \delta_i']'$, where $X_i = X_i' > 0$. The following vector dissipation inequality is instrumental in proving input tracking properties of the distributed attack detector (15):

$$\dot{V}_i + 2\alpha_i V_i + \delta_i' Q_i \delta_i + z_i' \bar{Q}_i z_i \leq \sum_{j \in \mathbf{V}_j} \pi_j V_j + \gamma^2 \|\mathbf{w}_i\|^2, \quad (21)$$

where π_i, π_j are constants selected so that the matrix

$$\begin{bmatrix} -2\alpha_1 & \pi_2\mathbf{a}_{12} & \dots & \pi_N\mathbf{a}_{1N} \\ \pi_1\mathbf{a}_{21} & -2\alpha_2 & \dots & \pi_N\mathbf{a}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_1\mathbf{a}_{N1} & \pi_2\mathbf{a}_{N2} & \dots & -2\alpha_N \end{bmatrix}$$

is diagonally dominant (and therefore it is Hurwitz [11]); here, $\mathbf{a}_{ij}$ is the element of the adjacency matrix of the graph $\mathbf{G}$. Indeed adding the inequalities (21) will result in

$$\begin{aligned} &\sum_{i=1}^{N}\dot{V}_i + \sum_{i=1}^{N}(\delta_i'Q_i\delta_i + z_i'\bar{Q}_iz_i) \\ &\quad\le \max\{-2\alpha_1 + q_1\pi_1, \dots, -2\alpha_N + q_N\pi_N\}\sum_{i=1}^{N}V_i \\ &\qquad + \gamma^2\sum_{i=1}^{N}(\|\xi\|^2 + \|\xi_i\|^2 + \|\nu_i\|^2). \end{aligned}$$

Selecting $\pi_i < \frac{2\alpha_i}{q_i}$ and letting $\varepsilon = \min\{2\alpha_1 - q_1\pi_1, \dots, 2\alpha_N - q_N\pi_N\} > 0$, $V = \sum_{i=1}^{N}V_i$, we then have

$$\begin{aligned} &\dot{V} + \sum_{i=1}^{N}(\delta_i'Q_i\delta_i + z_i'\bar{Q}_iz_i) \le \\ &\quad -\varepsilon V + \gamma^2\sum_{i=1}^{N}(\|\xi\|^2 + \|\xi_i\|^2 + \|\nu_i\|^2). \end{aligned} \tag{22}$$

This implies that when $\xi = 0$ and $f_i = 0$, $\xi_i = 0$ $\forall i$, then

$$\dot{V} < -\varepsilon V,$$

and provided $X_i > 0$, we have $z_i \to 0$, $\delta_i \to 0$ exponentially. That is, condition (i) of Problem 1 is established.

Also, when at least one of the signals ξ, ξ_i or f_i is not equal to zero (the latter is equivalent to $\nu_i \not\equiv 0$), then it follows from (22) that

$$\begin{aligned} \sum_{i=1}^{N}\int_0^T(\delta_i'Q_i\delta_i + z_i'\bar{Q}_iz_i)dt \le \sum_{i=1}^{N}\Big[V_i(z_i(0),\delta_i(0)) \\ + \gamma^2\int_0^T(\|\xi\|^2 + \|\xi_i\|^2 + \|\nu_i\|^2)dt\Big]. \end{aligned}$$

Note that $V_i(z_i(0),\delta_i(0)) = x_0'X_i^{11}x_0$, where X_i^{11} is the upper left block in the partition of X_i compatible with the dimensions of z_i and δ_i. Hence (19) also holds with $P = \gamma^{-2}\sum_{i=1}^{N}X_i^{11}$. It follows from this discussion that condition (21) ensures satisfaction of the conditions of Lemma 1. Therefore, to ensure that the distributed observer (15) can track the attack input f_i we need to determine coefficients $\tilde{L}_i$, $\tilde{K}_i$, F_i^η, and H_i^η for it so that (21) is satisfied.

To present conditions under which (21) holds, introduce the notation

$$\begin{aligned} &A_i^\mu = \begin{bmatrix} A & -[I\ 0] \\ 0 & \Omega_i \end{bmatrix}, \quad B_1^\mu = \begin{bmatrix} I \\ \Gamma_i \end{bmatrix}, \quad B_2^\mu = \begin{bmatrix} -B_2 & 0 \\ 0 & 0 \end{bmatrix}, \\ &D_{2i}^\mu = \begin{bmatrix} D_{2i} & \bar{D}_{2i} \end{bmatrix}, \quad C_{2i}^\mu = \begin{bmatrix} C_{2i} & 0 \end{bmatrix}, \quad H^\mu = \begin{bmatrix} I & 0 \end{bmatrix}, \\ &L_i^\mu = \begin{bmatrix} \tilde{L}_i \\ F_i^\eta \end{bmatrix}, \quad K_i^\mu = \begin{bmatrix} \tilde{K}_i \\ H_i^\eta \end{bmatrix}. \end{aligned} \tag{23}$$

Suppose D_{2i} and $\bar{D}_{2i}$ satisfy the condition

$$E_{2i} \triangleq D_{2i}^\mu(D_{2i}^\mu)' = D_{2i}D_{2i}' + \bar{D}_{2i}\bar{D}_{2i}' > 0. \tag{24}$$

The above assumption on E_{2i} is a standard assumption made in nonsingular H_∞ control problems [1].
Now let us introduce the matrix

$$Q_i^\mu = \begin{bmatrix} \bar{Q}_i & 0 \\ 0 & Q_i \end{bmatrix}, \tag{25}$$

where $Q_i = Q_i' > 0$. Also, $\bar{Q}_i = \bar{Q}_i'$ is selected to be positive definite when the aim is to design an attack observer to achieve asymptotic tracking of attack inputs. If L_2 tracking is acceptable, one can let $\bar{Q}_i = 0$. Given $\alpha_i > 0$, define $\pi_i = \frac{2\alpha_i}{q_i+1}$, where q_i is the out-degree of the graph node i. Clearly $\pi_i = \frac{2\alpha_i}{q_i+1} < \frac{2\alpha_i}{q_i}$.

Theorem 1: Suppose Assumption 1 holds and the digraph $\mathbf{G}$, the matrices $Q_i = Q_i' > 0$, $\bar{Q}_i = \bar{Q}_i' > 0$, $i = 1, \cdots, 6$ and the constants $\alpha_i > 0$, $i = 1, \cdots, N$ are such that the coupled linear matrix inequalities in (27) (on the next page) with respect to the variables $X_i = X_i' > 0$ and M_i, $i = 1, \cdots, N$ are feasible. Then choosing

$$\begin{aligned} K_i^\mu &= -X_i^{-1}M_i, \\ L_i^\mu &= (\gamma^2X_i^{-1}(C_{2i}^\mu)' - B_2^\mu(D_{2i}^\mu)')E_{2i}^{-1} \end{aligned} \tag{26}$$

ensures that the condition (21) holds.

Combined with Lemma 1 or Lemma 2, this theorem provides a complete result on the design of biasing attack detectors for the distributed observer (4).

IV. Conclusion

The paper is concerned with the problem of distributed attack detection is sensor networks. We consider a group of consensus-based distributed estimators and assume that the estimator dynamics are under attack. Then we propose a distributed H_∞ attack detector which allows for a broad range of uncertainty in the sensors and the plant model, as well as a quite broad range of bias attack inputs, and show that the proposed attack detector can track individual attack inputs at different sensors. A possible future direction is to construct a compensator to cancel the detected attack in the system.

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$$\begin{bmatrix} S_i & X_iB_1^\mu & X_iB_2^\mu\Big(I-(D_{2i}^\mu)'E_{2i}^{-1}D_{2i}^\mu\Big) & -M_iH^\mu & \dots & -M_iH^\mu \\ (B_1^\mu)'X_i & -\gamma^2 I & 0 & 0 & \dots & 0 \\ (I-(D_{2i}^\mu)'E_{2i}^{-1}D_{2i}^\mu)(B_2^\mu)'X_i & 0 & -\gamma^2 I & 0 & \dots & 0 \\ -(H^\mu)'M_i' & 0 & 0 & -\frac{2\alpha_{j_1}}{q_{j_1}+1}X_{j_1} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -(H^\mu)'M_i' & 0 & 0 & 0 & \dots & -\frac{2\alpha_{j_{p_i}}}{q_{j_{p_i}}+1}X_{j_{p_i}} \end{bmatrix} < 0, \qquad (27)$$

$$\begin{aligned} S_i = {} & X_i\left(A_i^\mu+\alpha_i I+B_2^\mu(D_{2i}^\mu)'E_{2i}^{-1}C_{2i}^\mu\right)+\left(A_i^\mu+\alpha_i I+B_2^\mu(D_{2i}^\mu)'E_{2i}^{-1}C_{2i}^\mu\right)'X_i \\ & +p_iM_iH^\mu+p_i(H^\mu)'M_i'+Q_i^\mu-\gamma^2(C_{2i}^\mu)'E_{2i}^{-1}C_{2i}^\mu. \end{aligned}$$

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