Sequential Relaxation of Unit Commitment with AC Transmission Constraints

Fariba Zohrizadeh, Mohsen Kheirandishfard, Adnan Nasir, and Ramtin Madani

Abstract—This paper proposes a sequential convex relaxation method for obtaining feasible and near-globally optimal solutions for unit commitment (UC) with AC transmission constraints. First, we develop a second-order cone programming (SOCP) relaxation for AC unit commitment. To ensure that the resulting solutions are feasible for the original non-convex problem, we incorporate penalty terms into the objective of the proposed SOCP relaxation. We generalize our penalization method to a sequential algorithm which starts from an initial point (not necessarily feasible) and leads to feasible and nearoptimal solutions for AC unit commitment. Once a feasible point is attained, the algorithm preserves feasibility and improves the objective value until a near optimal point is obtained. The experimental results on IEEE 57, IEEE 118, and IEEE 300 bus benchmark cases from MATPOWER [1] demonstrate the performance of the proposed method in solving challenging instances of AC unit commitment.

I. Introduction

The unit commitment (UC) is a classical problem in the area of power systems which involves determining the optimal schedule for power generating units throughout a given planning horizon. The main objective is to meet power demand with minimum production cost while respecting the limitations of generating units and network constraints. Due to the economic importance of the UC problem, it has been heavily investigated for decades and is proven to be computationally hard in general [2], [3]. The reader is referred to [4], [5] and the references therein, for detailed surveys of the conventional formulations and methods for solving unit commitment.

A general unit commitment problem can be formulated as a mixed-integer optimization whose solution specifies the optimal status of generating units as well as voltages and power flows throughout the planning horizon. Additionally, several papers have considered uncertainties of demand and renewable generation into consideration using stochastic and robust optimization frameworks [6]–[13]. The incorporation of several other power system optimization problems into unit commitment has been envisioned as well, such as the optimal power flow [14]–[16], network topology control [17], demand response [18], air quality control [19], and scheduling of deferrable loads [20].

Fariba Zohrizadeh and Mohsen Kheirandishfard are with the Department of Computer Science and Engineering, the University of Texas at Arlington, (email: fariba.zohrizadeh@uta.edu, mohsen.kheirandishfard@uta.edu). Adnan Nasir and Ramtin Madani are with the Department of Electrical Engineering, the University of Texas at Arlington (email: adnan.nasir@mavs.uta.edu, ramtin.madani@uta.edu). This work is in part supported by the NSF award 1809454 and a University of Texas System STARs award.

Various optimization methods have been used to approach the UC problem, such as branch-and-bound techniques [21]— [27] and convex relaxations [28]–[30]. In order to improve the efficiency of branch-and-bound searches, many papers have offered partial convex hull characterizations of UC feasible sets [31]-[34]. Conic inequalities are proposed in [14], [35]–[37] to strengthen convex relaxations in the presence of nonlinear cost functions. In [38], a combination of semidefinite programming relaxation and branch-and-bound is used to solve the day-ahead hydro unit commitment problem. In [39], [40], reformulation-linearization cuts are proposed to strengthen semidefinite programming relaxations of unit commitment. In [41], a decomposition method is developed based on second-order cone programming (SOCP) to solve network constrained unit commitment with AC power flow constraints. In [42], a family of valid inequalities are proposed to improve the quality of SOCP relaxations of unit commitment. In [43], a global search algorithm is proposed which solves a sequence of mixed-integer secondorder cone programming (MISOCP) problems, as well as nonlinear non-convex problems to lower- and upper-bound the globally optimal cost of unit commitment. In [44], [45], distributed frameworks on high-performance computing platforms are investigated for solving large-scale UC problems. Nevertheless, the improvements in run-time are reported to diminish with more than 15 parallel workers [46].

In this paper, we introduce a novel sequential convex relaxation for solving unit commitment with AC transmission constraints. We propose a penalization method which is guaranteed to recover feasible solutions for general nonconvex optimization problems under certain assumptions [47], [48]. The proposed penalized convex relaxation can be solved sequentially in order to find feasible and nearglobally optimal solutions. Our experimental results verify the effectiveness of this procedure in solving AC unit commitment problems on IEEE 57, IEEE 118, and IEEE 300 bus benchmark systems.

A. Notations

Throughout this paper, matrices, vectors, and scalars are represented by boldface uppercase, boldface lowercase, and italic lowercase letters, respectively. The symbols \mathbb{R} , \mathbb{C} , and \mathbb{H}_n denote the sets of real numbers, complex numbers, and $n \times n$ Hermitian matrices, respectively. The notation "i" is reserved for the imaginary unit. Notation $|\cdot|$ denotes either the absolute value of a scalar or the cardinality of a set, depending on the context. The symbols $(\cdot)^*$ and $(\cdot)^\top$ represent the conjugate transpose and transpose operators,

Unit Constraints:

$$x_{g,t} \in \{0,1\}$$
 (1a)

$$c_{g,t} = \alpha_g p_{g,t} + \beta_g p_{g,t}^2 +$$

$$\gamma_g x_{g,t} + \gamma_g^{\uparrow} (1 - x_{g,t-1}) x_{g,t} + \gamma_g^{\downarrow} x_{g,t-1} (1 - x_{g,t})$$
 (1b)

$$x_{g,\tau} - x_{g,\tau-1} \le x_{g,t} \qquad \forall \tau \in \{t - m_q^{\uparrow} + 1, \dots, t\}$$
 (1c)

$$x_{g,\tau-1} - x_{g,\tau} \leq 1 - x_{g,t} \qquad \qquad \forall \tau \in \{t - m_g^{\downarrow} + 1, \dots, t\} \tag{1d}$$

$$\underline{p}_g \, x_{g,t} \le p_{g,t} \le \bar{p}_g \, x_{g,t} \tag{1e}$$

$$\underline{q}_g x_{g,t} \le q_{g,t} \le \bar{q}_g x_{g,t} \tag{1f}$$

$$p_{g,t} - p_{g,t-1} \le r_g x_{g,t-1} + s_g (1 - x_{g,t-1})$$
(1g)

$$p_{g,t-1} - p_{g,t} \le r_g x_{g,t} + s_g (1 - x_{g,t}) \tag{1h}$$

AC Network Constraints:

$$\mathbf{d}_{\bullet,t} + \operatorname{diag}\{\mathbf{v}_{\bullet,t}\mathbf{v}_{\bullet,t}^*\mathbf{Y}^*\} = \mathbf{C}^{\top}(\mathbf{p}_{\bullet,t} + \mathrm{i}\mathbf{q}_{\bullet,t})$$
(2a)

$$\operatorname{diag}\{\vec{\mathbf{C}}\ \mathbf{v}_{\bullet,t}\mathbf{v}_{\bullet,t}^*\vec{\mathbf{Y}}^*\} = \vec{\mathbf{s}}_{\bullet,t}$$
 (2b)

$$\operatorname{diag}\{ \mathbf{\tilde{C}} \ \mathbf{v}_{\bullet,t} \mathbf{v}_{\bullet,t}^* \mathbf{\tilde{Y}}^* \} = \mathbf{\tilde{s}}_{\bullet,t}$$
 (2c)

$$\underline{\mathbf{v}} \le |\mathbf{v}_{\bullet,t}| \le \bar{\mathbf{v}} \tag{2d}$$

$$|\vec{\mathbf{s}}_{\bullet,t}|^2 \le \mathbf{f}_{\max:t}^2 \tag{2e}$$

$$|\mathbf{\bar{s}}_{\bullet,t}|^2 \le \mathbf{f}_{\max:t}^2 \tag{2f}$$

TABLE I: Unit and network constraints in power system scheduling.

respectively. For a given matrix A, the notations $A_{\bullet,k}$, $A_{j,\bullet}$, and A_{jk} refer to the k^{th} column, j^{th} row, and $(j,k)^{th}$ entry of the matrix A, respectively. The Notation $A \succeq 0$ means that A is symmetric/Hermitian and positive semidefinite.

II. PROBLEM FORMULATION

The unit commitment (UC) problem aims at finding the most reliable and cost-efficient schedule for a set of generating units throughout a discrete time horizon \mathcal{T} , subject to forecasted electricity demands and operational constraints. Let \mathcal{G} denote the set of generating units whose schedule needs to be determined. Define $x_{g,t} \in \{0,1\}$ as a binary variable indicating whether the generating unit $g \in \mathcal{G}$ is committed during the time slot $t \in \mathcal{T}$. If $x_{g,t} = 1$, the unit is active and generates power within its capacity limitations, otherwise, no power is produced by g during the time interval f. Define f0, and f1, respectively, as the amounts of active power and reactive power injections of generator f2 during the time interval f3.

Denoted $\mathcal V$ and $\mathcal E$ as the sets of buses and branches in the network, respectively. For every bus $k \in \mathcal V$, the demand forecast at time t is denoted as $d_{k,t} \in \mathbb C$, whose real and imaginary parts account for active and reactive power demands, respectively. Let $\mathbf C \in \{0,1\}^{|\mathcal G| \times |\mathcal V|}$ be the incidence matrix whose (g,k) entry is equal to 1, if and only if the generating unit g belongs to the bus k. Define the matrices $\vec{\mathbf C}, \dot{\vec{\mathbf C}} \in \{0,1\}^{|\mathcal E| \times |\mathcal V|}$ as the *from* and *to* incidence matrices, respectively. The (l,k) entry of $\vec{\mathbf C}$ is equal to one, if and only if the line $l \in \mathcal E$ starts at bus k, while the (l,k) entry of $\vec{\mathbf C}$ is equal to 1, if and only if the line l ends at bus

k. Additionally, define $\mathbf{Y} \in \mathbb{C}^{|\mathcal{V}| \times |\mathcal{V}|}$ as the nodal admittance matrices of the network and $\vec{\mathbf{Y}}$, $\vec{\mathbf{Y}} \in \mathbb{C}^{|\mathcal{E}| \times |\mathcal{V}|}$ as the *from* and *to* branch admittance matrices.

The feasible set of AC unit commitment can be described by unit constraints and AC network constraints. Unit constraints impose the minimum up and down time limits (1c)–(1d), generator capacities (1e)–(1f), as well as ramp limits (1g)–(1h). Define m_g^{\uparrow} and m_g^{\downarrow} , respectively, as the minimum up time and minimum down time limits for generating unit g. If the unit g is committed during the interval t, then its, active and reactive power injections must lie within the intervals $[p_g, \bar{p}_g]$ and $[q_g, \bar{q}_g]$, respectively. Additionally, denote r_g as the maximum variation of active power injection by unit g between two consecutive time slots in which the unit stays committed. Define s_g as the maximum amount of active power injection after start-up and prior to shutdown.

The network constraint (2a) accounts for nodal power balances. The constraint (2d) enforces voltage magnitude limits. Moreover, denote the line power flows at the starting and ending buses by $\vec{\mathbf{s}} \in \mathbb{C}^{|\mathcal{E}| \times |\mathcal{T}|}$ and $\vec{\mathbf{s}} \in \mathbb{C}^{|\mathcal{E}| \times |\mathcal{T}|}$, respectively. The constraints (2e) and (2f) enforce the thermal limits of lines.

Given the above definitions, the AC unit commitment problem can be formulated as the optimization

minimize
$$\sum_{g,t} c_{g,t}$$
 (3a)

subject to
$$(\mathbf{x}_{g,\bullet}^{\top}, \mathbf{p}_{g,\bullet}^{\top}, \mathbf{q}_{g,\bullet}^{\top}, \mathbf{c}_{g,\bullet}^{\top}) \in \mathcal{U}_g \quad \forall g \in \mathcal{G}, \quad (3b)$$

$$(\mathbf{p}_{\bullet,t}, \mathbf{q}_{\bullet,t}, \mathbf{v}_{\bullet,t}, \vec{\mathbf{s}}_{\bullet,t}, \vec{\mathbf{s}}_{\bullet,t}) \in \mathcal{N}_t \quad \forall t \in \mathcal{T}, \quad (3c)$$

with respect to the matrix variables $\mathbf{x} \triangleq [x_{g,t}]$, $\mathbf{p} \triangleq [p_{g,t}]$, $\mathbf{q} \triangleq [q_{g,t}]$, $\mathbf{c} \triangleq [c_{g,t}]$, $\mathbf{v} \triangleq [v_{k,t}]$, $\vec{\mathbf{s}} \triangleq [\vec{s}_{l,t}]$, and $\vec{\mathbf{s}} \triangleq [\vec{s}_{l,t}]$. The objective function (3a) is equal the sum of the production costs of all generating units throughout the time horizon \mathcal{T} . For any arbitrary generating unit g in time interval t, the production cost consists of the generation cost, start-up cost, shutdown cost, and a fixed cost. The generation cost is a quadratic function with respect to $p_{g,t}$ with nonnegative coefficients α_g and β_g . The start-up cost γ_g^{\uparrow} and shutdown cost γ_g^{\downarrow} are associated with every time slots at which the unit changes status. The fixed production cost γ_g is enforced if the unit is active.

Definition 1: For every generating units $g \in \mathcal{G}$, define $\mathcal{U}_g \subset \mathbb{R}^{|\mathcal{T}| \times 4}$ to be the set of all quadruplets $(\mathbf{x}_{g,\bullet}^\top, \mathbf{p}_{g,\bullet}^\top, \mathbf{q}_{g,\bullet}^\top, \mathbf{c}_{g,\bullet}^\top)$ that satisfy the constraints (1a)-(1h) throughout the entire planning horizon.

Definition 2: For every $t \in \mathcal{T}$, define $\mathcal{N}_t \subset \mathbb{R}^{|\mathcal{G}| \times 2} \times \mathbb{C}^{|\mathcal{V}|} \times \mathbb{C}^{|\mathcal{E}| \times 2}$ to be the set of all quintuplet $(\mathbf{p}_{\bullet,t}, \mathbf{q}_{\bullet,t}, \mathbf{v}_{\bullet,t}, \vec{\mathbf{s}}_{\bullet,t}, \vec{\mathbf{s}}_{\bullet,t})$ that satisfy the network constraints $(2\mathbf{a}) - (2\mathbf{f})$.

Problem (3a) – (3c) is a mixed-integer nonlinear optimization, due to the presence of binary variables and nonlinearity of the network constraints. In what follows, we will develop a convex relaxation to tackle the non-convexity of this problem.

Unit Constraints:

$$z_{g,t} = x_{g,t}, (4a)$$

$$c_{g,t} = \alpha_g p_{g,t} + \beta_g o_{g,t} + \gamma_g x_{g,t}$$

$$+\gamma_g^{\uparrow}(x_{g,t}-u_{g,t})+\gamma_g^{\downarrow}(x_{g,t-1}-u_{g,t}),\tag{4b}$$

(4c)

(4f)

$$x_{g,\tau} - x_{g,\tau-1} \le x_{g,t} \qquad \forall \tau \in \{t - m_g^{\uparrow} + 1, \dots, t\}$$

$$x_{g,\tau-1}-x_{g,\tau}\leq 1-x_{g,t} \qquad \forall \tau\in\{t-m_g^\downarrow+1,\ldots,t\} \tag{4d}$$

$$\underline{p}_g x_{g,t} \le p_{g,t} \le \overline{p}_g x_{g,t} \tag{4e}$$

$$p_{g,t} - p_{g,t-1} \le r_g x_{g,t-1} + s_g (1 - x_{g,t-1}) \tag{4g}$$

$$p_{g,t-1} - p_{g,t} \le r_g x_{g,t} + s_g (1 - x_{g,t})$$

$$(4h)$$

$$\begin{bmatrix} z_{g,t-1} & u_{g,t} \end{bmatrix} = \begin{bmatrix} x_{g,t-1} \\ \end{bmatrix} \begin{bmatrix} x_{g,t-1} & x_{g,t} \end{bmatrix} \succeq 0, \tag{4i}$$

$$\begin{bmatrix} z_{g,t-1} & u_{g,t} \\ u_{g,t} & z_{g,t} \end{bmatrix} - \begin{bmatrix} x_{g,t-1} \\ x_{g,t} \end{bmatrix} \begin{bmatrix} x_{g,t-1} & x_{g,t} \end{bmatrix} \succeq 0, \tag{4i}$$

$$\begin{bmatrix} z_{g,t} & b_{g,t} \\ b_{g,t} & o_{g,t} \end{bmatrix} - \begin{bmatrix} x_{g,t} \\ p_{g,t} \end{bmatrix} \begin{bmatrix} x_{g,t} & p_{g,t} \end{bmatrix} \succeq 0. \tag{4j}$$

AC Network Constraints:

 $q_q x_{q,t} \leq q_{q,t} \leq \bar{q}_q x_{q,t}$

$$\mathbf{d}_{\bullet,t} + \operatorname{diag}\{\mathbf{W}_t \mathbf{Y}^*\} = \mathbf{C}^{\top}(\mathbf{p}_{\bullet,t} + i\mathbf{q}_{\bullet,t}), \tag{5a}$$

$$\operatorname{diag}\{\vec{\mathbf{C}}\ \mathbf{W}_t\vec{\mathbf{Y}}^*\} = \vec{\mathbf{s}}_{\bullet,t},\tag{5b}$$

$$\operatorname{diag}\{\mathbf{\tilde{C}}\ \mathbf{W}_{t}\mathbf{\tilde{Y}}^{*}\} = \mathbf{\tilde{s}}_{\bullet t},\tag{5c}$$

$$\mathbf{v}^2 < \operatorname{diag}\{\mathbf{W}_t\} < \bar{\mathbf{v}}^2,\tag{5d}$$

$$|\vec{\mathbf{s}}_{\bullet,t}|^2 \le \vec{\mathbf{f}}_{\bullet,t} \le \mathbf{f}_{\max;t}^2,$$
 (5e)

$$|\mathbf{\bar{s}}_{\bullet,t}|^2 \le \mathbf{\bar{f}}_{\bullet,t} \le \mathbf{f}_{\max:t}^2$$
 (5f)

$$\mathbf{W}_{t} - \mathbf{v}_{\bullet,t} \mathbf{v}_{\bullet,t}^{*} \succeq_{\mathcal{C}} 0. \tag{5g}$$

TABLE II: Relaxed unit and AC network constraints.

III. CONVEX RELAXATION OF THE UC PROBLEM

The non-convex sets $\{\mathcal{U}_g\}_{g\in\mathcal{G}}$ and $\{\mathcal{N}_t\}_{t\in\mathcal{T}}$, are the sources of computational complexity. In this paper, we introduce convex surrogates $\{\mathcal{U}_q^{\text{conv}}\}_{g \in \mathcal{G}}$ and $\{\mathcal{N}_t^{\text{conv}}\}_{t \in \mathcal{T}}$, which lead to a class of computationally-tractable relaxations of the problem (3a)-(3c). To this end, define the auxiliary variables $\mathbf{u}, \mathbf{o}, \mathbf{r}, \mathbf{z}, \mathbf{b} \in \mathbb{R}^{|\mathcal{G}| \times |\mathcal{T}|}$, whose components account for monomials $x_{g,t-1}x_{g,t}$, $p_{g,t}^2$, $q_{g,t}^2$, $x_{g,t}^2$, and $x_{g,t}p_{g,t}$, respectively. Using the defined variables, non-convex constraints (1a) – (1b) can be convexified as (4a) – (4b). In addition, to relax the non-convexity of AC network constraints, we define the auxiliary variables $\vec{\mathbf{f}}_{\bullet,t}, \bar{\mathbf{f}}_{\bullet,t} \in \mathbb{R}^{|\mathcal{E}|}$ and $\mathbf{W}_t \in \mathbb{H}_{|\mathcal{V}|}$, accounting for $|\vec{s}_{ullet,t}|^2$, $|\vec{s}_{ullet,t}|^2$, and $\mathbf{v}_{ullet,t}\mathbf{v}_{ullet,t}^*$, respectively. Using the above auxiliary variables, the non-convex constraints (2a) - (2e) can be relaxed as (5a) - (5f).

In order to capture the binary requirements of the commitment decisions and enforce the relationship between the auxiliary variables and the corresponding monomials, we strengthen the proposed convex relaxation via conic constraints (4i) – (4j), and (5g), where C in (5g) is a pointed convex cone. Next, we define the convex surrogates $\{\mathcal{U}_q^{\text{conv}}\}_{q\in\mathcal{G}}$ and $\{\mathcal{N}_t^{\text{conv}}\}_{t\in\mathcal{T}}$.

Definition 3: For every $g \in \mathcal{G}$, define $\mathcal{U}_a^{\text{conv}} \subset \mathbb{R}^{|\mathcal{T}| \times 9}$ to be the set of all nonuplets $(\mathbf{x}_{g,\bullet}^{\top}, \mathbf{p}_{g,\bullet}^{\top}, \mathbf{q}_{g,\bullet}^{\top}, \mathbf{c}_{g,\bullet}^{\top}, \mathbf{u}_{g,\bullet}^{\top}, \mathbf{o}_{g,\bullet}^{\top}, \mathbf{r}_{g,\bullet}^{\top}, \mathbf{r}_{g,\bullet}^{\top}, \mathbf{z}_{g,\bullet}^{\top})$ that satisfy the constraints (4a)-(4j) throughout the entire planning horizon.

Definition 4: For every $t \in \mathcal{T}$, define $\mathcal{N}_t^{\text{conv}} \subset \mathbb{R}^{|\mathcal{G}| \times 2} \times$ $\mathbb{C}^{|\mathcal{V}|} \times \mathbb{C}^{|\mathcal{E}| \times 2} \times \mathbb{H}_{|\mathcal{V}|} \times \mathbb{R}^{|\mathcal{E}| \times 2}$ to be the set of all octuplets $(\mathbf{p}_{\bullet,t}, \mathbf{q}_{\bullet,t}, \mathbf{v}_{\bullet,t}, \vec{\mathbf{s}}_{\bullet,t}, \vec{\mathbf{s}}_{\bullet,t}, \mathbf{W}_t, \vec{\mathbf{f}}_{\bullet,t}, \vec{\mathbf{f}}_{\bullet,t})$ that satisfy the constraints (5a) - (5g).

The second-order cone programming (SOCP) relaxation of network constraints can be derived by incorporating the following convex set into the constraint (5g):

$$\hat{\mathcal{C}} \triangleq \{ \mathbf{H} \in \mathbb{H}_{|\mathcal{V}|} \mid H_{ii} \ge 0, \ H_{ii}H_{jj} \ge |H_{ij}|^2, \ \forall (i,j) \in \mathcal{E} \}.$$

The solution provided by the SOCP relaxation is a lowerbound for the globally optimal solution of AC unit commitment. In general, solutions obtained from convex relaxations are not necessarily feasible for the original non-convex problem. To remedy this shortcoming, we propose a novel penalization method to obtain feasible points. In the next section, we describe the proposed penalization method in details.

IV. PENALIZATION METHOD

We incorporate a linear penalty term $\kappa(\{\mathbf{W}_t\}_{t\in\mathcal{T}}, \mathbf{z}, \mathbf{o}, \mathbf{r},$ $(\mathbf{f}, \mathbf{f}, \mathbf{v}, \mathbf{x}, \mathbf{s}, \mathbf{s}, \mathbf{s})$ into the objective of the relaxed problem to enforce feasibility. Given an initial guess $\mathbf{y}^0 = (\mathbf{v}^0, \mathbf{x}^0, \mathbf{s}^0, \mathbf{s}^0, \mathbf{s}^0, \mathbf{s}^0)$ that is sufficiently close to the feasible set of the problem (3a)-(3c), the following choice of penalty function guarantees the feasibility of the resulting solution under the assumptions in [47], [48]:

$$\kappa_{\mathbf{M},\mathbf{y}_{0}}(\{\mathbf{W}_{t}\}_{t\in\mathcal{T}},\mathbf{z},\mathbf{o},\mathbf{r},\vec{\mathbf{f}},\dot{\mathbf{f}},\mathbf{v},\mathbf{x},\mathbf{s},\vec{\mathbf{s}},\dot{\mathbf{s}}) \triangleq \\
\sum_{t} (\operatorname{tr}\{\mathbf{W}_{t}\mathbf{M}\} - \mathbf{v}_{\bullet,t}^{0*} \mathbf{M} \mathbf{v}_{\bullet,t} - \mathbf{v}_{\bullet,t}^{*} \mathbf{M} \mathbf{v}_{\bullet,t}^{0} + \mathbf{v}_{\bullet,t}^{0*} \mathbf{M} \mathbf{v}_{\bullet,t}^{0} + \\
\mathbf{z}_{\bullet,t}^{\top} \mathbf{1} - 2 \mathbf{x}_{\bullet,t}^{\top} \mathbf{x}_{\bullet,t}^{0} + \mathbf{x}_{\bullet,t}^{0\top} \mathbf{x}_{\bullet,t}^{0} + \\
\mathbf{o}_{\bullet,t}^{\top} \mathbf{1} - 2 \mathbf{p}_{\bullet,t}^{\top} \mathbf{p}_{\bullet,t}^{0} + \mathbf{p}_{\bullet,t}^{0\top} \mathbf{p}_{\bullet,t}^{0} + \\
\mathbf{r}_{\bullet,t}^{\top} \mathbf{1} - 2 \mathbf{q}_{\bullet,t}^{\top} \mathbf{q}_{\bullet,t}^{0} + \mathbf{q}_{\bullet,t}^{0\top} \mathbf{q}_{\bullet,t}^{0} + \\
\vec{\mathbf{f}}_{\bullet,t}^{\top} \mathbf{1} - \vec{\mathbf{s}}_{\bullet,t}^{0*} \vec{\mathbf{s}}_{\bullet,t} - \vec{\mathbf{s}}_{\bullet,t}^{*} \vec{\mathbf{s}}_{\bullet,t}^{0} + \vec{\mathbf{s}}_{\bullet,t}^{0*} \vec{\mathbf{s}}_{\bullet,t}^{0} + \\
\vec{\mathbf{f}}_{\bullet,t}^{\top} \mathbf{1} - \vec{\mathbf{s}}_{\bullet,t}^{0*} \vec{\mathbf{s}}_{\bullet,t} - \vec{\mathbf{s}}_{\bullet,t}^{*} \vec{\mathbf{s}}_{\bullet,t}^{0} + \vec{\mathbf{s}}_{\bullet,t}^{0*} \vec{\mathbf{s}}_{\bullet,t}^{0}), \tag{6}$$

where $\mathbf{M} \in \mathbb{H}_{|\mathcal{V}|}$ is a fixed penalty matrix.

By augmenting the penalty term (6) into the objective function of the relaxed problem, the penalized convex relaxation of AC unit commitment can be formulated as:

min
$$g(\mathbf{c}) + \mu \kappa_{\mathbf{M}, \mathbf{y}_0}(\{\mathbf{W}_t\}_{t \in \mathcal{T}}, \mathbf{z}, \mathbf{o}, \mathbf{r}, \mathbf{f}, \mathbf{f}, \mathbf{v}, \mathbf{x}, \mathbf{p} + i\mathbf{q}, \mathbf{s}, \mathbf{\tilde{s}})$$
 (7a)
s.t. $(\mathbf{x}_{g, \bullet}^{\top}, \mathbf{p}_{g, \bullet}^{\top}, \mathbf{q}_{g, \bullet}^{\top}, \mathbf{c}_{g, \bullet}^{\top}, \mathbf{z}_{g, \bullet}^{\top}, \mathbf{o}_{g, \bullet}^{\top}, \mathbf{r}_{g, \bullet}^{\top}) \in \mathcal{U}_g^{\text{conv}} \quad \forall g \in \mathcal{G}, (7b)$
 $(\mathbf{p}_{\bullet}, t, \mathbf{q}_{\bullet}, t, \mathbf{v}_{\bullet}, t, \mathbf{\tilde{s}}_{\bullet}, t, \mathbf{\tilde{s}}_{\bullet}, t, \mathbf{W}_t, \mathbf{\tilde{f}}_{\bullet}, t, \mathbf{\tilde{f}}_{\bullet}, t) \in \mathcal{N}_t^{\text{conv}} \quad \forall t \in \mathcal{T}, (7c)$

with respect to decision variables $\mathbf{x} \triangleq [x_{g,t}], \mathbf{p} \triangleq [p_{g,t}],$ $\mathbf{q} \triangleq [q_{g,t}], \ \mathbf{c} \triangleq [c_{g,t}], \ \mathbf{z} \triangleq [z_{g,t}], \ \mathbf{o} \triangleq [o_{g,t}], \ \mathbf{r} \triangleq [r_{g,t}],$ $\mathbf{v} \triangleq [v_{k,t}], \ \mathbf{s} \triangleq [\vec{s}_{l,t}], \ \mathbf{\tilde{s}} \triangleq [\vec{s}_{l,t}], \ \mathbf{\tilde{f}} \triangleq [\vec{f}_{l,t}], \ \mathbf{\tilde{f}} \triangleq$ $[\tilde{f}_{l,t}]$, and $\{\mathbf{W}_t\}_{t\in\mathcal{T}}$. The nonnegative penalty parameter $\mu > 0$ sets the trade off between the objective and the penalty functions. The penalized convex relaxation (7a)-(7c) is said to be tight if it possesses a unique optimal solution $(\mathbf{x}, \mathbf{p}, \mathbf{q}, \mathbf{c}, \mathbf{z}, \mathbf{o}, \mathbf{r}, \mathbf{v}, \vec{\mathbf{s}}, \dot{\mathbf{f}}, \dot{\mathbf{f}}, \{\mathbf{W}_t\}_{t \in \mathcal{T}})$ such that $x_{g,t} \in \{0,1\}$ and $\mathbf{W}_t = \mathbf{v}_{\bullet,t} \mathbf{v}_{\bullet,t}^*$, for every $g \in \mathcal{G}$ and $t \in \mathcal{T}$.

The tightness of the penalization guarantees the recovery of a feasible point for AC unit commitment (3a)-(3c).

A. Choice of Penalty Matrix

Motivated by the previous literatures [49]–[51], we choose M such that the penalty term $\operatorname{tr}\{\mathbf{W}_t\mathbf{M}\}$ reduces the apparent power loss over the series admittance of every line in the network. Consider the standard π -model of line $l \in \mathcal{E}$, with series admittance $y_{\operatorname{srs},l} \triangleq g_{\operatorname{srs},l} + \mathrm{i}\, b_{\operatorname{srs},l}$ and total shunt susceptance $b_{\operatorname{prl},l}$, in series with a phase shifting transformer whose tap ratio has magnitude τ_l and phase shift angle θ_l [1]. The model is shown in Figure 1. In order to penalize the apparent power loss over all lines of the network, we choose matrix M as,

$$\mathbf{M} = \sum_{(i,j)\in\mathcal{E}} [\mathbf{e}_i, \mathbf{e}_j] (\mathbf{M}_{ij} + \alpha \mathbf{I}_2) [\mathbf{e}_i, \mathbf{e}_j]^{\mathsf{T}},$$

where $\mathbf{e}_1, \dots, \mathbf{e}_{|\mathcal{V}|}$ denote the standard basis for $\mathbb{R}^{|\mathcal{V}|}$, and α is a positive constant. Moreover, each \mathbf{M}_{ij} is a 2×2 positive semidefinite matrix defined as,

$$\mathbf{M}_{ij} = \zeta_{ij} (\vec{\mathbf{Y}}_{\mathbf{q};\,l} + \mathbf{\ddot{Y}}_{\mathbf{q};\,l}) + \frac{\eta}{1-\eta} (\vec{\mathbf{Y}}_{\mathbf{p};\,l} + \mathbf{\ddot{Y}}_{\mathbf{p};\,l}).$$

where $\eta>0$ sets the trade-off between active and reactive loss minimization, and

$$\begin{split} \vec{\mathbf{Y}}_{\mathrm{p};\,l} &\triangleq \begin{bmatrix} \frac{g_{\mathrm{srs},\,l}}{\tau_{l}^{2}} & \frac{e^{\mathrm{i}\theta_{l}}\,y_{\mathrm{srs},\,l}}{-2\tau_{l}}\\ \frac{y_{\mathrm{srs},\,l}}{2\tau_{l}e^{\mathrm{i}\theta_{l}}} & 0 \end{bmatrix}, \quad \vec{\mathbf{Y}}_{\mathrm{q};\,l} &\triangleq \begin{bmatrix} \frac{b_{\mathrm{srs},\,l}}{-\tau_{l}^{2}} & \frac{e^{\mathrm{i}\theta_{l}}\,y_{\mathrm{srs},\,l}}{2\tau_{l}\mathrm{i}}\\ \frac{y_{\mathrm{srs},\,l}}{2\tau_{l}\mathrm{i}e^{\mathrm{i}\theta_{l}}} & 0 \end{bmatrix}, \\ \vec{\mathbf{Y}}_{\mathrm{p};\,l} &\triangleq \begin{bmatrix} 0 & \frac{e^{\mathrm{i}\theta_{l}}\,y_{\mathrm{srs},\,l}}^{*}\\ \frac{y_{\mathrm{srs},\,l}}{-2\tau_{l}} & -2\tau_{l}\mathrm{i} \end{bmatrix}, \quad \vec{\mathbf{Y}}_{\mathrm{q};\,l} &\triangleq \begin{bmatrix} 0 & \frac{e^{\mathrm{i}\theta_{l}}\,y_{\mathrm{srs},\,l}}^{*}\\ \frac{y_{\mathrm{srs},\,l}}{-2\tau_{l}\mathrm{i}} & -b_{\mathrm{srs},\,l} \end{bmatrix}. \end{split}$$

Each $\zeta_{ij} \in \{-1, +1\}$ is determined based on the inductive or capacitive behavior of the line $l \in \mathcal{E}$. More precisely, we set $\zeta_{ij} = 1$ if the series admittance $y_{\mathrm{srs}, l}$ is inductive (i.e., $b_{\mathrm{srs}, l} \leq 0$), and $\zeta_{ij} = -1$, otherwise.

B. Sequential Penalized Relaxation

The penalized SOCP relaxation (7a)-(7c) is guaranteed to produce a feasible solution for AC unit commitment if the initial guess \mathbf{y}^0 is sufficiently close to the feasible set of the original problem (3a)-(3c). If a high quality initial point is not available, the proposed penalized SOCP relaxation can be solved sequentially until a feasible point for problem (3a)-(3c) is obtained. Once feasibility is attained, the sequential procedure improves the objective function while preserving the feasibility at each round until a near-optimal point is achieved. This sequential procedure is detailed by Algorithm 1.

V. EXPERIMENTAL RESULTS

In this section, we present the results of our experiment on IEEE 57 bus, IEEE 118 bus, and IEEE 300 bus systems from MATPOWER [1]. The numerical experiments are performed in MATLAB using a 64-bit computer with an Intel 3.0 GHz, 12-core CPU, and 256 GB RAM. Note that the experiments are all performed on a workstation with a single CPU. The

Algorithm 1 Sequential Penalized SOCP Relaxation.

Input: μ , M, $(\mathbf{v}_0, \mathbf{x}_0, \mathbf{s}_0, \vec{\mathbf{s}}_0, \vec{\mathbf{s}}_0)$

1: repeat

2: Solve problem (7a) – (7c) to obtain $(\mathbf{v}, \mathbf{x}, \mathbf{s}, \vec{\mathbf{s}}, \vec{\mathbf{s}})$

3: $(\mathbf{v}_0, \mathbf{x}_0, \mathbf{s}_0, \mathbf{s}_0, \mathbf{s}_0) \leftarrow (\mathbf{v}, \mathbf{x}, \mathbf{s}, \mathbf{s}, \mathbf{s})$

4: until stopping criteria satisfied

Output: best found solution $(\mathbf{v}, \mathbf{x}, \mathbf{s}, \vec{\mathbf{s}}, \vec{\mathbf{s}})$

CVX package version 3.0 [52] and MOSEK version 8.0 [53] are used to solve the proposed convex relaxations.

The details of data generation are taken from [54]. For each experiment, the cost coefficients α_g , β_g , γ_g , γ_g^{\downarrow} and γ_q^{\uparrow} are chosen uniformly between zero and \$1/(MW.h)^2, \$10/(MW.h), \$100, \$30 and \$50, respectively. The ramp limits of each generating unit are set to $r_q = s_q =$ $\max\{\bar{p}_q/4, p_q\}$. For each generating unit, the minimum up and down limits m_q^{\uparrow} and m_q^{\downarrow} are randomly selected in such a way that $m_q^{\uparrow} - 1$ and $m_q^{\downarrow} - 1$ have Poisson distribution with parameter 4. The initial status of generators at time period t = 0 is found by solving a single period economic dispatch problem corresponding to the demand at time t=1. For each generating unit $g \in \mathcal{G}$, it is assumed that the initial status has been maintained exactly since time period $t=-t_q^{(0)}$, where $t_q^{(0)}$ has Poisson distribution with parameter 4. For simplicity, all of the generating units with negative capacity are removed. Hourly load changes for the dayahead at all buses are considered proportional to the numbers reported in [55]. The changes in demand throughout the 24hour planning horizon are reported in Table III. For every time epoch, the corresponding demand factor at that time is multiplied by all loads in the system.

Table IV reports the results averaged over five Monte Carlo simulations for 24-hour scheduling. In this table, k_f denotes the average round number of Algorithm 1 at which the penalized relaxation produced a feasible solution with less than 10^{-6} per unit constraint violation.

In order to evaluate the resulting feasible solutions from Algorithm 1 we solved an unpenalized semidefinite programming (SDP) relaxation of AC unit commitment by replacing the set \mathcal{C} in (3a)–(3c) with the cone of $|\mathcal{V}| \times |\mathcal{V}|$ Hermitian positive semidefinite matrices. The SDP relaxation offers a lower bound for the globally optimal cost of AC unit commitment, using which we can calculate the quality of our feasible solutions from Algorithm 1 through the formula

$$GAP\% = 100 \times \frac{\sum_{g,t} (c_{g,t}^{\text{feasible}} - c_{g,t}^{\text{SDP-lower-bound}})}{\sum_{t,a,t} c_{g,t}^{\text{feasible}}}, \quad (8)$$

where $c_{g,t}^{\text{feasible}}$ denotes the optimal cost value of the generating unit $g \in \mathcal{G}$ at time $t \in \mathcal{T}$ at round 50 of the proposed sequential SOCP relaxation, and $c_{g,t}^{\text{SDP-lower-bound}}$ denotes the cost values obtained from unpenalized SDP relaxation of (3a)-(3c). The parameter t(s) reports the average run time of all 50 rounds of Algorithm 1 in seconds. The initial point of Algorithm 1 for all of the experiments is chosen as $\mathbf{v}_{\bullet,t}^0 = \mathbf{1}_{|\mathcal{V}|}$, $\mathbf{s}_{\bullet,t}^0 = \mathbf{p}_{\min}$, $\vec{\mathbf{s}}_{\bullet,t}^0 = \mathrm{diag}\{\vec{\mathbf{C}}\mathbf{v}_{\bullet,t}^0\mathbf{v}_{\bullet,t}^0\mathbf{v}_{\bullet,t}^{0}\mathbf{v}_{\bullet,t}^{0}\mathbf{v}_{\bullet,t}^{0}$, $\mathbf{v}_{\bullet,t}^{0}\mathbf$

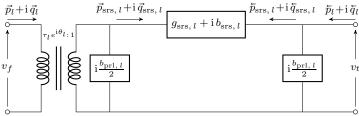


Fig. 1: Branch Model

TABLE III: Hourly Demand Factor.

Hour	Demand Factor	Hour	Demand Factor
12:00 AM	0.6843	12:00 PM	0.9460
01:00 AM	0.6451	01:00 PM	0.9516
02:00 AM	0.6198	02:00 PM	0.9721
03:00 AM	0.6044	03:00 PM	0.9992
04:00 AM	0.6057	04:00 PM	1.0000
05:00 AM	0.6269	05:00 PM	0.9638
06:00 AM	0.6773	06:00 PM	0.9608
07:00 AM	0.6937	07:00 PM	0.9271
08:00 AM	0.7297	08:00 PM	0.9270
09:00 AM	0.8084	09:00 PM	0.9089
10:00 AM	0.8930	10:00 PM	0.7654
11:00 AM	0.9223	11:00 PM	0.7641

TABLE IV: The performance of the proposed sequential penalized SOCP relaxation for 24-hour scheduling of IEEE benchmark systems.

Test Case	SOCP				
iest case	μ	α	k_f	GAP(%)	t(s)
case57	1e0	1	1	0.00	603.0
case118	1e0	10	1	2.27	1537.5
case300	1e1	10	12.4	5.52	4010.0

of the generators, for all $t \in \mathcal{T}$.

For all of the random experiments, Algorithm 1 successfully finds a fully feasible operating point. Moreover, the reported gaps in Table IV demonstrate the effectiveness of our method in solving large instances of AC unit commitment. Changes in the resulting cost values with respect to the round numbers for one of the random experiments of each benchmark case are illustrated in Figure 2.

VI. CONCLUSIONS

In this work, a sequential convex relaxation method is introduced for solving unit commitment with AC transmission constraints. We first, develop a second-order cone programming (SOCP) relaxation to convexity AC unit commitment problems. We then incorporate a penalty term into the objective of the proposed SOCP relaxation in order to find feasible solutions for the original non-convex AC unit commitment. The proposed penalized SOCP relaxations can be solved sequentially, to find feasible and near-globally optimal points. The experimental results on IEEE 57 bus, IEEE 118 bus, IEEE 300 bus systems demonstrate the effectiveness of the proposed approach in solving challenging instances of AC unit commitment.

REFERENCES

[1] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MAT-POWER: Steady-state operations, planning, and analysis tools for

- power systems research and education," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 12–19, 2011.
- [2] C. Tseng, On Power System Generation Unit Commitment Problems. University of California, Berkeley, 1996.
- [3] X. Guan, Q. Zhai, and A. Papalexopoulos, "Optimization based methods for unit commitment: Lagrangian relaxation versus general mixed integer programming," *Proc. of IEEE Power & Energy Society General Meeting*, vol. 2, no. 3, pp. 1095–1100, 2003.
- [4] B. Saravanan, S. Das, S. Sikri, and D. Kothari, "A solution to the unit commitment problem-a review," *Frontiers in Energy*, vol. 7, no. 2, pp. 223–236, 2013.
- [5] E. Allen and M. Ilic, Price-Based Commitment Decisions in the Electricity Market. Springer Science and Business Media, London, 2012.
- [6] E. Y. Bitar, R. Rajagopal, P. P. Khargonekar, K. Poolla, and P. Varaiya, "Bringing wind energy to market," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1225–1235, 2012.
- [7] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52–63, 2013.
- [8] D. T. Phan and A. Koc, "Optimization approaches to security-constrained unit commitment and economic dispatch with uncertainty analysis," Springer, Optimization and Security Challenges in Smart Power Grids, pp. 1–37, 2013.
- [9] Y. Yu and R. Rajagopal, "The impacts of electricity dispatch protocols on the emission reductions due to wind power and carbon tax," *Environ. Sci. Technol.*, vol. 49, no. 4, pp. 2568–2576, 2015.
- [10] A. Lorca and X. A. Sun, "Multistage robust unit commitment with dynamic uncertainty sets and energy storage," *IEEE Trans. Power* Syst., vol. 32, no. 3, pp. 1678–1688, 2017.
- [11] B. Zhao, A. J. Conejo, and R. Sioshansi, "Unit commitment under gas-supply uncertainty and gas-price variability," *IEEE Trans. Power* Syst., vol. 32, no. 3, pp. 2394–2405, 2017.
- [12] K. Sundar, H. Nagarajan, L. Roald, S. Misra, R. Bent, and D. Bienstock, "A modified benders decomposition for chance-constrained unit commitment with N-1 security and wind uncertainty," arXiv preprint arXiv:1703.05206, 2017.
- [13] Q. P. Zheng, J. Wang, and A. L. Liu, "Stochastic optimization for unit commitment-A review," *IEEE Trans. Power Syst.*, vol. 30, no. 4, pp. 1913–1924, 2015.
- [14] X. Bai and H. Wei, "Semi-definite programming-based method for security-constrained unit commitment with operational and optimal power flow constraints," *IET Gener. Transm. Dis.*, vol. 3, no. 2, pp. 182–197, 2009.
- [15] A. Castillo, C. Laird, C. A. Silva-Monroy, J.-P. Watson, and R. P. O'Neill, "The unit commitment problem with AC optimal power flow constraints," *IEEE Trans. Power Syst.*, vol. 31, no. 6, pp. 4853–4866, 2016
- [16] P. Lipka, S. S. Oren, R. P. O'Neill, and A. Castillo, "Running a more complete market with the SLP-IV-ACOPF," *IEEE Trans. Power Syst.*, vol. 32, no. 2, pp. 1139–1148, 2017.
- [17] K. W. Hedman, M. C. Ferris, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Co-optimization of generation unit commitment and transmission switching with N-1 reliability," *IEEE Trans. Power Syst.*, vol. 25, no. 2, pp. 1052–1063, 2010.
- [18] H. Wu, M. Shahidehpour, and M. E. Khodayar, "Hourly demand response in day-ahead scheduling considering generating unit ramping cost," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2446–2454, 2013.
- [19] P. Y. Kerl, W. Zhang, J. B. Moreno-Cruz, A. Nenes, M. J. Realff, A. G. Russell, J. Sokol, and V. M. Thomas, "New approach for optimal electricity planning and dispatching with hourly time-scale air quality

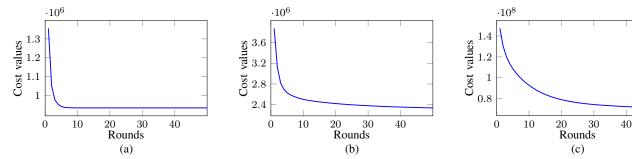


Fig. 2: Convergence behavior of the proposed sequential penalized SOCP relaxation. The resulting cost values per round numbers are shown for one of the random experiments on each of the (a) IEEE 57 bus; (b) IEEE 118 bus; (c) IEEE 300 bus systems.

- and health considerations," *Proc. Natl. Acad. Sci.*, vol. 112, no. 35, pp. 10884–10889, 2015.
- [20] A. Subramanian, M. J. Garcia, D. S. Callaway, K. Poolla, and P. Varaiya, "Real-time scheduling of distributed resources," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 2122–2130, 2013.
- [21] A. I. Cohen and M. Yoshimura, "A branch-and-bound algorithm for unit commitment," *IEEE Trans. Power App. Syst.*, no. 2, pp. 444–451, 1983.
- [22] M. G. Marcovecchio, A. Q. Novais, and I. E. Grossmann, "Deterministic optimization of the thermal unit commitment problem: A branch and cut search," *Comput. Chem. Eng.*, vol. 67, pp. 53–68, 2014.
- [23] B. Knueven, J. Ostrowski, and J. Wang, "The ramping polytope and cut generation for the unit commitment problem," *INFORMS J. Comput.*, 2017
- [24] T. S. Dillon, K. W. Edwin, H.-D. Kochs, and R. Taud, "Integer programming approach to the problem of optimal unit commitment with probabilistic reserve determination," *IEEE Trans. Power App. Syst.*, no. 6, pp. 2154–2166, 1978.
- [25] M. Carrión and J. M. Arroyo, "A computationally efficient mixedinteger linear formulation for the thermal unit commitment problem," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1371–1378, 2006.
- [26] E. Delarue and W. Dhaeseleer, "Adaptive mixed-integer programming unit commitment strategy for determining the value of forecasting," *Appl. Energy*, vol. 85, no. 4, pp. 171–181, 2008.
- [27] J. A. Muckstadt and R. C. Wilson, "An application of mixed-integer programming duality to scheduling thermal generating systems," *IEEE Trans. Power App. Syst.*, vol. PAS-87, no. 12, pp. 1968–1978, Dec 1968.
- [28] T. Li and M. Shahidehpour, "Price-based unit commitment: A case of Lagrangian relaxation versus mixed integer programming," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 2015–2025, 2005.
- [29] B. Knueven, J. Ostrowski, and J.-P. Watson. (2017) A novel matching formulation for startup costs in unit commitment. [Online]. Available: http://www.optimization-online.org/DBFILE/2017/03/5897.pdf
- [30] E. C. Finardi and M. R. Scuzziato, "A comparative analysis of different dual problems in the Lagrangian relaxation context for solving the hydro unit commitment problem," *Electr. Power Syst. Res.*, vol. 107, pp. 221–229, 2014.
- [31] J. Ostrowski, M. F. Anjos, and A. Vannelli, "Tight mixed integer linear programming formulations for the unit commitment problem," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 39–46, 2012.
- [32] Z. Geng, A. J. Conejo, and Q. Xia, "Alternative linearisations for the operating cost function of unit commitment problems," *IET Gener. Transm. Dis.*, vol. 11, no. 8, pp. 1992–1996, 2017.
- [33] P. Damci-Kurt, S. Küçükyavuz, D. Rajan, and A. Atamtürk, "A polyhedral study of production ramping," *Math. Program.*, vol. 158, no. 1-2, pp. 175–205, 2016.
- [34] J. Lee, J. Leung, and F. Margot, "Min-up/min-down polytopes," Discrete Optimization, vol. 1, no. 1, pp. 77–85, 2004.
- [35] M. S. Aktürk, A. Atamtürk, and S. Gürel, "A strong conic quadratic reformulation for machine-job assignment with controllable processing times," *Oper. Res. Lett.*, vol. 37, no. 3, pp. 187–191, 2009.
- [36] A. Frangioni and C. Gentile, "A computational comparison of reformulations of the perspective relaxation: SOCP vs. cutting planes," *Oper. Res. Lett.*, vol. 37, no. 3, pp. 206–210, 2009.
- [37] R. Jabr, "Rank-constrained semidefinite program for unit commitment," Int. J. Elec. Power, vol. 47, pp. 13–20, 2013.

- [38] M. Paredes, L. Martins, and S. Soares, "Using semidefinite relaxation to solve the day-ahead hydro unit commitment problem," *IEEE Trans. Power Syst.*, vol. 30, no. 5, pp. 2695–2705, 2015.
- [39] S. Fattahi, M. Ashraphijuo, J. Lavaei, and A. Atamtürk, "Conic relaxations of the unit commitment problem," *Energy*, vol. 134, pp. 1079–1095, 2017.
- [40] M. Ashraphijuo, S. Fattahi, J. Lavaei, and A. Atamtürk, "A strong semidefinite programming relaxation of the unit commitment problem," *Proc. IEEE Conf. Decision and Control*, pp. 694–701, 2016.
- [41] Y. Bai, H. Zhong, Q. Xia, C. Kang, and L. Xie, "A decomposition method for network-constrained unit commitment with AC power flow constraints," *Energy*, vol. 88, pp. 595–603, 2015.
- [42] R. Quan, J.-b. Jian, and Y.-d. Mu, "Tighter relaxation method for unit commitment based on second-order cone programming and valid inequalities," *Int. J. Elec. Power*, vol. 55, pp. 82–90, 2014.
- [43] J. Liu, A. Castillo, J.-P. Watson, and C. D. Laird, "Global solution strategies for the network-constrained unit commitment problem with AC transmission constraints," arXiv preprint arXiv:1801.07218, 2018.
- [44] A. Kargarian, Y. Fu, and Z. Li, "Distributed security-constrained unit commitment for large-scale power systems," *IEEE Trans. Power Syst.*, vol. 30, no. 4, pp. 1925–1936, 2015.
- [45] A. Papavasiliou, S. S. Oren, and B. Rountree, "Applying high performance computing to transmission-constrained stochastic unit commitment for renewable energy integration," *IEEE Trans. Power Syst.*, vol. 30, no. 3, pp. 1109–1120, 2015.
- [46] A. Papavasiliou and S. S. Oren, "A comparative study of stochastic unit commitment and security-constrained unit commitment using high performance computing," *Proc. IEEE Europ. Control Conf.*, pp. 2507– 2512, 2013.
- [47] M. Kheirandishfard, F. Zohrizadeh, and R. Madani, "Convex relaxation of bilinear matrix inequalities Part I: Theoretical results," in *Proc. IEEE Conf. Decision and Control*, 2018.
- [48] R. Madani, M. Kheirandishfard, J. Lavaei, and A. Atamturk, "Penalized conic relaxations for quadratically constrained quadratic programing," *Preprint: http://www.uta.edu/faculty/madanir/qcqp_conic.pdf*, 2018.
- [49] F. Zohrizadeh, M. Kheirandishfard, E. Quarm, and R. Madani, "Penalized parabolic relaxation for optimal power flow problem," in *Proc. IEEE Conf. Decision and Control*, 2018.
- [50] R. Madani, M. Ashraphijuo, and J. Lavaei, "Promises of conic relaxation for contingency-constrained optimal power flow problem," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1297–1307, 2016.
- [51] R. Madani, S. Sojoudi, and J. Lavaei, "Convex relaxation for optimal power flow problem: Mesh networks," *IEEE Trans. Power Syst.*, vol. 30, no. 1, pp. 199–211, 2015.
- [52] I. CVX Research, "CVX: MATLAB software for disciplined convex programming, version 2.0," http://cvxr.com/cvx, Aug. 2012.
- [53] MosekApS, "The MOSEK optimization toolbox for MATLAB," Manual Version 7.1 (Revision 28), p. 17, 2015.
- [54] R. Madani, A. Atamturk, and A. Davoudi, "A scalable semidefinite relaxation approach to grid scheduling," arXiv preprint arXiv:1707.03541, 2017.
- [55] A. Khodaei and M. Shahidehpour, "Transmission switching in security-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 25, no. 4, pp. 1937–1945, 2010.