On Stability Condition of Wireless Networked Control Systems under Joint Design of Control Policy and Network Scheduling Policy

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Abstract—In this paper, we study a wireless networked control system (WNCS) with $N \ge 2$ sub-systems sharing a common wireless channel. Each sub-system consists of a plant and a controller and the control message must be delivered from the controller to the plant through the shared wireless channel. The wireless channel is unreliable due to interference and fading. As a result, a packet can be successfully delivered in a slot with a certain probability. A network scheduling policy determines how to transmit those control messages generated by such N subsystems and directly influences the transmission delay of control messages. We first consider the case that all sub-systems have the same sampling period. We characterize the stability condition of such a WNCS under the joint design of the control policy and the network scheduling policy by means of 2^N linear inequalities. We further simplify the stability condition into only one linear inequality for two special cases: the perfect-channel case where the wireless channel can successfully deliver a control message with certainty in each slot, and the symmetric-structure case where all sub-systems have identical system parameters. We then consider the case that different sub-systems can have different sampling periods, where we characterize a sufficient condition for stability.

Index Terms—wireless networked control system (WNCS), control policy, network scheduling policy, stability condition, timely throughput.

I. INTRODUCTION

Networked Control Systems (NCSs) that exchange information between a plant and its controller through a shared communication network have been a topic of active research for decades in both the academia and the industry [2]–[4]. Existing communication networks employed in NCS include controllerarea network (CAN), Ethernet, and wireless networks (called wireless NCS (WNCS)) [2]. Among them, WNCS is widely used in many applications such as automated highway systems, factories, and unmanned aerial vehicles (UAVs), etc., because wireless communication can be easily deployed with low cost and low complexity [2], [5]–[7]. In this paper, we focus on systems based on WNCS.

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Traditional NCS researches typically focus on controltheoretic issues while highly abstracting the network-system performance in terms of transmission delay and packet dropout/loss [8]. For example, the behavior of a wireless communication network with finite capacity is commonly modeled by assuming that packets traveling through the shared communication network experience a fixed or random delay (with a known distribution) [9] or by a fixed packet dropout rate [10]. Using types of simplifications, many researches focus on how to design the control policy so as to stabilize the system [11], [12] or optimize the system performance [13]. In [14], the authors consider stabilization problems for NCSs with both packet dropout and transmission delay. By utilizing a delay-dependent algebraic Riccati equation, a necessary and sufficient stabilization condition is derived.

However, packet delay and packet dropout incurred in a shared communication network are results of network operations as defined by network protocol or network scheduling policy. To completely understand the behaviour and performance of NCSs, it is important to simultaneously consider both the control policy in the dynamic system and also the network scheduling policy in the network system. There are some existing works that consider such joint design for WNCS [8], [15], [16] (also see a survey [6] and the references therein). Reference [8] considers a WNCS with only one plant and one controller which exchanges information through a multi-hop wireless network. The authors jointly design the control policy and network scheduling policy to minimize the closed-loop loss function and propose a modular co-design framework to solve the problem. The authors in [16] analyze the system performance of WNCS with multiple plants and controllers when the wireless communication network adopts the standard IEEE 802.15.4 protocol. The authors in [15] also study a WNCS with multiple plants and controllers and they analyze the system performance by jointly considering control policy and cross-layer network design. However, to the best of our knowledge, there does not exist work on characterizing the stability condition of a WNCS with multiple plants and controllers sharing a common wireless channel.

We also remark that the joint design of control system and network system also influences the network scheduling policy design. Generally, the central performance metric of network system design is throughput in the delay-unconstrained case or timely throughput in the delay-constrained case. However, a control system may not only depend on the long-term throughput or timely throughput, but may also depend on the sampled paths, i.e., the short-term behaviors (see more details in later analysis), which also poses more challenges to the network system design.

In this paper, we study a WNCS with multiple sub-systems

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sharing a common wireless channel. Each sub-system consists of a plant and a controller and the control message must be delivered from the controller to the plant through the shared wireless channel. We characterize the stability condition of such a WNCS under the joint design of the control policy and the network scheduling policy. In particular, we make the following contributions, and summarize the main results on stability condition in Table I:

- For the stated WNCS with general system parameters so that all sub-systems could have different parameters (asymmetric-structure case in Table I) and the wireless channel could be imperfect (imperfect-channel case in Table I), we characterize the stability condition by means of 2^N linear inequalities, where N is the number of sub-systems.
- We simplify the general stability condition into only one linear inequality for two special cases of the considered WNCS: the perfect-channel case and the symmetric-structure case.
- For perfect-channel case, we show that the system can be stabilized if the sampling period is larger than a threshold. This result quantifies the effect of the sampling period on both the network system and the control system.
- We show a monotonic property of the stability region in terms of the wireless channel quality: if the WNCS can be stabilized under a channel quality vector, it can also be stabilized under a better channel quality vector. This result enables us to efficiently find the minimum channel quality to stabilize the system for the special symmetricstructure case.
- Our previous analysis assumes that all sub-systems have the same sampling period. With this assumption removed, we also characterize a sufficient condition for stability under the case of heterogenous sampling periods.

The rest of this paper is organized as follows. We first describe our system model in Sec. II. We next present the stability condition for a general WNCS in Sec. III. Then, in Sec. IV and Sec. V, we simplify the general stability condition into only one linear inequality for two special cases. In Sec. VI, we prove a monotonic result in terms of channel quality. In addition, we propose a sufficient condition for stability when different sub-systems have different sampling periods in Sec. VII. We use simulation to validate our theoretical analysis in Sec. VIII. Finally, we conclude our paper in Sec. IX. Throughout this paper, we define set $[C] \triangleq \{1, 2, \dots, C\}$ for any positive integer $C \in \mathbb{Z}^+$.

We also remark that as compared with our preliminary conference version [1], this paper presents more new results, including (i) the monotonic result in terms of channel quality in Sec. VI, (ii) a sufficient condition for stability when different sub-systems have different sampling periods in Sec. VII, and (iii) more simulation results in Sec. VIII (see Fig. 8, Fig. 9, and Fig. 10).

II. SYSTEM MODEL

We consider a WNCS with N sub-systems, indexed from 1 to N. An example of two sub-systems is shown in Fig. 1.

TABLE I MAIN RESULTS ON STABILITY CONDITION

	Symmetric Structure	Asymmetric Structure
Perfect	(34) for $h < N$, one inequality	(27) one inequality
Channel	(35) for $h \ge N$, one inequality	(27), one mequanty
Imperfect	(32) for $h < N$, one inequality	(22) $2N$ inequalities
Channel	(33) for $h \ge N$, one inequality	(22), 2 inequalities



Fig. 1. System model — an example of two sub-systems (i.e., N = 2).

Sub-system $i \in [N]$ has a plant (plant *i*) and a controller (controller *i*). Each plant has a sensor and an actuator. The sensor can sample and transmit their measurements to the controller over a dedicated channel without incurring packet loss and delay. The controller makes control decision based on sensor's measurements. The control message/packet of the controller is transmitted to the actuator of the plant to influence the dynamics of the plant over a shared wireless channel, which could incur packet loss and delay. A typical practical scenario of our model is the remote control of a fleet of unmanned aerial vehicles (UAVs) [17].

Sub-System Dynamics. The underlaying time system is continuous (starting from the initial time $t_0 = 0$) but we also create a slotted model (starting from slot 1) for the wireless transmission model where each slot spans $\Delta > 0$ units of time. Details of wireless transmission model will be provided below. Plant *i*'s underlaying state evolves according to the following continuous-time system:

$$\dot{x}^{i}(t) = A_{i}x^{i}(t) + B_{i}u^{i}(t), \tag{1}$$

where $x^i(t) \in \mathbb{R}$ is the state and $u^i(t) \in \mathbb{R}$ is the control input at time t. To guarantee that each sub-system is stabilizable, we assume that $A_i \ge 0$, and $B_i \ne 0$.¹ Moreover, we assume that each sensor samples the state at the beginning of every $h \in \mathbb{Z}^+$ slots (i.e., every $h\Delta$ units of time). Namely, we observe plant *i* every *h* slots. Starting from slot 1, every *T* slots forms a frame, as shown in Fig. 2. For example, frame *k* is from slot (k-1)h+1 to slot *kh*. We observe plant *i* at the beginning of each frame *k* (i.e., at slot (k-1)h+1), which is denoted as $x_k^i \in \mathbb{R}$. Controller *i* can instantaneously obtain plant *i*'s state x_i^k and then makes a control decision $u_k^i \in \mathbb{R}$. The control message/packet u_k^i needs to be transmitted to plant *i* through

¹In this paper, we consider the scalar-state case. It is interesting and important to extend our results to the general vector-state case.



Fig. 2. Control message/packet pattern.

a shared wireless channel. If u_k^i cannot be delivered before or at the end of frame k (i.e., slot kh), a packet dropout occurs. Denote by γ_k^i the random variable which is 1 if message u_k^i is delivered before/at the end of frame k and 0 otherwise. Then based on the analysis in [18], the sampled state of plant i evolves according to the following discrete-time stochastic system:

$$x_{k+1}^{i} = \bar{A}_{i}x_{k}^{i} + \gamma_{k-1}^{i}\bar{B}_{i}u_{k-1}^{i}, \quad k = 1, 2, \cdots$$
 (2)

where $\bar{A}_i = e^{A_i h \Delta} \ge 1$ and $\bar{B}_i = \int_0^{h \Delta} e^{A_i \tau} B_i d\tau \neq 0$.

Sub-system *i* is (mean-square) stable if for any initial conditions x_1^i , u_0^i and γ_0^i , the state x_k^i satisfies

$$\lim_{k \to \infty} \mathbb{E}\left[|x_k^i|^2 \right] = 0.$$
(3)

Our goal is to design a stabilizing controller $\{u_k^i : i \in [N], k = 1, 2, \dots\}$ to make all N sub-systems mean-square stable.

Wireless Channel and Scheduler. The wireless channel is shared by all sub-systems and there is a centralized scheduler to collect the control packets and then make scheduling decision to transmit them in some order over the wireless channel. We assume that only one packet can be transmitted over the wireless channel in each slot. As we mentioned before, each slot spans Δ units of time, which is the time length of transmitting a control packet from the scheduler to the plant and getting the acknowledge about whether the packet is delivered or not from the plant to the scheduler.

Wireless channel is usually unreliable because of interference and fading. We model such unreliability by a successful probability $p_i \in (0, 1]$. Namely, in a slot, if we transmit the control packet of sub-system *i*, the message will be delivered successfully with probability p_i .² Different plants are served with different channel quality depending on their distances from the scheduler and different ambient conditions. Thus, the successful probability p_i varies over sub-system index *i*.

Design Spaces. To make all sub-systems stable, our design spaces include two parts:

The control policy³ {uⁱ_k : i ∈ [N], k = 1, 2, ···}, which determines the control variable uⁱ_k for each frame k and each sub-system i;

• The scheduling policy, which determines the packet to transmit at each slot. Note that the distribution of random variable γ_k^i is completely determined by the scheduling policy.

Both the control policy and the scheduling policy influence the dynamics of the plants according to equation (2).

Assumption on Scheduling Policy. In principle, for any sub-system *i*, the joint distribution of random variables $\{\gamma_k^i : k = 1, 2, \cdots\}$ can be completely arbitrary because the scheduling policy is arbitrary. However, it would be difficult to design control policy to stabilize system (2) in the mean square sense when the joint distribution of random variables $\{\gamma_k^i : k = 1, 2, \cdots\}$ has no pattern. To the best of our knowledge, current literature on NCS only analyzes the case that $\{\gamma_k^i : k = 1, 2, \cdots\}$ are identical and independent distributed (i.i.d.) (see [11]). Therefore, to judiciously leverage the existing results on NCS and delay-constrained wireless communication (see our analysis in the next section), we only consider the set of scheduling policies such that $\{\gamma_k^i : k = 1, 2, \cdots\}$ are i.i.d. We call them *i.i.d. scheduling policies*.

III. STABILITY ANALYSIS

In this section, we leverage existing results in control system as shown in Sec. III-A and delay-constrained wireless communication as shown in Sec. III-B to characterize the stability region of our considered WNCS in Sec. III-C.

A. Maximum Dropout Rate of Control System [11]

According to [11, Theorem 3], for any sub-system i, if $\{\gamma_k^i : k = 1, 2, \dots, \}$ are i.i.d. with $P(\gamma_k^i = 0) = q_i$, where $q_i \in [0, 1]$ is called the packet dropout rate of sub-system i, then sub-system i is (mean-square) stable if and only if⁴

$$q_i < q_{\max}^i \left(A_i, h \right) \triangleq \frac{1}{\bar{A}_i^4 - \bar{A}_i^2 + 1} = \frac{1}{e^{4A_i h \Delta} - e^{2A_i h \Delta} + 1}.$$
(4)

Here $q_{\max}^i(A_i, h)$ is the maximum dropout rate of subsystem *i*, which depends on parameter A_i and sampling period h.⁵ Sub-system *i* can be stabilized if and only if its dropout rate q_i is strictly less than the maximum dropout rate $q_{\max}^i(A_i, h)$. Clearly, any i.i.d. scheduling policy induces a dropout rate q_i for any sub-system *i*. Thus to stabilize all sub-systems, we need to design an i.i.d. scheduling policy such that (4) holds for any sub-system $i \in [N]$.

Control Policy Design. For given dropout rate $q_i = P(\gamma_k^i = 0)$, we only mentioned the stability condition (4) of sub-system *i* but ignore the design of control policy. In fact, if q_i satisfies (4), we can solve the following delay-dependent algebraic Riccati equation (DARE) whose variable is $P_i > 0$,

$$P_i = \bar{A}'_i P_i \bar{A}_i + I - M'_i \Upsilon_i^{-1} M_i, \tag{5}$$

⁴For the scalar case, the stability of sub-system *i* does not depend on parameter B_i . The reason is that the control policy can determine the control variables $\{u_k^i\}$ to compensate the effect of parameter B_i (see system dynamics (1) and (2)).

⁵ In fact, $q_{\max}^{i}(A_{i},h)$ defined in (4) also depends on slot length Δ . However, for simplicity, we ignore it in the notation of $q_{\max}^{i}(A_{i},h)$ since we do not evaluate the effect of slot length Δ .

 $^{^{2}}$ If a message is delivered successfully, both the control packet and the acknowledgement packet have successfully reached their destinations.

³We consider linear control policies with respect to the predictive state in this paper [11].

with

$$\begin{split} \Upsilon_{i} = & (1 - q_{i})^{2} \bar{B}'_{i} P_{i} \bar{B}_{i} + q_{i} (1 - q_{i}) \bar{B}'_{i} \bar{A}'_{i} P_{i} \bar{A}_{i} \bar{B}_{i} \\ &+ q_{i} (1 - q_{i}) \bar{B}'_{i} \bar{B}_{i} + I, \\ M_{i} = & (1 - q_{i}) \bar{B}'_{i} P_{i} \bar{A}_{i}. \end{split}$$
(6)

Note that (5) and (6) are for general vector-state case. For our scalar-state case, we can simplify them by applying $\bar{A}'_i = \bar{A}_i$, $\bar{B}'_i = \bar{B}_i$, $M'_i = M_i$, and I = 1. Moreover, the stabilizing control policy is designed as

$$u_k^i = -\Upsilon_i^{-1} M_i \hat{x}_{k+1|k-1}^i, \tag{7}$$

where

$$\hat{x}_{k+1|k-1}^{i} \triangleq \bar{A}_{i} x_{k}^{i} + (1 - q_{i}) \bar{B}_{i} u_{k-1}^{i}$$
(8)

is the predicted state in frame k + 1 based on the observation of the state in frame k and the control variable in frame k - 1. Please refer to [11] for the details and proofs.

B. Timely Capacity Region of Network System [19]–[21]

In our network system, each packet has a hard deadline of h slots and it becomes useless if it cannot be delivered before its deadline. The major performance metric of such delay-constrained communication is *timely throughput*. In particular, the timely throughput of sub-system i is the long-term perframe average number of packets that are successfully delivered before their deadlines [19]–[21], i.e.,

$$R_i \triangleq \liminf_{K \to \infty} \frac{\mathbb{E}[\sum_{k=1}^K \gamma_k^i]}{K},\tag{9}$$

which depends on the scheduling policy. The (timely) capacity region is the set of timely throughput vector $\mathbf{R} = (R_1, R_2, \dots, R_N)$ such that there exists a scheduling policy under which the sub-system *i*'s timely throughput is at least R_i for any $i \in [N]$. Note that when we characterize the capacity region, we consider all possible scheduling polices (not necessarily i.i.d.). In addition, since the capacity region depends on both the channel quality vector $\mathbf{p} \triangleq (p_1, p_2, \dots, p_N)$ and the frame length (i.e. sampling period) h, we denote it as $\mathcal{R}(\mathbf{p}, h)$.

In the literature on delay-constrained wireless communication, there are two equivalent characterizations for the capacity region $\mathcal{R}(\boldsymbol{p}, h)$: one idle-time-based in [19], [20] and one MDP-based in [21].

Idle-Time-Based Approach. Denote Θ_i as the number of transmissions until one gets a successful delivery for subsystem *i*'s control message, which is a geometric random variable with mean $1/p_i$. Since we have in total *h* slots in a frame, the number of *idle slots* in a frame when we only schedule the control packets in sub-system set $S \subset [N]$ in any work-conserving manner is

$$I_{\mathcal{S}} \triangleq \max\left\{h - \sum_{i \in \mathcal{S}} \Theta_i, 0\right\}.$$
 (10)

Note that the distribution of I_S is the same for any workconserving scheduling policy [19]. Hou *et al.* in [19], [20] proved that the capacity region $\mathcal{R}(\boldsymbol{p},h)$ is the set of all timely throughput vectors $\boldsymbol{R} = (R_1, R_2, \cdots, R_N)$ satisfying

$$\sum_{i \in \mathcal{S}} \frac{R_i}{p_i} + \mathbb{E}[I_{\mathcal{S}}] \le h, \quad \forall \mathcal{S} \subset [N],$$
(11)

which contains 2^N linear inequalities.

Hou *et al.* in [19], [20] further proposed the largest-deficitfirst (LDF) scheduling policy and proved that LDF is feasibility optimal in the sense that it can achieve any input feasible timely throughput vector in the capacity region. However, LDF is frame-dependent and thus the induced $\{\gamma_k^i : k = 1, 2\cdots\}$ are not i.i.d. Therefore, it cannot be applied to our control system. This also illustrates that when we jointly consider the network system and control system, we introduce new challenge to design the network scheduling policy.

MDP-Based Approach. Deng *et al.* in [21] proposed another (equivalent) capacity region characterization based on the Markov Decision Process (MDP) theory. Denote the state of sub-system i at slot t as

$$S_t^i \triangleq \begin{cases} 1, & \text{If sub-system } i \text{ has a packet at slot } t; \\ 0, & \text{Otherwise.} \end{cases}$$
(12)

Then the state of the whole system at slot t is denoted as

$$\boldsymbol{S}_t \triangleq (S_t^1, S_t^2, \cdots, S_t^N).$$

The state space (the set of all possible states) is $S = \{0, 1\}^N$. The action at slot t, denoted as A_t , is to determine which sub-system's control packet to transmit. In particular, $A_t = i$ means to transmit sub-system *i*'s control packet at slot t. Then the action space is $\mathcal{A} = \{1, 2, \dots, N\} = [N]$. The reward function of sub-system *i* is denoted as

$$r_i(\boldsymbol{s}, a) = p_i \cdot 1_{\{\text{sub-system } i \text{ has a packet under state } \boldsymbol{s} \text{ and } a = i\}.$$
 (13)

It is also easy to compute the transition probability from state s to state s' if taking action a at slot t, which is denoted as $P_t(s'|s, a)$.

Deng *et al.* in [21] proved that the capacity region $\mathcal{R}(\boldsymbol{p}, h)$ is characterized by the following linear inequalities,

$$\sum_{a \in \mathcal{A}} x_{t+1}(s', a) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} x_t(s, a) P_t(s'|s, a),$$

$$\forall s' \in \mathcal{S}, t \in [h], \qquad (14a)$$

$$\sum_{a \in \mathcal{A}} x_1(s', a) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} x_h(s, a) P_t(s'|s, a),$$

$$\forall s' \in \mathcal{S}, \qquad (14b)$$

$$R_i \le \sum_{t \in [h]} \sum_{\boldsymbol{s} \in \mathcal{S}} \sum_{a \in \mathcal{A}} x_t(\boldsymbol{s}, a) r_i(\boldsymbol{s}, a), \quad \forall i \in [N], \qquad (14c)$$

$$\sum_{\boldsymbol{s}\in\mathcal{S}}\sum_{a\in\mathcal{A}}x_t(\boldsymbol{s},a)=1,\quad\forall t\in[h],$$
(14d)

$$x_t(\boldsymbol{s}, a) \ge 0, \quad \forall t \in [h], \boldsymbol{s} \in \mathcal{S}, a \in \mathcal{A}.$$
 (14e)

The capacity region $\mathcal{R}(\boldsymbol{p},h)$ is the set of all possible $\boldsymbol{R} = (R_1, R_2, \cdots, R_N)$ where there exists a set of $\{x_t(\boldsymbol{s}, a)\}$ such that (14) holds. Note that the MDP-based capacity region characterization (14) has $O(h \cdot N \cdot 2^N)$ linear equalities/inequalites.

As compared to the idle-time-based approach, another main result of the MDP-based approach (see [21, Theorem 2]) is that any feasible timely throughput vector \mathbf{R} can be achieved by an i.i.d. scheduling policy. Therefore, non-i.i.d. scheduling policies cannot enlarge the capacity region $\mathcal{R}(\mathbf{p}, h)$. In addition, Deng *et al.* in [21] also designed an i.i.d. scheduling policy such that it can achieve any input feasible timely throughput vector. More specifically, for any feasible timely throughput vector \mathbf{R} (i.e., \mathbf{R} satisfies (11) or (14)), we obtain a set of $\{x_t(s, a)\}$ such that (14) holds. Then the i.i.d. scheduling policy is

$$\begin{cases} P_{A_t|S_t}(a|s) = \frac{x_t(s,a)}{\sum_{a' \in \mathcal{A}} x_t(s,a')}, & \forall t \in [h];\\ P_{A_t|S_t}(a|s) = P_{A_{t-h}|S_{t-h}}(a|s), & \forall t > h, \end{cases}$$
(15)

where $P_{A_t|S_t}(a|s)$ is the probability to transmit sub-system a's control packet at slot t conditioning on that the system state is $S_t = s$ at slot t.

Since the traffic patten is frame-synchronized, the system state becomes $s = (1, 1, \dots, 1)$ in the beginning of each frame and the scheduling policy in (15) repeats every frame, the induced $\{\gamma_k^i : k = 1, 2, \}$ are i.i.d., which can thus be applied to our control system as discussed in Sec. III-A. In addition, according to the definition of timely throughput in (9), we can obtain that the induced dropout rate is

$$q_i = P(\gamma_k^i = 0) = 1 - P(\gamma_k^i = 1) = 1 - R_i, \quad (16)$$

which relates the timely throughput to the dropout rate.

C. Stability Condition of Our WNCS

We now present our main result on stability condition of our WNCS when both the control policy and the scheduling policy are taken into consideration.

Theorem 1: Suppose that we only consider those scheduling policies such that $\{\gamma_k^i : k = 1, 2, \cdots\}$ are i.i.d. Then given system parameters $\mathbf{A} = (A_1, A_2, \cdots, A_N)$, $\mathbf{B} = (B_1, B_2, \cdots, B_N)$, $\mathbf{p} = (p_1, p_2, \cdots, p_N)$, sampling period h, and slot length Δ , there exists a control policy and an i.i.d. network scheduling policy to make all sub-systems (mean-square) stable if and only if

$$\left(1-q_{\max}^{1}\left(A_{1},h\right),\cdots,1-q_{\max}^{N}\left(A_{N},h\right)\right)\in\operatorname{int}\left(\mathcal{R}(\boldsymbol{p},h)\right),\ (17)$$

where int(S) denotes the interior of a set S.

Proof: "If" Part. If (17) holds, then there exists a timely throughput vector $\mathbf{R} = (R_1, R_2, \cdots, R_N) \in \mathcal{R}(\mathbf{p}, h)$ such that

$$1 - q_{\max}^i(A_i, h) < R_i. \tag{18}$$

As we discussed in Sec. III-B, $\mathbf{R} = (R_1, R_2, \dots, R_N)$ can be achieved by an i.i.d. scheduling policy such that

$$R_i = P(\gamma_k^i = 1) = 1 - P(\gamma_k^i = 0) = 1 - q_i, \forall k = 1, 2, \cdots$$
(19)

Combining (18) and (19), we have

$$1 - q_{\max}^i(A_i, h) < R_i = 1 - q_i,$$

implying that

$$q_i < q^i_{\max}(A_i, h)$$

Thus, any sub-system i can be stabilized according to our discussion in Sec. III-A.

"Only If" Part. If the system can be stabilized under the condition that $\{\gamma_k^i : k = 1, 2, \cdots\}$ are i.i.d., then according to our discussion in Sec. III-A, we have

$$P(\gamma_k^i = 0) = q_i < q_{\max}^i(A_i, h).$$
 (20)

The scheduling policy such that $\{\gamma_k^i : k = 1, 2, \dots\}$ are i.i.d. with $P(\gamma_k^i = 0) = q_i$ achieves the timely throughput $R_i = P(\gamma_k^i = 1) = 1 - q_i$ for any sub-system *i*. Thus,

$$\mathbf{R} = (R_1, R_2, \cdots, R_N) = (1 - q_1, 1 - q_2, \cdots, 1 - q_N) \in \mathcal{R}(\mathbf{p}, h).$$
(21)

Combining (20) and (21), we prove that (17) holds.

The proof is completed.

Theorem 1 ensures that we only need to check condition (17) to determine whether a system can be stabilized. We have two different capacity region characterizations, but the idle-time-based one is easy to analyze. Thus, following from (11), the stability condition (17) is equivalent to the following inequalities

$$\sum_{i \in \mathcal{S}} \frac{1 - q_{\max}^i(A_i, h)}{p_i} + \mathbb{E}[I_{\mathcal{S}}] < h, \quad \forall \mathcal{S} \subset [N].$$
(22)

Note that in (22) we have in total 2^N linear inequalities.

If (22) holds, i.e., the system can be stabilized, we can input $\mathbf{R} = (1 - q_{\max}^1(A_i, h) + \epsilon, 1 - q_{\max}^2(A_i, h) + \epsilon, \cdots, 1 - q_{\max}^N(A_i, h) + \epsilon)$ where $\epsilon > 0$ is a sufficiently small positive real number into (14) and then we obtain a set of $\{x_t(s, a)\}$, which is further inserted to (15) to construct the i.i.d. scheduling policy.⁶ Inserting $q_i = q_{\max}^i(A_i, h) - \epsilon$ into (5) and (6), we obtain Υ_i and M_i and thus construct the control policy following from (7). The constructed control policy and i.i.d. scheduling policy make the whole system mean-square stable.

Next we simplify the stability condition (22) for two special cases: the perfect-channel case (Sec. IV) and the symmetric-structure case (Sec. V), both of which characterize the stability condition by means of only one linear inequality.

IV. THE PERFECT-CHANNEL CASE

Consider $p_i = 1$ for all $i \in [N]$, i.e., the wireless channel is perfect in the sense that it can successfully deliver a packet in each slot with certainty. In this case, we can simplify the stability condition. Note that $I_S = \max\{h - |S|, 0\}$ for perfect channel. Thus, (22) becomes

$$\sum_{i \in \mathcal{S}} \left[1 - q_{\max}^i(A_i, h) \right] + \max\{h - |\mathcal{S}|, 0\} < h, \quad \forall \mathcal{S} \subset [N],$$
(23)

If $|S| \le h$, then $\max\{h - |S|, 0\} = h - |S|$ and thus (23) becomes

$$\sum_{i \in \mathcal{S}} \left[1 - q_{\max}^i(A_i, h) \right] < |\mathcal{S}|, \quad \forall \mathcal{S} \subset [N],$$
(24)

⁶Though we use the idle-time-based approach to characterize the stability condition in (22), we need to use the MDP-based approach to construct the i.i.d. scheduling policy.

which is definitely true since $1 - q_{\max}^i(A_i, h) < 1, \forall i$.

If |S| > h, then max $\{h - |S|, 0\} = 0$ and thus (23) becomes

$$\sum_{i \in \mathcal{S}} (1 - q_{\max}^{i}(A_{i}, h)) \le h, \quad \forall \mathcal{S} \subset [N]$$
(25)

When $S = [N] = \{1, 2, \dots, N\}$, (25) becomes

$$\sum_{i=1}^{N} (1 - q_{\max}^{i}(A_{i}, h)) \le h,$$
(26)

which implies (25) for any other $S \subset [N]$. Therefore, in the perfect-channel case, the stability condition (22) becomes

$$\sum_{i=1}^{N} \left[1 - q_{\max}^{i}(A_{i},h) \right] = \sum_{i=1}^{N} \left[1 - \frac{1}{e^{4A_{i}h\Delta} - e^{2A_{i}h\Delta} + 1} \right] < h$$
(27)

Hence, we characterize the stability condition of the WNCS in the perfect-channel case by means of one linear inequality.

We further show a property for the perfect-channel case.

Theorem 2: There exists an $h_{\min} \in [N]$ such that (27) holds if the sampling period $h \ge h_{\min}$.

Proof: Please see Appendix A.

Theorem 2 shows that $h \ge h_{\min}$ is a sufficient condition for stability. Readers may wonder whether it is also necessary. The answer is negative as shown later in Fig. 6 in the simulation section. Note that the sampling period h influences both the control system and the network scheduling system. On one hand, the maximum dropout rate $q_{\max}^i(A_i, h)$ (see (4)) decreases as the sampling period h increases, meaning that it is more difficult to stabilize the control system when the sampling period increases. On the other hand, h is the frame length of the network system. It is straightforward to prove that the capacity region $\mathcal{R}(\mathbf{p}, h)$ increases as h increases. Thus, larger sampling period h can increase the delivery probability of a control message. Overall, the sampling period h balances the intensity of control messages and the delivery chance/quality of individual control messages. Theorem 2 shows that we prefer larger h from the perspective of the overall system: when we increase the sampling period h in the perfect-channel case, the benefit of increasing delivery chance/quality of individual control messages dominates the cost of decreasing the intensity of control messages.

V. THE SYMMETRIC-STRUCTURE CASE

The capacity region $\mathcal{R}(p, h)$ becomes much more complicated in the imperfect-channel case, i.e., when some $p_i < 1$. In this section, we analyze a special class of general channels (which could be perfect or imperfect), known as symmetricstructure case, such that $A_i = A$, $p_i = p$ for all $i \in [N]$. Thus,

$$q_{\max}^{i}(A_{i},h) = \frac{1}{e^{4Ah\Delta} - e^{2Ah\Delta} + 1} \triangleq q_{\max}(A,h), \forall i \in [N].$$
(28)

Due to the symmetric structure, if $|S_1| = |S_2|$, we have $\mathbb{E}[I_{S_1}] = \mathbb{E}[I_{S_2}]$. Then the stability condition (22) becomes

$$\frac{1 - q_{\max}(A, h)}{p} + \mathbb{E}[I_{\{1\}}] < h,$$
(29a)

$$\frac{2\left(1 - q_{\max}(A, h)\right)}{p} + \mathbb{E}[I_{\{1,2\}}] < h,$$
(29b)

$$\frac{N\left(1 - q_{\max}(A, h)\right)}{p} + \mathbb{E}[I_{\{1, 2, \cdots, N\}}] < h,$$
(29d)

which is equivalent to

. . .

h

$$\frac{1 - q_{\max}(A, h)}{p} < h - \mathbb{E}[I_{\{1\}}], \tag{30a}$$

$$\frac{1 - q_{\max}(A, h)}{p} < \frac{h - \mathbb{E}[I_{\{1,2\}}]}{2},$$
(30b)

$$\frac{1 - q_{\max}(A, h)}{p} < \frac{h - \mathbb{E}[I_{\{1, 2, \cdots, N\}}]}{N},$$
(30d)

We will further simplify (30) based on the following result. *Theorem 3:* In the symmetric-structure case, we have

$$-\mathbb{E}[I_{\{1\}}] \ge \frac{h - \mathbb{E}[I_{\{1,2\}}]}{2} \ge \dots \ge \frac{h - \mathbb{E}[I_{\{1,2,\dots,N\}}]}{N}.$$

Proof: See Appendix B.

Theorem 3 shows that the stability condition (30) can be further simplified into one linear inequality,

$$\frac{-q_{\max}(A,h)}{p} < \frac{h - \mathbb{E}[I_{\{1,2,\cdots,N\}}]}{N}.$$
 (31)

We next show how to calculate $h - \mathbb{E}[I_{\{1,2,\dots,N\}}]$. Note that $h - \mathbb{E}[I_{\{1,2,\dots,N\}}] = \mathbb{E}[h - I_{\{1,2,\dots,N\}}]$ is the expected number of transmissions in a frame of h slots when scheduling all subsystems' control packets. Denote random variable $X \triangleq h - I_{\{1,2,\dots,N\}}$. Then when $h \leq N$, we have that P(X = h) = 1. When h > N, we have that

$$P(X = N) = p^{N},$$

$$P(X = N + k) = {\binom{N+k-1}{k}}(1-p)^{k}p^{N}, \forall k \in [h-N-1]$$

$$P(X = h) = \sum_{i=0}^{N-1} {\binom{h}{i}}(1-p)^{h-i}p^{i} + {\binom{h-1}{h-N}}(1-p)^{h-N}p^{N}.$$

Then when $h \leq N$, stability condition (31) becomes,

$$\frac{1-q_{\max}(A,h)}{p} < \frac{h}{N}.$$
(32)

Otherwise, when h > N, stability condition (31) becomes,

$$\frac{1 - q_{\max}(A, h)}{p} < \frac{\mathbb{E}[X]}{N} = \frac{NP(X = N) + \sum_{k=1}^{h-N-1} (N+k)P(X = N+k) + hP(X = h)}{\sum_{k=1}^{N} \frac{N}{N} + \frac{Np^N + \sum_{k=1}^{h-N-1} (N+k)\binom{N+k-1}{k}(1-p)^k p^N}{N}}{N} + \frac{h\left(\sum_{i=0}^{N-1} \binom{h}{i}(1-p)^{h-i}p^i + \binom{h-1}{h-N}(1-p)^{h-N}p^N\right)}{N}.$$
(33)

Note that when q = 1, i.e., in the perfect-channel case, the stability condition (32) for $h \leq N$ becomes

$$N(1 - q_{\max}(A, h)) < h,$$
 (34)

which coincides with (27) under the symmetric structure; the stability condition (33) for h > N becomes

$$1 - q_{\max}(A, h) < 1,$$
 (35)

which always holds. This result again coincides with the analysis in Sec. IV that the system can always be stabilized when h > N.

Readers may wonder whether one can have a result similar to Theorem 2 for the non-perfect-channel case. However, it turns out that this is not possible, as shown later in Fig. 7 in the simulation section. This shows that the effect of the sampling period h to the control system and the network system in the symmetric-structure case is much more complicated than that in the perfect-channel case.

VI. A MONOTONIC PROPERTY IN TERMS OF CHANNEL QUALITY VECTOR

Intuitively, with better channel quality (i.e., p is larger), packets can have more chance to be delivered successfully and thus we can decrease the control packet dropout rate. Then for our control systems, it becomes easier to be stabilized. We thus present the following result.

Theorem 4: Consider two channel quality vectors $\boldsymbol{p} = (p_1, p_2, \cdots, p_N)$ and $\tilde{\boldsymbol{p}} = (\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_N)$ satisfying $\boldsymbol{p} \leq \tilde{\boldsymbol{p}}$, i.e., $p_i \leq \tilde{p}_i, \forall i \in [N]$. Then the capacity region under channel quality vector \boldsymbol{p} is a subset of the capacity region under channel channel quality vector $\tilde{\boldsymbol{p}}$, i.e., $\mathcal{R}(\boldsymbol{p}, h) \subset \mathcal{R}(\tilde{\boldsymbol{p}}, h)$.

Proof: With a straightforward induction procedure, instead of improving the quality of all channels in Theorem 4, it suffices to prove it when improving the quality of only one channel (say channel 1 without loss of generality). Therefore, we only need to prove the following result:

• (a) Given $\boldsymbol{p} = (p_1, p_2, \cdots, p_N)$ and $\tilde{\boldsymbol{p}} = (\tilde{p}_1, p_2, \cdots, p_N)$ where $p_1 \leq \tilde{p}_1$, we have $\mathcal{R}(\boldsymbol{p}, h) \subset \mathcal{R}(\tilde{\boldsymbol{p}}, h)$.

We use the idle-time-based capacity region characterization (11) to prove result (a). We reorganize (11) as follows,

$$\sum_{i \in \mathcal{S}} \frac{R_i}{p_i} \le h - \mathbb{E}[I_{\mathcal{S}}], \quad \forall \mathcal{S} \subset [N].$$
(36)

Note that both LHS and RHS of (36) depends on the channel quality vector $\boldsymbol{p} = (p_1, p_2, \cdots, p_N)$. Define

$$X_{\mathcal{S}} \triangleq h - I_{\mathcal{S}},\tag{37}$$

which is the number of active slots in a frame if we use a workconserving policy to scheduling all flows in S. According to our definition for I_S in (10), we can see that

$$X_{\mathcal{S}} = \min\left\{\sum_{i\in\mathcal{S}}\Theta_i, h\right\}.$$
(38)

We next show the following lemma.

Lemma 1: For any $\mathcal{S} \subset \{2, 3, \cdots, N\}$,

$$f(p) = p_1 \mathbb{E}[X_{1 \cup \mathcal{S}} - X_{\mathcal{S}}], \tag{39}$$

increases as p_1 increases⁷.

The proof of Lemma 1 is shown in Appendix C.

According to (36), the capacity region under channel quality vector $\boldsymbol{p} = (p_1, p_2, \cdots, p_N)$, i.e., $\mathcal{R}(\boldsymbol{p}, h)$ becomes

$$\sum_{i \in \mathcal{S}} \frac{R_i}{p_i} \le \mathbb{E}[X_{\mathcal{S}}], \quad \forall \mathcal{S} \subset [N].$$
(40)

Depending on whether S in (40) contains flow 1 or not, we can equivalently express (40) as

$$\sum_{i \in \mathcal{S}} \frac{R_i}{p_i} \le \mathbb{E}[X_{\mathcal{S}}], \quad \forall \mathcal{S} \subset \{2, 3, \cdots, N\},$$
(41a)

$$\frac{R_1}{p_1} + \sum_{i \in \mathcal{S}} \frac{R_i}{p_i} \le \mathbb{E}[X_{1 \cup \mathcal{S}}], \quad \forall \mathcal{S} \subset \{2, 3, \cdots, N\}, \quad (41b)$$

Under the improved channel quality vector $\tilde{\boldsymbol{p}} = (\tilde{p}_1, p_2, \cdots, p_N)$, we use \tilde{X}_S to denote the number of active slots in a frame if we use a work-conserving policy to scheduling all flows in S. Then the new capacity region $\mathcal{R}(\tilde{\boldsymbol{p}}, h)$ becomes,

$$\sum_{i \in \mathcal{S}} \frac{R_i}{p_i} \le \mathbb{E}[\tilde{X}_{\mathcal{S}}] = \mathbb{E}[X_{\mathcal{S}}], \quad \forall \mathcal{S} \subset \{2, 3, \cdots, N\} \quad (42a)$$
$$\frac{R_1}{\tilde{p}_1} + \sum_{i \in \mathcal{S}} \frac{R_i}{p_i} \le \mathbb{E}[\tilde{X}_{1 \cup \mathcal{S}}], \quad \forall \mathcal{S} \subset \{2, 3, \cdots, N\}, \quad (42b)$$

We now show that any $\mathbf{R} = (R_1, R_2, \dots, R_N)$ satisfying (41) also satisfies (42). Since \mathbf{R} satisfies (41a), \mathbf{R} also satisfies (42a). According to Lemma 1, since $p_1 \leq \tilde{p}_1$, we have

$$p_1 \mathbb{E}[X_{1 \cup \mathcal{S}} - X_{\mathcal{S}}] \le \tilde{p}_1 \mathbb{E}[X_{1 \cup \mathcal{S}} - X_{\mathcal{S}}], \tag{43}$$

Since from (41a) and (42a) we have

$$\sum_{i\in\mathcal{S}}\frac{R_i}{p_i} \le \mathbb{E}[\tilde{X}_{\mathcal{S}}] = \mathbb{E}[X_{\mathcal{S}}],\tag{44}$$

then

$$p_{1}\left(\mathbb{E}[X_{1\cup\mathcal{S}}] - \sum_{i\in\mathcal{S}}\frac{R_{i}}{p_{i}}\right) - \tilde{p}_{1}\left(\mathbb{E}[\tilde{X}_{1\cup\mathcal{S}}] - \sum_{i\in\mathcal{S}}\frac{R_{i}}{p_{i}}\right)$$
$$= p_{1}\mathbb{E}[X_{1\cup\mathcal{S}}] - \tilde{p}_{1}\mathbb{E}[\tilde{X}_{1\cup\mathcal{S}}] + (\tilde{p}_{1} - p_{1})\sum_{i\in\mathcal{S}}\frac{R_{i}}{p_{i}}$$
$$\leq p_{1}\mathbb{E}[X_{1\cup\mathcal{S}}] - \tilde{p}_{1}\mathbb{E}[\tilde{X}_{1\cup\mathcal{S}}] + (\tilde{p}_{1} - p_{1})\mathbb{E}[X_{\mathcal{S}}]$$
$$= p_{1}\mathbb{E}[X_{1\cup\mathcal{S}} - X_{\mathcal{S}}] - \tilde{p}_{1}\mathbb{E}[\tilde{X}_{1\cup\mathcal{S}} - \tilde{X}_{\mathcal{S}}]$$
$$\leq 0. \tag{45}$$

Therefore, we have

$$p_1\left(\mathbb{E}[X_{1\cup\mathcal{S}}] - \sum_{i\in\mathcal{S}}\frac{R_i}{p_i}\right) \le \tilde{p}_1\left(\mathbb{E}[\tilde{X}_{1\cup\mathcal{S}}] - \sum_{i\in\mathcal{S}}\frac{R_i}{p_i}\right).$$
(46)

⁷Here for simplicity, we use X_1 to represent $X_{\{1\}}$, i.e., $S = \{1\}$ in (37). Similarly, we use $X_{1\cup S}$ to represent $X_{\{1\}\cup S}$.



Fig. 3. An example for heterogenous sampling periods.

Note that (41b) can be reorganized as

$$R_1 \le p_1 \left(\mathbb{E}[X_{1\cup\mathcal{S}}] - \sum_{i\in\mathcal{S}} \frac{R_i}{p_i} \right).$$
(47)

Due to (46), we thus have

$$R_1 \le \tilde{p}_1 \left(\mathbb{E}[\tilde{X}_{1\cup\mathcal{S}}] - \sum_{i\in\mathcal{S}} \frac{R_i}{p_i} \right), \tag{48}$$

which is equivalent to (42b). Therefore, (42b) also holds and thus (42) holds.

Therefore, any $\mathbf{R} = (R_1, R_2, \cdots, R_N)$ satisfying (41) also satisfies (42), implying $\mathcal{R}(\mathbf{p}, h) \subset \mathcal{R}(\mathbf{\tilde{p}}, h)$.

This completes the proof.

Theorem 1 and Theorem 4 show that if the WNCS can be stabilized under channel quality vector p, it can also be stabilized under a better channel quality vector \tilde{p} where $p \leq \tilde{p}$. As a by-product, in the symmetric-structure case, we can use a binary-search scheme to find the minimum channel quality p such that the WNCS can be stabilized. In addition, we remark that Theorem 4 itself is a new result for the delay-constrained wireless communication problem with frame-synchronized traffic pattern [19], [20].

VII. HETEROGENOUS SAMPLING PERIODS

In our model, we assume that all sub-systems are sampled every h slots. It is more practical to consider heterogenous sampling periods. Suppose that sub-system i is sampled every $h_i \in \mathbb{Z}^+$ slots. An example of $N = 3, h_1 = 1, h_2 = 2, h_3 = 3$ is shown in Fig. 3. Again, our aim is to study the stability condition of the whole system.

Since the traffic pattern is no longer frame-synchronized, we cannot use the idle-time-based capacity region characterization [19], [20]. However, the MDP-based approach [21] can be applied to general traffic patterns including our case with heterogenous sampling periods. In particular, consider the common period

$$H = \text{Least Common Multiple}(h_1, h_2, \cdots, h_N).$$
 (49)

Then every H slots forms a *big frame*. Namely, big frame 1 is from slot 1 to slot H; big frame 2 is from slot H + 1 to slot 2H; and so on (see Fig. 3). Clearly, in a big frame, sub-system i will be sampled H/h_i times. Thus, every big frame has $n_i = H/h_i$ periods of sub-system i. Then we can characterize the timely capacity region $\mathcal{R}(\boldsymbol{p}, \boldsymbol{h})$ by some linear equalities/inequalities (see [21, Equ. (11)]), similar to



Fig. 4. Illustration of the notations in heterogenous sampling periods.

(14) for frame-synchronized traffic pattern. In addition, for any achievable timely throughput vector $\mathbf{R} \in \mathcal{R}(\mathbf{p}, \mathbf{h})$, we can design a *cyclo-periodic* scheduling policy to achieve it. Since we consider heterogenous sampling periods, the constructed cyclo-periodic scheduling policy cannot ensure that $\{\gamma_k^i : k = 1, 2, \cdots\}$ are i.i.d. Instead, the constructed cycloperiodic scheduling policy satisfies the following conditions,

- (a) The random variables $\{\gamma_j^i : j = 1, 2, \cdots\}$ are independent (but not identical) distributed.
- (b) For any $i \in [N]$ and any $j \in [n_i]$, the random variables $\{\gamma_{(k-1)n_i+j}^i : k = 1, 2, \cdots\}$ are i.i.d. i.e., the delivery probability of the control message is the same every n_i periods, i.e.,

$$P(\gamma_{(k-1)n_i+j}^i = 1) = \delta_j^i, \quad \forall k = 1, 2, \cdots.$$
 (50)

Therefore, random variables $\{\gamma_j^i : j = 1, 2, \cdots\}$ are no longer i.i.d now. We call that $\{\gamma_j^i : q = 1, 2, \cdots\}$ are *independent and periodic distributed* (i.p.d) with period n_i if they satisfy (a) and (b) above. We also denote $q_j^i = P(\gamma_{(k-1)n_i+j}^i = 0) = 1 - \delta_j^i$, which is the dropout rate of sub-system *i* in the *j*-th period (and any $((k-1)n_i + j)$ -th period). We illustrate the notations introduced above in Fig. 4. We now characterize a sufficient condition to stabilize subsystem *i* under i.p.d. dropout out events.

Theorem 5: Suppose that random variables $\{\gamma_j^i : j = 1, 2, \cdots\}$ are i.p.d. with period n_i . Then sub-system *i* is (mean-square) stable if $q_j^i < q_{\max}^i(A_i, h_i), \forall j \in [n_i]$, where $q_{\max}^i(A_i, h_i)$ is defined in (4).

Proof: Please see Appendix D.

Theorem 5 requires that the dropout rate of any period is strictly less than $q_{\max}^i(A_i, h_i)$. This adds further restrictions to the design of the network scheduling policy. To further propose a sufficient condition to stabilize the whole system, we need to utilize and modify the details of the MDP-based capacity region characterization in [21].

Theorem 6: The system with heterogenous sampling periods h_1, h_2, \dots, h_N can be stabilized if there exists a set of



Fig. 5. State evolution of three sub-systems with system parameters in (52).



Fig. 6. Stability condition for perfect-channel case with system parameters in (53).



Fig. 7. Stability condition for symmetric-structure case with system parameters in (54).

 $\{x_t(s,a)\}\$ such that the following linear inequalities hold,

$$\sum_{a \in \mathcal{A}} x_{t+1}(s', a) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} x_t(s, a) P_t(s'|s, a),$$

$$\forall s' \in \mathcal{S}, t \in [H-1],$$
(51a)

$$\sum_{a \in \mathcal{A}} x_1(\mathbf{s}', a) = \sum_{\mathbf{s} \in \mathcal{S}} \sum_{a \in \mathcal{A}} x_H(\mathbf{s}, a) P_H(\mathbf{s}' | \mathbf{s}, a),$$

$$\forall \mathbf{s}' \in \mathcal{S},$$
(51b)

$$1 - q_{\max}^{i}(A_{i}, h_{i}) < \sum_{\substack{t=(j-1)h_{i}+1}}^{jh_{i}} \sum_{\boldsymbol{s}\in\mathcal{S}} \sum_{\boldsymbol{a}\in\mathcal{A}} x_{t}(\boldsymbol{s}, \boldsymbol{a}) r_{i}(\boldsymbol{s}, \boldsymbol{a}),$$

$$\forall i \in [N] \ i \in [n_{i}]$$
(51c)

$$\sum r_t(s, a) = 1 \quad \forall t \in [H]$$
(51d)

$$\sum_{\boldsymbol{s}\in\mathcal{S}}\sum_{a\in\mathcal{A}}x_t(\boldsymbol{s},a)=1,\quad\forall t\in[H],$$
(51)

$$x_t(\boldsymbol{s}, a) \ge 0, \quad \forall t \in [H], \boldsymbol{s} \in \mathcal{S}, a \in \mathcal{A}.$$
 (51e)

Proof: If (51) holds, we obtain a set of $\{x_t(s, a)\}$. Then we construct the cyclo-periodic policy according to [21, Equ. (12)], similar to (15) for frame-synchronized traffic pattern. It is straightforward to show that $\{\gamma_j^i : j = 1, 2, \cdots\}$ are independent because the state of any sub-system *i* starts over with state $S_i = 1$ at the beginning of each period. In addition, since the state of the whole system starts over with $s = (1, 1, \cdots, 1)$ at the beginning of each big frame and the scheduling policy is cyclo-periodic, the induced $\{\gamma_{(k-1)n_i+j}^i : k = 1, 2, \cdots\}$ are i.i.d. for any $i \in [N]$ and any $j \in [n_i]$. Thus, random variables $\{\gamma_j^i : j = 1, 2, \cdots\}$ are i.p.d. with period n_i . In addition, due to (51c), the dropout rate of the sub-system *i*'s control message in any period $j \in [n_i]$ satisfies

$$q_j^i = 1 - \sum_{t=(j-1)h_i+1}^{jh_i} \sum_{\boldsymbol{s}\in\mathcal{S}} \sum_{a\in\mathcal{A}} x_t(\boldsymbol{s}, a) r_i(\boldsymbol{s}, a) < q_{\max}^i(A_i, h_i).$$

Therefore, according to Theorem 5, the whole system is (mean-square) stable.

Theorem 6 characterizes a sufficient condition for stability in the case of heterogenous sampling periods. But we should note that it is not a necessary condition. How to find the exact stability condition in the case of heterogenous sampling periods is an interesting and promising future direction.

VIII. SIMULATION

In this section, we use simulation to confirm our theoretic analysis.

General System. We consider a general (imperfect-channel asymmetric-structure) system with system parameters,

$$N = 3, h = 5, \Delta = 0.01,$$

$$A_1 = 6.5137, A_2 = 5.8265, A_3 = 8.8964,$$

$$B_1 = 1, B_2 = 1, B_3 = 1,$$

$$p_1 = 0.7690, p_2 = 0.7277, p_3 = 0.2846.$$
 (52)

According to our stability condition (22), we can check that the system can be stabilized. We then construct the network scheduling policy and the control policy to get the per-frame state of each sub-system, which is shown in Fig. 5. As we can see, indeed, the states of all three sub-systems converge to 0 and thus all three sub-systems are stabilized. This verifies our stability condition (22) for general system.

Perfect-Channel Case. We further consider a perfectchannel case with system parameters,

$$N = 6, \Delta = 0.0114,$$

$$A = (A_1, A_2, A_3, A_4, A_5, A_6)$$

$$= (3.7482, 8.7512, 7.7711, 8.5482, 6.8823, 5.6830),$$

$$p = (p_1, p_2, p_3, p_4, p_5, p_6) = (1, 1, 1, 1, 1, 1).$$
 (53)

We change the sampling period h from 1 slot to 10 slots. The stability result is shown in Fig. 6. We can see that there exists an $h_{\min} = 3 \in \{1, 2, 3, 4, 5, 6\}$ such that the system can be stabilized if $h \ge h_{\min}$. This is consistent with Theorem 2. From this figure, we can also see that $h \ge h_{\min}$ is not necessary for stability in (27), because the system can be stabilized when $h = 1 < h_{\min} = 3$.

Symmetric-Structure Case. We next consider a symmetricstructure (imperfect-channel) case with parameters,

$$N = 2, \Delta = 0.1, A_1 = A_2 = A = 1.$$
(54)

We change the sampling period h from 1 slot to 10 slots and consider three different levels of channel quality $p_1 = p_2 =$ $p \in \{0.300, 0.425, 0.500\}$. The stability result is shown in Fig. 7. We can see that the system is unstable for all sampling periods when the channel quality is bad, i.e., p = 0.30. However, the system can be stabilized for all sampling periods when the channel quality is good, i.e., p = 0.5. When the channel quality is medium, i.e., p = 0.425, the system can be stabilized when the sampling period $h \in \{2, 7, 8, 9, 10\}$ and unstable otherwise. Thus, for such imperfect-channel case, we do not have a similar result like Theorem 2 for perfect-channel case. Instead, the stability result becomes more complicated when we consider the effect of channel quality.

Monotonic Result. We verify our monotonic result (Theorem 4) by considering the following three systems with different N's,

• System 1 with N = 2:

$$N = 2, h = 3, \Delta = 0.01,$$

$$A = (A_1, A_2) = (1.9047, 6.1553),$$

$$p_1 = p_2 = p.$$
(55)

• System 2 with N = 3:

$$N = 3, h = 3, \Delta = 0.01,$$

$$A = (A_1, A_2, A_3) = (1.9047, 6.1553, 7.9464),$$

$$p_1 = p_2 = p_3 = p.$$
(56)

• System 3 with N = 4:

$$N = 4, h = 3, \Delta = 0.01,$$

$$\mathbf{A} = (A_1, A_2, A_3, A_4) = (1.9047, 6.1553, 7.9464, 9.3456)$$

$$p_1 = p_2 = p_3 = p_4 = p.$$
(57)

We change p from 0.001 to 1. The stability result is shown in Fig. 8. As we can see, for each system, we have a monotonic property in terms of channel quality p. This verifies Theorem 4.

Minimum Channel Quality. Our monotonic result, i.e., Theorem 4, further enables us to find the minimum channel quality to stabilize the whole system for the symmetricstructure case. Namely, we can use a binary-search scheme to efficiently find the minimum channel quality

 $p_{\min} \triangleq \inf\{p : \text{system can be stabilized under channel quality } p\}.$ (58)

We evaluate the effect of parameter $A_1 = A_2 = \cdots = A_N = A$ and the effect of N, i.e., the number of sub-systems. First, we consider the following system parameters in the symmetric-structure case:

$$N = 3, \Delta = 0.01, h = 5.$$
(59)

We change A from 1 to 5 and the minimum channel quality is shown in Fig. 9. We can see that when A increases, the channel quality also needs to be improved to stabilize the system. This is because control-system parameter A (see (1)) represents the degree of self-amplification and larger A means larger self-instability. Second, we consider the following system parameters in the symmetric-structure case:

$$A = 1, \Delta = 0.01, h = 5.$$
(60)

We change N from 1 to 5 and the minimum channel quality is shown in Fig. 10. As we can see, the channel quality needs to be improved to stabilize the system when the number of subsystems increases. This is because when we have more subsystems to be stabilized, virtually we need to allocate more communication resources to the new sub-systems; thus, we need to improve the cannel quality to create more "communication resources".

IX. CONCLUSION

In this paper, we characterize the stability condition of a WNCS with multiple plants and controllers sharing a common wireless channel under the joint design of control policy and network scheduling policy. To solve our WNCS problem, we have leveraged the recent results in the research area of delayconstrained wireless communication [19]–[21]. In the future, it is interesting and important to generalize our system in several aspects. First, we have only considered the scalar-state case and thus the general vector-state case is worth studying. Second, we assume that the hard deadline of each control message is equal to the sampling period corresponding to d = 1 in [18]. It would also be interesting to investigate the case of d > 1. Finally, our results are based on the assumption that the dropout random variables $\{\gamma_k^i : k = 1, 2, \dots\}$ are i.i.d. or i.p.d., which simplifies the design of the control policy but shrinks the design space. It would be challenging to consider the full design space where $\{\gamma_k^i : k = 1, 2, \dots\}$ may not be i.i.d or i.p.d.

REFERENCES

- L. Deng, C. Tan, and W. S. Wong, "On stability condition of wireless networked control systems under joint design of control policy and network scheduling policy," in *Proc. IEEE CDC*, 2018.
- [2] R. A. Gupta and M.-Y. Chow, "Networked control system: Overview and research trends," *IEEE Transactions on Industrial Electronics*, vol. 57, no. 7, pp. 2527–2535, 2010.
- [3] J. Baillieul and P. J. Antsaklis, "Control and communication challenges in networked real-time systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 9–28, 2007.
- [4] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.
- [5] A. W. Al-Dabbagh and T. Chen, "Design considerations for wireless networked control systems," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 9, pp. 5547–5557, 2016.
- [6] P. Park, S. C. Ergen, C. Fischione, C. Lu, and K. H. Johansson, "Wireless network design for control systems: A survey," *IEEE Communications Surveys & Tutorials*, vol. PP, no. 99, pp. 1–36, 2017.
- [7] H. Cheng, Y. Chen, W. S. Wong, Q. Yang, L. Shen, and J. Baillieul, "Stabilizing and tracking control of multiple pendulum-cart systems over a shared wireless network," in *Proc. CCC*, 2012.
- [8] B. Demirel, Z. Zou, P. Soldati, and M. Johansson, "Modular design of jointly optimal controllers and forwarding policies for wireless control," *IEEE Transactions on Automatic Control*, vol. 59, no. 12, pp. 3252– 3265, 2014.
- [9] H. Gao, T. Chen, and J. Lam, "A new delay system approach to networkbased control," *Automatica*, vol. 44, no. 1, pp. 39–52, 2008.
- [10] S. Hu and W. Yan, "Stability robustness of networked control systems with respect to packet loss," *Automatica*, vol. 43, no. 7, pp. 1243–1248, 2007.
- [11] C. Tan, L. Li, and H. Zhang, "Stabilization of networked control systems with both network-induced delay and packet dropout," *Automatica*, vol. 59, pp. 194–199, 2015.
- [12] J. Xiong and J. Lam, "Stabilization of linear systems over networks with bounded packet loss," *Automatica*, vol. 42, no. 1, pp. 80–87, 2007.
- [13] H. Chen, J. Gao, T. Shi, and R. Lu, "H_∞ control for networked control systems with time delay, data packet dropout and disorder," *Neurocomputing*, vol. 179, pp. 211–218, 2016.
- [14] C. Tan and H. Zhang, "Necessary and sufficient stabilizing conditions for networked control systems with simultaneous transmission delay and packet dropout," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 4011–4016, 2017.
- [15] X. Liu and A. Goldsmith, "Wireless network design for distributed control," in *Proc. IEEE CDC*, 2004.
- [16] P. Park, J. Araújo, and K. H. Johansson, "Wireless networked control system co-design," in *Proc. IEEE ICNSC*, 2011.



Fig. 8. The monotonic property in terms of channel quality with system parameters in (55)-(57).



Fig. 9. The minimal channel quality to stabilize the system (v.s. A) with system parameters in (59).



Fig. 10. The minimal channel quality to stabilize the system (v.s. N) with system parameters in (60).

- [17] Z. Jiang, H. Cheng, Z. Zheng, X. Zhang, X. Nie, W. Li, Y. Zou, and W. S. Wong, "Autonomous formation flight of uavs: Control algorithms and field experiments," in *Proc. CCC*, 2016.
- [18] C. Tan, W. S. Wong, and H. Zhang, "Integrated control over software defined network," in *Proc. CCC*, 2017.
- [19] I.-H. Hou, V. Borkar, and P. R. Kumar, "A theory of QoS for wireless," in *Proc. IEEE INFOCOM*, 2009.
- [20] I.-H. Hou and P. R. Kumar, "Queueing systems with hard delay constraints: a framework for real-time communication over unreliable wireless channels," *Queueing Systems*, vol. 71, no. 1-2, pp. 151–177, 2012.
- [21] L. Deng, C.-C. Wang, M. Chen, and S. Zhao, "Timely wireless flows with general traffic patterns: capacity region and scheduling algorithms," *IEEE/ACM Transactions on Networking*, vol. 25, no. 6, pp. 3473–3486, 2017.
- [22] I.-H. Hou, A. Truong, S. Chakraborty, and P. Kumar, "Optimality of periodwise static priority policies in real-time communications," in *Proc. IEEE CDC*, 2011.

APPENDIX

A. Proof of Theorem 2

The stability condition (27) is equivalent to

$$f(h) \triangleq \sum_{i=1}^{N} \left[1 - \frac{1}{e^{4A_i h \Delta} - e^{2A_i h \Delta} + 1} \right] - h < 0.$$
(61)

Instead of considering integer h, we consider $h \in \mathbb{R}$ for function f(h). First, it is easy to see that f(0) = 0 and f(h) < 0 for all $h \ge N$. We trace back f(h) from h = N to h = 0and find the first h such that f(h) = 0 and we denote it as h^* , i.e.,

$$h^* \triangleq \max\{h \in \mathbb{R} : 0 \le h \le N, f(h) = 0\}.$$
(62)

Note that h^* is well-defined because f(0) = 0 and $h^* \in [0, N)$ because f(N) < 0. Since f(h) is continuous and f(N) < 0, we have that

$$f(h) < 0, \quad \forall h \in (h^*, N]. \tag{63}$$

In addition, since f(h) < 0 for all $h \ge N$, we have that

$$f(h) < 0, \quad \forall h \in (h^*, \infty) \tag{64}$$

Then we denote

$$h_{\min} = \begin{cases} h^* + 1, & \text{if } h^* \text{ is an integer;} \\ \lceil h^* \rceil, & \text{otherwise.} \end{cases}$$
(65)

Since $h^* \in [0, N)$, we obtain that $h_{\min} \in [N]$. Clearly, f(h) < 0 when $h \ge h_{\min}$. The proof is completed.

B. Proof of Theorem 3

Define function

$$f(\mathcal{S}) \triangleq h - \mathbb{E}[I_{\{\mathcal{S}\}}], \quad \forall \mathcal{S} \subset [N].$$
(66)

Then to prove Theorem 3, we need to prove the following inequality,

$$f(\{1\}) \ge \frac{f(\{1,2\})}{2} \ge \dots \ge \frac{f(\{1,2,\dots,N\})}{N}$$
(67)

Clearly $f(\emptyset) = 0$ and

$$f(\mathcal{S}_1) = f(\mathcal{S}_2), \text{ if } |\mathcal{S}_1| = |\mathcal{S}_2| \tag{68}$$

due to the symmetry. The authors in [22] show that f(S) is a submodular function. Therefore, for any $S_1 \subset [N], S_2 \subset [N]$, we have

$$f(\mathcal{S}_1) + f(\mathcal{S}_2) \ge f(\mathcal{S}_1 \cup \mathcal{S}_2) + f(\mathcal{S}_1 \cap \mathcal{S}_2), \quad (69)$$

Now we prove (67) by induction.

First, when setting $S_1 = \{1\}, S_2 = \{2\}$ in (69), we have

$$f(\{1\}) + f(\{2\}) \ge f(\{1\} \cup \{2\}) + f(\{1\} \cup \{2\})$$

= $f(\{1,2\}) + f(\emptyset) = f(\{1,2\}).$ (70)

In addition, due to (68), we have $f(\{2\}) = f(\{1\})$. This implies that

$$f(\{1\}) \ge \frac{f(\{1,2\})}{2}.$$
(71)

Second, suppose that

$$\frac{f(\{1, 2, \cdots, k-1\})}{k-1} \ge \frac{f(\{1, 2, \cdots, k\})}{k}$$
(72)

holds for $k \geq 2$. Now in (69), we consider two sets $S_1 = \{1, 2, \dots, k\}$ and $S_2 = \{2, 3, \dots, k, k + 1\}$. Clearly both S_1 and S_2 are of size k and thus $f(S_2) = f(S_1) = f(\{1, 2, \dots, k\})$. In addition, we have $S_1 \cup S_2 = \{1, 2, \dots, k + 1\}$ and $S_1 \cap S_2 = \{2, 3, \dots, k\} \triangleq S_3$. Since S_3 is of size k - 1, we have $f(S_3) = f(\{1, 2, \dots, k - 1\})$. Thus, we have

$$2f(\{1, 2, \cdots, k\}) = f(\mathcal{S}_1) + f(\mathcal{S}_2) \ge f(\mathcal{S}_1 \cup \mathcal{S}_2) + f(\mathcal{S}_3)$$

= $f(\{1, 2, \cdots, k+1\}) + f(\{1, 2, \cdots, k-1\})$
 $\ge f(\{1, 2, \cdots, k+1\}) + \frac{k-1}{k}f(\{1, 2, \cdots, k\}),$ (73)

where the last inequality follows from hypothesis (72). Rearranging (73), we have

$$\frac{f(\{1, 2, \cdots, k\})}{k} \ge \frac{f(\{1, 2, \cdots, k+1\})}{k+1}, \qquad (74)$$

which shows that (72) also holds for k+1. Thus we complete the proof for (67).

C. Proof of Lemma 1

Clearly, we have

$$X_{1\cup\mathcal{S}} = \min\{X_1 + X_{\mathcal{S}}, h\}.$$
(75)

Thus,

$$\mathbb{E}[X_{1\cup\mathcal{S}} - X_{\mathcal{S}}] = \sum_{k=1}^{h} \mathbb{E}[X_{1\cup\mathcal{S}} - X_{\mathcal{S}}|X_{\mathcal{S}} = k]P(X_{\mathcal{S}} = k)$$

$$= \sum_{k=1}^{h} \mathbb{E}[\min\{X_{1} + X_{\mathcal{S}}, h\} - X_{\mathcal{S}}|X_{\mathcal{S}} = k]P(X_{\mathcal{S}} = k)$$

$$= \sum_{k=1}^{h} \mathbb{E}[\min\{X_{1} + k, h\} - k]P(X_{\mathcal{S}} = k)$$

$$= \sum_{k=1}^{h} \mathbb{E}[\min\{X_{1}, h - k\}]P(X_{\mathcal{S}} = k)$$

$$= \sum_{k=1}^{h-1} \mathbb{E}[\min\{X_{1}, h - k\}]P(X_{\mathcal{S}} = k), \quad (76)$$

Note that since $1 \notin S$, $P(X_S = k)$ does not depends on p_1 . On the contrast, $\mathbb{E}[\min\{X_1, h - k\}]$ only depends on p_1 . For any $k \in \{1, 2, \dots, h - 1\}$, we next compute $\mathbb{E}[\min\{X_1, h - k\}]$ as follows,

$$\mathbb{E}[\min\{X_{1}, h - k\}] = \sum_{x=1}^{h-k-1} xP(X_{1} = x) + (h-k)P(X_{1} \ge h-k) \\
= \sum_{x=1}^{h-k-1} x(1-p_{1})^{x-1}p_{1} + (h-k)(1-p)^{h-k-1} \\
= p_{1} \sum_{x=1}^{h-k-1} \frac{d}{dp_{1}} \left[-(1-p_{1})^{x} \right] + (h-k)(1-p)^{h-k-1} \\
= -p_{1} \cdot \frac{d}{dp_{1}} \left[\sum_{x=1}^{h-k-1} (1-p_{1})^{x} \right] + (h-k)(1-p)^{h-k-1} \\
= -p_{1} \cdot \frac{d}{dp_{1}} \left[\sum_{x=1}^{h-k-1} (1-p_{1})^{x} \right] + (h-k)(1-p)^{h-k-1} \\
= -p_{1} \cdot \frac{d}{dp_{1}} \left[\frac{(1-p_{1})-(1-p_{1})^{h-k}}{p_{1}} \right] + (h-k)(1-p)^{h-k-1} \\
= -p_{1} \cdot \left[\frac{\left[(h-k)(1-p_{1})^{h-k-1} - 1 \right]p_{1} - \left[(1-p_{1}) - (1-p_{1})^{h-k} \right]}{p_{1}^{2}} \right] \\
+ (h-k)(1-p)^{h-k-1} \\
= \frac{1-(1-p_{1})^{h-k}}{p_{1}}.$$
(77)

Therefore,

$$p_1 \mathbb{E}[\min\{X_1, h-k\}] = 1 - (1 - p_1)^{h-k},$$
 (78)

increases as p_1 increase. Thus,

$$f(p) = p_1 \mathbb{E}[X_{1 \cup S} - X_S]$$

= $\sum_{k=1}^{h-1} p_1 \mathbb{E}[\min\{X_1, h-k\}] P(X_S = k),$ (79)

increases as p_1 increases. This completes the proof. This completes the proof.

D. Proof of Theorem 5

It follows from Theorem 1 in [11], when $q_j^i < q_{\max}^i(A_i, h_i)$, $j = 1, \dots, n_i$, the scalar DARE (5) has a unique positive solution $P_i > 0$. We construct the following control policy

$$u_{k}^{i} = -\Upsilon_{i}^{-1} M_{i} \hat{x}_{k+1|k-1}^{i}$$

= $-\Upsilon_{i}^{-1} M_{i} \left[\bar{A}_{i} x_{k}^{i} + (1 - q_{i}) \bar{B}_{i} u_{k-1}^{i} \right], \qquad (80)$

where

$$q_i = \max_{j \in [n_i]} q_j^i < q_{\max}^i(A_i, h_i).$$
(81)

For sub-system i, define the Lyapunov function as

$$V_k(x_k^i) = \mathbb{E}\left[(P_i + \Upsilon_i^{-1}(M_i)^2)(x_k^i)^2 - \Upsilon_i^{-1}(M_i)^2 x_k^i \hat{x}_{k|k-2}^i \right],$$
(82)

where $\hat{x}_{k|k-2}^{i}$ is the predicted state in frame k based on the observation of the state in frame k-1 and the control variable in frame k-2 (see (8)).

By utilizing the orthogonality of x_k^i and $x_k^i - \hat{x}_{k|k-2}^i$, (82) can be equivalently rewritten as

$$V_k(x_k^i) = \mathbb{E}\left[P_i(x_k^i)^2 + \Upsilon_i^{-1}(M_i)^2(x_k^i - \hat{x}_{k|k-2}^i)^2\right] \\ \ge \mathbb{E}\left[P_i(x_k^i)^2\right] \ge 0.$$

Assume $k = mn_i + j$. Then, it follows that

$$\begin{split} V_{k}(x_{k}^{i}) &- V_{k+1}(x_{k+1}^{i}) \\ = \mathbb{E}\left[(P_{i} + \Upsilon_{i}^{-1}(M_{i})^{2})(x_{k}^{i})^{2} - \Upsilon_{i}^{-1}(M_{i})^{2}x_{k}^{i}\hat{x}_{k|k-2}^{i} \\ &- (P_{i} + \Upsilon_{i}^{-1}(M_{i})^{2})(\bar{A}_{i}x_{k}^{i} + \gamma_{k-1}^{i}\bar{B}_{i}u_{k-1}^{i})^{2} \\ &- (\bar{A}_{i}x_{k}^{i} + \gamma_{k-1}^{i}\bar{B}_{i}u_{k-1}^{i})\Upsilon_{i}^{-1}(M_{i})^{2}(\bar{A}_{i}x_{k}^{i} + (1 - q_{j}^{i})\bar{B}_{i}u_{k-1}^{i})) \\ = \mathbb{E}\left[(x_{k}^{i})^{2} + (u_{k-1}^{i})^{2} - \Upsilon_{i}(u_{k-1} + \Upsilon_{i}^{-1}M_{i}\hat{x}_{k|k-2}^{i})^{2} \\ &+ 2(q_{j}^{i} - q_{i})\bar{A}_{i}P_{i}\bar{B}_{i}(\hat{x}_{k|k-2}^{i})u_{k-1}^{i} \\ &+ ((1 - q_{i})^{2} - (1 - q_{j}^{i})^{2})P_{i}(\bar{B}_{i})^{2}(u_{k-1}^{i})^{2} \\ &+ (q_{i}(1 - q_{i}) - q_{j}^{i}(1 - q_{i}^{i}))(P_{i} + \Upsilon_{i}^{-1}(M_{i})^{2})(\bar{B}_{i})^{2}(u_{k-1}^{i})^{2} \right]. \end{split}$$

Applying the control policy (80) to the above equation, we obtain

$$V_{k}(x_{k}^{i}) - V_{k+1}(x_{k+1}^{i})$$

$$= \mathbb{E}\left[(x_{k}^{i})^{2} + (u_{k-1}^{i})^{2}\right] + \frac{q_{i} - q_{j}^{i}}{1 - q_{i}} \mathbb{E}\left[\Upsilon_{i}^{-1}(M_{i})^{2}(\hat{x}_{k|k-2})^{2}\right]$$

$$+ \mathbb{E}\left[\left(\frac{q_{i} - q_{j}^{i}}{1 - q_{i}} + (q_{i} - q_{j}^{i})(1 - q_{j}^{i})\Upsilon_{i}^{-1}(M_{i})^{2}(\bar{B}_{i})^{2}\right) \times (\Upsilon_{i}^{-1}M_{i}\hat{x}_{k|k-2})^{2}\right],$$

which implies that $V_k(x_k) - V_{k+1}(x_{k+1}) > 0$ for $0 < q_j^i \le q_i < 1$ when state x_k is not zero for finite k. Therefore, based on Lyapunov stability theory, sub-system i is stabilizable in the mean square sense.



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