Distributed Noise Covariance Matrices Estimation in Sensor Networks

Jiahong Li, Nan Ma, and Fang Deng

Abstract-Adaptive algorithms based on in-network processing over networks are useful for online parameter estimation of historical data (e.g., noise covariance) in predictive control and machine learning areas. This paper focuses on the distributed noise covariance matrices estimation problem for multi-sensor linear time-invariant (LTI) systems. Conventional noise covariance estimation approaches, e.g., auto-covariance least squares (ALS) method, suffers from the lack of the sensor's historical measurements and thus produces high variance of the ALS estimate. To solve the problem, we propose the distributed auto-covariance least squares (D-ALS) algorithm based on the batch covariance intersection (BCI) method by enlarging the innovations from the neighbors. The accuracy analysis of D-ALS algorithm is given to show the decrease of the variance of the D-ALS estimate. The numerical results of cooperative target tracking tasks in static and mobile sensor networks are demonstrated to show the feasibility and superiority of the proposed D-ALS algorithm.

I. INTRODUCTION

Recent advances in machine learning and information fusion have led to the formulation of increasingly demanding distributed estimation and inference problems, as discussed in [1]. The distributed estimation fusion methods in [2], and especially the batch covariance intersection (BCI) approach (see [3] and references therein) provided an upper bound on estimation accuracy without assuming any knowledge on the correlation between the estimates of sensors. [4] proposed a average consensus estimation algorithm based on a new BCI strategy. However, the fusion methods above lack the consideration of the exact knowledge of the noise statistics, which is not plausible due to the mismatch of the nominal system or invalidity of offline calibration in many practical systems, e.g., low-cost integrated GPS/INS positioning systems [5], energy-based source localization [6] and fault tolerant systems [7].

One effective approach is to use the historical openloop data, which can be divided into several categories, e.g., correlation techniques [8]–[16], Bayesian [17], [18], maximum likelihood [19], [20], covariance matching [21], methods based on the minimax approach [22], subspace methods [23] and prediction error methods [24]. An alternative approach that directly estimates the gain of a linear estimator has been developed in [13], [15], [24], [25]. The connections between two approaches were discussed in [16]. The Bayesian and maximum likelihood methods are

Jiahong Li and Nan Ma are now with the School of Robotics, Beijing Union University, Beijing 100101, China. jqrjiahong@buu.edu.cn, xxtmanan@buu.edu.cn Fang Deng is with the School of Automation, Beijing Institute of well suited to multi-model approaches, but are costly in terms of computation. Covariance matching is a technique to provide biased estimates of the true covariances based on the residuals of the state estimates. The minimax approach provides a fixed system whose worst performance among an assumed possible uncertainty set is the best possible. The advantages and disadvantages of the approach have been discussed in [26], [27]. The subspace methods formulate the estimation problem as projections of Hankel matrices and the model can be retrieved from the row and column spaces of the projected data matrix. The prediction error methods reduces the parameter identification problem to the minimization of empirical average losses. Among all the methods, the correlation methods can provide unbiased estimates with acceptable computational requirements even for high-dimensional systems [16]. The correlation methods were firstly proposed by Mehra and Bélanger in [8] and [9] as a three-step procedure, and were reformulated to a single-step procedure called the auto-covariance leastsquares (ALS) method in [10]. In the ALS method, the correlations between routine operating data formed a least-squares problem of the noise covariance matrix, whose solution was guaranteed by solving the semi-definite programming (SDP) problem. The necessary and sufficient conditions for the uniqueness of the variance estimates for dependent state and measurement noise were presented in [11]. The ALS problem with the estimation of a state noise disturbance structure was formulated in [12]. The optimal weight was formulated in the least-squares objective to ensure minimum variance in [16]. However, the performance of the correlation methods would become poor if the time window size of open-loop measurements is small.

In this paper, the distributed noise covariance estimation problem over networks is formulated, and the distributed auto-covariance least squares (D-ALS) algorithm are proposed based on batch covariance intersection (BCI) method. The estimation accuracy of the proposed algorithm can increase by fusing the innovations from the neighboring agents. The theoretical analysis of the algorithm is also provided to shown the efficiency. The simulation results of cooperative target tracking case show the superiority of the ALS-BCI algorithm in terms of the mean square error criterion.

II. PRELIMINARIES

We consider a connected sensor network of M agents modeled as an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where the vertices set $\mathcal{V} = 1, \ldots, M$ corresponds to the agents and the edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ represents the communication links between the pairs of agents. Agent *i* can communicate with its neighbors

Technology, Beijing 100081, China. dengfang@bit.edu.cn.

whose indexes are in the set $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}, i \neq j\}$ with cardinality $M_i = \|\mathcal{N}_i\|$.

In the sensor network, each agent observes the linear discrete time-invariant dynamic system $x_{k+1} = Fx_k + w_k$ with linear time-invariant measurement model $z_{i,k} = H_i x_k + v_{i,k}$. where the vector $x_k \in \mathbb{R}^{n_x}$ and $z_{i,k} \in \mathbb{R}^{n_z}$ represent the state and the measurement of the i^{th} agent at time instant $k \in \mathbb{N}^+$. $F \in \mathbb{R}^{n_x \times n_x}$ and $H_i \in \mathbb{R}^{n_z \times n_x}$ are state-transitional and measurement-transitional matrix. The variables w_k and $v_{i,k}$ represent the process noise and measurement noise respectively, and are mutually independent following the zero-mean Gaussian statistics with probability $w_k \sim \mathcal{N}(0_{n_x \times 1}, Q)$ and $v_{i,k} \sim \mathcal{N}(0_{n_z \times 1}, R_i)$ with unknown covariance matrices $Q \in \mathbb{R}^{n_x \times n_x}$ and $R_i \in \mathbb{R}^{n_z \times n_z}$. I_n and 0_n denote the identity matrix and the zeros matrix of dimension n respectively.

According to the conventional distributed linear filtering algorithm, each agent updates local state estimate $\hat{x}_{i,k} = F\hat{x}_{i,k-1} + K_i e_{i,k}$ and state covariance estimate $P_{i,k} = (I - K_i H_i)(FP_{i,k-1}^{-1}F^{T}+Q)$ with estimation gain $K_i \in \mathbb{R}^{n_x \times n_z}$, and transmits them to its neighbors to fuse the global ones. where $e_{i,k} = z_{i,k} - H_i F \hat{x}_{i,k-1}$ denote the innovation. K_i is designed as Kalman gain $K_i = P_{i,k-1}H_i^{T}(H_i(FP_{i,k-1}^{-1}F^{T}+Q)H_i^{T}+R_i)^{-1}$ in terms of minimum mean square error (MMSE). Noting that the noise covariance matrices Q and R_i are unknown and estimated by the auto-covariance least-squares method below.

Denote the residuals and residuals covariance of the i^{th} agent as $\varepsilon_{i,k} = x_{i,k} - F\hat{x}_{i,k-1}$ and $P_{\varepsilon,i,k} = E[\varepsilon_{i,k}\varepsilon_{i,k}^{\mathrm{T}}]$ respectively, then the estimator of the residuals is deduced as

$$\varepsilon_{i,k} = \underbrace{(F - K_i H_i F)}_{\bar{F}_i} \varepsilon_{i,k-1} + \underbrace{[I_{n_x} - K_i H_i, -K_i]}_{G_i} \underbrace{\begin{bmatrix} w_k \\ v_{i,k} \end{bmatrix}}_{\bar{w}_{i,k}}$$
(1)

$$P_{\varepsilon,i,k} = \bar{F}_i P_{\varepsilon,i,k-1} \bar{F}_i^{\mathrm{T}} + G_i \Sigma_i G_i^{\mathrm{T}}$$
(2)

where $\Sigma = E(\bar{w}_{i,k}\bar{w}_{i,k}^{\mathrm{T}}) = \begin{bmatrix} Q & 0_{n_x \times n_z} \\ 0_{n_z \times n_x} & R_i \end{bmatrix}$. According to the Lyapunov equation $P_{\varepsilon} = \bar{F}P_{\varepsilon}\bar{F}^{\mathrm{T}} + G\Sigma G^{\mathrm{T}}$ in [28], the steady-state residual covariance solution exists if \bar{F} is stable. To ensure \bar{F} is stable, the residual covariance P_{ε} should satisfy $(P_{\varepsilon})_s = \left((I - \bar{F} \otimes \bar{F})^{-1}G \otimes G\right)\Sigma_s$ through the vectorization, where \otimes denotes the Kronecker product, A_s denotes the columnwise stacking of the matrix A into a vector.

The innovations $e_{i,k}$ is deduced as $e_{i,k} = H_i \varepsilon_{i,k} + v_{i,k}$. Then denote the auto-covariance $C_{e,0}^i = \mathbb{E}[e_{i,k}e_{i,k}^{\mathrm{T}}]$ and $C_{e,l}^i = \mathbb{E}[e_{i,k+l}e_{i,k}^{\mathrm{T}}]$ of the i^{th} agent's innovation as

$$C_{e,l}^{i} = H_{i}\bar{F}^{l}P_{\varepsilon}H_{i}^{\mathrm{T}} - H_{i}\bar{F}^{l-1}FK_{i}R_{i} \quad l = 1, 2, \dots, N-1$$
(3)

where $C_{e,0}^i = H_i P_{\varepsilon,i} H_i^{\mathrm{T}} + R_i$, N is a user-defined parameter defining the maximum time-window lag. It can be derived as

$$\mathcal{A}_i \theta_i = b_i \tag{4}$$

where $\theta = [Q_s^{\mathrm{T}}, (R_i)_s^{\mathrm{T}}]^{\mathrm{T}}$ and $b = (C_e(N))_s$ with $C_e(N) = [C_{e,0}, C_{e,1}^{\mathrm{T}}, \dots, C_{e,N-1}^{\mathrm{T}}]^{\mathrm{T}}$. \mathcal{A}_i satisfies

$$\mathcal{A}_{i} = [D_{i}, D_{i}(FK_{i} \otimes FK_{i}) + (I_{n_{x}} \otimes \Gamma_{i})]$$

$$D_{i} = (H_{i} \otimes \mathcal{O}_{i})(I_{n_{x}^{2}} - \bar{F}_{i} \otimes \bar{F}_{i})^{-1}$$

$$\mathcal{O}_{i} = [H_{i}^{\mathrm{T}}, (H_{i}\bar{F}_{i})^{\mathrm{T}}, \dots, (H_{i}\bar{F}_{i}^{N-1})^{\mathrm{T}}]^{\mathrm{T}}$$

$$\Gamma_{i} = [I_{n_{z}}, -(H_{i}FK_{i})^{\mathrm{T}}, \dots, -(H\bar{F}_{i}^{N-2}FK_{i})^{\mathrm{T}}]^{\mathrm{T}}$$
(5)

The parameters θ is computed as the solution of semidefinite constrained least squares problem

$$\hat{\theta}_i = \operatorname*{argmin}_{\theta_i} \|\mathcal{A}_i \theta_i - b_i\|_2^2 \quad s.t., \quad Q, R_i \ge 0$$
(6)

where the matrix inequalities $Q, R_i \ge 0$ can be handled by adding a logarithmic barrier function to the objective. [12] proves the uniqueness of the solution to the problem is guaranteed if and only if \mathcal{A} has full column rank. Furthermore, if (F, H) is observable and F is non-singular, the optimization in (6) has a unique solution if and only if dim[null(D)] = 0.

It should be noted that when the dimension of the state x is large and the window size of auto-covariance is small, the equation (6) is easy to fall into overfitting problem. To alleviate it, the L_2 regularization term is applied to (6), then

$$\hat{\theta}_i = \underset{\theta_i}{\operatorname{argmin}} \|\mathcal{A}_i \theta_i - b_i\|_2^2 + \mu \|\theta_i\|_2^2 \tag{7}$$

where $\|\theta\|_2$ can be replaced by the trace of process noise covariance tr(Q) for simplicity. μ is the regularization term, and a good value of μ is such that tr(Q) is small and any further decrease in value of tr(Q) causes significant increase. When the matrix inequality holds, $\hat{\theta}_i$ is estimated in the minimum mean-square error sense as

$$\hat{\theta}_i = (\mathcal{A}_i^{\mathrm{T}} \mathcal{A}_i + \mu I)^{-1} \mathcal{A}_i^{\mathrm{T}} \hat{b}_i = \mathcal{A}_i^+ \hat{b}_i$$
(8)

where $\hat{b} = (\hat{C}_e(N))_s$ is the unbiased estimate of the vector b and computed as the empirical mean of the i^{th} agent's auto-covariance innovations $\hat{C}_{e,i,l}$ is computed by using the ergodic property of the *L*-innovations from the given set of data $\hat{C}_{e,i,l} = \frac{1}{\tau - l} \sum_{k=1}^{\tau - l} e_{i,k+l} e_{i,k}^{\mathrm{T}}$.

It is shown in [16] that the optimal estimator gain K_i can be determined as $K_i^{\star} = \operatorname{argmin}_{K_i} f(\mathcal{J}(K_i))$ by minimizing the upper bound of the variance of the ALS estimate according to Isserlis' theorem, denoted as $\hat{\theta} P_{\hat{\theta}} = cov[\hat{\theta}] = \mathbb{E}[(\theta - \hat{\theta})(\theta - \hat{\theta})^{\mathrm{T}}] = \mathcal{A}^+ cov[\hat{b}]\mathcal{A}^{+\mathrm{T}}$. where $f(\cdot)$ is a suitable function, e.g., the trace. $\mathcal{J}(K_i)$ is the known criterion of K_i defined in [16].

III. DISTRIBUTED ALS METHOD

The ALS estimator of the noise covariance matrices is proven to be unbiased and converging asymptotically to the true values with increasing number of data τ in [10]. But in sensor network, each agent has a limited storage capacity and suffers from the lack of innovations, i.e., τ is small. Besides, with the increase of τ , the computation and stoage burden of each agent will become heavier. Therefore, it is necessary to reformulate the distributed ALS method to balance the tradeoff between the state estimation accuracy and the computation capacity. One effective approach is that each agent enlarges the number of input data τ by receiving the auto-covariance from its neighbors $j \in N_i \cup i$. The equation (7) turns into a joint cost function, as shown below.

$$\hat{\theta}_i = \underset{\theta_i}{\operatorname{argmin}} \sum_{j \in N_i \cup i} \left(\|\mathcal{A}_j \theta_j - b_j\|_2^2 + \mu \|\theta_j\|_2^2 \qquad (9)$$

The empirical mean of the auto-covariance innovations term $\hat{C}_{e,i,l}$ is reformulated as the mean of the neighbors and itself

$$\hat{C}_{e,i,l} = \frac{1}{\tau - l} \frac{1}{M_i + 1} \sum_{j \in N_i \cup i} \sum_{k=1}^{\tau - l} e_{j,k+l} e_{j,k}^{\mathrm{T}} \qquad (10)$$

We do experiments on a linear time-invariant system with 10 sensors to be deployed in a fully connected network. The variance of the ALS-estimates $var(\theta_i) = var(Q_i)$ denotes the state estimation accuracy. F = -0.8, $H_i =$ $1, i = 1, \ldots, 10$, and with true but unknown noise variances Q = 8 and $R = [1, 2, \dots, 10]$. The time intervals are set as $\tau = [55, 60, \dots, 100]$. The system is simulated for 10^4 steps. The relationship between $var(\hat{Q}_i)$ and the number of sensors \mathcal{N}_i to be fused plus the different values of τ is shown in Fig. 1.

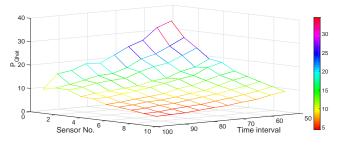


Fig. 1. Relationship between $var(\hat{Q}_i)$ and (\mathcal{N}_i, τ)

As is indicated from Fig. 1, the variance of ALS estimate var(Q) decreases with the increase of the number of sensors \mathcal{N}_i and the number of innovations. var(Q) decreases from 34.4 to 10.25 as the number of innovations increases from 55 to 100 when the number of sensors is 1, and var(Q)decreases from 10.25 to 4.97 as the number of fused sensors increases from 1 to 10 when the number of innovations is 100. Therefore, it is possible for each agent to reduce the variance of the ALS estimate by receiving the innovations from its neighbors instead of increasing the number of innovations.

The empirical mean of the auto-covariance innovations term $C_{e,i,l}$ in (10) is only the fusion of b_i . To derive the optimal fused noise covariance estimate denoted as $\hat{\theta}_{F}^{\star}$, the fused residual $\varepsilon_{k,F}$ and residual covariance $P_{\varepsilon,k,F}$ is computed based on the batch covariance intersection (BCI) method:

$$P_{\varepsilon,k,F}^{-1} = \sum_{j \in N_i \cup i} w_j P_{\varepsilon,k,j}^{-1} \tag{11}$$

$$P_{\varepsilon,k,F}^{-1}\varepsilon_{k,F} = \sum_{j\in N_i\cup i} w_j P_{\varepsilon,k,j}^{-1}\varepsilon_{k,j}$$

$$\sum_{i=1}^N w_i = 1, w_i \in [0,1], i = 1, 2, \dots, N$$
(12)

where the weights w_i can be determined by using some sub-optimal methods such as minimizing the trace of fused residual covariance $P_{\varepsilon,k,F}^{-1}$ in [29].

$$w_i = \frac{1/tr(P_{\varepsilon,k,i})}{\sum\limits_{i \in N_i \cup i} 1/tr(P_{\varepsilon,k,j})}$$
(13)

$$P_{\varepsilon,k,i} = \left(\sum_{j \in N_i \cup i} \frac{1/tr(P_{\varepsilon,k,i})}{\sum_{j \in N_i \cup i} 1/tr(P_{\varepsilon,k,j})} P_{\varepsilon,k,j}^{-1}\right)^{-1}$$
(14)

Denote the matrices \mathcal{A}_F and \hat{b}_F as $\mathcal{A}_F = \bigoplus_{i=1}^{M_i} \mathcal{A}_i$ and $\hat{b}_F = \hat{b}_i \otimes I_{M_i}$, where $\hat{b}_i = [\hat{C}_{e,i,0}^{\mathrm{T}}, \hat{C}_{e,i,1}^{\mathrm{T}}, \dots, \hat{C}_{e,i,N-1}^{\mathrm{T}}]^{\mathrm{T}}$. Then the solution to the problem in (9) can be solved by

solving the regularized LS problem.

$$\hat{\theta}_F^{\star} = \operatorname*{argmin}_{\theta_F} \|\mathcal{A}_F \theta_F - \hat{b}_F\|_2^2 + \mu \|\theta_F\|_2^2 \qquad (15)$$

where $\hat{\theta}_F = \hat{\theta}_i \otimes I_{M_i}$. The problem is solved as

$$\hat{\theta}_F^{\star} = (\mathcal{A}_F^{\mathrm{T}} \mathcal{A}_F + \mu I)^{-1} \mathcal{A}_F^{\mathrm{T}} \hat{b}_F = \mathcal{A}_F^+ \hat{b}_F$$
(16)

Then the ALS method combined with the BCI algorithm is summarized in Alg. 1.

Algorithm 1 Solving problem (9) by D-ALS algorithm
Input: $\mu = 0.01, \nu = 5 \times 10^{-3}, \tau = 100, N_{sim} = 10^3.$
Initialize: $k = 0$, \hat{x}_0 , Q_0 , $z_{i,1:N_{sim}}$, $R_{0,i}$, $P_{\varepsilon,0}$ and K_0 , $i =$
$1, \cdots, M.$

Output: \hat{Q}^* .

while in loop and $\hat{Q}_{k+1} - \hat{Q}_{k+1} > \nu$ do

- 1) Update $\hat{x}_{i,k+1}$, $\varepsilon_{i,k+1}$, $P_{\varepsilon,i}$ and K_i in (1) to (2), and then calculate the fused residual and its covariance $P_{\varepsilon,k,F}$ by BCI method in (11) and (12). Update the matrix \mathcal{A}_i , \hat{b}_i , \mathcal{A}_F and \hat{b}_F in .
- 2) Update the global optimal noise covariance $\hat{\theta}^{\star}$ in (15) to (16), and set $\hat{Q}_{k+1} = \hat{\theta}_1^{\star}$.

Remark 1: It is easily derived that the augmented matrix \mathcal{A}_F and the permutation matrix \hat{b}_F for i^{th} sensor has dimensions of $M_i N n_x \times (n_x + M_i n_z)$ and $M_i N n_x (n_x + M_i n_z)$ $M_i n_z) \times 1$ respectively. The computation complexity of the D-ALS algorithm is $O(M_i N^2 n_x)$. Expanding the number of the auto-covariance of innovations would increase the computation time and even lead to the intractable computation. Therefore, the number of sensors and the window size should be made from a tradeoff between the accuracy and the computation burden.

Then the variance of the fused noise covariance matrix $\hat{\theta}_F$ is lower than the variance of each agent's noise covariance matrix $\hat{\theta}_i$, as is proved in Theorem 1.

Theorem 1: (Accuracy Analysis of D-ALS algorithm) The relations between local and fused residuals covariance $P_{\varepsilon,k,i}, P_{\varepsilon,k,0}, \bar{P}_{\varepsilon,k,F}$ and $P_{\varepsilon,k,F}$ are shown as follows.

$$tr(P_{\varepsilon,k,0}) \le tr(P_{\varepsilon,k,F}) \le tr(P_{\varepsilon,k,F}) \le tr(P_{\varepsilon,k,i})$$
(17)

Then the relations between the fused noise covariance matrix $\hat{\theta}_F$ and the noise covariance matrix for each agent are shown below.

$$var(\hat{\theta}_F) \le var(\hat{\theta}_i), \quad i = 1, \dots, M_i$$
 (18)

Proof: Using the unbiasedness of $\hat{\varepsilon}_{k,i}$ for each agent *i*, it can be derived that $\hat{\varepsilon}_{F,k}$ is a linear unbiased estimate. Since $\hat{\varepsilon}_{k,0}$ is the best linear unbiased estimate, then $P_{\varepsilon,k,0} \leq \bar{P}_{\varepsilon,k,F}$ holds. The inequality $\bar{P}_{\varepsilon,k,F} \leq P_{\varepsilon,k,F}$ is proved as the consistency property in [30]. Because the operator of trace is monotonically increasing function, then the inequalities $tr(P_{\varepsilon,k,0}) \leq tr(\bar{P}_{\varepsilon,k,F})$ and $tr(\bar{P}_{\varepsilon,k,F}) \leq tr(P_{\varepsilon,k,F})$ hold. When the parameters are set to $w_i = 1$ and $w_j = 1, j \neq i$, $tr(P_{\varepsilon,k,F}) = tr(P_{\varepsilon,k,i})$. Because the parameter w is determined by minimizing the trace of $P_{\varepsilon,k,F}$, as shown in (19), it is easily derived that $P_{\varepsilon,k,F} \leq P_{\varepsilon,k,i}$.

$$w = \underset{w}{\operatorname{argmin}} tr \Big[(\sum_{i=1}^{M_i} w_i P_{\varepsilon,k,i}^{-1})^{-1} \Big]$$
(19)

Then it can easily be derived that \mathcal{A}_F is the best estimate of \mathcal{A} . As shown in (10), the number of data to compute \hat{b}_F is larger than that to compute \hat{b}_i . As is proven in [10], The ALS estimate of the noise covariance matrix converges asymptotically to the true values with increasing number of data. Therefore, the variance of $\hat{\theta}_F$ is smaller than the variance of $\hat{\theta}_i$. Q.E.D.

IV. SIMULATION RESULTS

A. Static sensor networks

Consider fully connected sensor network The linear timeinvariant system is modeled as $x_{k+1} = 0.8x_k + w_k$ measured by 3 sensors modeled as $y_{i,k+1} = H_i x_k + v_{i,k}$ with $H_1 = [1,0], H_2 = I_2, H_3 = [1,0]. w_k$ and $v_{i,k}$ are zero-mean Gaussian noise with unknown covariance Q and R_i , where the real values are set as $Q = 4, R_1 = 0.81,$ $R_2 = diag(4, 0.64)$, and $R_3 = 2.25$. Here, to guarantee the unbiasedness, we ran $N_s = 10^4$ Monte Carlo simulations for each simulation data set. The estimation performance for the i_{th} sensor is measured as the mean square error (MSE):

$$MSE_{i,k} = \frac{1}{N_s} \sum_{t=k}^{k+N_s} (\varepsilon_{i,t} - \hat{\varepsilon}_{i,t})^2, \quad i = 1, 2, 3, M_i \quad (20)$$

Using the steady-state Kalman filter, the gain K_i is computed as

$$K_1 = [1.68, 0.81], K_2 = \begin{bmatrix} 0.36 & 1.22\\ 0.06 & 0.84 \end{bmatrix}, K_3 = [1.40, 0.61]$$
(21)

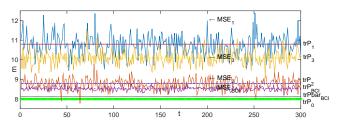


Fig. 2. Comparisons of MSE curve and the trace of residual covariance between fused and single sensor.

TABLE I Comparisons of the trace of residual covariance between fused and single sensor.

•	trP_1	trP_2	trP_3	trP_F	$tr\bar{P}_F$	$tr\bar{P}_0$
	10.791	10.083	8.771	8.512	8.102	7.985

According to the equations (11), the steady state covariances of each agent and the cross covariance matrices are

$$P_{\varepsilon,1} = \begin{bmatrix} 5.05 & 4.94 \\ 4.94 & 5.73 \end{bmatrix}, \quad P_{\varepsilon,2} = \begin{bmatrix} 3.73 & 2.78 \\ 2.78 & 5.05 \end{bmatrix}$$
$$P_{\varepsilon,3} = \begin{bmatrix} 5.79 & 3.65 \\ 3.65 & 4.29 \end{bmatrix}, \quad P_{\varepsilon,1,2} = \begin{bmatrix} 1.30 & -0.22 \\ -0.22 & 0.38 \end{bmatrix}$$
$$P_{\varepsilon,2,3} = \begin{bmatrix} 0.75 & 0.21 \\ 0.21 & 0.45 \end{bmatrix}, \quad P_{\varepsilon,1,3} = \begin{bmatrix} 0.69 & 1.21 \\ 1.21 & 4.19 \end{bmatrix}$$
(22)

The comparisons between MSE and the trace of $P_{\varepsilon,k,i}$, $P_{\varepsilon,k,0}$, $\bar{P}_{\varepsilon,k,F}$ and $P_{\varepsilon,k,F}$ are shown in Fig. 2 and Table I. As indicated from the figure, the true accuracy of the BCI fused residual covariance is similar to the linear optimal residual covariance, because $tr(\bar{P}_{\varepsilon,k,F}) = 8.102$ is near to $tr(\bar{P}_{\varepsilon,k,0}) = 7.985$. Besides, the variance of the fused estimate $tr\bar{P}_F$ is lower than others, illustrating that the proposed D-ALS algorithm outperforms than the ALS method.

B. Mobile sensor networks

Cooperative target tracking in mobile sensor networks (MSNs) is an important task in many applications, e.g., the unmanned aerial vehicle (UAV). Compared with the target tracking case in static sensor networks (SSNs) in section IV-A, each agent node in this case is mobile and versatile, and is required to be deployed in any scenario with rapid topology changes. Therefore, the ALS-BCI algorithm is also applied to the target tracking in MSNs to show its efficiency.

Consider 10 sensor nodes tracking the maneuver target in a $110m \times 90m$ square. The target is driven by a turning rate model:

$$X_{k+1} = \begin{bmatrix} 1 & \frac{\sin(\eta T_s)}{\eta} & 0 & -\frac{1-\cos(\eta T_s)}{\eta} \\ 0 & \cos(\eta T_s) & 0 & -\sin(\eta T_s) \\ 0 & \frac{1-\cos(\eta T_s)}{\eta} & 1 & \frac{\sin(\eta T_s)}{\eta} \\ 0 & \sin(\eta T_s) & 0 & \cos(\eta T_s) \end{bmatrix} X_k + Gw_k$$
(23)

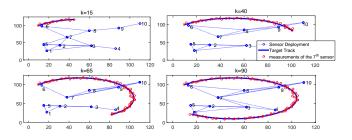


Fig. 3. The diagram of the target tracking in the time-varying sensor deployment.

where $X_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^{\mathrm{T}}$ is the states to be estimated at time k. The states includes the position $[x_k, y_k]$ and the velocity $[\dot{x}_k, \dot{y}_k]$, and the initial values are $[10m,2m/s,100m,2m/s].\;\eta$ is the turn rate and is set to $\frac{\pi}{60} rad/s$. w_k is Gaussian white noise with covariance matrix $Q = diag[Q_x, Q_{\dot{x}}, Q_y, Q_{\dot{y}}]$, where $Q_x, Q_y, Q_{\dot{x}}, Q_{\dot{y}}$ are unknown scalar variable to be estimated and the real ones are set to $10m^2$, 0, $10m^2$ and 0 respectively. T_s is the sampling time and is set to 1s. $G = \begin{bmatrix} \frac{1}{2}T_s^2 & T_s & 0 & 0\\ 0 & 0 & \frac{1}{2}T_s^2 & T_s \end{bmatrix}^{\mathrm{T}}$. The measurements of each agent is given by:

$$Z_{i,k} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} X_k + v_{i,k} \quad i = 1, \dots, M \quad (24)$$

where $Z_{i,k} = [zx_k, zy_k]^T$ is the measurement of the position of the target. $v_{i,k}$ is Gaussian white noise with unknown covariance matrix $R_i = diag[R_{x,i}, R_{y,i}]$. $R_{x,i}$ and $R_{y,i}$ are unknown parameter to be estimated and the real ones are set to $2m^2$ and $2m^2$ respectively. The motion of each agent is described by the following kinematic equations:

$$qx_{i}(k) = qx_{i}(k-1) + qv_{i} * T_{s} * \cos\theta_{k}$$

$$qy_{i}(k) = qy_{i}(k-1) + qv_{i} * T_{s} * \sin\theta_{k}$$
(25)

where $(qx_i(k), qy_i(k))$ is the position of the i^{th} sensor at time k. $\theta_k = \arctan \frac{z_{i,k} - qy_i(k-1)}{z_{i,k} - qx_i(k-1)}$ is the measurement of angular position of the i^{th} sensor towards the target. qv_i is the constant speed of the i^{th} sensor and is set to 0.5m/s. The initial position of the sensors are set to (18, 27), (31, 43), (62, 41), (86, 33), (15, 45), (13, 98), (38, 105),(60, 99), (89, 93), (110, 106)], and the unit is meter. The communication ranges and the sensing ranges of the sensors are all set to $r_c = 45m$ and $r_s = 60m$ respectively.

100 Monte Carlo simulations are run on the simulated model. For comparison we ran ALS and ALS-BCI on the same data sets. The initial parameters of the algorithm 9 are set to $Q_A = diag[5, 0, 5, 0]$ and $R_A = diag[1, 1]$. The diagram of the target tracking in the time-varying sensor deployment is shown in Fig. 3. The connectivity of the i^{th} sensor is denoted as the sum of the adjacent matrix $|A_{ij}|$, as shown in Fig. 4.

As is shown from the above figures, the 7^{th} sensor only measures the target in the sensing range, and the communication topology is varying due to the mobility of the sensors.

The comparisons between MSE and the trace of the 7^{th} sensor's residual covariance P_7 and its fused residual

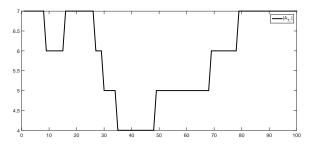


Fig. 4. The connectivity of the 7^{th} sensor.

covariance P_{BCI} and \bar{P}_{BCI} along the x-axis are shown in Fig. 5 and Table II.

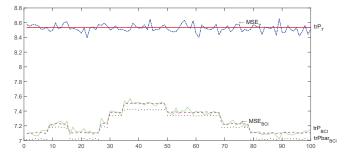


Fig. 5. Comparisons of MSE curve and the residual covariance between fused and 7th sensor.

TABLE II COMPARISONS OF THE RESIDUAL COVARIANCE BETWEEN FUSED AND 7th SENSOR.

+r D -	$tr P_{BCI}$ k=15 k=43		$tr\bar{P}_{BCI}$		
017	k=15	k=43	k=15	k=43	
8.536	7.223	7.502	7.185	7.411	

As is indicated from the figure and the table, the fused residual covariance trP_{BCI} and the MSE values MSE_{BCI} of 7^{th} sensor is less than that of its own trP_7 and MSE_7 . Besides, it is noticed that trP_{BCI} and $tr\bar{P}_{BCI}$ decreases with the increase of the number of neighbors.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper proposes the distributed auto-covariance least squares algorithm based on BCI to solve the distributed estimation problem with unknown noise covariance over networks. The efficiency of the algorithm is proven because the fused error covariances converges to the true values faster and the variance of the ALS estimate is smaller. The numerical results are illustrated to show the performance of the algorithm.

B. Future Works

In real-time applications, the latency and limited power are the main problems in wireless sensor network. Since the ALS-BCI algorithm still needs some time to compute the matrix A, it is necessary to compute the a-priori estimate of the lower bound of the variance of the fused ALS estimate $\hat{\theta}_F$. Future work is needed to derive the lower bound of the fused noise covariance estimate $\hat{\theta}_F$.

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