Direct vs Indirect Methods for Behavior-based Attack Detection

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Abstract-We study the problem of data-driven attack detection for unknown LTI systems using only input-output behavioral data. In contrast with model-based detectors that use errors from an output predictor to detect attacks, we study behavior-based data-driven detectors. We construct a behavior-based chi-squared detector that uses a sequence of inputs and outputs and their covariance. The covariance of the behaviors is estimated using data by two methods. The first (direct) method employs the sample covariance as an estimate of the covariance of behaviors. The second (indirect) method uses a lower dimensional generative model identified from data to estimate the covariance of behaviors. We prove the consistency of the two methods of estimation and provide finite sample error bounds. Finally, we numerically compare the performance and establish a tradeoff between the methods at different regimes of the size of the data set and the length of the detection horizon. Our numerical study indicates that neither method is invariable superior, and reveals the existence of two regimes for the performance of the two methods, wherein the direct method is superior in cases with large data sets relative to the length of the detection horizon, while the indirect method is superior in cases with small data sets.

I. INTRODUCTION

Cyber-physical systems are growing in complexity and size since the advent of better communication and higher computation power. This has also introduced a greater possibility for adversarial attacks. Several attacks, such as the ones on the power grid in Ukraine in 2015, the Maroochy attack in 2000, and others mentioned in [1], have exposed the vulnerability of CPSs. Detection and mitigation of attacks has broadly been addressed using model-based [2]-[8] and data-driven techniques [9]-[15]. Model-based attack detection and their fundamental limitations have been understood well [2], [3], model-based techniques assume knowledge of the underlying system or the knowledge of the statistics of the measured signals. Therefore the implementation of model-based methods need prior system identification, which could be difficult to achieve due to the complexity of the system or lack of data. In contrast, data-driven methods usually operate with no knowledge of the system, yet offer scalable and accurate detection. However, their limitations are not fully understood. We study the problem of data-driven attack detection in stochastic systems and, in particular, the tradeoffs between direct and indirect methods of detection.

Related work: Machine Learning approaches to attack detection [9]-[11] have gained popularity recently due to their ease of implementation. However, these techniques lack an insight into their functioning. System theoretic approaches provide more explainable solutions, and they can broadly be classified into direct and indirect methods. Direct methods map observations to nominal or attacked conditions [12] using past data. Indirect methods [13]-[15] utilize information about the underlying system to detect attacks. [13] considers the noiseless case where future outputs are predicted from past data and predictions are compared with measurements, while [14], [15] address the noisy case. [14] proposes a datadriven filter for fault detection and isolation. [15] analyses the conditions for successful attacks when the adversary does not have the knowledge of the system parameters. Differently from these works, this paper focuses on a finite-sample analysis and tradeoffs between direct and indirect methods.

A promising model-based approach that can be adapted for the data-driven problem is the χ^2 anomaly detector [4]–[6]. These works use the detector in conjunction with innovations from a state estimator and assume the knowledge of the system. We provide a modified data-driven approach using a χ^2 detector constructed from the measured system behaviors.

Contributions: This paper contributes a data-driven attack detector that uses the behaviors of an unknown LTI system to differentiate between nominal operation and attacked operation. To implement the detector, we propose two methods to estimate the covariance of the behaviors, namely the direct and indirect method. The direct method is motivated by discriminative classification where we use data to differentiate between nominal and attacked conditions. The indirect method is motivated by generative classification and the fact that an underlying lower dimensional system generates the observed behaviors. We analytically show that both methods are consistent. We also provide finite sample error bounds for the estimates of the two methods. Finally, we numerically compare the two methods at different noise levels using the ROC curve as a metric for performance, and establish different regimes where one outperforms the other.

II. DATA-DRIVEN ATTACK DETECTION

We consider the following discrete-time stochastic system:

$$x_{t+1} = Ax_t + Bu_t + w_t + B^a u_t^a, y_t = Cx_t + v_t + G^a y_t^a,$$
(1)

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ are the system matrices, $x_t \in \mathbb{R}^n$, $u_t \in \mathbb{R}^m$, and $y_t \in \mathbb{R}^p$ are the state, control input and the output, respectively, u_t^a and y_t^a are the malicious input and false sensor measurement injected by an

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adversary as attacks on the system. The attacks are affecting the system through the matrices B^a and G^a , respectively. We assume that the system is stable, controllable, and observable. The process noise w_t and measurement noise v_t are i.i.d. Gaussian processes with zero mean. The data-driven attack detection problem is posed as follows.

Data-driven attack detection: Given inputs $\boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}_0^\top, \dots, \boldsymbol{u}_{T-1}^\top \end{bmatrix}^\top$ and outputs $\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1^\top, \dots, \boldsymbol{y}_T^\top \end{bmatrix}^\top$ over the detection horizon T, determine if $(\boldsymbol{u}^a, \boldsymbol{y}^a) \neq 0$, where $\boldsymbol{u}^a = \begin{bmatrix} \boldsymbol{u}_0^{\top}, \dots, \boldsymbol{u}_{T-1}^a \end{bmatrix}^\top$ and $\boldsymbol{y}^a = \begin{bmatrix} \boldsymbol{y}_1^{\top}, \dots, \boldsymbol{y}_T^a \end{bmatrix}^\top$.

To perform attack detection, we have input-output data from N attack-free experiments $(\boldsymbol{u}^{(i)}, \boldsymbol{y}^{(i)})$, where $i \in \{1, 2, ..., N\}$. Each experiment runs over a horizon of length T, with $x_0 = 0$. The inputs for the experiments are generated by a Gaussian random process $\mathcal{N}(0, \Sigma_u)$.

We begin by noting that the outputs y are generated by inputs u, noises w and v, and attacks u^a and y^a as,

$$\boldsymbol{y} = \boldsymbol{\mathcal{C}}\boldsymbol{u} + \boldsymbol{\mathcal{C}}'\boldsymbol{w} + \boldsymbol{v} + \boldsymbol{\mathcal{C}}^{a}\boldsymbol{u}^{a} + F^{a}\boldsymbol{y}^{a}.$$
 (2)

Here w and v are the process and measurement noise w_t and v_t over the horizon T. The matrix C is defined as

$$C = \begin{bmatrix} CB & 0 & \dots & 0\\ CAB & CB & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ CA^{T-1}B & CA^{T-2}B & \dots & CB \end{bmatrix}.$$
 (3)

C' is obtained by replacing B with the identity matrix I in C, and C^a is obtained by replacing B with B^a in C. Note that in the attack-free case, the assumptions made earlier ensure that y as well as the behavior Z are stationary Gaussian processes. In general, control inputs need not be generated by a Gaussian distribution. However, any output stabilizing controller will generate stationary measurements when the system is in operation.

Although we have access to the inputs \boldsymbol{u} and outputs \boldsymbol{y} over the detection horizon, we have access to neither the system matrices nor the Markov parameters CA^iB , making it difficult to perform attack detection using methods that rely on a model based χ^2 detector or KL-divergence [3]. We propose to modify the usage of the χ^2 detector as follows,

$$g = Z^{\top} S^{-1} Z \underset{H_1}{\overset{H_0}{\leqslant}} \lambda, \tag{4}$$

where $Z = [\boldsymbol{u}^{\top}, \boldsymbol{y}^{\top}]^{\top}$ is the behavior of the system (1) over the time horizon T. The covariance matrix $S = \mathbb{E}[ZZ^{\top}]$. Note that $\mathbb{E}[Z] = 0$ under nominal operation. Further, hypothesis H_1 denotes an alarm, whereas H_0 denotes nominal operation. λ is an arbitrary threshold used to differentiate nominal and attacked operation. To implement the above detector, we need to estimate S for which we propose the direct and indirect methods.

A. Direct method

The direct method is motivated by discriminative methods of classification, where a measured signal is mapped to nominal or attacked operation using data. We propose to use the data from N experiments to estimate the covariance S without considering the structure or characteristics of the samples. In particular, we use the sample covariance as a direct estimate,

$$\hat{S}^{d} = \frac{1}{N} \sum_{i=0}^{N} \boldsymbol{z}^{(i)} \boldsymbol{z}^{(i)^{\top}} = \frac{1}{N} \boldsymbol{Z} \boldsymbol{Z}^{\top}.$$
 (5)

Here, $\boldsymbol{z}^{(i)} = [\boldsymbol{u}_0^{(i)^{\top}}, \dots, \boldsymbol{u}_{T-1}^{(i)^{\top}}, \boldsymbol{y}_1^{(i)^{\top}}, \dots, \boldsymbol{y}_T^{(i)^{\top}}]^{\top} \in \mathbb{R}^{T(m+p)\times 1}$, and $\boldsymbol{Z} = [\boldsymbol{z}^{(1)} \ \boldsymbol{z}^{(2)} \ \dots \ \boldsymbol{z}^{(N)}] \in \mathbb{R}^{T(m+p)\times N}$. The direct method is computationally simple. In cases with large data sets, the sample covariance is a good estimator of the true covariance matrix S. As the dimension of the covariance matrix S grows with the detection horizon T, the sample covariance needs more data for accurate estimation. Further, when N < T(m+p), the sample covariance is not a good estimator. Therefore, in the sequel, we propose the indirect method which can potentially outperform the direct method in cases with little data.

B. Indirect method

In the indirect method, we identify a lower dimensional generative model that gives rise to the behaviors. We first break the behavior Z into smaller behaviors called *minor* behaviors, each of length L denoted by f_t . In particular, $f_t = [u_{t-L}^{\top} \ u_{t-L+1}^{\top} \ \dots \ u_t^{\top} | y_{t-L}^{\top} \ y_{t-L+1}^{\top} \ \dots \ y_t^{\top}]^{\top}$. Note that we can construct T - L + 1 minor behaviors from Z. We seek to regress future minor behaviors on past minor behaviors of the form, $f_{t+1} = \mathcal{M}f_t + \epsilon_t$ using data. \mathcal{M} defines the linear evolution of the minor behaviors while ϵ_t is additive noise entering the system which is uncorrelated to the behaviors. By exploiting the nature of the underlying dynamics of the minor behaviors, we can potentially estimate the covariance matrix of the behaviors with less data. We first note that the minor behaviors follow a stationary process owing to our initial assumptions. Let the covariance of the minor behaviors be P, and the covariance of the noise ϵ_i be Σ_{ϵ} . Now, the covariance of the behaviors can be computed as,

$$\operatorname{Cov} (f_{i+1}) = \operatorname{Cov} (\mathcal{M}f_i + \epsilon_i),$$

$$P = \mathcal{M}P\mathcal{M}^{\top} + \Sigma_{\epsilon}.$$
 (6)

It is clear that equation (6) is in the form of a Lyapunov equation. Therefore, the covariance matrix P can be computed from (6) if \mathcal{M} and Σ_{ϵ} are available. If \mathcal{M} is regressed from data, Σ_{ϵ} can be estimated from the residuals of the regression using the sample covariance. In turn, P can be estimated by solving the Lyapunov equations using the estimates of \mathcal{M} and Σ_{ϵ} . We propose to estimate \mathcal{M} using Ordinary Least Squares (OLS) as,

$$\hat{\mathcal{M}} = F'F^{\top} \left(FF^{\top}\right)^{-1},\tag{7}$$

where F and F' are data matrices constructed from the minor behaviors obtained from the experimental data:

$$F = \begin{bmatrix} f_1^{(1)} & \dots & f_{T-L}^{(1)} & f_1^{(2)} & \dots & f_{T-L}^{(N)} \end{bmatrix} \in \mathbb{R}^{L(p+m) \times N(T-L)},$$

$$F' = \begin{bmatrix} f_2^{(1)} & \dots & f_{T-L+1}^{(1)} & f_2^{(2)} & \dots & f_{T-L+1}^{(N)} \end{bmatrix}.$$

If $\hat{\epsilon}_i$ are the residuals from the regression (7), Σ_{ϵ} can be estimated using the sample covariance.

$$\hat{\epsilon}_{j}^{(i)} = f_{j+1}^{(i)} - \hat{\mathcal{M}} f_{j}^{(i)}, \qquad (8)$$

$$\hat{\Sigma}_{\epsilon} = \frac{1}{N_{id}} E E^{\top}.$$
(9)

Here $E = [\hat{\epsilon}_1^{(1)} \ \hat{\epsilon}_2^{(1)} \dots \ \hat{\epsilon}_{T-L}^{(N)}]$ and $N_{id} = N(T-L)$. $f_j^{(i)}$ represents the minor behavior f_j from the experiment *i* with $j \in \{1, 2, \dots, T-L+1\}$. We now have the estimates of \mathcal{M} and Σ_{ϵ} to solve equation (6). Therefore, P is estimated by solving $\hat{P} = \hat{M}\hat{P}\hat{M} + \hat{\Sigma}_{\epsilon}$.

We next describe the estimation of S using the covariance of the minor behaviors P. Let $D = \begin{bmatrix} f_1^\top & f_2^\top & \dots & f_{T-L+1}^\top \end{bmatrix}^\top$. Then Z = KD, where K is a known sparse matrix that reconstructs Z from D. Let the covariance of D be $\mathbb{E} \begin{bmatrix} DD^\top \end{bmatrix} = \Sigma_D$. Therefore $S = \mathbb{E}[ZZ^\top] = K\Sigma_D K^\top$. Let the matrix \mathcal{F} be given as,

$$\mathcal{F} = \begin{bmatrix} I & \mathcal{M} & \dots & \mathcal{M}^{T-L} \\ \mathcal{M} & I & \dots & \mathcal{M}^{T-L-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{M}^{T-L} & \mathcal{M}^{T-L-1} & \dots & I \end{bmatrix}.$$

Then the covariance Σ_D is,

$$\Sigma_D = \mathcal{F} \otimes P, \tag{10}$$

where \otimes denotes the Kronecker product. Therefore, Σ_D can be estimated as $\hat{\Sigma}_D = \hat{\mathcal{F}} \otimes \hat{P}$, where $\hat{\mathcal{F}}$ is obtained by replacing \mathcal{M} with its estimate $\hat{\mathcal{M}}$. Finally, S is estimated as $\hat{S}^{id} = K \hat{\Sigma}_D K^{\top}$.

III. CONSISTENCY AND FINITE-SAMPLE ANALYSIS

In this section, we establish that both the direct and indirect methods are consistent estimators of the covariance matrix S. We also provide finite sample analysis on the deviation of the estimated covariance matrices \hat{S}^d and \hat{S}^{id} from the true covariance.

Theorem 3.1: (Consistency) Let the covariance matrix S be estimated as \hat{S}^d and \hat{S}^{id} using the direct and indirect methods, respectively, then,

$$\lim_{N \to \infty} \hat{S}^d = \lim_{N \to \infty} \hat{S}^{id} = S.$$

We refer the reader to Appendix A for the proof. This result establishes that with infinite data, both the methods estimate the covariance matrix S perfectly. We next establish a finite sample error bound for the two methods. We denote the spectral norm of a matrix by $\|\cdot\|$.

Theorem 3.2: (Finite-sample bound on error - Direct method) If the covariance S is estimated as \hat{S}^d using the direct method, then for any $\theta \ge 0$, and $r = \frac{\text{Tr}(S)}{\|S\|}$,

$$\|S - \hat{S}^d\| \le \left(\sqrt{\frac{2\theta(r+1)}{N}} + \frac{2\theta r}{N}\right) \|S\|$$
(11)

with probability at least $1 - 2T(m+p)e^{-\theta}$, for any N > T(m+p).

We provide the proof in Appendix B. Theorem 3.2 characterizes a finite sample bound on the error in estimation arising from using the direct method. This bound grows larger with the dimension of S as r is a strictly increasing function of the size of S. This contributes to high sample complexity of the direct method, leading to poor estimates at low data regimes for big S matrices. Further, the above bound converges to 0 at a rate of the order $O(\frac{1}{\sqrt{N}})$.

For the indirect method, it is evident from equation (10) that solving the OLS problem introduces errors in the matrices \mathcal{F} and P. Define the errors arising from the OLS solution as $\Delta_{\mathcal{M}} = \mathcal{M} - \hat{\mathcal{M}}$ and $\Delta_{\Sigma_{\epsilon}} = \Sigma_{\epsilon} - \hat{\Sigma}_{\epsilon}$. Using result from Theorem 1 of [16], we obtain with probability at least $1 - \theta$,

$$\|\Delta_{\mathcal{M}}\| \le \sqrt{\frac{k}{N_{id}}} \gamma_s\left(\mathcal{M}, \frac{\theta}{4}\right),\tag{12}$$

for any $N_{id} \ge \max\left(N_{\eta}(\theta), N_{s}(\theta)\right)$. Here k is an absolute constant. We define $\gamma_{s}(\mathcal{M}, \frac{\theta}{4}), N_{\eta}(\theta)$ and $N_{s}(\theta)$ as follows,

$$\gamma_{s}(\mathcal{M},\theta) = \sqrt{8L(m+p)\left(\log\frac{5}{\theta} + \frac{\log 4\mathrm{Tr}(\Gamma_{N}(\mathcal{M})) + 1}{2}\right)}$$
$$N_{\eta}(\theta) = k\log\frac{2}{\theta} + (L(m+p)\log5),$$
$$N_{s}(\theta) = k\left(L(m+p)\log(\mathrm{Tr}(\Gamma_{N}(\mathcal{M})) + 1) + 2L(m+p)\log\frac{5}{\theta}\right),$$
$$\Gamma_{N}(\mathcal{M}) = \sum_{j=0}^{N_{id}} \mathcal{M}^{j} \mathcal{M}^{j^{\top}}.$$

Equation (12) is a tight probabilistic bound on the error arising from the OLS problem. The error in covariance of the residuals $\Delta_{\Sigma_{\epsilon}}$ follows the same distribution as established in equation (11) because the residuals of the OLS solution are uncorrelated Gaussian random variables. Therefore both $\|\Delta_{\mathcal{M}}\|$ and $\|\Delta_{\Sigma_{\epsilon}}\|$ converge to 0 with rate $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$.

Before we bound the error in estimation from the indirect method, we first provide a sensitivity analysis for matrices \mathcal{F} and P when they are estimated.

Lemma 3.3: (Sensitivity of \mathcal{F} and P) Given estimated matrices $\hat{\mathcal{F}}$ and \hat{P} , let the errors in estimation of P be $\Delta_P = \hat{P} - P$, and \mathcal{F} be $\Delta_{\mathcal{F}} = \hat{\mathcal{F}} - \mathcal{F}$. Then,

(a) Sensitivity of P is,

$$\begin{split} \|\Delta_{P}\| &\leq \left(\sqrt{L(m+p)} \|I \otimes I - \mathcal{M}^{\top} \otimes \mathcal{M}^{\top}\|\right) \\ &\left((1 + \|\mathcal{M} + \Delta_{\mathcal{M}}\|)^{2} \left(\frac{\|\Delta_{\Sigma_{\epsilon}}\|}{\|\Sigma_{\epsilon} + \Delta_{\Sigma_{\epsilon}}\|}\right) \\ &+ 2(\|\mathcal{M}\| + \|\Delta_{\mathcal{M}}\|)^{2} \left(\frac{\|\Delta_{\mathcal{M}}\|}{\|\mathcal{M} + \Delta_{\mathcal{M}}\|}\right)\right). \end{split}$$
(13)

if Δ_P and $\Delta_{\Sigma_{\epsilon}}$ are positive semi-definite.

(b) Sensitivity of \mathcal{F} is,

$$\begin{aligned} \|\Delta_{\mathcal{F}}\| &\leq \|\mathcal{F}\| + 1 + 2\big(\|\mathcal{M} + \Delta_{\mathcal{M}}\| \\ &+ \|\mathcal{M} + \Delta_{\mathcal{M}}\|^2 + \dots + \|\mathcal{M} + \Delta_{\mathcal{M}}\|^{T-L}\big). \end{aligned}$$
(14)

We provide the proof in Appendix C. The above sensitivity analysis allows us to provide the rate of convergence for both Δ_P and Δ_F . Since $\|\Delta_{\mathcal{M}}\|$ and $\|\Delta_{\Sigma_{\epsilon}}\|$ both converge at the rate of $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$, $\|\Delta_P\|$ converges to 0 at the rate of $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$, and $\|\Delta_F\|$ converges to a system dependent constant at the rate of $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$. Following Lemma 3.3, we provide a probabilistic bound

Following Lemma 3.3, we provide a probabilistic bound on the estimation error of the indirect method obtained from finite samples. We bound the estimation error $||S - \hat{S}^{id}||$ as a function of Δ_P and Δ_F . Recall that $S = K \Sigma_D K^{\top}$ and $\hat{S}^{id} = K \hat{\Sigma}_D K^{\top}$. Here, the matrix K has at most one identity matrix per column as it picks elements from the vector D to construct Z, therefore ||K|| = 1. Using the properties of the Kronecker product and the triangle inequality, we obtain the error bound.

$$\|S - \hat{S}^{id}\| = \|K(F \otimes P)K^{\top} - K((F + \Delta_F) \otimes (P + \Delta_P))K^{\top}\|$$

$$\leq \|F \otimes P - (F + \Delta_F) \otimes (P + \Delta_P)\|$$

$$\leq \|\mathcal{F}\| \|\Delta_P\| + \|\Delta_{\mathcal{F}}\| \|P\| + \|\Delta_{\mathcal{F}}\| \|\Delta_P\|.$$
(15)

The indirect method scales at the rate $\mathcal{O}(L)$ while the direct method scales at the rate $\mathcal{O}(T)$. The data set expands for the indirect method to N(T-L) as each experiment $z^{(i)}$ is broken into minor behaviors $f^{(i)}_j$ with $j \in \{1, \ldots, T-L+1\}$. This makes the indirect method better suited for cases with low data sizes and longer detection horizons as $L \leq T$. The performance of the indirect method is also dependent on the choice of L. When L = n, the representation of the minor behaviors in equation (6) is exact as proven in Appendix A. However, choosing a large value for L increases the model complexity, thereby overfitting the data. Contrary to this, choosing a small L can underfit the data. The direct method is free of design parameters and can potentially outperform the indirect method when the size of the available data set N is large.

IV. SIMULATIONS

In this section we numerically compare and establish a tradeoff in performance of the detector using the two proposed methods of estimation. To make a comparison, we consider the case when the system is operating nominally as well as when the system is attacked by an adversary. As the performance of the detector depends on both nominal and attacked conditions, we compare the False Positive Rate (FPR) and True Positive Rate (TPR) of the two methods. We also investigate the effect of the noise on the performance of the two methods by varying the signal to noise ratio. We consider a stable SISO system with n = 3 with randomly generated A, B, and C matrices. Also, for experimental data we use $u_t \sim \mathcal{N}(0, \sigma_u I)$, $w_t \sim \mathcal{N}(0, \sigma_w I)$, and $v_t \sim \mathcal{N}(0, \sigma_v I)$. We vary σ_u , σ_v and σ_w to compare the performance of the estimates under various signal to noise ratios using ROC curves.

Under nominal conditions, the performance of the detector is determined by the FPR, which is defined as $\mathbb{P}(g \geq \lambda | (\boldsymbol{u}^a, \boldsymbol{y}^a) \equiv 0)$. Under an attack, the performance is determined by the TPR defined as $\mathbb{P}(g \geq \lambda | (\boldsymbol{u}^a, \boldsymbol{y}^a) \neq 0)$. To compute the TPR, we introduce a detectable attack where $u_t^a \sim \mathcal{N}(0, 1.5)$, and $y_t^a \sim \mathcal{N}(0, 1.5)$. The TPR and FPR in the following comparisons have been averaged over 50 trials. For the indirect method, L = n.

1) Comparison 1: In this comparison, we set $\sigma_u = \sigma_v = \sigma_w = 1$ and compare the ROC curves of the two methods by plotting the logarithm of the FPR versus the logarithm of the TPR. In Figure 1, it is evident that the direct method performs poorly at low data-regimes while the indirect method performs better. However, as the data size N increases, the direct method outperforms the indirect method. As the detection horizon increases to T = 14 in Figure 1 (e)-(h), the direct method takes longer to outperform the indirect method. However, both methods feature higher TPR as T increases.

2) Comparison 2: In this comparison, we vary the the signal to noise ratio in the data. Similar to the first comparison, we compare the ROC curves of the two methods. We first set $\sigma_u = 0.5$ and $\sigma_v = \sigma_w = 1$. From Figure 2(a)-(d), we can see that the performance of both the methods deteriorates as the SNR decreases. However, the indirect method is consistently better when the size of the data set is small. The direct method is affected more by the decrease in SNR with a decrease in TPR. Further, it takes more data for the direct method to outperform the indirect method. Next, we set $\sigma_u = 2$ and $\sigma_v = \sigma_w = 1$, thereby increasing the SNR. In Figure 2(e)-(h), the gap between the direct and indirect methods decreases across all data sizes. Further, with more data the direct method outperforms the indirect method quicker.

V. CONCLUSION AND FUTURE WORK

In this paper, we proposed a data-driven χ^2 attack detector that uses the behaviors of the system to differentiate between nominal and attacked operation. The proposed detector requires estimation of the covariance of input-output behaviors of the system. This is achieved by the two proposed methods, the direct and indirect method. We analytically showed the consistency of the two methods and established a probabilistic finite-sample error bounds. After a numerical study of the performance of the two methods, it was evident that the neither method is invariably superior. The direct method performs well in cases with more data and shorter detection horizons. However, as the attack detection horizon increases, the direct method starts to perform poorly. The indirect method, with its reliance on an underlying generative model of the system, outperforms the direct method in small data

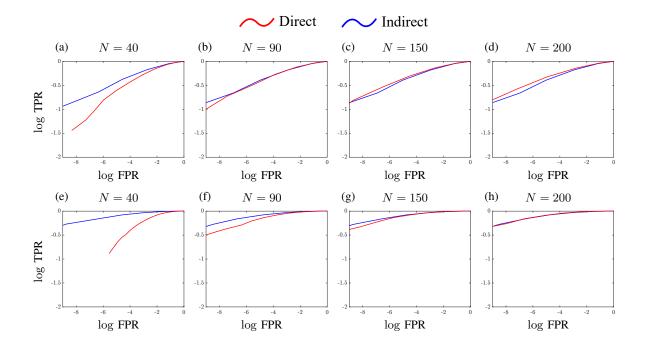


Fig. 1. ROC curves for the two proposed methods. For (a)-(d), detection horizon T = 7. The direct method performs worse than the indirect method in regimes of low data and soon outperforms the indirect method as the size of the data set N increases. For (e)-(h) detection horizon T = 14. Both methods show better performance, however the gap between the two methods increases when the data size is low and direct method takes more data to outperform the indirect method.

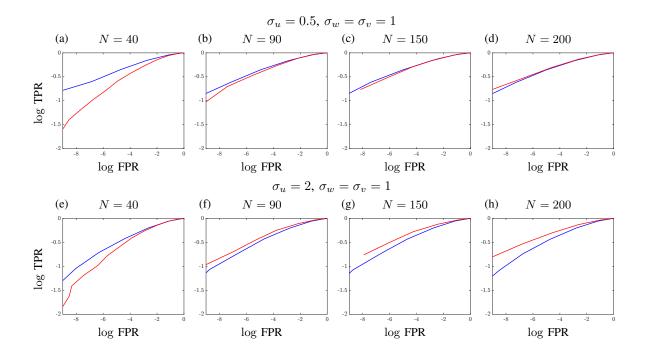


Fig. 2. ROC curves for different SNR with detection horizon T = 7. For (a)-(d) $\sigma_u = 0.5$, $\sigma_w = \sigma_v = 1$, which means that SNR is low. The direct method performs worse for the same amount of data with low SNR. For figures (e)-(h) $\sigma_u = 2$, $\sigma_w = \sigma_v = 1$, which means that SNR is high. The direct method performs better and takes fewer samples to outperform the indirect method. However, the indirect method is affected less by the change in SNR.

regimes. This study shows that neither method has invariably superior performance and that the choice of method must be based on the size and characteristics of the available data set.

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APPENDIX

A. Proof of Theorem 3.1

We provide a sketch of the proof in two parts. We first prove the consistency of the direct method using the

properties of the Wishart distribution [17]. Next, we prove the consistency of the indirect method using properties of the OLS solution.

1) Direct method: The sample covariance matrix is an unbiased estimator of the true covariance. Since $z^{(i)}$ are i.i.d. gaussian experiments, ZZ^{\top} in fact follows a Wishart distribution [17]. By the properties of the Wishart distribution, for every element \hat{S}_{ij}^d ,

$$\mathbb{V}\mathrm{ar}\left[\hat{S}_{ij}^{d}\right] = \frac{1}{N} \left(S_{ij}^{2} - S_{ii}S_{jj}\right)$$

Therefore, as $N \to \infty$, $\mathbb{V}ar\left[\hat{S}_{ij}^d\right] \to 0$. By applying the Chebyshev inequality for every element \hat{S}_{ij}^d , it follows that the sample covariance is a consistent estimator of the covariance S.

2) Indirect method: First, consider the augmented behavior $h_t = [f_t^\top | \zeta_t^\top]^\top$, where ζ_t defined as,

$$\zeta_t = [w_{t-L}^{\top} \ w_{t-L+1}^{\top} \ \dots \ w_t^{\top} | v_{t-L}^{\top} \ v_{t-L+1}^{\top} \ \dots \ v_t^{\top}]^{\top}.$$

When L = n, h_t follows a stationary Vector Auto-Regressive(1) process such that $h_{t+1} = \mathcal{G}h_t + e_t$, where e_t is a vector of external noises. We divide the matrix \mathcal{G} into block matrices G_{11}, G_{12}, G_{21} , and G_{22} . $G_{11} \in \mathbb{R}^{L(p+m) \times L(p+m)}$ captures the dependence of future minor behaviors f_{t+1} on past minor behaviors f_t which is in the Brunovsky canonical form. G_{12} captures the dependence of f_{t+1} on the past noises ζ_t . G_{21} is a zero matrix as the noises ζ_t do not depend on the behaviors f_t . Lastly, G_{22} is the matrix that captures the dependence of future noise ζ_{t+1} on past noises ζ_t . Let the covariance of the augmented behaviors be $\mathbb{E}[h_t h_t^{\top}] = \mathcal{H}$. Then, $\mathcal{H} = \operatorname{Cov}[\mathcal{G}h_t + e_t] = \mathcal{GHG}^\top + \Sigma_e$, which is in the form of a Lyapunov equation. Therefore, it is sufficient to know \mathcal{G} and Σ_e to compute \mathcal{H} . However, we are only interested in the first $L(m+p) \times L(m+p)$ block of \mathcal{H} as it captures the covariance of the minor behaviors f_t given by P from equation (6). We now describe how P can be computed from the Lyapunov equation $\mathcal{H} = \mathcal{GHG}^{\top} + \Sigma_e$.

Define the (1,1), (1,2), (2,1) and (2,2) blocks of \mathcal{H} as P_{11}, P_{12}, P_{21} and P_{22} of appropriate size. It is crucial to note that $P_{11} = P$ in equation (6) as it captures the covariance of the minor behaviors. Then P_{11} can be computed by solving the Lyapunov equation as,

$$P = (\mathcal{G}_{11} + \mathcal{G}_{12}P_{21}P^{-1})P(\mathcal{G}_{11} + \mathcal{G}_{12}P_{21}P^{-1})^{\top} + \Sigma_{e11} + \mathcal{G}_{12}(\mathcal{H}/P)\mathcal{G}_{12}^{\top}.$$
(16)

Let $\tilde{\mathcal{G}} = \mathcal{G}_{11} + \mathcal{G}_{12}P_{21}P^{-1}$ and $\tilde{\Sigma} = \Sigma_{e11} + \mathcal{G}_{12}(\mathcal{H}/P)\mathcal{G}_{12}^{\top}$. Then equation (16) can be rewritten as $P = \tilde{\mathcal{G}}P\tilde{\mathcal{G}} + \tilde{\Sigma}$. This implies that there exists a VAR(1) system such that $f_{t+1} = \tilde{\mathcal{G}}f_t + h_t$, where $h_t \sim \mathcal{N}(0, \tilde{\Sigma})$.

Next, we prove that regressing future minor behaviors over past minor behaviors using OLS yields \tilde{G} . We seek to solve the following problem.

$$\min_{\hat{G}} \mathbb{E}\left[\|f_{t+1} - \hat{G}f_t\|^2\right]$$
(17)

Note that $f_{t+1} = \mathcal{G}_{11}f_t + G_{12}\zeta + \tilde{e}_t$, where \tilde{e}_t is the first L(m+p) elements of the vector e_t . Then,

$$\begin{split} \mathbb{E}[\|f_{t+1} - \hat{G}f_t\|] &= \mathbb{E}\left[\|\left((\mathcal{G}_{11} - \hat{G})f_t + \mathcal{G}_{12}\zeta_t + \tilde{e}_t\right)\|\right] \\ &= \mathbb{E}\left[\left((\mathcal{G}_{11} - \hat{G})f_t + \mathcal{G}_{12}\zeta_t + \tilde{e}_t\right)^\top \\ \left((\mathcal{G}_{11} - \hat{G})f_t + \mathcal{G}_{12}\zeta_t + \tilde{e}_t\right)\right] \\ &= \mathbb{E}\left[\mathrm{Tr}\left(\left((\mathcal{G}_{11} - \hat{G})f_t + \mathcal{G}_{12}\zeta_t + \tilde{e}_t\right) \\ \left((\mathcal{G}_{11} - \hat{G})f_t + \mathcal{G}_{12}\zeta_t + \tilde{e}_t\right)^\top\right)\right] \\ &= \mathrm{Tr}\left((\mathcal{G}_{11} - \hat{G})P(\mathcal{G}_{11} - \hat{G})^\top \\ &+ (\mathcal{G}_{11} - \hat{G})P_{12}\mathcal{G}_{12}^\top + \mathcal{G}_{12}P_{21}(\mathcal{G}_{11} - \hat{G})^\top\right) \end{split}$$

Optimality condition for the problem occurs when the derivative of the cost function with respect to \hat{G} is 0. Differentiating the cost function using above expression with respect to \hat{G} and setting it to 0, we obtain,

$$\frac{d \mathbb{E}\left[\|f_{t+1} - \hat{G}f_t\|^2\right]}{d\hat{G}} = (\mathcal{G}_{11} - \hat{G})P - \mathcal{G}_{12}P_{21} = 0$$
$$\hat{G} = \mathcal{G}_{12}P_{21}P^{-1} + \mathcal{G}_{11} = \tilde{G}.$$

Therefore, as $N \to \infty$, $\hat{G} \to \tilde{G}$. Residuals of the least squares regression are now $f_{t+1} - \tilde{G}f_t$. By computing the covariance of the residuals, we obtain

$$\mathbb{E}\left[(f_{t+1} - \tilde{G}f_t)(f_{t+1} - \tilde{G}f_t)^{\top}\right] = \Sigma_{e_{11}} + \mathcal{G}_{12}(\mathcal{H}/P)\mathcal{G}_{12}^{\top}.$$

Therefore the ordinary least squares problem is consistent with equation (16). Further, from equation (6) we have that, $\Sigma_{\epsilon} = \Sigma_{e11} + \mathcal{G}_{12}(\mathcal{H}/P)G_{12}^{\top} = \tilde{\Sigma}$. This also proves that the residuals are uncorrelated with the regressors f_t .

B. Proof of Theorem 3.2

We provide a sketch of the proof. We follow the approach from [18]. Firstly, the i.i.d. experiments $z^{(i)}$ satisy the Bernstein moment condition. The Bernstein moment condition states that, for any sequence of random variables $\xi_i \sim \mathcal{N}(0, \Sigma_{\xi})$, and for any $l \geq 2$, and H > 0

$$\mathbb{E}\left[(\xi_i\xi_i^{\top})^l\right] \preceq \frac{l!}{2}H^{l-2}J,$$

where J is a positive definite matrix. Then it follows from Theorem 3.6 in [19] that, for any $\theta > 0$,

$$\lambda_1(\sum_i \xi_i \xi_i^{\top}) \ge N\lambda_1(S) + \sqrt{2N\theta\lambda_1(J)} + \theta H, \quad (18)$$

$$\lambda_d(\sum_i \xi_i \xi_i^\top) \le N\lambda_d(S) - \sqrt{2N\theta\lambda_d(J)} + \theta H, \quad (19)$$

with probability not more than $de^{-\theta}$. Here λ_1 and λ_d denote the largest and smallest eigenvalues, respectively, d is the length of the vector ξ_i and N is the sample size. Next, to provide error bounds we follow the Bernstein moment condition for $(\boldsymbol{z}^{(i)}\boldsymbol{z}^{(i)^{\top}} - S)$. Using Lemma 4 of [18] we get:

$$\mathbb{E}\left[\boldsymbol{z}^{(i)}\boldsymbol{z}^{(i)^{\top}} - S\right)^{l} \leq \frac{l!}{2}H^{l-2}J,$$
$$\mathbb{E}\left[S - \boldsymbol{z}^{(i)}\boldsymbol{z}^{(i)^{\top}}\right] \leq \frac{l!}{2}H^{l-2}J,$$

where J = Tr(S)S and H = 2Tr(S). Using $r = \frac{Tr(S)}{S}$, we have that $||J|| \le (r+1)||S||^2$ and H = 2r||S||. From equations (18) and (19), we get for any $\theta > 0$,

$$\left\|\frac{1}{N}\boldsymbol{Z}\boldsymbol{Z}^{\top} - S\right\| \ge \left(\sqrt{\frac{2\theta(r+1)}{N}} + \frac{2\theta r}{N}\right) \|S\|$$
(20)

with probability not greater than $2T(p+m)e^{-\theta}$. Note that equation (20) is a lower bound on $\|\frac{1}{N}ZZ^{\top} - S\|$. The upper bound on $\|\frac{1}{N}ZZ^{\top} - S\|$ follows with probability $1 - 2T(p+m)e^{-\theta}$.

C. Proof of Lemma 3.3

We provide a sketch of the proof for brevity.

(a) Sensitivity of P: We obtain this result from the characterizing the sensitivity of the Lyapunov equation (6). We use the result from Corollary 2.7 of [20]

(b) Sensitivity of \mathcal{F} : We exploit the structure of $\hat{\mathcal{F}}$ to obtain the bound. We have the same elements in the off-diagonals of the matrix.

$$\begin{split} \hat{\mathcal{F}} &= \begin{bmatrix} I & \mathcal{M} & \dots & \mathcal{M}^{T-L} \\ \hat{\mathcal{M}} & I & \dots & \hat{\mathcal{M}}^{T-L-1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathcal{M}}^{T-L} & \hat{\mathcal{M}}^{T-L-1} & \dots & I \end{bmatrix} = I \\ &+ \hat{\mathcal{M}} \otimes \left(\begin{bmatrix} 0 & I & 0 & \dots & 0 \\ 0 & 0 & I & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & I \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} + \begin{bmatrix} 0 & I & 0 & \dots & 0 \\ 0 & 0 & I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}^{\top} \right) \\ &+ & \vdots \\ &+ \hat{\mathcal{M}}^{T-L} \otimes \left(\begin{bmatrix} 0 & \dots & 0 & I \\ 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} + \begin{bmatrix} 0 & \dots & 0 & I \\ 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix}^{\top} \right) . \end{split}$$

Here, all the matrices with only the identity matrices on the off diagonal positions have an operator norm equal to 1 as they have utmost 1 identity per column. We use the property that for any arbitrary matrices X and Y of appropriate dimensions, $||X \otimes Y|| = ||X|| ||Y||$. Consequently,

$$\begin{aligned} \|\hat{\mathcal{F}}\| &\leq \|1+2\|\hat{\mathcal{M}}\|+2\|\hat{\mathcal{M}}^2\|+\dots+2\|\hat{\mathcal{M}}^{T-L}\| \\ &\leq 1+2\|\hat{\mathcal{M}}\|+2\|\hat{\mathcal{M}}\|^2+\dots+2\|\hat{\mathcal{M}}\|^{T-L} \end{aligned} \blacksquare$$