# Double Shock Control Bump Design Optimization Using Hybridised Evolutionary Algorithms

DongSeop Lee, Jacques Periaux, Jordi Pons-Prats, Gabriel Bugeda and Eugenio Oñate

Abstract—The paper investigates two advanced optimisation methods for solving active flow control device shape design problem and also compares their optimisation efficiency in terms of computational cost and design quality. The first optimisation method Hierarchical Asynchronous Parallel Multi-Objective Evolutionary Algorithm (HAPMOEA) and the second uses Hybridized EA with Nash-Game strategies. Both optimisation method are based on a canonical evolution strategy and incorporates the concepts of parallel computing and asynchronous evaluation. For the practical test case, one of active flow control devices named Shock Control Bump (SCB) is considered and it is applied to Natural Laminar Flow (NLF) aerofoil. The concept of SCB is to decelerate supersonic flow on upper/lower surface of transonic aerofoil that leads delay of shock occurrence. Such active flow technique reduces a total drag at transonic speeds.

Numerical results clearly show that Hybrid-Game helps EA to accelerate optimisation process, and also applying SCB on the suction and pressure sides significantly reduces transonic wave drag and improves lift on drag (L/D) value when compared to the baseline design.

*Keywords*—Active Flow Control, Shock Control Bump, Shape design optimisation, Hybrid-Game, Nash Equilibrium, Evolutionary Algorithm.

## I. INTRODUCTION

DEVELOPING efficient optimisation techniques is the most challenging task in the field of Evolutionary Algorithms (EAs) research due to the complexity of modern design problems. One of emerging techniques to improve an optimisation performance can be the use of Nash-equilibrium concept which will be acting as a pre-conditioner of global

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optimizer.

Lee et al. [1] studied the concept of Hybrid-Game (Pareto + Nash) coupled to a well-known MOEA; Non-dominating Sort Genetic Algorithm II (NSGA-II) [2] to solve Unmanned Aerial System (UAS) multi-objective Mission Path Planning System (MPPS) design problems where the Hybrid-Game improves the NSGA-II performance by 80% when compared to the original NSGA-II. In addition, Lee et al. [3] hybridised NSGA-II with Nash-Game strategy to study a role of Nash-Players in Hybrid-Game by solving multi-objective mathematical design problems convex, discontinuous, etc. And Hierarchical Asynchronous Parallel Multi-Objective Evolutionary Algorithm (HAPMOEA) [4] is also hybridised to solve a real-world robust multidisciplinary design problem. Numerical results show that the Hybrid-Game improves 70% of HAPMOEA performance while producing better Pareto optimal solutions. References [1, 3, 5] clearly describes merits of using Hybrid-Game coupled to MOEA for engineering design applications which consider a complex geometry or a large number of design variables. Hybrid-Game has two major characteristics; the first is a decomposition of design problem, for instance, a multi-objective design problem will be split into several simpler single-objective problems corresponding to Nash-Players which have their own design search space. The second is that Nash-Players are synchronised with Global/Pareto-Player as a pre-conditioner hence Pareto-Player can accelerate the optimisation process by using a set of elite designs obtained by the Nash-Players during

The main goal of this paper is to investigate an efficiency of Hybrid-Game for a single-objective design problem. The search space herein will be decomposed respect to Nash-Players. In this paper, HAPMOEA is hybridized with Nash game strategy to improve optimisation efficiency. Both optimisation methods are implemented to Active Flow Control (AFC) device shape design optimisation and their performance are compared in terms of computational cost and design quality.

Recent advances in design tools, materials, electronics and actuators offer implementation of flow control technologies to improve aerodynamic efficiency [6 -10]. Such aerodynamic improvement saves mission operating cost while condensing critical aircraft emissions. The main benefits of using ACF

techniques on current transonic aircraft are the improvement of the aerodynamic efficiency and the reduction of the manufacturing cost when compared to designing new airfoil or wing planform shape.

In this paper, one of active flow control devices; double Shock Control Bump (SCB) [8-10] on the suction and pressure sides of Natural Laminar Flow (NLF) aerofoil; RAE 5243 [10, 11] is investigated in order to reduce a transonic total drag, especially a wave drag at the critical flight conditions where two shocks occur.

The rest of paper is organised as follows; Section II describes optimisation method. Aerodynamic analysis tools are demonstrated and validated at Section III. Section IV demonstrates the use of SCB. Section V considers double SCB design optimisation using HAPMOEA and Hybrid-Game. Section VI delivers conclusion and future works.

#### II. OPTIMISATION METHODS

The evolutionary algorithm used in this paper is based on Covariance Matrix Adaptation Evolutionary Strategies (CMA-ES) [12, 13] which incorporates an asynchronous parallel computation and a Pareto tournament selection [14 -16]. The first method; HAPMOEA uses the concept of hierarchical multi-population topology which can handle different models including precise, intermediate and approximate models. Each node (Node0 ~ Node6) belonging to the different hierarchical layer can be handled by a different EA code. The second method hybridises HAPMOEA by applying a concept of Nash-Equilibrium instead of the concept of hierarchical multi-population topology [4, 17]. The Nash-Game players choose their own strategy to improve their own objective. Both HAPMOEA and Hybridised EA are coupled to the aerodynamic analysis tool. Details and validations of HAPMOEA and Hybrid-Game can be found in references [1, 3, 17].

#### III. AERODYNAMIC ANALYSIS TOOLS

In this paper, Euler + Boundary layer: MSES written by Drela [18] is utilised. The MSES software is a coupled viscous/inviscid Euler method for the analysis and design of multi-element/single-element aerofoils. It is based on a streamline-based Euler discretization and a two-equation integral boundary layer formulation coupled through the displacement thickness and solved simultaneously by a full Newton method. To obtain a prescribed lift coefficient  $C_l$ , the angle of attack ( $\alpha$ ) of the aerofoil is adapted or the other way around. Figure 1 a) shows the mesh (36 × 213) obtained by MSES that will be applied to optimisation.

The validation test is conducted on  $4 \times 2.8$  GHz CPU. The flight conditions are Mach number  $(M_{\infty}) = 0.729$ , angle of attack  $(\alpha) = 2.31^{\circ}$  and Reynolds number  $(Re) = 6.5 \times 10^{6}$ . Figure 1 b) compares the results obtained by MSES to the wind tunnel experimental data [19]. It can be seen there is good agreement between Cp distributions obtained by MSES and the wind tunnel data.

## IV. WAVE DRAG REDUCTION VIA SHOCK CONTROL BUMP

At transonic speed, the flow over the high camber wing causes shock waves where there is a large amount of gas property changes and the flow becomes irreversible.

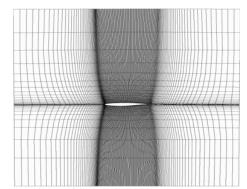


Fig. 1 a). RAE 2822 aerofoil mesh obtained by MSES.

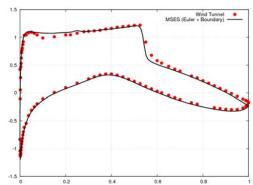


Fig. 1 b). Comparison of *Cp* distributions obtained by MSES (line) and wind tunnel experimental data (dots).

Through the shock, total pressure decreases and entropy increases which means there is a loss of energy. In other words, there is an increment of wave drag. To cope with this problem, Ashill et al. 1992 [7] proposed the concept of a transonic bump which so-called Shock Control Bump (SCB) by using geometry adaption on an aerofoil. As illustrated in Figure 2, the typical design variables for SCB are: length, height and peak position and, the center of SCB will be located at sonic point where the flow speed transits from supersonic to subsonic on the transonic aerofoil design.

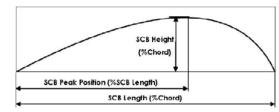


Fig. 2. Design components of Shock Control Bump.

Figure 3 illustrates the concept and benefit of using SCB. The transonic flow over normal aerofoil without SCB accelerates to supersonic and the pressure forms a strong shock

that leads a high wave drag ( $Cd_{Wave}$ ). However, the pressure difference over the SCB causes a deceleration of supersonic flow which delays shock occurrence. SCB cannot totally remove a shock, but it produces a weaker shock or breaks into isentropic compression waves (lower  $Cd_{Wave}$ ).

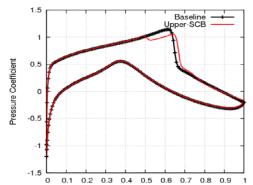


Fig. 3.  $\it{Cp}$  distributions obtained by RAE 2822 (dots and line) and with SCB (line).

Table I compares the aerodynamic performance obtained by RAE 2822 and with SCB. Even though applying SCB on RAE 2822 produces 5% higher viscous drag ( $\Delta Cd_{Viscous} = 0.0005$ ), it reduces 60% wave drag ( $\Delta Cd_{Wave} = 0.0036$ ) while improving 19% of L/D when compared to RAE 2822 aerofoil.

TABLE I AERODYNAMIC CHARACTERISTICS

Aerofoil	$Cd_{Total}$	$Cd_{Viscous}$	$Cd_{Wave}$	L/D
RAE 2822	0.0153	0.0093	0.0060	34.34
with SCB	0.0123 (-20%)	0.0098 (+5%)	0.0024 (-60%)	42.6 (+24%)

Note:  $M_{\infty} = 0.77$ , Re = 17.93 ×10<sup>6</sup> and *Cl* is fixed to 0.524.

Applying SCB on either upper or lower surface of aerofoil will produces slightly thicker thickness ratio (t/c) which causes increment of viscous drag ( $Cd_{Viscous}$ ) however the use of SCB is still beneficial due to  $Cd_{Wave}$  reduction especially when the Mach number is higher than critical Mach number where the shock starts appearing.

In Section V, the shape of SCB is optimized at critical flight conditions where two shocks occur on the suction and pressure sides of aerofoil. This flight conditions make a suitable application for Hybrid-Game (Global + Nash) since two SCBs are required. The aerodynamic characteristics of baseline with optimal double SCB are also investigated at normal flight conditions where a single shock is on the upper surface of aerofoil.

#### V. SCB DESIGN OPTIMISATION ON RAE 5243

For baseline design, a natural laminar flow aerofoil RAE 5243 is selected as shown in Figure 4 a). The problem considers the critical flow conditions;  $M_{\infty} = 0.8$ ,  $C_l = 0.175$ ,  $Re = 18.63 \times 10^6$  where two shocks occur on the suction and pressure sides of RAE 5243 aerofoil as shown in Figure 4 b).

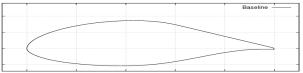


Fig. 4 a). Baseline design (RAE 5243) geometry (Note:  $\max t/c = 0.14$  at 41%c and  $\max \text{ camber} = 0.018$  at 54%c).

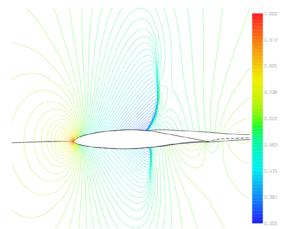


Fig. 4 b).  $P/P_0$  contour of RAE 5243.

The upper and lower sonic points are occurred at 62.6% and 58.1% of chord respectively. In following sections, double SCB design optimization using HAPMOEA and Hybrid-Game are conducted to minimize the total drag ( $Cd_{Total}$ ). The aerodynamic analysis tool; MSES will run two times at each function evaluation; the first run will analyse upper SCB and then upper and lower SCB will be analysed at the second run.

## A. Evaluation Mechanism for HAPMOEA and Hybrid-Game

Figure 5 a) shows the evaluation mechanism for HAPMOEA which consists of hierarchical multi-population (Node 0  $\sim$  Node 6) based on multi-resolution. Each population will run aerodynamic analysis tool two times to evaluation double SCB design in different resolution conditions.

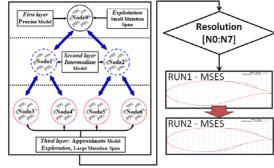


Fig. 5 a). Evaluation mechanism of HAPMOEA.

Figure 5 b) shows the evaluation mechanism for Hybrid-Game which employs three players; Global-Player and Nash-Player 1 and 2. Solely Global-Player runs aerodynamic analysis tool two times since its optimisation domain includes

both upper and lower SCB. However, analysis tool will run only once for Nash-Player 1 and 2 due to the Nash-Game characteristics; decomposition of problem. For Hybrid-Game, double SCB design problem becomes two single SCB design problems; Nash-Game 1 will only optimize upper SCB with elite lower SCB obtained by Nash-Player 2 while Nash-Player 2 will optimise lower SCB with elite upper SCB design from Nash-Player 1. The elite designs obtained by Nash-Players will be seed to the population of the Global-Player that will allow Global-Player to accelerate optimisation process.

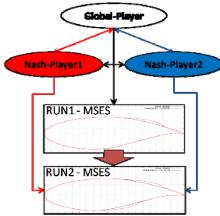


Fig. 5 b). Evaluation mechanism of Hybrid-Game.

## B. SCB Design Optimisation using HAPMOEA

## Problem Definition

This test case considers a single objective double SCB design optimisation using HAPMOEA to minimize total drag ( $Cd_{Total}$ ) which consists of viscous drag ( $Cd_{Viscous}$ ) and wave drag ( $Cd_{Total}$ ). The flow conditions are  $M_{\infty}=0$ . 8,  $C_l=0.175$ ,  $Re=18.63\times10^6$ . The fitness function is shown in equation (1).

$$f(U\_SCB, L\_SCB) = \min(Cd_{Total})$$
 (1)

where  $Cd_{Total} = Cd_{Viscous} + Cd_{Wave}$ .

## Design Variables

The design variable bounds for both SCB on the suction and pressure sides are illustrated in Table II. In total, six design variables are considered for double SCB.

TABLE II SCB DESIGN VARIABLES AND BOUNDS

DCD D	SCD DESIGN VARIABLES AND BOUNDS				
Design Variables	Lower bound	Upper bound			
Length (% chord)	15	30			
Height (% chord)	0.15	0.65			
Peak position	0	100			

Note: Peak position is in % of SCB length.

The centre of SCB (50% of SCB length) will be positioned where the flow speed transits from supersonic to subsonic.

## **Implementation**

The following conditions are for multi-resolution/population hierarchical populations.

- $1^{st}$  Layer: Population size of 10 with a computational grid of  $36 \times 213$  points (Node0).
- $2^{nd}$  Layer: Population size of 20 with a computational grid of  $24 \times 131$  points (Node1, Node2).
- $3^{rd}$  Layer: Population size of 20 with a computational grid of  $36 \times 111$  points (Node3 ~ Node6).

Note: these grid conditions produce less than 5% accuracy error compared to precise model at the 1<sup>st</sup> layer (Node0).

#### Numerical Results

As illustrated in Figure 6, the algorithm was allowed to run for 24 hours and 2,508 function evaluations using single  $4 \times 2.8$  GHz processor. The algorithm converged at 1,053 function evaluations with  $Cd_{Total} = 0.03441$  after ten hours.

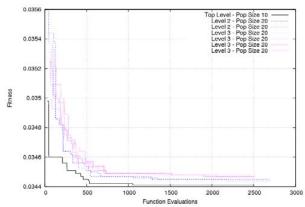


Fig. 6. Convergence objective using HAPMOEA

Table III compares the aerodynamic characteristics obtained by the baseline design (RAE 5243) and the baseline design with upper and lower SCBs. Applying SCB on upper surface of RAE 5243 aerofoil saves the wave drag by 8% which leads 12% of total drag reduction. This optimal double SCB improves L/D by 13.0%.

TABLE III

AEROD I NAMIC CHARACTERISTICS				
Aerofoil	Aerofoil $Cd_{Total}$ $Cd_{Wave}$			
Baseline	0.03898	0.0088	4.49	
with SCB	0.03441 (- 12%)	0.0081 (-8%)	5.08 (+ 13%)	

Note: Cl is fixed to 0.175.

TABLE IV

Variables	Peak Position		
U_SCB	23.31	0.649	84.95
L_SCB	26.38	0.477	75.98

Note: *Peak position* is in % of SCB length. The U\_SCB starts from x and y coordinates (0.5084, 0.0838) to (0.7416, 0.0480) and L\_SCB is positioned from (0.4397, -0.05269) to (0.7035, -0.0258).

The optimal double SCB shape of is described in Table IV. Figure 7 compares the geometry of the baseline design and baseline with the optimal SCB which has same t/c while the max camber is increased by 0.0005 and its position is moved 16%c towards to the trailing edge when compared to the baseline design.

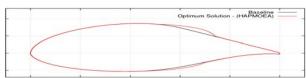


Fig. 7. Baseline design with the optimal SCB obtained by HAPMOEA (Note:  $\max t/c = 0.14$  at 41%c and  $\max$  camber = 0.0209 at 69.8%c).

Figure 8 shows the contour of baseline design with the optimal double SCB. It can be seen that the strong shocks on the baseline design shown in Figure 4 b) get weaker by adding double SCB.

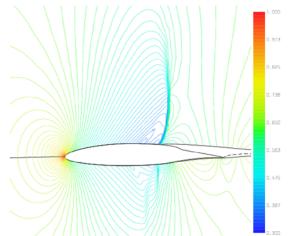


Fig. 8.  $P/P_0$  contour of the optimal SCB solution obtained by HAPMOEA.

Figure 9 compares *Cp* distribution obtained by the baseline design and the baseline design with upper and lower SCB. It can be seen that the total drag is reduced by 9% while the double SCB reduces 12% of total drag. The shock on upper surface is delayed while the shock on lower surface becomes weak isentropic waves.

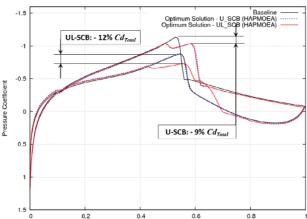


Fig. 9. *Cp* distributions obtained by the baseline (black line) and the baseline with the optimal double SCB solution (broken line: only upper SCB, red line: upper and lower SCBs).

## C. SCB Design Optimisation using Hybrid-Game

#### Problem Definition

This test case considers a single objective double SCB design optimisation using Hybrid-Game on MOEA to minimize total drag at flow conditions  $M_{\infty} = 0.8$ ,  $C_l = 0.175$ ,  $Re = 18.63 \times 10^6$ . Hybrid-Game consists of three players; one Global-Player (*GP*), two Nash-Players (*NP1* and *NP2*) instead of hierarchical multi-population/resolution (Node 0 ~ Node 6). The fitness functions for Hybrid-Game are shown in equation (2).

$$f_{GP}(U\_SCB, L\_SCB) = \min(Cd_{Total})$$

$$f_{NP1}(U\_SCB, L\_SCB^*) = \min(Cd_{Total})$$

$$f_{NP2}(U\_SCB^*, L\_SCB) = \min(Cd_{Total})$$
(2)

where  $Cd_{Total} = Cd_{Viscous} + Cd_{Wave}$ .  $U\_SCB$  and  $L\_SCB$  represent upper and lower SCB, and \* is elite SCB design obtained by Nash-Players.  $U\_SCB^*$  and  $L\_SCB^*$  are elite SCB designs obtained by Nash-Player 1 and 2. These elite SCB designs will be seeded to the population of Global-Player at every 10 function evaluations and will act as a pre-conditioner.

### Design Variables

The design variable bounds for the upper and lower SCB geometry are illustrated in Table II. Table V shows design variable distribution for Hybrid-Game. It can be seen that the Nash-Players 1 and 2 consider only 3 design variables while the Global-Player of Hybrid-Game considers 6 design variables.

TABLE V
DESIGN VARIABLE DISTRIBUTION FOR HYBRID-GAME

Type of		Hybrid-Game	;	HAPMOEA
SCB	GP	NP1	NP2	(Node 0 ~ 6)
U_SCB	<b>√</b>	<b>V</b>		√
L SCB	√		√	√

Note: GP, NP1 and NP2 represent global player and Nash-Players 1 and 2.

## **Implementation**

The following conditions are for Hybrid-Game; Global-Player, Nash-Player 1 and Nash-Player 2.

- GP: Population size of 10 with a grid of  $36 \times 213$ .
- NP1: Population size of 10 with a grid of  $36 \times 213$ .
- NP2: Population size of 10 with a grid of  $36 \times 213$ .

## Numerical Results

As illustrated in Figure 10, the algorithm was allowed to run for 5 hours and 1,775 function evaluations using single  $4 \times 2.8$  GHz processor. The algorithm converged at 683 function evaluations (approximately 1.9 hours) with  $Cd_{Total} = 0.03437$  which HAPMOEA could not capture even after 24 hours shown in previous test Section B. To compare the computational efficiency of HAPMOEA and Hybrid-Game, the fitness value is chosen to  $Cd_{Total} = 0.03441$  which HAPMOEA captured after 10 hours. The Hybrid-Game took 1.48 hours which is only 15 % of HAPMOEA computational cost. In other words, Nash-Game improves performance of EA by 85% as shown in Figure 11.

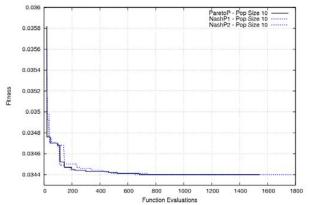


Fig. 10. Convergence objective using Hybrid-Game.

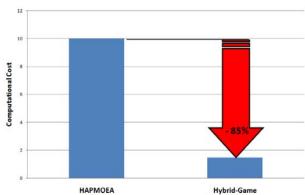


Fig. 11. Performance comparison between HAPMOEA and Hybrid-Game.

Table VI compares the aerodynamic characteristics obtained by the baseline design (RAE 5243) and the baseline design with upper SCB. Applying SCB on upper surface of RAE 5243 aerofoil saves the wave drag by 8% which leads 12% of total drag reduction. This optimal double SCB improves *L/D* by 13.0%.

TABLE VI AERODYNAMIC CHARACTERISTICS

Aerofoil	$Cd_{Total}$	$Cd_{Wave}$	L/D
Baseline	0.03898	0.0088	4.49
with SCB	0.03437 (- 12%)	0.0081 (-8%)	5.09 (+ 13%)

Note: Cl is fixed to 0.175.

The optimal double shape of double SCB obtained by Hybrid-Game is described in Table VII. It can be seen that the upper SCBs obtained by Hybrid-Game and HAPMOEA (Table IV) have almost same shape while the lower SCB from Hybrid-Game is 10% shorter than the lower SCB obtained by HAPMOEA.

 $TABLE\ VII$  Optimal SCB Design Variables obtained by Hybrid-Game

Variables	Length (%c)	Height (%c)	Peak Position
U_SCB	23.65	0.649	84.99
L_SCB	23.88	0.384	80.35

Note: *Peak position* is in % of SCB length. U\_SCB and L\_SCB represent SCB on the suction and pressure sides of RAE 5243 aerofoil. The U\_SCB starts from x and y coordinates (0.5067, 0.0839) to (0.7432, 0.04774) and L\_SCB position is located from (0.4521, -0.0528) to (0.6910, -0.0277).

Figure 12 illustrates the geometry of the baseline design and baseline with the optimal double SCB from HAPMOEA and Hybrid-Game. The baseline design with double SCB obtained by Hybrid-Game has same t/c while the max camber is increased by 0.00055 and its position is moved 15%c towards to the trailing edge when compared to the baseline design.

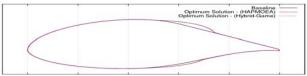


Fig. 12. Baseline design with the optimal double SCB (Note: max t/c = 0.14 at 41%c and max camber = 0.0214 at 69.0%c).

Figure 13 shows the pressure contour of baseline design with the optimal double SCB obtained by Hybrid-Game. It can be seen that the upper shock is moved toward to the trailing edge while lower shock becomes weak isentropic waves.

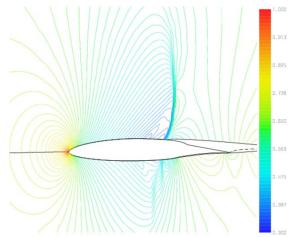


Fig. 13.  $P/P_{\theta}$  contour of the optimal double SCB solution obtained by Hybrid-Game.

Figures 14 a) and 14 b) compare a total drag ( $Cd_{Total}$ ) and a wave drag ( $Cd_{Wave}$ ) distributions obtained by the baseline design and with the optimal double SCB from both HAPMOEA and Hybrid-Game. The flow conditions are  $M_{\infty} \in [0.5:0.85]$  with constant  $Cl_{Fixed} = 0.175$  and  $Re = 18.63 \times 10^6$ . It can be seen that both optimal double SCB obtained by HAPMOEA and Hybrid-Game perform almost same drag along the Mach numbers. The baseline design with the optimal double SCB starts to produce lower total drag when Mach number is higher than 0.71. One thing should be noticed from Figure 14 b) is that the critical Mach number ( $M_C = 0.65$ ) for baseline design is extended to 0.71 by adding double SCB. The maximum total drag reduction (- 26%) is observed at  $M_{\infty} = 0.75$  shown in Figure 14 a) due to 88% of wave drag reduction shown in Figure 14 b) when compared to the baseline design.

The baseline design with the optimal double SCB obtained by HAPMOEA and Hybrid-Game is also tested at five different flight conditions. The histogram showed in Figure 15 a) compares the total drag. It can be seen that the double SCB optimized at critical flight conditions reduces more total drag by 15 to 44% while improving the lift to drag ratio by 13.5 to 80% as shown in Figure 15 b) at the normal flight conditions.

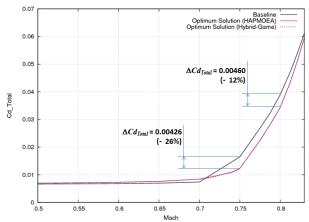


Fig. 14 a).  $Cd_{Total}$  vs. Mach numbers.

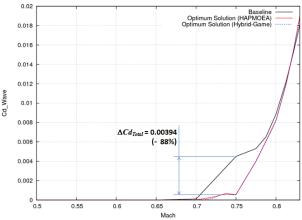


Fig. 14 b).  $Cd_{Wave}$  vs. Mach numbers.

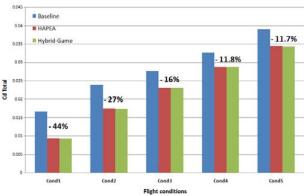


Fig. 15 a). Drag reduction obtained by the optimal double SCB at five different flight conditions.

Note: Cond, represents *i*th flight conditions.

 $\begin{aligned} & \operatorname{Cond_1:} M_{\infty} = 0.705, \, Cl = 0.690, \, Re = 18.63 \times 10^6 \\ & \operatorname{Cond_2:} M_{\infty} = 0.730, \, Cl = 0.560, \, Re = 18.63 \times 10^6 \\ & \operatorname{Cond_3:} M_{\infty} = 0.750, \, Cl = 0.430, \, Re = 18.63 \times 10^6 \\ & \operatorname{Cond_4:} M_{\infty} = 0.775, \, Cl = 0.300, \, Re = 18.63 \times 10^6 \\ & \operatorname{Cond_5:} M_{\infty} = 0.800, \, Cl = 0.175, \, Re = 18.63 \times 10^6 \end{aligned}$ 

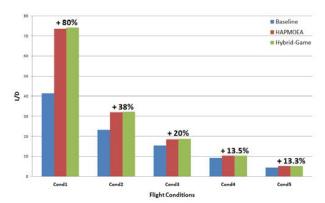


Fig. 15 b).  $L\!/\!D$  obtained by the optimal double SCB at five different flight conditions.

One example ( $Cond_1$ ) is shown in Figures 16 a) and 16 b) where the pressure ratio contours obtained by the baseline design and optimal solution from the Hybrid-Game are illustrated. Even though the double SCB is optimized at the critical flight condition, the optimal double SCB moves the normal strong shock shown in Figure 16 a) towards to the trailing edge by 10%c and reduce total drag by 44% which leads to 80% improvement of L/D.

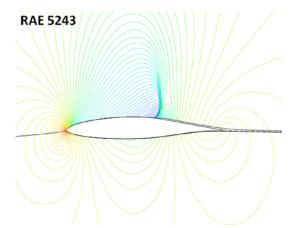


Fig. 16 a).  $P/P_{\theta}$  contour of the baseline design at Cond<sub>1</sub> (Fig. 15a).

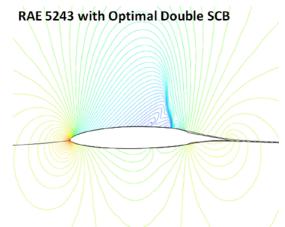


Fig. 16 b).  $P/P_{\theta}$  contour of the optimal double SCB obtained by Hybrid-Game at Cond<sub>1</sub> (Fig. 15a).

To summarise the optimisation test case, a double SCB on RAE 5243 is optimised using HAPMOEA and Hybrid-Game to reduce a transonic drag at the critical flight conditions. The use of optimal double SCB is beneficial at both normal and critical flow conditions. In addition, Hybrid-Game significantly reduces the computational cost for double SCB design optimisation while generating high quality optimal solution when compared to HAPMOEA.

The design engineer will choose the optimal double SCB obtained by Hybrid-Game which has 10% shorter length than SCB from HAPMOEA. In other words, the double SCB from Hybrid-Game will require less modification in current manufacturing system as well as less material.

#### VI. CONCLUSION

In this paper, two advanced optimisation techniques have been demonstrated and implemented as a methodology for active flow control bump named Shock Control Bump shape design optimisation. Analytical research clearly shows the benefits of using Hybrid-Game in terms of computational cost and design quality. In addition, the use of SCB on current aerofoil reduces significantly transonic drag. In long term view, the use of SCB will save not only operating cost but also critical aircraft emissions due to less fuel burn.

Future work will focus on robust multi-objective design optimization of SCB (Taguchi method) which can produce the model with better performance and stability at variability of operating conditions and transition positions. In forthcoming research, other evolutionary optimiser including Strength Pareto Evolutionary Algorithm 2 (SPEA2), Self-adaptive Pareto Differential Evolution (SPDE) will be hybridized with Nash-Game strategy and their results will be compared in terms of solution quality and computational cost.

#### ACKNOWLEDGMENT

The authors gratefully acknowledge Dr. Eric J. Whitney, and Mourad Sefrioui, Dassault Aviation for fruitful discussions on Hierarchical EAs and their contribution to the optimisation procedure and, also to M. Drela at MIT for providing *MSES* software.

This work has been supported by the Spanish Ministerio de Ciencia e Innovación through project DPI2008-05250.

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