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Heuristic Search Towards the Invention of an Optimal-Ignition Internal Combustion Engine

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***Abstract*—Most internal combustion engines are built on compression or spark ignition, which is far from optimal and the problem of which is more than optimization. This paper first improves a genetic algorithm (GA) for such an application, aiming at the potential invention of a homogeneous charge microwave ignition (HCMI) engine. For an HCMI system, search for optimal emitters under the intrinsic constraints of resonant frequencies forms a coupled constraint optimization problem and poses an intractable challenge to the GA and virtual prototyping for the invention. A predefined GA (PGA) is then developed to handle appropriate frequency ranges for this problem so as to allow the parameters of the emitter, as well as its structure, to be optimized in an evolutionary process. The heuristic search is compared with the deterministic NM simplex and the nondeterministic conventional GA. Results show that while the NM and GA heuristics find an insufficient mode, the PGA often finds the global maximum, with a higher convergence rate and independent of the algorithm's initial settings. When the complexity of the problem increases with the number of variables, the PGA also delivers a robust performance while the NM and the GA yield divergent results. This application confirms the viability and power of evolutionary heuristics in inventing novel real-world solutions if properly adapted.**

Keywords*—heuristic algorithm; coupled constraint optimization; homogeneous charge microwave ignition; internal combustion engine; evolutionary algorithm

I. Introduction

Like the principles of computational intelligence algorithms, microwave ignition was also first proposed in the 1950s [1]. It was to replace the spark ignition for a petrol or gasoline internal combustion engine (ICE) with volumetric ignition. An electromagnetic field, instead of an electric spark, is generated by microwave resonance, instead of a high voltage between electrodes of the sparkplug. Academic research into the potential invention of such an engine has then studied both engine and microwave aspects. Recently, University of Glasgow [2] have developed into a homogeneous charge microwave ignition (HCMI) system. The homogeneous charge air-fuel mixture can burn more thoroughly and faster with improved thermal efficiency and reduced emissions.

The success of HCMI primarily relies on the adequacy of the resonant power and the resonant frequency of the electromagnetic (EM) wave emitted into the engine cylinder. Theoretical analysis and simulations have shown that 100W input power is enough to generate an EM field of which 82% volume is above the required ignition field strength [3]. An extra complication is the unmatched impedance and hence unwanted microwave reflection, which reduces the EM field intensity and could also harm the microwave source. In [4], emitter parameters are isolated and optimized in simulation for impedance matching. On the other hand, a change in the emitter, as well as the piston position, air-fuel-ratio (AFR), cylinder diameter, etc., also changes the resonant frequency [3, 5]. Hence the emitter design in HCMI system should consider both the resonant frequency and impedance matching.

There are usually multiple parameters that define an HCMI emitter. Adjusting the parameters by a human engineer based on a usual trial-and-error approach would cost an impractical amount of time and scope. Using an automated exhaustive search with a computer based simulation module will save physical prototyping tests, but requires an exponential time and hence is intractable in reality. However, evolutionary computation can help find globally near-optimal parameters and their structure in a nondeterministic polynomial time. Such heuristic virtual prototyping allows the exploration of HCMI with a shortened design cycle.

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There exist various heuristic methods that have been widely studied for various optimization problems. The optimization problem arising from the design of an HCMI system is significantly challenging and different from many other application. The evaluation of optimal parameters depends on the resonant mode and frequency, leading to constraints coupled with arguments. Further, the sensitivity of resonant frequency implies that the search resolution on the frequency should be small, which creates a significant challenge to a generic GA.

In this paper, the problem of HCMI design with cylinder and emitter models are first described in Section II. In Section III, heuristic search methods adaptable to the presented problem are discussed first, including the Nelder-Mead (NM) simplex method and the conventional GA, and then a 'predefined genetic algorithm' (PGA) is developed. Two case studies are presented in Sections IV, with the three methods applied and compared in Section V. Conclusions are drawn in Section **Error! Reference source not found.**.

II. HCMI Design Evaluation and Virtual Prototyping Through Simulation

A. *Models of the Emitter and Cylinder*

The geometry model in Figure 1 is a basic model of cylinder with a single antenna in the center of the cylinder. This model consists of four parts: the cylinder head, the cylinder body, the emitter and a coaxial transmission line. The inly changeable variable of the emitter is the length of the emitter, which is denoted by a_1. There are four different materials used for this emitter design. The material inside the cylinder head and the cylinder body represents the homogeneous air-fuel. The inner material of the emitter, in this case part 3, is copper with a radius of 2.25 mm. The bound on the outside is steel with a fixed width of 2mm for this model. A dielectric material fill the gap between the steel and copper, according to a standard coaxial cable.

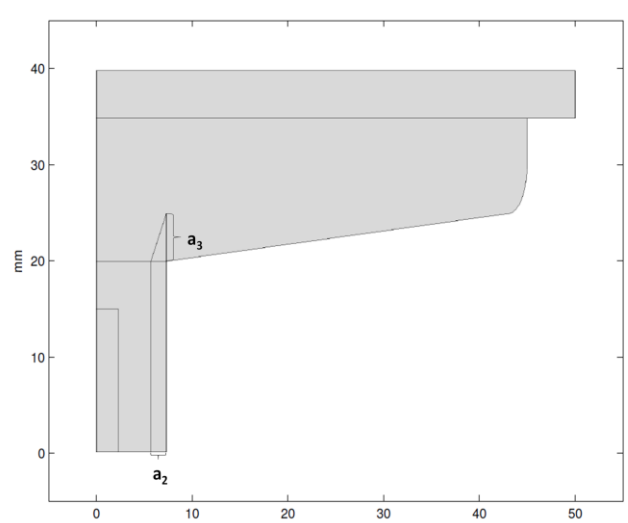

Figure 2 Extended model of an emitter

The antenna is extended by additional antenna designs in Figure 2 with two more variables, emitter width as $a_2$emitter height as a_3. The additional antenna is made of steel and connected to the outer shield of the emitter. The emitter width is limited, due to the dimensions of the cylinder and the emitter radius, in this case, to a maximum of 7.35mm. The emitter height starts at the top of the plug and can be in the range from 0.5 mm to 15 mm (the height of the cylinder head).

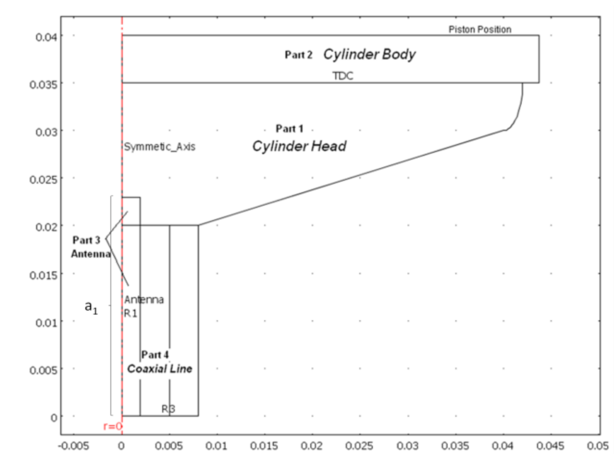

Figure 1 Default geometry model of the emitter and the cylinder

In order to obtain best propagation performance inside the combustion chamber, search methods are applied for emitter design. Ideally, the EM field inside the cylinder needs to reach a maximum electrical field. The simulation is built with COMSOL software and EM field intensity is calculated through the FEM solver.

The major challenge in the design process is the search resolution in the simulations. In the past, trial and error method is used to find optimal parameters: try one set of parameters and run the simulation to get resonant frequency and maximum EM intensity, then try another set of parameters to search for improvement, and so on. In this paper, an auto search based on optimization algorithm in Matlab is implemented. Values of parameters are the output of Matlab while input of simulation models in COMSOL. And maximum EM field intensity is feedback to optimization algorithm from simulation in COMSOL to get the next set of parameters. This optimization algorithm needs to be able to communicate with the simulation model to exchange data and perform a detailed search.

At each search iteration, once the parameters are determined, the frequency search is adopted in COMSOL to search for resonant frequency with the emitter parameters from optimization algorithm. In frequency search, different frequency microwave would generate EM field with different intensity. By search the maximum EM field intensity, the corresponding frequency is the resonant frequency. For this case, FEM simulation for EM field would be implemented multiple times for one set of parameters to determine maximum EM filed intensity and resonant frequency. For optimal parameters design, the FEM simulation could be the major cost of optimizer overhead.

B. *Coupled Constraint Optimization Problem*

The HCMI and its emitter design is a multivariable optimization problem. Their performance evaluation is mainly the EM field intensity. For optimal emitter parameters, the EM field intensity would need to be the maximum. Meanwhile, for each fixed set of parameters, resonant state of electromagnetic waves is reached if and only if the input frequency is a resonant frequency. The objective of this evaluation is to maximize the EM field intensity. Meanwhile, the size of the emitter is restricted by the structure of the cylinder head. This multivariable optimization problem can be described initially as:

$$E_{\max} = max\, E\ (a_1, a_2, a_3, f)$$
$$s.t.\quad g_i(a_i) \geq 0,\ i = 1, 2, 3$$
$$g_4(f) \geq 0 \tag{1}$$

where E is the intensity of the EM field, $a_i, i = 1,2,3$, are the emitter parameters, f is the frequency of the input microwave, and $g_i(a_i) \geq 0,\ and\ g_4(f) \geq 0$ are boundary conditions depending on the size of the cylinder and the frequency range.

For different sets of $a_i, i = 1,2,3$, the resonant frequency is different. In order to evaluate emitter parameters, the resonant frequency for each set of parameters much be determined first. For each set of $a_i, i = 1,2,3$, frequency of input microwave is resonant frequency if and only if the intensity of EM field is at the maximum. Thus f in optimization problem (1) is the implicit function of $a_i, i = 1,2,3$. Problem (1) is hence expressed as a 'coupled constraint optimization problem':

$$E_{\max} = max\, E\ (f | a_1, a_2, a_3)$$
$$s.t.\quad g_4(f) \geq 0 \tag{2}$$

$$E_{\max} = max\, E\ (a_1, a_2, a_3)$$
$$s.t.\quad g_i(a_i) \geq 0,\ i = 1, 2, 3 \tag{3}$$

To solve problem (3), problem (2) needs to be solved first. The EM field intensity, $E_{\max}$, is calculated using a finite element method (FEM). Hence the computational cost depends on searching times and solving process of FEM. For a cylinder with a pre-determined shape, the solving time of FEM for different emitter parameters varies little. In (2), the search range of frequency is given as $g_4(f) \geq 0$.

In the application to an HCMI system, it is desirable to have as few function evaluations as possible, due to the high computing cost in evaluating the EM virtual prototyping. If the search range of frequency is too small, the search result might reach to local extreme. If the search range of frequency is too broad, the computational cost would be high. The common search methods are with determined search range which becomes a dilemma for this specific application. It is necessary to improve current search method for tractable performance. To proceed, however, models of emitter and cylinder are introduced first.

III. HEURISTIC METHODS AND IMPROVED GA SEARCH

A. *Existing Heuristic Methods Tested*

The optimization problem described in section II can be divided into two sub-optimization problems: the search for resonant frequencies with fixed emitter parameters and the search for optimal emitter parameters. Both of the sub-problems have the same evaluation of intensity of the EM field. Hence the solution to such an optimization problem is:

Step 1: Generate emitter parameters as input variables by the search algorithm;

Step 2: Evaluate HCMI field strengths on the FEM models;

Step 3: Go back to Step 1 if search is not finished;

Step 4: Output the resonant frequency, emitter parameters and field intensity of the final set of candidates.

Figure 3 gives a diagram of a general solution to this optimization problem. The frequency is considered as input variables. There are various heuristic search methods that have been widely employed in various fields targeting similar optimization problems, whether they are deterministic or nondeterministic.

A typical deterministic example is the NM simplex method, which is a posteriori algorithm first proposed by Nelder and Mead in 1965 [6] . This method makes it possible to search and calibrate several design parameters at once. It finds a locally optimal solution to a problem with *N* variables if the objective function varies smoothly [7]. Though the NM search method is a local search method, it only requires a few function evaluations per iteration, in comparison to an

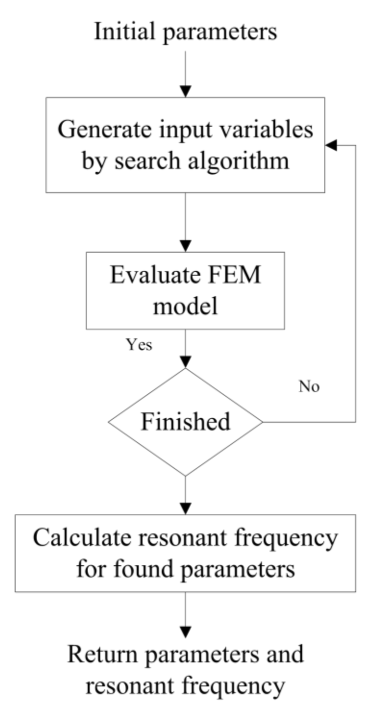

Figure 3 Optimal heuristic search

evolutionary algorithm (EA). The NM search requires a rough knowledge of the solution range along multioptimization. When resonant frequency is involved in optimizing the HCMI system, the performance of the NM method is inadequate.

An EA, on the other hand, is a non-deterministic search algorithm, with its idea originated from the 1950s. The EA includes three operators: selection, crossover and mutation. The EA begins with the creation of a random population of a defined number of individuals. Search performance also depends on how initial parameters are assigned, as they can affect the convergence towards global extrema.

B. Improved GA Search

The definition of initial parameters includes initial values and the search range of the parameters. If the search range is too small, the results could be local extreme, not a globe one. However, if the search range is too wide, the search would cost a lot of time. In the application to HCMI, the time of solving the FEM relates to the time of search iterations. Thus, a suitable definition of initial parameters is important.

To address this issue, a ‘predefined genetic algorithm’ is developed to narrow down the search range for each search of the emitter parameters. The PGA will generate an initial population, as well as calculate the frequency range before starting the search for emitter parameters. To pre-search the frequency range, the algorithm will select characteristic values out of the initial population and locate the resonance frequency for these values. Additional to the characteristic values, the algorithm will also select a defied number of random parameters out of the initial population and evaluate their corresponding resonant frequency. The frequency range will be calculated from the found minimum and maximum and expanded by a defined boundary. Therefore, the defined frequency range will be appropriate to the minimum required range for the given search parameters. Once the search range of frequency is determined, the GA is applied to search for optimal emitter parameters. Detailed in Figure 4, a full implementation procedure of PGA is presented as follows.

1) ***Initialization****:* Generate an initial population$\{A_0^i, i = 1,2,\cdots,N\}$, which are drawn from the given input parameters. N is the population size. $A_0^i = \{a_j, j = 1,2,3\}$, a_j are the emitter parameters and $g_j(a_j) \geq 0$, A_0^i is regarded as an individual.

2) ***Calculate the frequency range***: Obtain the frequency range $[f_{min}, f_{max}]$.

a) Select characteristic values λ out of $\{\mathrm{A_0^i}, \mathrm{i} = 1,2,\cdots,\mathrm{N}\}$ and locate the resonance frequency f_{r1} for λ;

b) Select a defined number of random parameters out of $\{\mathrm{A_0^i}, \mathrm{i} = 1,2,\cdots,\mathrm{N}\}$ and evaluate their corresponding resonance frequency f_{r2};

c) Calculate the frequency range: $f_{min} = \min(f_{r1}, f_{r2})$, $f_{max} = \max(f_{r1}, f_{r2})$ and $g(f) \geq 0$.

3) ***Execute the GA****:* Obtain optimal emitter parameters $\{\hat{\mathrm{a}}_\mathrm{j}, \mathrm{j} = 1,2,3\}$.

a) Evaluate the fitness of each individual with regard to the given objective function. The given objective function is the evaluation of EM field intensity (2), which is calculated using a FEM.

b) Select the individual with higher fitness as the parent individuals, the roulette wheel method is chosen.

c) Perform the crossover and mutation operator to produce the offspring individuals;

d) Repeat the whole process until the termination condition is reached.

4) ***Return Parameters****.*

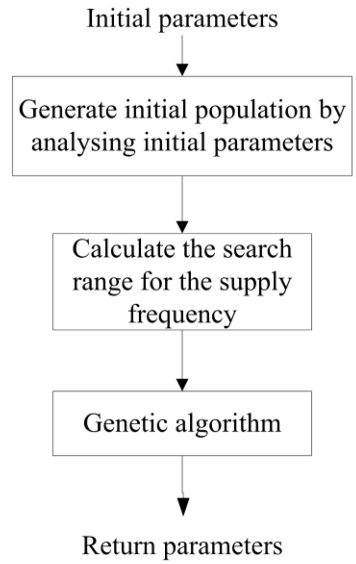

Figure 4 Flow chart of the PGA

Compared with the NM and the generic GA, the impact of the initial parameters of PGA can be reduced by decoupling the frequency range. Further, the PGA improves the search efficiency and optimality of the GA.

IV. CASE STUDIES

The NM, the generic GA and the PGA methods are all applied to solve this optimization problem with three case studies. For both NM and GA, it is easy to be trapped into local extrema and fail. However, in PGA, the initial range of frequency has been predefined, which helps not only shorten the search time but also make sure it reaches global extrema.

A. Case 1: Coupled Resonant Frequencies and Emitter Lengths

For deterministic optimization as the NM algorithm, the initial value of parameters is given as a starting step for the search. The starting conditions for the NM search are given in Table 1. This optimization algorithm is available from the MATLAB Optimization Toolbox. The optimization settings and stopping criteria for the algorithm used were set as in Table 2.

For non-deterministic search like the GA or PGA, initial search ranges are given as initial conditions. Table 3: Starting conditions of GA searchesTable 3 gives the initial ranges of input variables for the GA and Table 5 for the PGA. It is not required to define a starting value for the supply frequency here because the minimum necessary frequency range can be search for in a PGA. The default settings of the NM and GA are adopted from the MATLAB Optimization Toolbox. For the GA, the optimization setting and stopping criteria for the used algorithm were set as per Table 4. For the PGA, the optimization settings and stopping criteria for the NM part of the combined search algorithm were set the same as in Table 2. The optimization setting and stopping criteria for the GA part of the combined search algorithm were set the same as in Table 4.

TABLE 1: STARTING CONDITIONS OF NM SEARCHES

Search	Frequency (GHz)	Emitter length (mm)
1st	2.5	2
2nd	2.59	2
3rd	2.5	10
4th	2.59	10

TABLE 2 ALGORITHM SETTING IN MATLAB OPTIMIZATION TOOLBOX FOR NM SEARCH

Display	final
MaxFunEvals	500
MaxIter	500
TolFun	1E-4
TolX	1E-4

TABLE 3: STARTING CONDITIONS OF GA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)
1st	2.55-2.65	0-10
2nd	2.5-2.7	0-20

TABLE 4 ALGORITHM SETTING IN MATLAB OPTIMIZATION TOOLBOX FOR GA SEARCH

Display	final
CrossoverFcn	crossoverscattered
CrossoverFraction	0.8
MutationFraction	0.2
EliteCount	2
Generations	40
PopulationSize	101
SelectionFcn	selectionstochunif
StallGenLimit	20
TimeLimit	Inf
TolFun	1E-6

TABLE 5: STARTING CONDITION OF PGA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)
1st	Not required	0-10
2nd	Not required	0-20

B. *Case 2: Coupled Frequencies, Emitter Lengths, Emitter Heights and Emitter Widths*

Two variables are added to the extended emitter models, i.e., the emitter heights and emitter widths. The default values of the height and width of the simplest emitter are set to zero.

Similar to Case 1, the starting conditions of NM searches are given in Table 6. The optimization algorithm is available from the MATLAB Optimization Toolbox. Table 7 gives the initial range of the input variables for the GA and Table 8 gives the initial range of the input variable for the PGA. It is not required to define a starting value for the supply frequency here because the minimum necessary frequency range will be calculated during the search process.

The default settings of the NM and the GA are adopted from the MATLAB Optimization Toolbox with the same settings as in Case 1.

TABLE 6: STARTING CONDITIONS OF NM SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Emitter height (mm)	Emitter width (mm)
1st	2.55	5	5	2
2nd	2.59	10	7	5
3rd	2.55	5	5	2
4th	2.59	10	7	5

TABLE 7: STARTING CONDITIONS OF GA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Emitter height (mm)	Emitter width (mm)
1st	2.55-2.6	0-5	0-6	0-5
2nd	2.55-2.6	0-10	0-6	0-5

TABLE 8: STARTING CONDITION OF PGA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Emitter height (mm)	Emitter width (mm)
1st	Not required	0-10	0-6	0-5
2nd	Not required	0-20	0-6	0-5

V. VIRTUAL PROTOTYPING RESULTS AND COMPARISON

A. *Virtual Prototyping for Case 1with a Default Emitter*

a. *Search results of the NM method*

As the NM search algorithm is a deterministic heuristic method, it is unnecessary to run multiple simulations to achieve a reliable performance of the convergence speed. Figure 5 shows the search trace of the maximum EM field intensity with the search of the 1st set of initial condition. It takes about 100 times of searches to reach the maximum point. Table 9 lists the gathered search results for the different initial parameters. These results show that the initial values are highly relevant for the search to attain acceptable results. Furthermore, this confirms that the NM algorithm can only reach a local extreme value. In the results, resonant frequency from four searches remains at approximately 2.596 GHz while the emitter length varies for every single search.

TABLE 9: SEARCH RESULTS OF NM SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Maximum EM field intensity (V/m)
1st	2.596058	2.407	1.335×10^8
2nd	2.596060	4.422	0.757×10^8
3rd	2.596058	6.477	0.339×10^8
4th	2.596058	6.503	0.335×10^8

b. Search results of the generic GA

Figure 6 shows a typical result of the search trace of the

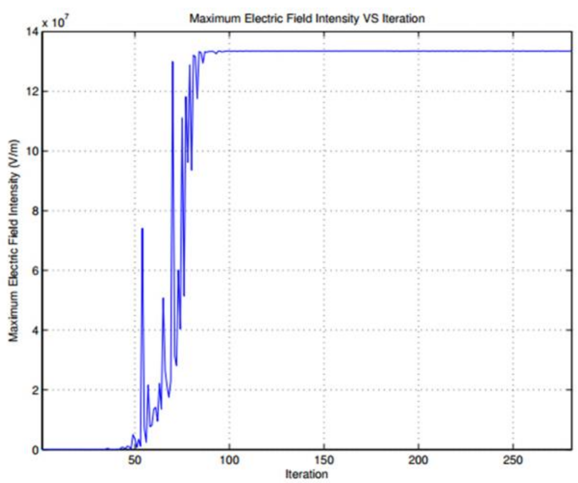

Figure 6 NM search trace for the maximum EM field intensity in Case 1

maximum EM field intensity. It takes about 15 generations for the GA to reach an acceptable result, with the 2nd search of the initial condition, which has the maximum EM intensity in GA search for Case 1. Table 10 gives the results of the different searches. The resulting frequency of both optimization searches is located around 2.596 GHz. However, the emitter lengths vary between the searches, which influence the EM field distribution inside the cavity. Compared with the results of the NM method, the EM field intensity is much smaller, which could imply that the nondeterministic GA is an inferior heuristic search method for such an application.

TABLE 10: SEARCH RESULTS OF GA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Maximum EM field intensity (V/m)
1st	2.596120	9.881	0.882×10^6
2nd	2.595678	13.446	1.920×10^6

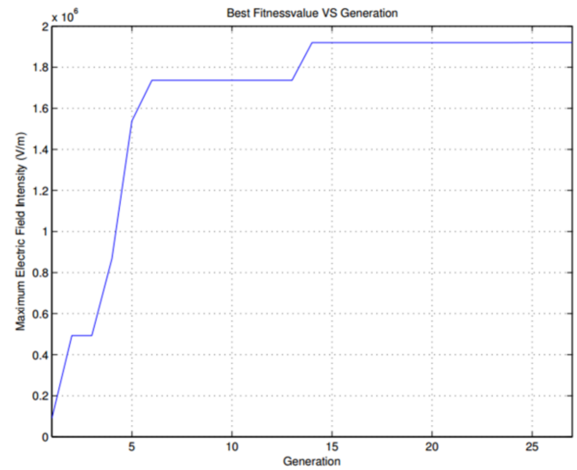

Figure 5: GA search trace for the maximum EM field intensity in Case 1

c. Search results of the PGA

It also takes about 15 generations for the PGA to reach an acceptable result, with the 2nd search of the initial condition, as a typical result shown in Figure 7. Compared with the results of the GA, the convergence speed was similar. However, in Table 11, the results of the different searches show a consistency in PGA search. The resulting frequency of the optimal searches is found at 2.596059 GHz. The best emitter length is found to be 1.154mm in 1st and 1.401mm in 2nd searches, with the maximum EM field intensity approximate 1.47×108 V/m. Compared with the results of PGA with NM and GA, it is the highest of all.

TABLE 11: SEARCH RESULTS OF PGA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Maximum EM field intensity (V/m)
1st	2.596059	1.154	1.466×10^8
2nd	2.596059	1.401	1.473×10^8

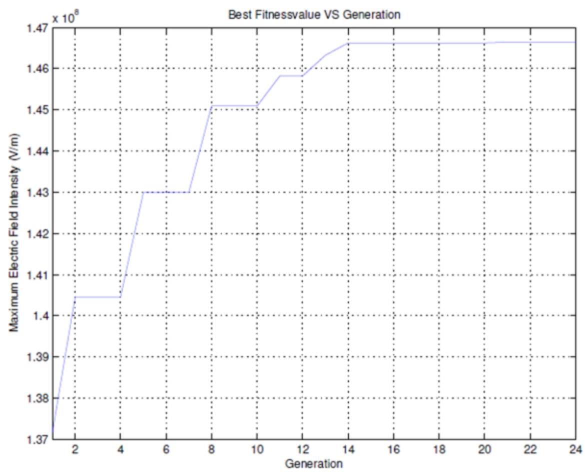

Figure 7: PGA search trace for the maximum EM field intensity in Case 1

B. *Virtual Prototyping for Case 2 with an Extended Emitter*

a. *Search results of the NM simplex*

Figure 8 shows the search trace of the maximum EM field intensity with the 1st set of initial condition. It takes

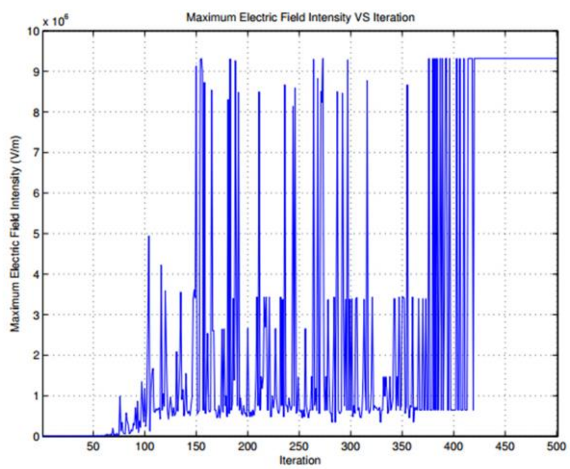

Figure 9: NM search trace for the maximum EM field intensity in Case 2

about 420 times of searches to reach the maximum point. As it is shown in Figure 8, the EM field intensity varies much during the search. That's because the EM field intensity is

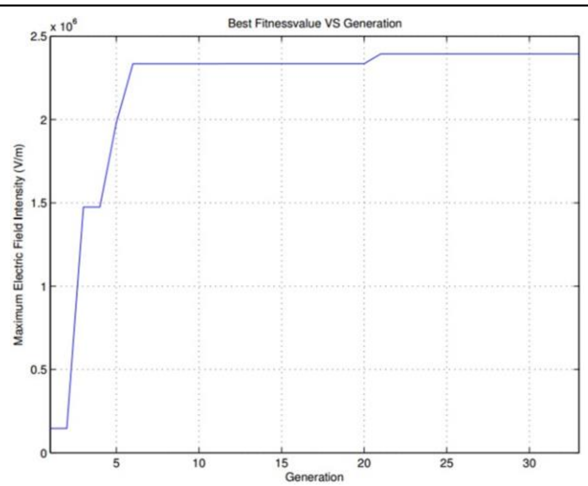

Figure 10: GA search trace for the maximum EM field intensity in Case 2

quite sensitive to frequency. In Case 2, the number of input variables increases to 3. It would take more time to complete the search than in Case 1. Table 12 lists the gathered search results for the different initial parameters. These results are all local extremes. In the results, both frequency and emitter parameters are divergent.

b. *Search results of the GA*

The GA search is a non-deterministic optimization method and therefore the results will be inconsistent between different optimization searches. Figure 9 shows that it takes about 40 generations for the GA to reach an acceptable result, with the 2nd search of the initial condition due to the higher complexity of Case 2. Table 13 gives the typical results of the different searches. Both resonant frequency and maximum EM field intensity are divergent, which proves that the results are not a global extremum. Furthermore, the EM field intensity is much smaller here than using the NM search.

c. *Search results of the PGA*

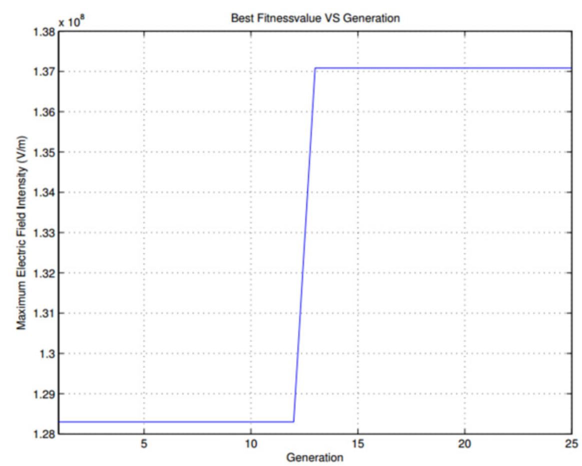

Figure 8: PGA search trace for the maximum EM field intensity in Case 2

It takes about 15 generations for the PGA to reach an acceptable result with the 1st search of the initial condition in Figure 10. In Table 14, the results of the different searches show a consistency, unlike with the GA. The resulting frequency and the emitter length are the same as in Case 1. The emitter height is 0.0 mm, while the emitter width is 1.42mm and 3.33 mm in two searches, also delivering the highest filed strength as shown in the table.

The PGA heuristic search combines a deterministic and a non-deterministic methods, hence the results being inconsistent with multiple searches. The PGA exhibits a significant better performance than the GA. The found maximum EM field intensity is nearly identical to that of the default emitter model in Case 1.

VI. CONCLUSION

This paper has developed an improved GA heuristic method for a challenging real-world application – the potential invention of an HCMI engine. During the optimization for this coupled constraint problem, the NM simplex search method and the conventional GA have failed due to the high influence of the incident EM frequency. It has been found that the improved GA, i.e., the PGA, offers a higher convergent speed and reaches global extrema in various tests. Furthermore, the selection of initial values has little impact on the final results of the PGA. When the complexity of the problem is increased with the number of input variables, the PGA has also offered a consistent performance, despite that the NM and the GA yield divergent results.

TABLE 12: SEARCH RESULTS OF NM SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Emitter height (mm)	Emitter width (mm)	Maximum EM field intensity (V/m)
1st	2.537480	5.32	4.77	1.98	9.318×10^6
2nd	2.462754	10.68	7.03	5.00	1.067×10^6
3rd	2.535684	5.25	4.84	2.03	3.413×10^6
4th	2.532301	18.11	8.99	-1.07	1.459×10^6

TABLE 13: SEARCH RESULTS OF GA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Emitter height (mm)	Emitter width (mm)	Maximum EM field intensity (V/m)
1st	2.594545	4.85	0.64	0.53	7.294×10^5
2nd	2.583839	9.78	1.96	1.63	2.394×10^6

TABLE 14: SEARCH RESULTS OF PGA SEARCHES

Search	Frequency (GHz)	Emitter length (mm)	Emitter height (mm)	Emitter width (mm)	Maximum EM field intensity (V/m)
1st	2.596060	1.154	0.0	1.42	1.371×10^8
2nd	2.596059	1.401	0.0	3.33	1.274×10^8

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