Revisiting the Optimal Scheduling Problem

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***Abstract*— In this paper, we revisit the problem of determining the minimum-length schedule that satisfies certain traffic demands in a wireless network. Traditional approaches for the determination of minimum-length schedules are based on a collision channel model, in which neighboring transmissions cause destructive interference if and only if they are within the "interference region" of the receiving nodes. By contrast, here we adopt a more realistic model for the physical layer by requiring that a threshold be exceeded by the signal-to-interference-plus-noise ratio (SINR) for a transmission to be successful. Further we include aspects of the routing problem and utilize column generation for carrying out the computations.**

I. Introduction

Wireless channel interference has often been modeled using fixed communication and interference ranges, where nodes within the communication range can communicate with each other, while any transmission within the interference range resulted in packet collisions. The *Protocol Interference Model* [6] describes interference constraints according to a conflict graph, where nodes within a certain distance can communicate as long as the receiver is separated by at least a distance d from any other active transmitter. However, these models do not take the cumulative effects of interference due to simultaneous transmissions into account. On the other hand, the *Physical Interference Model* directly considers the signal-to-interference-plus-noise ratio (SINR) constraints at the receivers by accounting for all the secondary transmissions as interference. While the physical interference model is considered to be more accurate, it is also more complicated.

As an alternative to contention-based mechanisms such as Aloha, link scheduling, i.e., the problem of identifying sets of links that can be simultaneously activated as well as the corresponding duration of activation, has been studied extensively in the context of wireless networks, as early as in [7] by Hajek and Sasaki. They presented a strongly polynomial-time algorithm for the problem of finding a minimum-length schedule in a wireless network that satisfies a set of link traffic requirements. The network mapped into an undirected graph, and the only constraints that the schedule had to satisfy were the transmission constraints, i.e., two wireless links could not be active simultaneously if they shared a node; thereby ignoring the interference constraints among the active links. Other models include the so-called "disk model" and those that are based on graph coloring methods (e.g., [8], [10]) and conflict graphs [6]. Recently, a more accurate model that considers the cumulative interference in the form of SINR has recently gained wider acceptance. Borbash and Ephremides showed in [3] that the general problem of determining a minimum-length schedule that satisfies given link demands in a wireless network, and subject to SINR constraints is at least as hard as the "MAX-SIR-MATCHING" problem. Furthermore, they provided examples of a special case where the traffic demand vector satisfied a "superincreasing" property, to be tractable. Bjorklund *et al.*, showed in [2] that even the most basic planning problems in wireless networks such as node and link assignment are NP-hard. They formulated the so-called node and link assignment optimization problems, which assign at least one time slot to each node or link such the the number of time slots is minimized using set-covering formulations, and developed a column generation approach for solving the resulting linear programming relaxations. However, specific traffic demands on links were not taken into account. Furthermore, although heuristic algorithms were developed in the past (e.g., [5]), it is important to design efficient algorithms that can provide theoretical guarantees of optimality.

Scheduling, as an access control method, avoids collisions and retransmissions that are typical in contention-based methods. Whereas scheduling methods, such as time division multiple access (TDMA) schemes, can guarantee such delay bounds, their efficiency can be further improved both in terms of delay guarantees as well as achieving higher capacities by allowing the TDMA time-slots to be shared by simultaneous transmissions that are geographically separated. This improvement is appropriately termed Spatial-TDMA or STDMA [11]. In this paper, we attempt to determine the minimum-length schedule that is required to satisfy a set of specified link demands in a wireless network, such that a given SINR is exceeded at the receivers of all simultaneously active links. It should be understood that the minimum schedule length is indicative of the ability of a wireless network to carry a given amount of traffic.

The objective of the minimum-length scheduling problem that is presented in this paper is to compute the shortest schedule that can satisfy the traffic demands for a set of chosen links, under the SINR criterion. That is, in addition to the standard transmission constraints discussed earlier, we impose a constraint on SINR at each receiving node. This set of schedulable links could constitute valid paths between source-destination pairs, in which case we can imagine that the paths are chosen *a priori*. However, in wireless networks, prior selection without explicit physical layer considerations is clearly sub-optimal. Therefore, we extend our investigation of resource allocation and STDMA-based link scheduling optimization in multi-hop wireless networks, by jointly determining routing between source-destination pairs and the minimum-length schedule of link activation in order to satisfy end-to-end traffic demands. We present a cross-layer formulation of the problem that incorporates multi-path routing at the network layer, while concurrently generating "matchings" to address the media-access control problem. Each such matching consists of a set of links that can simultaneously be active, without violating the specified signal-to-interference-plus-noise ratio (SINR) requirement. After considering a problem formulation that is restricted to the use of the same transmission power by all nodes, we further extend our model to incorporate power control at the transmitters with the goal of reducing interference and maximizing spatial reuse.

In Section II, we discuss the network and communication model that is used in the formulation of the minimum-length scheduling problem, which is presented in Section III. We then propose a column-generation based solution procedure in Section IV and we present various extensions to the fixed transmit power model in Section V. We extend the minimum-length scheduling problem to include routing in Section VI, and in Section VII, we provide final conclusions.

II. Network and Communication Model

We model a multi-hop wireless network as a set of stationary nodes $\mathcal{N}$. A set of (directed) links $\mathcal{E}$ constitutes the network topology, and link $\{i,j\} \in \mathcal{E}$ exists if node i can communicate directly with node j, i.e., the corresponding signal-to-noise ratio (SNR) in the absence of any other interference source exceeds a specific threshold. Therefore, the graph representation of the wireless network is based on whether a node can reach another node when transmitting in isolation for a given power, noise level and channel gain.

Let P_i be the transmission power for node i, G_{ij} the gain of the radio channel between nodes i and j, and η_j the thermal noise at receiver j. The SINR at receiver j due to transmission from node i in the presence of other transmissions is given by:

$$SINR_{ij} = \frac{P_i G_{ij}}{\eta_j + \sum_{k \neq i,j} P_k G_{kj}}. \tag{1}$$

Here, the channel gain is calculated by the widely used free-space model (without fading) $G_{ij} = d_{ij}^{-\alpha}$, where d_{ij} is the distance between nodes i and j, and α is the path loss index, but in fact any arbitrary propagation model can be subtituted.

The capacity of the wireless channel associated with a link (i,j) is a complicated and unknown quantity. We assume that data is coded separately for each link and the receivers consider unintended receptions as noise. In that case, a simplified view of each link (i,j) consists of a single-user Gaussian channel, the Shannon capacity of which, over a frequency band W, is given by:

$$c_{ij} = W \; log_2(1 + SINR_{ij}). \tag{2}$$

In practice however, it is understood that most communication schemes will achieve lower rates. That depends on target bit error-rate, modulation and coding schemes. We are not concerned here with the capacity issue and use Equation (2) only selectively for bounding purpose.

Given a set of links M, all links in M can be activated concurrently if such simultaneous activation does not violate the minimum SINR required for communication, i.e., the SINR threshold γ is satisfied at the receivers of all links in M, as shown in (3).

$$SINR_{ij} \geq \gamma. \tag{3}$$

A set M satisfying this condition is called a "feasible matching", or simply, a *matching*. Therefore, the communication model that is used in this paper, directly considers the SINR constraints at the receivers by accounting for all the secondary transmissions as interference.

A schedule is defined as a finite indexed collection $\mathcal{S} = (M^s, \lambda^s, s \in Z^+)$, where the continuous quantity $\lambda^s \geq 0$ is the duration associated with the matching M^s

for each s. Therefore, the length τ of the schedule $\mathcal{S}$ is defined as

$$\tau = \sum_{s} \lambda^s. \quad (4)$$

Each link $\{i, j\} \in \mathcal{E}$ has a certain non-negative traffic demand that needs to be satisfied by the schedule, and a link may be active in one or several time slots based on how many matchings contain this link. The goal is to minimize τ, given the location of the nodes and the link traffic demands.

III. The Minimum-Length Scheduling Problem

Each link $\{i, j\} \in \mathcal{E}$ has a specific traffic demand of f_{ij} bits per frame that need to be transmitted across the link, where the frame length is not specified *a priori*. The entire information transfer across all the links can be completed in a time interval of length τ as follows. Each matching M^s indexed by $s \in Z^+$, is active for a duration of λ^s, and each link $\{i, j\}$ that is part of the matching M^s transmits at a rate of c^s_{ij} bits/sec, which is computed based on the SINR at receiver j, as described in Equation (2), or via other appropriate formulas. Thus a link $\{i, j\}$ is active during all the slots for which $\{i, j\} \in M^s$, and the overall data that is transmitted in the duration for which the link $\{i, j\}$ is active, must be at least f_{ij}.

The minimum-length scheduling problem, therefore, involves computing the schedule $\mathcal{S} = (M^s, \lambda^s, s \in Z^+)$ that minimizes the schedule length $\tau = \sum_s \lambda^s$, such that the traffic demands f_{ij} of all the wireless links $\{i, j\} \in \mathcal{E}$ are satisfied. Note that the traffic demand can also be expressed in terms of bit rate rather than bit volume with minor adjustments to the model.

Minimization of frame length is useful because it permits a larger number of frames per unit time (and hence higher overall data rate, if the amount of data that must be transmitted per frame is fixed). Alternatively, if the data rate of the specified traffic that is to be scheduled is fixed, minimization of the time required to transmit all of a frame's data permits more of the network's resourced to be used for other traffic.

Given the set of all possible feasible matchings denoted by $\mathcal{M}$, (i.e., any $M^s \in \mathcal{M}$), the Minimum-Length Scheduling Problem [MLSP] is formulated as follows.
[MLSP]:

$$\textbf{Minimize}: \tau = \sum_{1 \le s \le |\mathcal{M}|} \lambda^s \quad (5)$$

subject to:

$$\sum_{1 \le s \le |\mathcal{M}|} c^s_{ij} \lambda^s \ge f_{ij}, \quad \forall \{i, j\} \in \mathcal{E} \quad (6)$$

$$\lambda^s \ge 0, \quad \forall s = 1, ..., |\mathcal{M}|.$$

Note that even though [MLSP] is a linear programming (LP) problem having a very simple constraint structure, the complexity of the problem lies in the computation of the set of all feasible matchings $\mathcal{M}$. The total number of matchings that would have to be enumerated in order to compute an optimum may be as large as $2^{|\mathcal{E}|}$. Therefore, a straightforward solution of [MLSP] is not computationally efficient. However, this complexity can be reduced by eliminating those matchings from the problem formulation that are either infeasible or inefficient (and thus unlikely to be used in the optimal schedule). For example, nodes could be limited to either send or receive in a matching but not both at the same time, due to high self interference, and nodes could be restricted from transmitting simultaneously to multiple nodes and/or receiving from multiple nodes in a matching. Such observations could significantly reduce the number of feasible matchings, thereby reducing the problem complexity. Other heuristic approaches, (see [4]), could also be used to generate valid matchings that have a very high chance of being used in the optimal solution.

Alternatively, [MLSP] can be solved in such a way that the matchings, which constitute columns in the linear program, are not explicitly enumerated, but are computed in an iterative manner, such that, the newer columns thus generated have a potential to improve the objective function. In the next section, we show how [MLSP] can be solved by selectively enumerating only those matchings that contribute towards an optimum, using a column generation approach.

IV. Column Generation based Solution Procedure

A. Column Generation

Column generation is an iterative approach for solving huge linear or integer programming problems, where the number of variables are too large to be considered explicitly. In the column generation approach, the original problem is decomposed into a master problem and a subproblem. The master problem and subproblem could be either linear or integer programs depending on the problem formulation. The strategy of this decomposition procedure is to operate iteratively on two separate, but easier-to-solve, problems. During each iteration, the algorithm tries to determine whether any variables exist that have a *negative reduced cost* (in the case of a minimization problem) and adds the variable with the "most negative reduced cost" to the master problem. The key idea of the solution approach is to sequentially improve the current solution by solving the subproblem that identifies a single new variable (a column) during every iteration, and adding it to the master problem, until the algorithm terminates at or close to the optimal solution.

B. Master Problem

The master problem [MP] is a restriction of the original problem [MLSP], which uses only a subset of columns (matchings) indexed by $s \in \{1, ..., |\mathcal{M}|\}$. Recall that the formulation of [MLSP] includes the entire set of feasible matchings $\mathcal{M}$, even though, we know that most of the matchings will not be a part of the optimal solution, and will have duration $\lambda^s = 0$. Therefore, [MP] is first initialized with any feasible schedule $\mathcal{S}$ that satisfies the link demands of all the links in $\mathcal{E}$. Section IV-C.2 discusses different ways of initializing the schedule $\mathcal{S}$. The master problem [MP] can now be formulated as follows.

[MP($\mathcal{S}$)]:

$$\textbf{Minimize}: \tau = \sum_{s \in \mathcal{S}} \lambda^s \tag{7}$$

subject to:

$$\sum_{s \in \mathcal{S}} c_{ij}^s \lambda^s \geq f_{ij}, \quad \forall \{i,j\} \in \mathcal{E} \tag{8}$$

$$\lambda^s \geq 0, \quad \forall s \in \mathcal{S}.$$

Since this formulation optimizes over a subset $\mathcal{S}$ of all feasible matchings, it is a restriction of the original problem [MLSP]. Hence, an optimal solution to [MP] provides an upper bound UB for the [MLSP].

C. Generating Feasible Matchings

During every iteration, when the master problem [MP] is solved, we need to either conclude that the current solution is optimal, or else identify a *new* matching that can improve the current solution, i.e., we need to identify a new column to enter into the basis. Recall that each matching constitutes one column in [MP]. Based on the theory of linear programming and the revised simplex algorithm [1], this can be achieved by examining whether any new column (that is not currently in [MP]), has a negative reduced cost. Denoting the dual variables corresponding to (8) by $(\bar{\omega}_{ij})$, the reduced cost $\bar{z}_k$ for any column k in [MP] can be expressed as:

$$\bar{z}_k = 1 - \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^k. \tag{9}$$

Therefore, in order to find a new column having the most negative reduced cost, we solve the subproblem defined as

$$\underset{k \in \mathcal{M} \backslash \mathcal{S}}{\textbf{Minimize}} \ \bar{z}_k,$$

or equivalently,

$$\underset{k \in \mathcal{M} \backslash \mathcal{S}}{\textbf{Maximize}} \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^k. \tag{10}$$

Here, the term $\mathcal{M} \backslash \mathcal{S}$ refers to the set of all columns that are in $\mathcal{M}$ and are not a part of $\mathcal{S}$. This subproblem can be referred to as the scheduling subproblem, because it aids in identifying a new matching that could be a part of the optimal schedule. Based on the optimal solution to the scheduling subproblem, a non-negative reduced cost implies that current solution to [MP] is indeed the optimal solution to [MLSP]. Otherwise, the new matching that is identified by the subproblem is included in the current schedule $\mathcal{S}$, and [MP] is re-optimized. We first consider the case of no power control.

1) Fixed Transmit Power: In this scheme, the source nodes of all active links in the matching use their maximum RF transmission power P_{max}, with the condition that the SINR of all the active links in the matching exceeds a *fixed* SINR threshold γ. The SINR threshold γ is a an increasing function of rate c_{ij} over link $\{i, j\}$. In turn c_{ij} is a function of factors such as modulation scheme, coding scheme, and the specified bit error rate requirement.

Given a set of dual variables (ω_{ij}) (obtained from the master problem), a new matching can be generated by solving the corresponding subproblem shown below. The formulation of the subproblem is explained in greater detail in [9].

$$\textbf{Maximize}: \sum_{\{i,j\} \in \mathcal{E}} \omega_{ij} c_{ij} x_{ij}$$

subject to:

$$(\eta_j + \sum_{k \neq i,j} G_{kj} P_{max} - \gamma^{-1} G_{ij} P_{max}) x_{ij}$$

$$+ \sum_{k,m \neq i,j} G_{kj} P_{max} x_{km} \leq \sum_{k \neq i,j} G_{kj} P_{max}, \ \forall \{i,j\} \in \mathcal{E}$$

$$\sum_{j:\{i,j\} \in \mathcal{E}} x_{ij} + \sum_{j:\{j,i\} \in \mathcal{E}} x_{ji} \leq 1, \quad \forall i \in \mathcal{N}$$

$$x_{ij} \in \{0,1\}, \quad \forall \{i,j\} \in \mathcal{E}.$$

2) Generating the Initial Feasible Solution: In order to pass down a set of cost coefficients from the master problem [MP] to the subproblem, the initial set of matchings in $\mathcal{S}$ must provide a feasible solution to the original problem [MLSP]. For this purpose, one can initialize $\mathcal{S}$ with a set of matchings, where each matching contains exactly one single link. This corresponds to the traditional TDMA scheduling. Other heuristic and greedy approaches can also be applied with the aim of generating a variety of possible matchings.

V. Extensions to the Fixed Transmit Power Model

A. Transmit Power Control

It is possible to relax the assumption of fixed transmission power. In this case, the source nodes of all active links in the matching can transmit *up to* their maximum power P_{max} while satisfying the SINR constraints i.e., the SINR of all the active links exceeds a *fixed* threshold γ. As in the fixed power case, the formulation for the case of variable transmit power is explained in greater detail in [9]. The resulting subproblem is as follows:

Maximize: $\sum_{\{i,j\}\in\mathcal{E}} \omega_{ij} c_{ij} x_{ij}$

subject to:

$$\gamma(\eta_j + \sum_{k\neq i,j} G_{kj}P_{max})x_{ij} + \gamma \sum_{k\neq i,j} G_{kj}P_k - G_{ij}P_i \leq \gamma \sum_{k\neq i,j} G_{kj}P_{max}, \quad \forall\{i,j\}\in\mathcal{E}$$

$$\sum_{j:\{i,j\}\in\mathcal{E}} x_{ij} + \sum_{j:\{j,i\}\in\mathcal{E}} x_{ji} \leq 1, \quad \forall i\in\mathcal{N}$$

$$x_{ij}\in\{0,1\}, \quad \forall\{i,j\}\in\mathcal{E}$$

$$0\leq P_i\leq P_{max}, \quad \forall i\in\mathcal{N}.$$

B. Variable Transmission Rate

We can extend the previous scheme so that nodes can choose the best transmission rate for communication, from a finite set of rates $\{c^{(1)},..,c^{(t)},..,c^{(T)}\}$, depending on the SINR that can be achieved at the receivers. Associated with the transmit rate $c^{(t)}$ is an SINR threshold $\gamma^{(t)}$.

Defining a new binary variable $x_{ij}^{(t)}$ for each link $\{i,j\}\in\mathcal{E}$, where

$$x_{ij}^{(t)} = \begin{cases} 1, & \text{if link } \{i,j\} \text{ transmits at rate } c^{(t)} \\ 0, & \text{otherwise,} \end{cases}$$

and rewriting the constraints of the subproblem, we have,

Maximize: $\sum_{\{i,j\}\in\mathcal{E}} \omega_{ij} \sum_t c^{(t)} x_{ij}^{(t)}$

subject to:

$$\gamma^{(t)}(\eta_j + \sum_{k\neq i,j} G_{kj}P_{max})x_{ij}^{(t)} + \gamma^{(t)} \sum_{k\neq i,j} G_{kj}P_k - G_{ij}P_i \leq \gamma^{(t)} \sum_{k\neq i,j} G_{kj}P_{max}, \quad \forall\{i,j\}\in\mathcal{E}$$

$$\sum_t x_{ij}^{(t)} \leq 1, \quad \forall\{i,j\}\in\mathcal{E}$$

$$\sum_t \sum_{j:\{i,j\}\in\mathcal{E}} x_{ij}^{(t)} + \sum_t \sum_{j:\{j,i\}\in\mathcal{E}} x_{ji}^{(t)} \leq 1, \quad \forall i\in\mathcal{N}$$

$$x_{ij}^{(t)}\in\{0,1\}, \quad \forall t, \forall\{i,j\}\in\mathcal{E}$$

$$0\leq P_i\leq P_{max}, \quad \forall i\in\mathcal{N}.$$

VI. Joint Routing and Link Scheduling

In this section, we extend our investigation of link scheduling optimization by jointly determining routing between source-destination pairs and the minimum-length schedule of link activation in order to satisfy end-to-end traffic demands.

Consider L concurrent sessions, each of which corresponds to a source-destination pair in the network. The traffic demand for each session l, $1\leq l\leq L$, is given by R_l which is to be transmitted from the source node s_l to destination node d_l, along a set of links that constitute paths for each session l. As in Section III, the traffic demand R_l is expressed in terms of bits per frame. In order to relay this traffic demand for each session, we take advantage of the availability of multiple paths between source-destination pairs, and allow the source to split the data into multiple sub-flows if necessary. Extending the definition of f_{ij} as described in Section III to include different sessions flowing through the link, we now denote the data rate associated with the l-th session on link (i,j) by f_{ij}^l. The flow balancing equations for each session can now be expressed as follows.

If a node i is the source node of session l (i.e., $i=s_l$), the following condition holds:

$$\sum_{j\in\mathcal{N}(i)} f_{ij}^l = R_l, \tag{11}$$

where $\mathcal{N}(i)$ denotes the set of all nodes having links that originate at node i, i.e., one-hop neighbors of node i.

If node i is an intermediate relay node for the l-th session (meaning, $i\neq s_l$ and $i\neq d_l$), then, the flow balance equations can be written as follows:

$$\sum_{j \in \mathcal{N}(i)} f_{ij}^l - \sum_{j : i \in \mathcal{N}(j)} f_{ji}^l = 0. \quad (12)$$

Finally, if node i is the destination node, $i = d_l$, we have

$$-\sum_{j : i \in \mathcal{N}(j)} f_{ji}^l = -R_l. \quad (13)$$

Constraints (11)-(13) for all l, $1 \leq l \leq L$ can be concisely written as follows:

$$\mathbf{Af} = \mathbf{R}, \quad (14)$$

where $A \in Z^{|\mathcal{N}|L \times |\mathcal{E}|L}$ is an integer matrix whose entries are either 1, -1 or 0, and $\mathbf{R}$ is the rate vector with zero-valued entries for intermediate relay nodes. The vector **f** represent the set of data rate variables $\{f_{ij}^l\}$.

A. Minimum-Length Scheduling Problem with Routing

As defined earlier in Section II, a schedule $\mathcal{S} = (M^s, \lambda^s, s \in S)$ is a collection of matchings M^s and corresponding non-negative values λ^s, such that each of the matchings is feasible, and the end-to-end demands of all sessions are satisfied. Our objective is to minimize the schedule length $\tau = \sum_s \lambda^s$. A source-destination pair associated with a session l has R_l bits of information to be transferred from source to destination in each STDMA frame. This information can be split and sent across multiple routes, and the traffic demand on link $\{i, j\}$ resulting from all such sessions $1 \leq l \leq L$ is given by $f_{ij} = \sum_{1 \leq l \leq L} f_{ij}^l$, which is the amount of information that needs to be transmitted in every frame, for each link $\{i, j\}$.

Given the set $\mathcal{M}$ of all possible feasible matchings for a network, where $M^s \in \mathcal{M}$, the *Minimum-Length Scheduling Problem with Routing* [MLSPR] can be formulated as follows:

[MLSPR]:

$$\textbf{Minimize}: \tau = \sum_{1 \leq s \leq |\mathcal{M}|} \lambda^s \quad (15)$$

subject to:

$$\mathbf{Ar} = \mathbf{R} \quad (16)$$

$$\sum_{1 \leq s \leq |\mathcal{M}|} c_{ij}^s \lambda^s \geq \sum_{1 \leq l \leq L} f_{ij}^l, \quad \forall \{i, j\} \in \mathcal{E} \quad (17)$$

$$f_{ij}^l \geq 0, \quad \forall \{i, j\} \in \mathcal{E}, 1 \leq l \leq L$$

$$\lambda^s \geq 0, \quad \forall s = 1, ..., |\mathcal{M}|.$$

[MSLPR] is a linear programming problem with a constraint structure that is similar to [MLSP], and therefore, it can be solved in a similar fashion using column generation. In the case of [MLSPR], the master problem would also be a restriction of the original problem that uses only a subset of matchings. It should be noted that because of the similarities in the problem formulation, the subproblems associated with [MLSPR] are identical to those of [MLSP] discussed in Sections IV-C.1 and V. Also, the functionality of the subproblem, which is the generation of feasible matchings, is the same in both cases as well.

VII. Conclusion

In this paper, we reviewed the problem of minimum-length scheduling in wireless networks in the presence of SINR constraints. We formulated the problem as a cross-layer optimization problem with consideration of link layer and physical layer parameters, and incorporated dynamic power and rate control, in order to generate feasible matchings. We proposed a solution procedure based on column generation, and showed that this method actually converges to an optimal solution.

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