

Relay Beamforming Strategies for Physical-Layer Security

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Abstract—¹ In this paper, collaborative use of relays to form a beamforming system and provide physical-layer security is investigated. In particular, amplify-and-forward (AF) relay beamforming designs under total and individual relay power constraints are studied with the goal of maximizing the secrecy rates when perfect channel state information (CSI) is available. In the AF scheme, not having analytical solutions for the optimal beamforming design under both total and individual power constraints, an iterative algorithm is proposed to numerically obtain the optimal beamforming structure and maximize the secrecy rates. Robust beamforming designs in the presence of imperfect CSI are investigated for decode-and-forward (DF) based relay beamforming, and optimization frameworks are provided.

Index Terms: amplify-and-forward relaying, decode-and-forward relaying, physical-layer security, relay beamforming, robust beamforming.

I. INTRODUCTION

The broadcast nature of wireless transmissions allows for the signals to be received by all users within the communication range, making wireless communications vulnerable to eavesdropping. The problem of secure transmission in the presence of an eavesdropper was first studied from an information-theoretic perspective in [1] where Wyner considered a wiretap channel model. Wyner showed that secure communication is possible without sharing a secret key if the eavesdropper's channel is a degraded version of the main channel, and identified the rate-equivocation region and established the secrecy capacity of the degraded discrete memoryless wiretap channel. The secrecy capacity is defined as the maximum achievable rate from the transmitter to the legitimate receiver, which can be attained while keeping the eavesdropper completely ignorant of the transmitted messages. Later, Wyner's result was extended to the Gaussian channel in [3] and recently to fading channels in [4] and [5]. In addition to the single antenna case, secrecy in multi-antenna models is addressed in [6] and [7]. Regarding multiuser models, Liu *et al.* [8] presented inner and outer bounds on secrecy capacity regions for broadcast and interference channels. The secrecy capacity of the multi-antenna broadcast channel is obtained in [9]. Additionally, it is well known that even if they are equipped with single-antennas individually, users can cooperate to form a distributed multi-antenna system by performing

relaying. When channel side information (CSI) is exploited, relay nodes can collaboratively work similarly as in a MIMO system to build a virtual beam towards the receiver. Relay beamforming research has attracted much interest recently (see e.g., [10]–[14] and references therein). Cooperative relaying under secrecy constraints was also recently studied in [15]–[18]. In [15], a decode-and-forward (DF) based cooperative protocol is considered, and a beamforming system is designed for secrecy capacity maximization or transmit power minimization. For amplify-and-forward (AF), suboptimal closed-form solutions that optimize bounds on secrecy capacity are proposed in [16]. However, in those studies, the analysis is conducted only under total relay power constraints and perfect CSI. In our recent work [18], we studied the problem of DF beamforming under both total and individual power constraints. It is shown that the total power constraint leads to a closed-form solution. The design under individual relay power constraints is formulated as an optimization problem which is shown to be easily solved using two different approaches, namely semidefinite programming and second-order cone programming. In this paper, we study amplify-and-forward (AF) relay beamforming under both total and individual power constraints. We also extend our previous study by considering two robust beamforming design methods for DF relaying under imperfect CSI.

II. CHANNEL MODEL

We consider a communication channel with a source S , a destination D , an eavesdropper E , and M relays $\{R_m\}_{m=1}^M$ as depicted in Figure.1. We assume that there is no direct link between S and D , and S and E . We also assume that relays work synchronously by multiplying the signals to be transmitted with complex weights $\{w_m\}$ to produce a virtual beam point to destination and eavesdropper. We denote the channel fading coefficient between S and R_m as $g_m \in \mathbb{C}$, the fading coefficient between R_m and D as $h_m \in \mathbb{C}$, and the fading coefficient between R_m and E as $z_m \in \mathbb{C}$. In this model, the source S tries to transmit confidential messages to D with the help of the relays while keeping the eavesdropper E ignorant of the information. It's obvious that our channel is a two-hop relay network. In the first hop, the source S transmits x_s to relays with power $E[|x_s|^2] = P_s$. The received signal at

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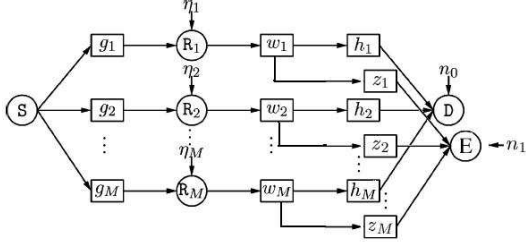


Fig. 1. Channel Model

R_m is given by

$$y_{r,m} = g_m x_s + \eta_m \quad (1)$$

where η_m is the background noise that has a Gaussian distribution with zero mean and variance of N_m . In first part of the paper, we focus on amplify-and-forward relaying. Robust design for DF will be discussed subsequently.

In the AF scenario, the received signal at the m^{th} relay R_m is directly multiplied by $l_m w_m$ without decoding, and forwarded to D . The relay output can be written as

$$x_{r,m} = w_m l_m (g_m x_s + \eta_m). \quad (2)$$

The scaling factor,

$$l_m = \frac{1}{\sqrt{|g_m|^2 P_s + N_m}}, \quad (3)$$

is used to ensure $E[|x_{r,m}|^2] = |w_m|^2$. There are two kinds of power constraint for relays. First one is a total relay power constraint in the following form: $\|\mathbf{w}\|^2 = \mathbf{w}^\dagger \mathbf{w} \leq P_T$ where $\mathbf{w} = [w_1, \dots, w_M]^T$, P_T is the maximum total power for all relays. $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and conjugate transpose, respectively, of a matrix or vector. In a multiuser network such as the relay system we study in this paper, it is practically more relevant to consider individual power constraints as wireless nodes generally operate under such limitations. Motivated by this, we can impose $|w_m|^2 \leq p_m \forall m$ or equivalently $\|\mathbf{w}\|^2 \leq \mathbf{p}$ where $\|\cdot\|^2$ denotes the element-wise norm-square operation and \mathbf{p} is a column vector that contains the components $\{p_m\}$. p_m is the maximum power for the m^{th} relay node.

The received signals at the destination D and eavesdropper E are the superposition of the messages sent by the relays. These received signals are expressed, respectively, as

$$y_d = \sum_{m=1}^M h_m w_m l_m (g_m x_s + \eta_m) + n_0, \text{ and} \quad (4)$$

$$y_e = \sum_{m=1}^M z_m w_m l_m (g_m x_s + \eta_m) + n_1. \quad (5)$$

where n_0 and n_1 are the Gaussian background noise components at D and E , respectively, with zero mean and variance

N_0 . Now, it is easy to compute the received SNR at D and E as

$$\Gamma_d = \frac{|\sum_{m=1}^M h_m g_m l_m w_m|^2 P_s}{\sum_{m=1}^M |h_m|^2 l_m^2 |w_m|^2 N_m + N_0}, \text{ and} \quad (6)$$

$$\Gamma_e = \frac{|\sum_{m=1}^M z_m g_m l_m w_m|^2 P_s}{\sum_{m=1}^M |z_m|^2 l_m^2 |w_m|^2 N_m + N_0}. \quad (7)$$

The secrecy rate is now given by

$$R_s = I(x_s; y_d) - I(x_s; y_e) \quad (8)$$

$$= \log(1 + \Gamma_d) - \log(1 + \Gamma_e) \quad (9)$$

$$= \log \left(\frac{\sum_{m=1}^M |z_m|^2 l_m^2 |w_m|^2 N_m + N_0}{\sum_{m=1}^M |h_m|^2 l_m^2 |w_m|^2 N_m + N_0} \times \frac{|\sum_{m=1}^M h_m g_m l_m w_m|^2 P_s + \sum_{m=1}^M |h_m|^2 l_m^2 |w_m|^2 N_m + N_0}{|\sum_{m=1}^M z_m g_m l_m w_m|^2 P_s + \sum_{m=1}^M |z_m|^2 l_m^2 |w_m|^2 N_m + N_0} \right). \quad (10)$$

where $I(\cdot; \cdot)$ denotes the mutual information. In this paper, we address the joint optimization of $\{w_m\}$ with the aid perfect CSI and hence identify the optimum collaborative relay beamforming (CRB) direction that maximizes the secrecy rate in (10).

III. OPTIMAL BEAMFORMING FOR AF CASE

Let us define

$$\mathbf{h}_g = [h_1^* g_1^* l_1, \dots, h_M^* g_M^* l_M]^T, \quad (11)$$

$$\mathbf{h}_z = [z_1^* g_1^* l_1, \dots, z_M^* g_M^* l_M]^T, \quad (12)$$

$$\mathbf{D}_h = \text{Diag}(|h_1|^2 l_1^2 N_1, \dots, |h_M|^2 l_M^2 N_M), \text{ and} \quad (13)$$

$$\mathbf{D}_z = \text{Diag}(|z_1|^2 l_1^2 N_1, \dots, |z_M|^2 l_M^2 N_M). \quad (14)$$

where superscript $*$ denotes conjugate operation. Then, the received SNR at the destination and eavesdropper can be reformulated, respectively, as

$$\Gamma_d = \frac{P_s \mathbf{w}^\dagger \mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0} = \frac{P_s \text{tr}(\mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w} \mathbf{w}^\dagger)}{\text{tr}(\mathbf{D}_h \mathbf{w} \mathbf{w}^\dagger) + N_0}, \text{ and} \quad (15)$$

$$\Gamma_e = \frac{P_s \mathbf{w}^\dagger \mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0} = \frac{P_s \text{tr}(\mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w} \mathbf{w}^\dagger)}{\text{tr}(\mathbf{D}_z \mathbf{w} \mathbf{w}^\dagger) + N_0} \quad (16)$$

where $\text{tr}(\cdot)$ represent the trace of a matrix. It is obvious that we only have to maximize the term inside the logarithm function in (10). With these notations, we can write the objective function of the optimization problem as

$$\frac{1 + \Gamma_d}{1 + \Gamma_e} = \frac{1 + \frac{P_s \mathbf{w}^\dagger \mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0}}{1 + \frac{P_s \mathbf{w}^\dagger \mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0}} \quad (17)$$

$$= \frac{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0 + P_s \mathbf{w}^\dagger \mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0 + P_s \mathbf{w}^\dagger \mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w}} \times \frac{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0}{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0} \quad (18)$$

$$= \frac{N_0 + \text{tr}((\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger) \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}((\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger) \mathbf{w} \mathbf{w}^\dagger)} \times \frac{N_0 + \text{tr}(\mathbf{D}_z \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}(\mathbf{D}_h \mathbf{w} \mathbf{w}^\dagger)}. \quad (19)$$

If we denote $t_1 = \frac{N_0 + \text{tr}((\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger) \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}((\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger) \mathbf{w} \mathbf{w}^\dagger)}$, $t_2 = \frac{N_0 + \text{tr}(\mathbf{D}_z \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}(\mathbf{D}_h \mathbf{w} \mathbf{w}^\dagger)}$, and define $\mathbf{X} \triangleq \mathbf{w} \mathbf{w}^\dagger$ using the similar semidefinite programming method as described in [18], we can express the optimization problem as

$$\begin{aligned} \max_{\mathbf{X}, t_1, t_2} \quad & t_1 t_2 \\ \text{s.t.} \quad & \text{tr}(\mathbf{X}(\mathbf{D}_z - t_2 \mathbf{D}_h)) \geq N_0(t_2 - 1) \\ & \text{tr}\left(\mathbf{X}\left(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_1 \left(\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger\right)\right)\right) \geq N_0(t_1 - 1) \\ & \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad (\text{and/or } \text{tr}(\mathbf{X}) \leq P_T) \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (20)$$

where $\mathbf{X} \succeq 0$ means that \mathbf{X} is a symmetric positive semi-definite matrix. Since \mathbf{X} by definition is a rank one matrix, finding the optimal weights is in general a nonconvex optimization problem. Thus, we above ignore the rank constraint, and hence employ semidefinite relaxation (SDR) [13]. If the matrix \mathbf{X}_{opt} obtained by solving the above optimization problem happens to be rank one, then its principal component will be the optimal solution to the original problem.

Notice that this formulation is applied to both total relay power constraint and individual relay power constraint which are represented by $\text{tr}(\mathbf{X}) \leq P_T$ and $\text{diag}(\mathbf{X}) \leq \mathbf{p}$, respectively. When there is only total power constraint, we can easily compute the maximum values of t_1 and t_2 separately since now we have Rayleigh quotient problems.

$$t_{1,u} = \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \frac{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0 + P_s \mathbf{w}^\dagger \mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0 + P_s \mathbf{w}^\dagger \mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w}} \quad (21)$$

$$= \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \frac{\mathbf{w}^\dagger (\mathbf{D}_h + \frac{N_0}{P_T} + P_s \mathbf{h}_g \mathbf{h}_g^\dagger) \mathbf{w}}{\mathbf{w}^\dagger (\mathbf{D}_z \mathbf{w} + \frac{N_0}{P_T} + P_s \mathbf{h}_z \mathbf{h}_z^\dagger) \mathbf{w}} \quad (22)$$

$$= \lambda_{\max} \left(\mathbf{D}_h + \frac{N_0}{P_T} \mathbf{I} + P_s \mathbf{h}_g \mathbf{h}_g^\dagger, \mathbf{D}_z + \frac{N_0}{P_T} \mathbf{I} + P_s \mathbf{h}_z \mathbf{h}_z^\dagger \right) \quad (23)$$

where $\lambda_{\max}(\mathbf{A}, \mathbf{B})$ is the largest generalized eigenvalue of the matrix pair (\mathbf{A}, \mathbf{B}) ².

Similarly, maximum values of t_2 under total power constraint is

$$t_{2,u} = \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \frac{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0}{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0} \quad (24)$$

$$= \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \frac{\mathbf{w}^\dagger (\mathbf{D}_z + \frac{N_0}{P_T}) \mathbf{w}}{\mathbf{w}^\dagger (\mathbf{D}_h + \frac{N_0}{P_T}) \mathbf{w}} \quad (25)$$

$$= \lambda_{\max} \left(\mathbf{D}_z + \frac{N_0}{P_T} \mathbf{I}, \mathbf{D}_h + \frac{N_0}{P_T} \mathbf{I} \right). \quad (26)$$

When there are individual power constraints imposed on the relays, The maximum values $t_{1,i,u}$ and $t_{2,i,u}$ ³ for t_1 and t_2

²For a Hermitian matrix $\mathbf{A} \in \mathbb{C}^{n \times n}$ and positive definite matrix $\mathbf{B} \in \mathbb{C}^{n \times n}$, (λ, ψ) is referred to as a generalized eigenvalue – eigenvector pair of (\mathbf{A}, \mathbf{B}) if (λ, ψ) satisfy $\mathbf{A}\psi = \lambda \mathbf{B}\psi$ [19].

³Subscripts i in $t_{1,i,u}$ and $t_{2,i,u}$ are used to denote that these are the maximum values in the presence of individual power constraints.

is obtained by solving following optimization problem:

$$\begin{aligned} \max_{\mathbf{X}, t_1} \quad & t_1 \\ \text{s.t.} \quad & \text{tr}\left(\mathbf{X}\left(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_1 \left(\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger\right)\right)\right) \geq N_0(t_1 - 1) \\ & \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0, \end{aligned} \quad (27)$$

$$\begin{aligned} \text{and} \\ \max_{\mathbf{X}, t_2} \quad & t_2 \\ \text{s.t.} \quad & \text{tr}(\mathbf{X}(\mathbf{D}_z - t_2 \mathbf{D}_h)) \geq N_0(t_2 - 1) \\ & \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (28)$$

In fact, for any value of t_1 , the feasible set in (27) is convex. If, for any given t_1 , the convex feasibility problem

$$\begin{aligned} \text{find } \mathbf{X} \\ \text{s.t.} \quad & \text{tr}\left(\mathbf{X}\left(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_1 \left(\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger\right)\right)\right) \geq N_0(t_1 - 1) \\ & \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0, \end{aligned} \quad (29)$$

is feasible, then we have $t_{1,i,u} \geq t_1$. Conversely, if the convex feasibility optimization problem (29) is not feasible, then we conclude $t_{1,i,u} < t_1$. Therefore, we can check whether the optimal value $t_{1,i,u}$ of the quasiconvex optimization problem in (27) is smaller than or greater than a given value t_1 by solving the convex feasibility problem (29). If the convex feasibility problem (29) is feasible then we know $t_{1,i,u} \geq t_1$. If the convex feasibility problem (29) is infeasible, then we know that $t_{1,i,u} < t_1$. Based on this observation, we can use a simple bisection algorithm to solve the quasiconvex optimization problem (27) by solving a convex feasibility problem (29) at each step. We assume that the problem is feasible, and start with an interval $[l, u]$ known to contain the optimal value $t_{1,i,u}$. We then solve the convex feasibility problem at its midpoint $t_1 = (l + u)/2$ to determine whether the optimal value is larger or smaller than t . We update the interval accordingly to obtain a new interval. That is, if t_1 is feasible, then we set $l = t_1$, otherwise, we choose $u = t_1$ and solve the convex feasibility problem again. This procedure is repeated until the width of the interval is smaller than the given threshold. Then, we conclude that $t_{1,i,u} = l$.

Similarly, $t_{2,i,u}$ can be obtained with the same bisection algorithm by repeatedly solving the following feasibility problems:

$$\begin{aligned} \text{find } \mathbf{X} \\ \text{s.t.} \quad & \text{tr}(\mathbf{X}(\mathbf{D}_z - t_2 \mathbf{D}_h)) \geq N_0(t_2 - 1) \\ & \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (30)$$

To solve the convex feasibility problem, one can use the well-studied interior-point based methods as well. We use the well-developed interior point method based package SeDuMi [22], which produces a feasibility certificate if the problem is feasible, and its popular interface Yalmip [23].

Note that for both total and individual power constraints, the maximum values of t_1 and t_2 are obtained separately

above, and these values are in general attained by different $\mathbf{X} = \mathbf{w}\mathbf{w}^\dagger$. Now, the following strategy can be used to obtain achievable secrecy rates. For those \mathbf{X} values that correspond to $t_{1,i,u}$ and $t_{1,u}$ (i.e., the maximum t_1 values under individual and total power constraints, respectively), we can compute the corresponding $t_2 = \frac{N_0 + \text{tr}(\mathbf{D}_z \mathbf{w}\mathbf{w}^\dagger)}{N_0 + \text{tr}(\mathbf{D}_h \mathbf{w}\mathbf{w}^\dagger)}$ and denote them as $t_{2,i,l}$ and $t_{2,l}$ for individual and total power constraints, respectively. Then, $\log(t_{1,i,u} t_{2,i,l})$ and $\log(t_{1,u} t_{2,l})$ will serve as our amplify-and-forward achievable rates for individual and total power constraints, respectively. With the achievable rates, we propose the following algorithm to iteratively search over t_1 and t_2 to get the optimal $t_{1,o}$ and $t_{2,o}$ that maximize the product $t_1 t_2$ by checking the following feasibility problem.

$$\begin{aligned} & \text{find } \mathbf{X} \succeq 0 \\ & \text{s.t } \text{tr}(\mathbf{X}(\mathbf{D}_z - t_2 \mathbf{D}_h)) \geq N_0(t_2 - 1) \\ & \text{tr}\left(\mathbf{X}\left(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_1 (\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger)\right)\right) \geq N_0(t_1 - 1) \\ & \text{and } \text{tr}(\mathbf{X}) \leq P_T \quad \text{if there is total power constraint,} \\ & \text{or } \text{diag}(\mathbf{X}) \leq \mathbf{p} \quad \text{if there is individual power constraint.} \end{aligned} \quad (31)$$

A. Proposed Algorithm

Define the resolution $\Delta t = \frac{t_{1,u}}{N}$ or $\Delta t = \frac{t_{1,i,u}}{N}$ for some large N for total and individual power constraints, respectively.

- 1) Initialize $t_{1,o} = t_{1,u}$, $t_{2,o} = t_{2,l}$ when total power constraint is imposed, and $t_{1,o} = t_{1,i,u}$, $t_{2,o} = t_{2,i,l}$ when individual power constraint is imposed. Initialize the iteration index $i = N$.
- 2) Set $t_1 = i\Delta t$. If $t_1 t_{2,u} < t_{1,o} t_{2,o}$ (total power constraint) or $t_1 t_{2,i,u} < t_{1,o} t_{2,o}$ (individual power constraint), then go to Step (3). Otherwise,
 - a) Let $t_2 = \frac{t_{1,o} t_{2,o}}{t_1}$. Check the feasibility problem (31). If it is infeasible, go to step (3). If it is feasible, use the bisection algorithm in (31) with t_1 to get the maximum possible values of t_2 and denote this maximum as $t_{2,m}$. The initial interval in the above bisection algorithm can be chosen as $[\frac{t_{1,o} t_{2,o}}{t_1}, t_{2,u}]$ or $[\frac{t_{1,o} t_{2,o}}{t_1}, t_{2,i,u}]$ depending on the power constraints.
 - b) Update $t_{1,o} = t_1$, $t_{2,o} = t_{2,m}$, $i = i - 1$. Go back to step (2).
- 3) Solve the following convex problem to get the optimal \mathbf{X}

$$\begin{aligned} & \min_{\mathbf{X}} \quad \text{tr}(\mathbf{X}) \\ & \text{s.t } \text{tr}(\mathbf{X}(\mathbf{D}_z - t_{2,o} \mathbf{D}_h)) \geq N_0(t_{2,o} - 1) \\ & \quad \text{tr}\left(\mathbf{X}\left(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_{1,o} (\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger)\right)\right) \\ & \quad \geq N_0(t_{1,o} - 1) \\ & \quad \mathbf{X} \succeq 0 \text{ and} \\ & \quad \text{tr}(\mathbf{X}) \leq P_T \quad \text{if there is total power constraint,} \\ & \quad \text{diag}(\mathbf{X}) \leq \mathbf{p} \quad \text{if there is individual power constraint.} \end{aligned} \quad (32)$$

IV. ROBUST BEAMFORMING DESIGN FOR DF CASE

In the second hop, we can also employ decode and forward transmission scheme. In this scheme, each relay R_m first decodes the message x_s and normalizes it as $x'_s = x_s / \sqrt{P_s}$. Subsequently, the normalized message is multiplied by the weight factor w_m to generate the transmitted signal $x_r = w_m x'_s$. The output power of each relay R_m is given by

$$E[|x_r|^2] = E[|w_m x'_s|^2] = |w_m|^2. \quad (33)$$

The received signals at the destination D and eavesdropper E are the superpositions of the signals transmitted from the relays. These signals can be expressed, respectively, as

$$y_d = \sum_{m=1}^M h_m w_m x'_s + n_0 = \mathbf{h}^\dagger \mathbf{w} x'_s + n_0, \quad \text{and} \quad (34)$$

$$y_e = \sum_{m=1}^M z_m w_m x'_s + n_1 = \mathbf{z}^\dagger \mathbf{w} x'_s + n_1. \quad (35)$$

Additionally, we have defined $\mathbf{h} = [h_1^*, \dots, h_M^*]^T$, $\mathbf{z} = [z_1^*, \dots, z_M^*]^T$. The metrics of interest are the received SNR levels at D and E , which are given by

$$\Gamma_d = \frac{|\sum_{m=1}^M h_m w_m|^2}{N_0} \quad \text{and} \quad \Gamma_e = \frac{|\sum_{m=1}^M z_m w_m|^2}{N_0}. \quad (36)$$

It has been proved that the secrecy rate R_s over the channel between the relays and destination is

$$R_s = I(x_s; y_d) - I(x_s; y_e) \quad (37)$$

$$= \log(1 + \Gamma_d) - \log(1 + \Gamma_e) \quad (38)$$

$$= \log\left(\frac{N_0 + |\sum_{m=1}^M h_m w_m|^2}{N_0 + |\sum_{m=1}^M z_m w_m|^2}\right). \quad (39)$$

By using the semidefinite relaxation (SDR) approach to approximate the problem as a convex semidefinite programming (SDP) problem, the beamforming design for decode-and-forward under perfect CSI with individual power constraint can be formed as the following optimization problem [18]:

$$\begin{aligned} & \max_{\mathbf{X}, t} \quad t \\ & \text{s.t } \text{tr}(\mathbf{X}(\mathbf{h}\mathbf{h}^\dagger - t\mathbf{z}\mathbf{z}^\dagger)) \geq N_0(t - 1), \\ & \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (40)$$

which can be solved efficiently by interior point methods with a bisection algorithm [18].

Systems robust against channel mismatches can be obtained by two approaches. In most of the robust beamforming methods, the perturbation is modeled as a deterministic one with bounded norm which leads to a worst case optimization. The other approach applied to the case in which the CSI error is unbounded is the statistical approach which provides the robustness in the form of confidence level measured by probability.

We define $\hat{\mathbf{H}} = \hat{\mathbf{h}}\hat{\mathbf{h}}^\dagger$ and $\hat{\mathbf{Z}} = \hat{\mathbf{z}}\hat{\mathbf{z}}^\dagger$ as the channel estimators, and $\tilde{\mathbf{H}} = \mathbf{H} - \hat{\mathbf{H}}$ and $\tilde{\mathbf{Z}} = \mathbf{Z} - \hat{\mathbf{Z}}$ as the estimation errors. First, consider the worst case optimization. In the worst case assumption, $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{Z}}$ are bounded in their Frobenius norm as

$\|\hat{\mathbf{H}}\| \leq \epsilon_H$, $\|\tilde{\mathbf{Z}}\| \leq \epsilon_Z$, where ϵ_H, ϵ_Z are assumed to be upper bounds on the channel uncertainty. Based on the result of [20], the robust counterpart of the previously discussed optimization problem can be written as

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{s.t.} \quad & \text{tr}(\mathbf{X}((\hat{\mathbf{H}} - \epsilon_H \mathbf{I}) - t(\tilde{\mathbf{Z}} + \epsilon_Z \mathbf{I})) \geq N_0(t-1), \\ & \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (41)$$

Note that the total power constraint $\text{tr}(\mathbf{X}) \leq P_T$ can be added into the formulation or substituted for the individual power constraint in (41). This problem can be solved the same way as discussed in [18].

However, the worst-case approach requires the norms to be bounded, which is usually not satisfied in practice. Also, this approach is too pessimistic since the probability of the worst-case may be extremely low. Hence, statistical approach is a good alternative in certain scenarios. In our case, we require the probability of the non-outage for secrecy transmission is greater than the predefined threshold ε by imposing

$$\begin{aligned} & Pr \left(\frac{N_0 + \text{tr}((\hat{\mathbf{H}} + \tilde{\mathbf{H}})\mathbf{X})}{N_0 + \text{tr}((\tilde{\mathbf{Z}} + \tilde{\mathbf{Z}})\mathbf{X})} \geq t \right) \\ & = Pr \left(\text{tr}(\mathbf{X}(\hat{\mathbf{H}} + \tilde{\mathbf{H}} - t(\tilde{\mathbf{Z}} + \tilde{\mathbf{Z}})) \geq N_0(t-1)) \right) \geq \varepsilon. \end{aligned} \quad (42)$$

Now, the optimization problem under imperfect CSI can be expressed as

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{s.t.} \quad & Pr \left(\text{tr}(\mathbf{X}(\hat{\mathbf{H}} + \tilde{\mathbf{H}} - t(\tilde{\mathbf{Z}} + \tilde{\mathbf{Z}})) \geq N_0(t-1)) \right) \geq \varepsilon, \\ & \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p} \text{ (or } \text{tr}(\mathbf{X}) \leq P_T), \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (43)$$

If relays are under individual power constraints, we use $\text{diag}(\mathbf{X}) \leq \mathbf{p}$. Otherwise, for the case of total power constraint, we use $\text{tr}(\mathbf{X}) \leq P_T$. We can also impose both constraints in the optimization. For simplicity of the analysis we assume that the components of the Hermitian channel estimation error matrices $\hat{\mathbf{H}}$ and $\tilde{\mathbf{Z}}$ are independent, zero-mean, circularly symmetric, complex Gaussian random variables with variances $\sigma_{\hat{H}}^2$ and $\sigma_{\tilde{Z}}^2$. Now, we can rearrange the probability in the constraint as

$$Pr \left(\text{tr}((\hat{\mathbf{H}} - t\tilde{\mathbf{Z}} + \tilde{\mathbf{H}} - t\tilde{\mathbf{Z}})\mathbf{X}) \geq (t-1)N_0 \right). \quad (44)$$

Let us define $y = \text{tr}((\hat{\mathbf{H}} - t\tilde{\mathbf{Z}} + \tilde{\mathbf{H}} - t\tilde{\mathbf{Z}})\mathbf{X})$. For given \mathbf{X} , $\hat{\mathbf{H}}$, and $\tilde{\mathbf{Z}}$, we know from the results of [21] that y is a Gaussian distributed random variable with mean $\mu = \text{tr}((\hat{\mathbf{H}} - t\tilde{\mathbf{Z}})\mathbf{X})$ and variance $\sigma_y^2 = (\sigma_{\hat{H}}^2 + t^2\sigma_{\tilde{Z}}^2)\text{tr}(\mathbf{X}\mathbf{X}^\dagger)$.

Then, the non-outage probability can be written as

$$\begin{aligned} Pr(y \geq (t-1)N_0) &= \int_{(t-1)N_0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{(y-\mu)^2}{2\sigma_y^2}\right) dy \\ &= \frac{1}{2} - \frac{1}{2} \text{erf}\left(\frac{(t-1)N_0 - \mu}{\sqrt{2}\sigma_y}\right) \geq \varepsilon, \end{aligned} \quad (45)$$

or equivalently as,

$$\frac{(t-1)N_0 - \mu}{\sqrt{2}\sigma_y} \leq \text{erf}^{-1}(-2\varepsilon + 1). \quad (47)$$

Note that ε should be close to one for good performance. Thus, both $-2\varepsilon + 1$ and $\frac{(t-1)N_0 - \mu}{\sqrt{2}\sigma_y}$ should be negative valued. Note further that we have $\text{tr}(\mathbf{X}\mathbf{X}^\dagger) = \|\mathbf{X}\|^2$, and hence $\sigma_y = \sqrt{\sigma_{\hat{H}}^2 + t^2\sigma_{\tilde{Z}}^2}\|\mathbf{X}\|$. Then, this constraint can be written as

$$\|\mathbf{X}\| \leq \frac{(t-1)N_0 - \mu}{\sqrt{2(\sigma_{\hat{H}}^2 + t^2\sigma_{\tilde{Z}}^2)}\text{erf}^{-1}(-2\varepsilon + 1)}. \quad (48)$$

As a result, the optimization problem becomes

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{s.t.} \quad & \|\mathbf{X}\| \leq \frac{(t-1)N_0 - \mu}{\sqrt{2(\sigma_{\hat{H}}^2 + t^2\sigma_{\tilde{Z}}^2)}\text{erf}^{-1}(-2\varepsilon + 1)}, \\ & \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p} \text{ (or } \text{tr}(\mathbf{X}) \leq P_T), \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (49)$$

Using the same bisection search, we can solve this optimization numerically.

V. NUMERICAL RESULTS

We assume that $\{g_m\}$, $\{h_m\}$, $\{z_m\}$ are complex, circularly symmetric Gaussian random variables with zero mean and variances σ_g^2 , σ_h^2 , and σ_z^2 respectively. Moreover, each figure is plotted for fixed realizations of the Gaussian channel coefficients. Hence, the secrecy rates in the plots are instantaneous secrecy rates

In Fig. 2, we plot the secrecy rate for amplify-and-forward collaborative relay beamforming system for both individual and total power constraints. We also provide the result of suboptimal achievable secrecy rate for comparison. The fixed parameters are $\sigma_g = 10$, $\sigma_h = 2$, $\sigma_z = 2$, $N_m = 1$, $N_0 = 1$ and $M = 10$. Since the AF secrecy rates depend on both the source and relay powers, the rate curves are plotted as a function of P_T/P_s . We assume that the relays have equal powers in the case in which individual power constraints are imposed, i.e., $p_i = P_T/M$. It is immediately seen from the figure that the achievable rates for both total and individual power constraints are very close to the corresponding optimal ones. Thus, the achievable beamforming scheme is a good alternative in the amplify-and-forward relaying case due to the fact that it has much less computational burden. Moreover, we interestingly observe that imposing individual relay power constraints leads to only small losses in the secrecy rates with respect to the case in which we have total relay power constraints.

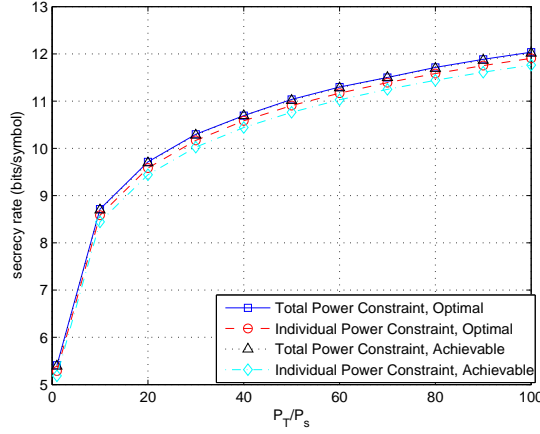


Fig. 2. AF secrecy rate vs. P_T/P_s . $\sigma_g = 10$, $\sigma_h = 2$, $\sigma_z = 2$, $M = 10$.

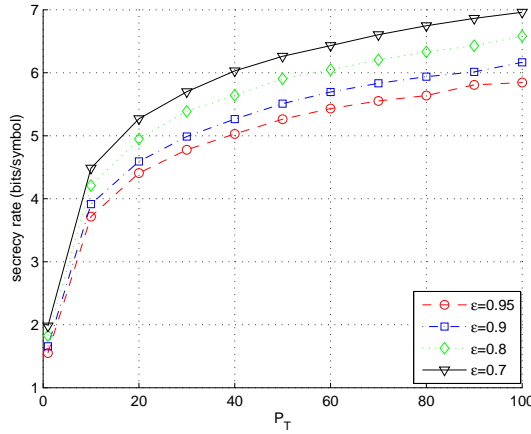


Fig. 3. DF second secrecy rate vs. P_T under different ϵ .

In Fig. 3, we plot the maximum second hop secrecy rate of decode-and-forward that we can achieve for different power P_T and non-outage probability ϵ values. In this simulation, we fix $M = 5$. $\hat{\mathbf{h}}$ and $\hat{\mathbf{z}}$ are randomly picked from Rayleigh fading with $\sigma_{\hat{h}} = 1$ and $\sigma_{\hat{z}} = 2$, and we assume that estimation errors are inversely proportional to P_T . More specifically, in our simulation, we have $\sigma_{\hat{H}}^2 = 0.1/P_T$ and $\sigma_{\hat{Z}}^2 = 0.2/P_T$. We also assume the relays are operating under equal individual power constraints, i.e., $p_i = \frac{P_T}{M}$. It is immediately observed in Fig. 3 that smaller rates are supported under higher non-outage probability requirements. In particular, this figure illustrates that our formulation and the proposed optimization framework can be used to determine how much secrecy rate can be supported at what percentage of the time. For instance, at $P_T = 100$, we see that approximately 7 bits/symbol secrecy rate can be attained 70 percent of the time (i.e., $\epsilon = 0.7$) while supported secrecy rate drops to about 5.8 bits/symbol when $\epsilon = 0.95$.

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