

# On the Optimal Placement of Underwater Sensors in a Tree Shaped Multi-hop Hierarchical Network

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**Abstract:** We analyze the optimal location of underwater acoustic sensor nodes, with respect to the capacity of the wireless links between the nodes, in a multi-hop hierarchical tree structure based on the information each node has to transfer between underwater levels. A novel algorithm is proposed to calculate the optimal distances between sensor nodes for the proposed hierarchical network. Numerical results for different channel conditions; such as an ideal channel, an imperfect channel with a certain BER, and a channel with interference, are provided. In addition, our algorithm is used to obtain the optimal frequency for each level in the tree shaped network for the above-mentioned conditions.

**Index Terms** — physical layer, tree structured topology, acoustic wireless, underwater sensor communication, channel capacity, acoustic channel, BFSK, CDMA.

## I. INTRODUCTION

The problem of sensor location is crucial for system identification, control, and damage detection, which require accurate system response. There is some extant work in the area of wireless sensors' placements in order to improve overall wireless communication system performance at the physical layer [1]. However, most of the previous work has been done in the terrestrial field, whereas very little research has been done for our domain of interest: underwater communication. The main challenges for the transmission of an acoustic signal in the marine environment are path loss, limited bandwidth, power consumption, and acoustic noise [2].

An initial effort in the placement of mobile data collectors in three-dimensional underwater wireless sensor networks (UWSNs) is given in [3], where two schemes are presented: one is for the delay tolerant placement and routing and, another is for delay constrained placement and routing. Another approach to optimal sensor placement problem is to deploy multiple surface-level gateways as in [4], where a polynomial time approximation algorithm has been proposed. Given a set of candidate locations (where gateways, whose role is to communicate UWSNs with a base station (BS), can be placed), the objective of [4] is to place the minimum number of surface gateways from all the candidate locations such that this minimal subset is connected from any underwater sensor network to a terrestrial BS so that the performance of UWSNs would be guaranteed.

Another interesting algorithm based on total capacity of sensors network is reported in [6]. Their algorithm [6] focuses on the transmission power, path loss exponent, and the expected throughput capacity of the network.

Recent work on optimal placement [5] focuses on tree network topologies, which construct spatial graphs with large characteristic path lengths and small clustering coefficients. So far, the only research utilizing a tree-shaped topology has been done to find the optimal placement of wireless sensor nodes in order to minimize the destructive effects of shadow zones [7]. In contrast with the work mentioned in [7], **we focus on the underwater wireless sensor nodes' optimal frequency and placement from the information theoretic point of view.**

We assume that our network contains a number of sensor nodes, distributed at different water depth levels, and that the nodes communicate with each other for purposes of monitoring and tracking. Specifically, we focus on a symmetric tree like multi-hop hierarchical routing topology, which can potentially cover larger areas as we go deeper in the water than flat placement of UWSNs.

## II. OUR CONTRIBUTIONS

In this paper, we *investigate the locations* of underwater acoustic sensor nodes in a multi-hop hierarchical tree structure *considering the channel capacity* of the wireless links between the nodes based on the information each node has to transfer to different levels from the bottom level to the water surface. Setting the distances between sensor nodes according to a transferred information rate criterion, leads to increased coverage compared to the previous situation when the *same number* of sensor nodes was used (all the distances between sensor nodes were *equal* to the least distance determined from the highest rate of transferring information). In addition, based on the acquired distances, we calculate the optimal communication frequency for each level. Our results are given via our algorithm.

The paper is organized as the follows. In Section III we introduce our underwater channel model and the tree shaped topology. The interference free underwater communication channel model is presented in Section IV, together with our proposed algorithm. Section V considers the interference free case with imperfect channel conditions, whereas the interference case is discussed in Section VI. Numerical

results are presented in Section VII. Conclusions and future work are summarized in Section VIII.

### III. UNDERWATER CHANNEL MODEL AND TOPOLOGY

An underwater acoustic channel is characterized by an attenuation that depends on both the distance  $r$  between the transmitter and the receiver and the signal frequency  $f$ :

$$A(r, f) = r^k a(f)^r \quad (1)$$

where  $k$  is the spreading factor ( $1 \leq k \leq 2$ ) and  $a(f) \geq 1$  is the absorption coefficient as defined in [9] and [10]. We consider the empirical model for  $a(f)$  as defined as in [9] and [10] with in  $f$  kHz, and via Thorp's formula we have the empirical expression (for "higher" frequencies) for the acoustic path loss:

$$10 \log a(f) = 0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{4100+f} + (2.75)10^{-4} f^2 + 0.003. \quad (2)$$

The noise of the acoustic underwater channel under discussion is complex additive colored (frequency dependent) Gaussian noise with zero mean and power spectral density (psd)  $N_o(f)$  that decays linearly on the logarithmic scale in the frequency region 1–20 kHz. For the psd of the noise of an acoustic channel we use the same approximation as given in [9], [11]:

$$10 \log N_o(f) = N_1 - \eta \log(f) \quad (3)$$

for the positive constant  $N_1 = 50$  dB re  $\mu$  pa [19] and  $\eta = 18$  dB/decade [9]. Obviously, the noise psd of a narrowband underwater network depends on the carrier frequency but it is flat over the communication band.

Let us consider a symmetric tree where the total number of nodes is  $N$ , each parent node has  $M$  children, the network has  $K$  levels (each sensor node at the bottom level  $K$ , does not have any children, and is called a leaf sensor node), and the information rate of each node is equal to  $\mu$ , as in Fig. 1 ( $M = 2$ ,  $K = 4$ ). The nodes in the tree shaped network are denoted by  $n_i$  ( $i = 1, 2, \dots, N$ ). The edges in the network connecting the nodes represent the acoustic links  $L_j$  from level  $i$  to  $i-1$ ; each link has a bandwidth denoted by  $BW$  for communication between the sensor nodes.

Every sensor node in the network monitors the area around it through some measurements and the obtained information is sent to the base station on the surface of the water for post processing. In this network structure, the processed information must be appropriately routed through a number of intermediate sensor nodes (multi-hop) to the

base station. As shown in Fig. 1, the network has a root sensor node  $n_1$  which is the only sensor node in the network that communicates with the base station and the information of the other sensor nodes has to be routed through the root sensor node.

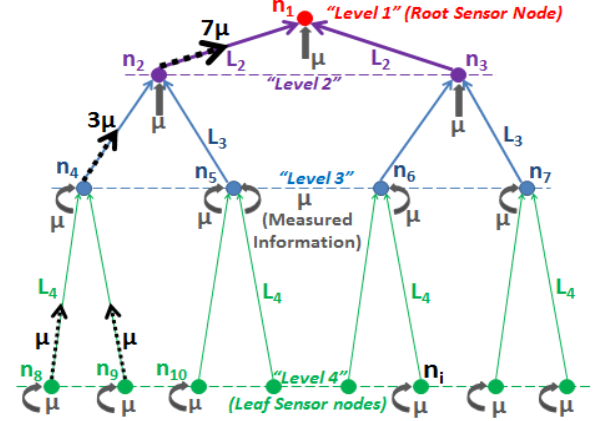


Fig. 1 Symmetric tree structured network

In the tree structured network, each node  $n_i$  ( $i > 1$ ) at level  $j$  is directly connected to one and only one node above it, a.k.a., its parent, through one of the wireless links  $L_j$  with distance  $r_j$ . Every node measures information at the rate of  $\mu$ , and may also receive information from its children (if the node is not the root node  $n_1$ ) and this information has to be routed to its parent. Therefore, the links' direction in this network are upwards (i.e., the information flow is on the uplink from a child to its parent). The total rate of information from node  $n_i$  ( $i > 1$ ) to its parent through the acoustic link  $L_j$  is represented by  $R_j$ . Using a graph representation of the network topology, models covering random graphs, small-world and scale-free networks, would be other avenues for future work which can be considered as the interplay between topology and networks' robustness against failures, and more generally, network performance.

Clearly, the total information rate per node at level  $K$  is equal to  $\mu$ , at level  $K-1$  is equal to  $(M+1)\mu$  (from the  $M$  children and the node itself), and at level 1 is equal to  $\frac{M^K - 1}{M - 1} \mu$ . In general, the information rate at level  $j$  is equal to:  $\frac{M^{(K-j+1)} - 1}{M - 1} \mu$  (via the geometric series).

Since the nodes in the symmetric tree structured network are identical, we assume that all the nodes transmit at fixed power  $P$ . Clearly, the placement of the nodes can be adjusted by using an intelligent transmission power to achieve the desired  $SNR$  level. Therefore, the assumption of fixed power can be relaxed for optimal node placement and is not considered here, but left for future work.

It is worth noting that in an asymmetric tree shaped topology a parent node can have a random number of children nodes different from that of other parent nodes, as represented in Fig. 2.

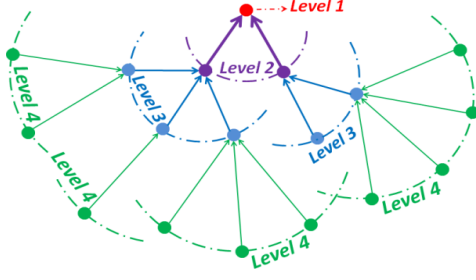


Fig. 2 Asymmetric tree structured network

#### IV. INTERFERENCE FREE COMMUNICATIONS

Under the interference free scenario, we look into the case in which the children of a node do not cause interference on the signal of their siblings received signal at the receiver of their parent node. In other words, a parent node has a number of independent receivers each one specific to one of its children. Later in this paper, we relax this assumption and investigate more realistic scenarios based on the algorithm proposed in this section. Additionally, it has been assumed that the information bits are transmitted un-coded over the acoustic links and binary phase shift keying (BPSK) modulation is used. We call this an **ideal channel** (no coding!).

By using (1)-(3) and the below relation from [8]:

$$\begin{aligned} SNR(r, f) &= \frac{S_r(f)}{A(r, f)N_o(f)} \\ &= \frac{P}{(BW)A(r, f)N_o(f)} \end{aligned} \quad (4)$$

where  $S_r(f)$  is the power spectral density of the transmitted signal. After same manipulations we obtain:

$$\begin{aligned} S_r(f) &= 50 + SNR(r, f) + 10\log(r) \\ &\quad + a(f) \cdot r \cdot 10^{-3} - 18\log(f). \end{aligned} \quad (5)$$

The communication channel model of the link  $L_j$  ( $1 \leq j \leq K$ ) is assumed to be Gaussian and, for a reliable communication, the information rate over the link  $L_j$  is bounded by the Shannon capacity (i.e.  $R_j \leq C_j$ ):

$$\begin{aligned} C &= (BW) \log_2(1 + SNR) \\ &= (BW) \log_2 \left( 1 + \frac{P}{(BW)A(r, f)N_o(f)} \right) \end{aligned} \quad (6)$$

We can also consider two children nodes as the transmitters or users. In general case for  $M$  users, which are using time sharing, and a receiver that jointly demodulates and decodes all the users' signals then the maximum of the combined rates for  $M$  users (which is 2 here in our model) would be [17], [18]

$$\begin{aligned} R &= (BW) \log_2(1 + SNR) \\ &= (BW) \log_2 \left( 1 + \frac{MP}{(BW)A(r, f)N_o(f)} \right) \end{aligned} \quad (7)$$

in bits per second. From (6), given the required rate of the link  $R$ , the bandwidth  $BW$  and the frequency  $f$  we obtain  $A(r, f)$  as:

$$A(r, f) = \frac{P}{(BW)N_o(f)(2^{R/BW} - 1)}. \quad (8)$$

This is a key equation used in our algorithm in order to provide the optimum distance  $r_j$ . Please note that the attenuation also depends on the communication frequency (1).

Regarding the optimal communication frequency  $f_{opt}(r_j)$  we make the following observation. If in (6) the parameters  $R$ ,  $BW$  and  $P$  are fixed, then the product  $A(r, f)N_o(f)$  is also fixed and constant (it is not depending of the frequency, anymore). Considering the above assumption and, by using (4), then  $SNR(r, f)$  is directly proportional with  $S_r(f)$ . By maximizing  $SNR(r, f)$  which is equivalent to maximizing the capacity, leads to the maximization of  $S_r(f)$ . We do not need to use the Karush Kuhn Tucker conditions and the two step optimization procedure as proposed in [11]. Therefore the optimal communication frequency,  $f_{opt}(r_j)$ , can be calculated by solving the next equation which is the same as in [11, eq.(12)]:

$$\begin{aligned} \frac{\partial S_r(f)}{\partial f} &= \left[ \frac{2.2 \cdot 10^{-4}}{(1 + f^2)^2} + \frac{360.8}{(4100 + f^2)^2} + 5.5 \cdot 10^{-7} \right] f \cdot r - \\ &\quad - \frac{18}{f \cdot \ln(10)} = 0. \end{aligned} \quad (9)$$

#### Optimal distance/frequency algorithm

- **Input** [  $K$  (levels),  $M$  (children),  $BW$  (channel bandwidth),  $P$  (transmission power),  $\mu$

(information rate measured by each node),  $\alpha$  (spreading factor)].

- **Output** [ $r_j$  (optimal distance between children node and their parents),  $f_{opt}$  (optimal communication frequency)].
1. **Start from the leaf sensor nodes in the tree shaped network.**
  2. **The required rate  $R_j$  ( $1 \leq j \leq k$ ) of the link  $L_j$  above the sensor node  $n_j$  is equal to the aggregate rate of the measured information  $\mu_j$  and the rate of the incoming links from each child of the node  $n_j$ .**
  3. **Given the rate  $R_j$ , the path loss can be obtained from (8).**
  4. **The optimal placement of each node from its parent can be calculated from (1) based on (8).**
  5. **Based on the estimated distance, the optimum communication frequency,  $f_{opt}(r_j)$  for that level is obtained from (9).**
  6. **Repeat the procedure from step 2 for all the siblings (nodes on the same level).**

Note that we assume that children nodes do not interact with each other (the assumption that has been also used in reference [10]). Our algorithm provides the optimum distances between children nodes and their parent nodes since information transmission is only upward or perfectly directional as in [10]. Note, that since the children are on the same level, and the upward distance is determined by algorithm, they are symmetric about the midpoint of the vertical line from the parent.

This algorithm applies to both asymmetric as well as symmetric tree shaped hierarchical networks, but as we discussed before, here just the symmetric model has been considered.

## V. INTERFERENCE FREE WITH IMPERFECT CHANNEL

We consider the case in which the bit error rate over the acoustic links in the symmetric tree is less than a small-predetermined value  $\varepsilon$  (e.g.,  $\varepsilon = 0.001$ ).

The bit error rate of the BPSK modulation at level  $j$  follows [16], [19]:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right) = Q\left(\sqrt{\frac{2SNR}{I_j/BW_j}}\right) \quad (10)$$

$$= Q\left(\sqrt{\frac{2P(M-1)}{\mu A(r_j, f) N_o(f) (M^{K-j+1} - 1)}}\right)$$

After simple manipulations we obtain:

$$A(r, f) = \frac{2P(M-1)}{\mu N_o(f) (M^{(K-j+1)} - 1) (Q^{-1}(\varepsilon))^2} \quad (11)$$

then we obtain the optimal length for the acoustic links at level  $j$  for a bit error rate of  $\varepsilon$  ( $\alpha = 1$ ) [9] and the optimum frequency would be achieved by replacing the distance in (9).

## VI. UNDERWATER CHANNELS WITH INTERFERENCE

Now, let us look at the situation where there is interference. We obtain results, but the optimization is left for future work. Due to the large path-loss of acoustic links, we assume that the children do not interfere with nodes above their parent level. Additionally, we ignore multi-path propagation. The other assumption in the previous section are still valid; for example, the information bits are transmitted un-coded over the acoustic link and BPSK modulation is used in a symmetric tree shaped hierarchical topology underwater acoustic sensor network.

Then, based on our new assumptions, the average one-sided total noise power spectral density becomes [20]:

$$I_o = (M-1) \frac{P}{BW} + N_o(f). \quad (12)$$

We calculate the parameter  $E_b/I_o$  (bit energy-to-noise density ratio) for each water deep level  $K$  according to [20]:

$$\left(\frac{E_b}{I_o}\right)_j = \frac{P/A(r_j, f)}{I_o R_j} = \frac{P/A(r_j, f) R_j}{N_o(f) + (M-1)P/BW} \quad (13)$$

The optimum distances between nodes and the water levels can be calculated by combining (10) and (13). Consequently, the optimum communication frequency for each level can be calculated from (8).

## VII. NUMERICAL EXAMPLES

We consider a tree shaped topology with number of children  $M = 2$  with  $K = 4$  (Fig. 3) and  $K = 7$  (Fig. 4) levels. Note, for  $K = 4$  we only analyze the ideal channel. We assume that the channel bandwidth  $BW = 5$  KHz, the measurement information rate at each sensor node is  $\mu = 5$  bps, and the spreading factor of  $\alpha = 1$  [9]. As seen in Fig. 3 and Fig. 4 and roughly speaking, the optimal distances in case of the ideal channel scenario are almost two times longer than the optimal distances in the case of imperfect channel scenario. Moreover, according to Fig. 4 and for the ideal channel scenario, the optimal distance for the leaf

sensor nodes successfully transmit the measured information to their parent nodes can be up to 16 Km. According to the results, using CDMA could improve the system performance and the distances could be even longer indicating that for the same amount of transferring information, a channel with CDMA utilization needs less capacity than a regular channel.

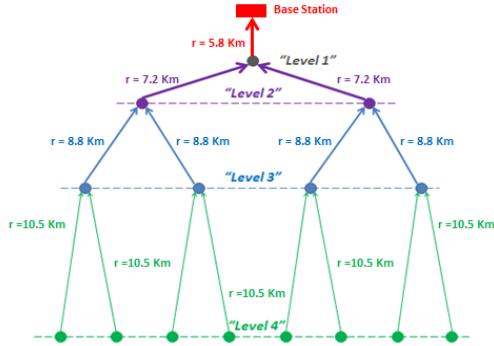


Fig.3 A 4 level tree structured network with 2 children nodes

However, this placement may not provide adequate coverage at those depths since every sensor can cover up to specific distance. Thus, there is a tradeoff between optimal distance in terms of reliable communication and optimal distance in terms of coverage for the nodes at deeper water. In summary, the optimal placement for the nodes closer to the water surface should follow the non-uniform distance needed for reliable communication and the optimal placement for the nodes farther away from the water surface should follow the coverage rule.

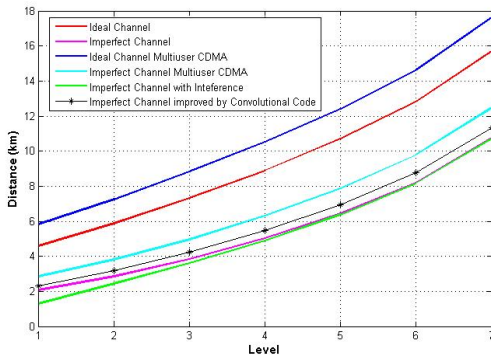


Fig. 4. Optimal distance (on the top) for ideal and imperfect channels including BER and interference for a tree shaped topology with 7 levels

As it can be seen from Fig. 4, the distances for transmitting the same amount of information at each level can be longer in the case of ideal channel comparing to the same level for imperfect channel. For both scenarios, the optimum frequency at higher level (corresponding to lower transmitting data rate) would be much more than the required frequency at lower levels.

Bearing in mind all the channel characteristics for the 7 level symmetric tree structured sensor network, the optimum

transmission frequencies have been numerically calculated and illustrated in Fig. 5 under the case of ideal channel scenario. Figure 6 depicts the optimal frequency in the case of imperfect channel with BER of  $\varepsilon = 0.001$ .

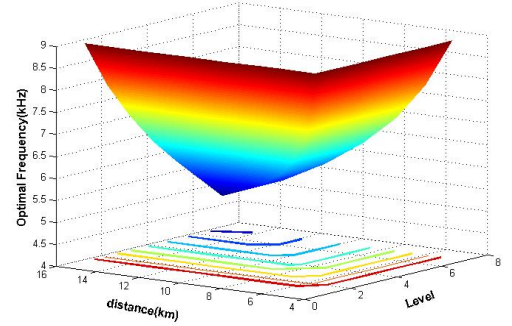


Fig. 5. Optimal frequency at each distance associated with a specific level for an ideal channel in a 7-level tree structured network

On the other hand, the improved system by using convolutional codes has been shown in Fig.7 [14], [15]. Figure 7 shows the effect of convolutional code on optimal frequency of a tree structured underwater wireless sensor network in which  $f_{opt}$  is closer to that one corresponding to the ideal channels. The highest optimal frequency in the case of ideal channel is 8.9 kHz at level 7, while this goes up to 13.69 kHz in the case of imperfect condition and after utilization of convolutional coding this amount decreases slightly to 12.8 kHz.

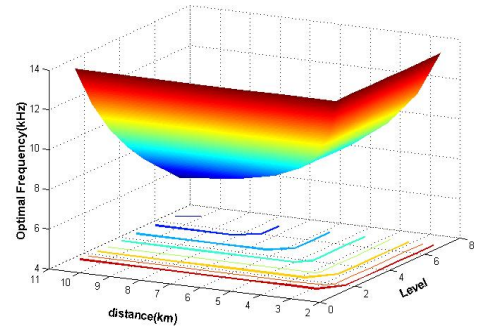


Fig. 6. Optimal frequency at each distance associated with a specific level for an imperfect channel (BER = 0.001) in a 7-level tree structured UWSN

Finally, Fig. 8 shows the variation of the path-loss in tree structured wireless sensor network as a function of frequency and networking level in the described model.



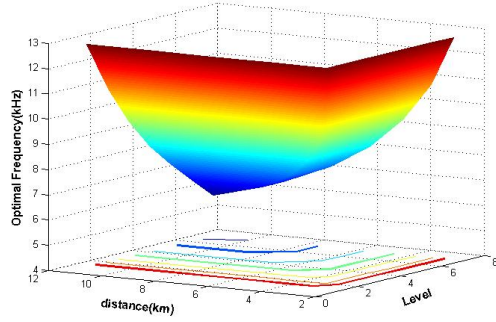


Fig. 7. Optimal frequency at each distance associated with a specific level for an imperfect channel (BER = 0.001) improved with convolutional code in a 7-level tree structured network [12], [13]

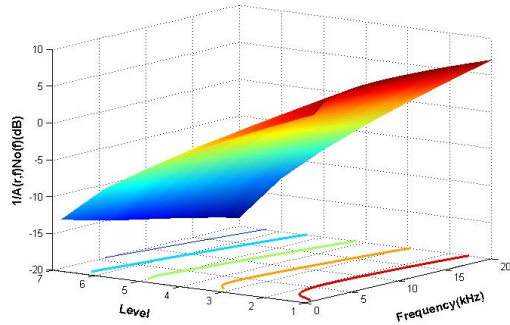


Fig. 8. Frequency dependent portion of narrow-band SNR,

$$\frac{1}{A(r,f)N_o(f)}$$

## VIII. CONCLUSIONS

The problem of sensor nodes' placements for a tree shaped topology multi-hop underwater wireless system, under different scenarios such as an ideal channel, imperfect channel with no interference and imperfect channel considering the interference has been investigated. The optimum communication frequencies at each level for mentioned scenarios have been calculated. The improvements of the sensors' locations after using convolutional codes have been considered, as well. For the future work, we will focus on solving the problem considering Doppler spread. We are also going to use our results to explore optimum placement in various clustering problems.

## Acknowledgements

The first two authors thank for support received from the Department of Defense (DoD) Grant W911NE-11-1-0144.

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