$\mathcal{O}(1)$ Communication for Distributed SGD through Two-Level Gradient Averaging

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Abstract

Large neural network models present a hefty communication challenge to distributed Stochastic Gradient Descent (SGD), with a communication complexity of $\mathcal{O}(n)$ per worker for a model of n parameters. Many sparsification and quantization techniques have been proposed to compress the gradients, some reducing the communication complexity to $\mathcal{O}(k)$, where $k \ll n$. In this paper, we introduce a strategy called two-level gradient averaging (A2SGD) to consolidate all gradients down to merely two local averages per worker before the computation of two global averages for an updated model. A2SGD also retains local errors to maintain the variance for fast convergence. Our theoretical analysis shows that A2SGD converges similarly like the default distributed SGD algorithm. Our evaluation validates the theoretical conclusion and demonstrates that A2SGD significantly reduces the communication traffic per worker, and improves the overall training time of LSTM-PTB by $3.2\times$ and $23.2\times$, respectively, compared to Top-K and QSGD. To the best of our knowledge, A2SGD is the first to achieve $\mathcal{O}(1)$ communication complexity per worker for distributed SGD.

1 Introduction

Deep learning has found great success in image classification, speech recognition, and language processing [1, 2], etc. The demand for more powerful and accurate Deep Neural Networks (DNNs) leads to large and complex models with more than 1 Billion parameters, such as GPT-2 (1.5B) [3] and Transformer (6B) [4]. Such large-scale models require distributed Stochastic Gradient Descent (SGD) algorithms for training. Distributed SGD typically adopts data parallelism, in which P workers hold the same model $w \in \mathbb{R}^n$ of n parameters and train it in parallel through many iterations. At the t-th iteration, weight w is updated as follows based on the learning rate η and the gradients g:

$$w_{t+1} = w_t - \eta_t \frac{1}{P} \sum_{p=1}^{P} g_t^p, \tag{1}$$

where a worker computes local gradients (of the same size *n*) for the model using its fraction of a mini-batch, and exchanges the gradients across all workers for an updated global model. Such a global exchange and synchronization problem imposes a hefty requirement on both the latency and bandwidth of distributed systems, and hampers the scalability of distributed SGD [5–9]. Various strategies have been proposed to tackle this problem by increasing the mini-batch sizes [10–12], reducing the rounds of communication [13–15], or pruning the neural networks [16–19].

Particularly, there exists a fundamental bottleneck, i.e., the need to transfer $\mathcal{O}(n)$ local gradients for each worker. Many studies have proposed to compress the gradients through quantization [20–23, 6, 15] and/or sparsification [24–27]. Quantization enables lossy compression of gradients by reducing the precision of their representation to a varying degree, from 1BitSGD [28, 6] with only a sign bit, TernGrad [20] with three numerical levels $\{-1,0,1\}$, to QSGD [21] that supports multiple quantization levels. These quantization techniques can reduce the communication volume by at most 32 times, assuming gradients are single-precision float-point numbers.

Gradient sparsification can achieve higher compression by selecting only k out of n gradients to reduce the communication traffic per worker [25-27]. The selection criteria can be based on a user-defined threshold (Top-K) [26], a gaussian-estimated threshold (Gaussian-K) [25], or simple randomization (Rand-K) [27]. Prior results [27, 24] have shown that, theoretically, sparsified SGD can converge within the same upper bound as the original distributed SGD (dense SGD) algorithm, which exchanges full gradients. In practice, they have different convergence behaviors, for which Shi et al. [25] have performed a theoretical analysis to distinguish them.

In this paper, we propose a novel algorithm different from both sparsification and quantization. Our algorithm two-level gradient averaging (A2SGD) consolidates all local gradients down to merely two local means and achieves a communication complexity of $\mathcal{O}(1)$ per worker. It then aggregates the local means into two global means across all workers for an updated model. The key idea behind A2SGD is to not drop or quantize any gradient, but average all local gradients and record the difference between the gradients and the resulting means locally at each worker. In doing so, A2SGD retains local errors to maintain the same variance across gradients as dense SGD, avoiding any potential variance blowup or any increase on the number of iterations. A2SGD does not require complex sampling or sorting of gradients but only simple calculations for the two means and their differences with the gradients. Our theoretical analysis shows that A2SGD converges similarly like dense SGD. Our evaluation validates the theoretical conclusion and demonstrates that A2SGD significantly improves the execution time per iteration and the overall training time, by $3.2\times$ and 23.2×, respectively, compared to Top-K and QSGD for LSTM-PTB, a big model with around 66 million parameters. Compared to the default dense SGD algorithm, A2SGD improves the overall training time of LSTM-PTB by $1.72\times$. Compared to other techniques such as Top-K, Gaussian-K and QSGD, A2SGD achieves the best overall performance in terms of convergence accuracy, execution time, and scaling efficiency.

Our Contributions. In summary, by examining the scalability challenge of gradient synchronization in distributed SGD and analyzing its computation and communication complexities, we have proposed a two-level gradient averaging algorithm A2SGD for distributed workers to exchange only two means globally. Our theoretical analysis and experimental results have confirmed the convergence of A2SGD and demonstrated that A2SGD achieves an overall improvement compared to other sparsification and quantization algorithms [25, 27, 21]. Our results also show that A2SGD achieves fast computation complexity. To the best of our knowledge, A2SGD is the first to achieve $\mathcal{O}(1)$ communication complexity per worker for distributed SGD. It can be further integrated with quantization [28, 6, 20, 21, 29, 22, 30, 31] or communication reduction [15, 13] techniques for additional improvements on the scalability of distributed SGD.

2 Related Work

Gradient Quantization. Gradient quantization takes advantage of the fact that distributed SGD can still converge with low-precision gradients instead of 32-bit floating-point representations. A wide variety of quantization techniques have tried to represent gradients in 16 bits [32–34], 8 bits [35], 2.8 bits [21], 2 bits [36], or even 1 bit [28, 6]. In addition, Wen et al. [20] have quantized gradients from workers to the server using ternary values {-1, 0, 1}. Furthermore, some studies have provided theoretical analysis on the convergence guarantees of quantization techniques [21, 29, 15, 22, 30, 31]. Notwithstanding the compulsory cost for quantizing the gradients, quantization is inherently limited by its optimization scope, i.e., the number of bits representing the gradients. Thus it can reduce the network traffic by at most 32x compared to 32-bit numbers. The overall improvement on the time per iteration or the total training time is further limited.

Gradient Sparsification. Compared to gradient quantization, sparsification examines the total number (n) of gradients and selectively transfers only a small number (k) of them while still allowing

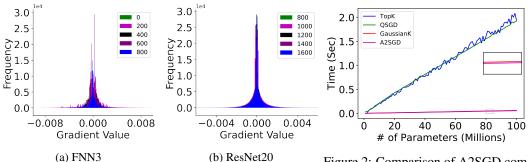


Figure 1: Progression of Gradient Distribution.

Figure 2: Comparison of A2SGD computation time with other algorithms.

DNN models to converge. Because k can be several orders of magnitude smaller than n, sparsification techniques [6, 26, 37, 38, 27, 24, 15, 39] have been shown to be much more effective than quantization in reducing the communication traffic. Several studies [6, 26] have differentiated gradient values by magnitude and purged the small ones under a threshold. Lin et al. [37] adopted a number of optimizations to achieve very high sparsity in the exchanged gradients and carefully tuned the hyperparameters of DNN models to avoid any loss of accuracy. Wangni et al. [38], Stich et al. [27], Alistarh et al. [24] theoretically analyzed the performance of sparsification and established various bounds on the convergence rate. Nonetheless, it is imperative for these techniques to process all gradients, at certain computation costs, to reach their desired sparsity levels. As an alternative, Shi et al. [25] have recently proposed to take advantage of the gaussian distribution property of gradients and statistically pinpoint a threshold to select the top k gradients at low computation costs.

3 Design of Distributed SGD with Two-Level Gradient Averaging

As mentioned in §1, gradient synchronization imposes a fundamental scalability challenge for dataparallel distributed SGD due to the requirement for all workers to exchange their gradients. While the sparsity and quantization levels are important to the communication complexity of gradient synchronization, the computation efficiency of sparsification and quantization can be critical to the scalability of gradient synchronization as well. Shi et al. [25] reported that, while Top-K sparsification reduces the communication traffic, its computation overhead can offset the overall benefit, resulting in a suboptimal improvement on the execution time per iteration. On systems with high-bandwidth communication networks at 100 Gbps or higher, the computation costs from Top-K sparsification can overshadow its gains on communication efficiency, as we have observed in our experimental evaluation (§4). The same tradeoff happens to quantization techniques such as QSGD. Shi et al. [25] proposed Gaussian-K to avoid costly sorting and selection of top K elements across all gradient values. Gaussian-K assumes a gaussian distribution of gradient values and estimates a statistical threshold for the selection of gradient values. It has demonstrated the importance of low computation for sparsification. Sparsification and quantization can also be combined and generalized as compression techniques for the improvement of gradient synchronization [15, 40, 22]. All these studies have mitigated the computation costs of gradient costs while allowing the models to converge. But all of them require the workers in distributed SGD to exchange some fraction of their gradients.

We propose an alternative to sparsification. Instead of selecting a top fraction of gradient values, we can exchange the mean across the distributed workers. To avoid over simplification caused by a unified mean, we arrange the gradient values into two groups: positive (≥ 0) and negative, and compute their absolute means accordingly. Then all workers can exchange these two means for gradient synchronization. A global negative mean is then computed by averaging the negative means from all workers; and a global positive mean by averaging the positive means. We refer to our algorithm as *Two-level Gradient Averaging (A2SGD)*. It effectively reduces the communication traffic down to two values, achieving the communication complexity of $\mathcal{O}(1)$. While this is different from gradient clipping in prior studies [20, 37, 22], it would also lead to some distorted gradients for the workers. Several prior studies [22, 28, 6, 27, 29] pursued the idea of error-feedback to correct either the momentum or the variance, or both to improve the convergence accuracy of training. To limit the impact on variance, we equip A2SGD with a similar error-feedback mechanism through a local error vector. We compute the difference between the gradients and the two means, and use a local error vector to store the difference. When the global synchronization completes, the error vector is

added back to the global means, according to the corresponding positions of the original positive and negative gradients, to generate the updated gradients.

Figure 1 shows the frequency distribution of gradients and its progression with an increasing number of iterations, for two representative models: FNN-3 and ResNet-20. Most of the values are close on either side of zero, following a normal distribution. Besides, as the models finish more iterations of the training, more gradient values are converging to the center around zero. These distribution plots from one representative worker provide a pictorial visualization on the convergence of gradients across all workers.

For an initial assessment on A2SGD's computation cost, we have measured its computation time with an increasing number of parameters, compared to Top-K, Gaussian-K and QSGD. A2SGD, Top-K, Gaussian-K use PyTorch [41] APIs with GPU support, and QSGD is implemented in Numpy [42] (see §4.1 for more details on our experimental setup). Figure 2 shows that A2SGD and Gaussian-K have much lower computation complexity than QSGD and Top-K. A2SGD has a slightly lower computation cost than Gaussian-K because Gaussian-K has to estimate a threshold [25] before gradient selection. We have elaborated the detailed computation complexity for these algorithms in §4.3. These initial results on the computation time suggest that A2SGD is very promising to support efficient gradient synchronization because of its significant reduction on communication traffic at very low computation costs.

3.1 Details of A2SGD

For a gradient vector $v = \{v_1, v_2, ..., v_n\} \in \mathbb{R}^n$, we denote $\mu_+(v) = I\!\!E\{(v_i)\} \ \forall \ v_i \geq 0$ as the absolute mean of all positive values v_i in v, and $\mu_-(v) = I\!\!E\{|v_i|\} \ \forall \ v_i < 0$ the absolute mean of all negative values in v. We introduce a new operator enc below to transform the values of v.

$$\mathbf{enc}(v) = pos(v) \cdot \mu_{+}(v) - neg(v) \cdot \mu_{-}(v) \tag{2}$$

where pos(v) and $neg(v) \in \mathbb{R}^n$ are vectors with values $\in \{1, 0\}$. The former has 1 in the corresponding positions $\forall v_i \geq 0, i \in [1, n]$, and the latter with $1, \forall v_i < 0, i \in [1, n]$,

Algorithm 1 Parallel A2SGD Algorithm at Worker p

```
Input: Initial learning rate \eta_0, weight w_0 = \vec{0}
 1: for t \leftarrow 1 to T - 1 do
           g_t \leftarrow SGD_t^p through training with M_t^p
                                                                                   \triangleright training M_t^p, portion of mini-batch for p.
           \mu_{t,+} \leftarrow \mu_+(g_t) and \mu_{t,-} \leftarrow \mu_-(g_t)
           \epsilon_t \leftarrow g_t - \mathbf{enc}(g_t)
                                                                                             \triangleright Store the errors in a local vector \epsilon_t
           (\bar{\mu}_{t,+}, \bar{\mu}_{t,-}) \leftarrow \mathbf{Allreduce}((\mu_{t,+}, \mu_{t,-}), \mathbf{average})
                                                                                                  g_t \leftarrow \epsilon_t + pos(g_t) \cdot \bar{\mu}_{t,+} - neg(g_t) \cdot \bar{\mu}_{t,-}
                                                                                                    \triangleright Aggregate the error vector \epsilon_t.
 7:
                                                                                                                     ▶ Update the model.
           w_{t+1} \leftarrow w_t - \eta_t \cdot g_t
 8: end for
 9: g_{T-1} \leftarrow Allreduce(g_{T-1}, average)
10: w_T \leftarrow w_{T-1} - \eta_{T-1} \cdot g_{T-1}
```

Algorithm 1 describes the proposed A2SGD algorithm in detail. Worker p starts with a learning rate η_0 and an initial weight w_0 . At any iteration t, Worker p computes its stochastic gradients g_t by training SGD_t^p with its portion of mini-batch M_t^p (Line 2). It then extracts the means for positive and negative gradients (Line 3) and subtracts the vector constructed from the means (Line 4). The errors are stored in a local error vector ϵ_t . All workers call the Allreduce operation to exchange their local means and get back the global means (Line 5). The global means are then combined with the errors stored in ϵ_t into the new gradients g_t (Line 6). Finally, the model weight is updated at Line 7 using the new gradient and the current learning rate. At the end of the training iterations, one more iteration is performed to synchronize the model across all workers (Lines 9 and 10).

3.2 Convergence Analysis

Distributed SGD can be analyzed in the framework of online learning systems. The convergence analysis of A2SGD follows the convergence proof of GOGA in Bottou [43], similarly as previous studies [20]. Assumption 1, Assumption 2 and Lemma 1 are also adapted from the same.

Assumption 1. C(w) has a single minimum w^* and gradient $-\nabla(w)$ always points to w^*

$$\forall \epsilon > 0, \inf_{\|w - w^*\| > \epsilon} (w - w^*)^T \nabla_w C(w) > 0$$
(3)

Convexity is a subset of Assumption 1, and we can easily find non-convex functions satisfying it.

Assumption 2. Learning rate η_t is positive and constrained as follows:

$$\begin{cases}
\sum_{t=0}^{+\infty} \eta_t^2 < +\infty \\
\sum_{t=0}^{+\infty} \eta_t = +\infty
\end{cases}$$
(4)

These constraints ensure that η_t changes neither very fast nor very slow. We define the square of distance between the current weight w_t and the minimum w^* below:

$$h_t \stackrel{\Delta}{=} \| w_t - w^* \|^2 \tag{5}$$

where $\| \cdot \|$ is l_2 norm. We also define the set of all random variables before step t as follows:

$$\mathcal{D}_t \stackrel{\Delta}{=} (z_{1\dots t-1}, b_{1\dots t-1}) \tag{6}$$

Under Assumption 1 and 2, using Lyapunov process and Quasi-Martingales convergence theorem, Bottou [43] proved the Lemma below.

Lemma 1. If $\exists \mathbb{A}, \mathbb{B} > 0$ s.t.

$$\mathbb{E}\{(h_{t+1} - (1 + \eta_t^2 \mathbb{B})h_t)|\mathcal{D}_t\} \le -2\eta_t (w_t - w^*)^T \nabla_w C(w_t) + \eta_t^2 \mathbb{A},\tag{7}$$

then C(z,w) converges almost surely toward minimum w^* i.e. $\mathbb{P}(\lim_{t\to +\infty} w_t = w^*) = 1$.

In each iteration, gradient g_t substracts its own means and then gains back the global means. We can denote $\bar{\mu}_t = pos(g_t) \cdot \bar{\mu}_{t,+} - neg(g_t) \cdot \bar{\mu}_{t,-}$ as the vector composed global means, and $\nabla \mu_t = \bar{\mu}_t - \mathbf{enc}(g_t)$ as the net gain. We draw another assumption similar to Eq. 4.34 of Bottou [43].

Assumption 3. (Gradient Bound)

$$\mathbb{E}\{\parallel g_t + \nabla \mu_t \parallel^2\} \le \mathbb{A} + \mathbb{B} \parallel w - w^* \parallel^2 \tag{8}$$

Theorem 1. When the learning system is updated as follows:

$$w_{t+1} = w_t - \eta_t (g_t + \nabla \mu_t), \tag{9}$$

then, it **converges almost surely** toward minimum w^* , *i.e.* $\mathbb{P}(\lim_{t\to+\infty} w_t = w^*) = 1$. The proof of Theorem 1 is provided in the Appendix.

4 Empirical Results

In this section, we first describe our experimental setup and then present our evaluation results validating the convergence of A2SGD. In addition, we compare its performance with dense SGD (Dense for short), two sparsification techniques Top-K [27] and Gaussian-K [25] and one quantization technique QSGD [21]. Our performance evaluation covers several aspects, including convergence accuracy, computation and communication complexities, scaling efficiency, and execution time.

4.1 Experimental Setup

Table 1: Experimental Setup

Model	Dataset	# Parameters	BatchSize	LR	Policy
FNN-3	MNIST	199,210	128	0.01	LS(1 x) + GW + PD
VGG-16	CIFAR10	14,728,266	128	0.1	LS(1.5 x) + GW + PD + LARS
ResNet-20	CIFAR10	269,722	128	0.1	LS(1 x) + GW + PD
LSTM-PTB	PTB	66,034,000	128	22	PD

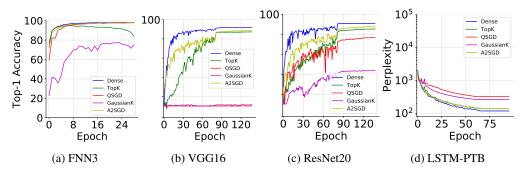


Figure 3: Comparison of Convergence Accuracy with 8 Workers

We have implemented A2SGD on top of PyTorch [41] v1.3.0 with CUDA [44] v10.1, and utilized Horovod [45] v0.19.1 with Allreduce [46] for data-parallel implementation of different models. Top-K and Gaussian-K implementations are adapted from a GitHub repository [47]. Both implementations use the PyTorch Tensor API. We have adapted the QSGD implementation from a GitHub implementation [42]. We have conducted all our experiments with 16 Nvidia V100 GPUs. Each node in this system is equipped with 256 GB CPU memory and 1 V100 GPUs per node with 16 GB GPU memory. Furthermore, all nodes in the system are connected with a high-bandwidth 100-Gbps InfiniBand network.

In our tests, we have employed four different DNN models, including (1) FNN-3 which is a Feed-forward Neural Network (FNN) with three hidden fully connected layers; 2) two types of Convolutional Neural Networks (CNNs), i.e., VGG-16 and ResNet-20 using CIFAR10 dataset; and 3) LSTM-PTB, i.e., the Long Short Term Memory (LSTM) model using Penn Treebank (PTB) dataset. We have used LARS [11], Linear Scaling(LS), Gradual Warmup (GW) and Polynomial Decay (PD) of learning rate (LR) for the Large Batch experiments. In all figures, we label Top-K and Gaussian-K without the hyphen for brevity. Table 1 lists the detailed hyperparameters for these models.

4.2 Convergence Accuracy

To demonstrate the convergence of A2SGD, we run all four models, with 30 epochs for FNN-3, 150 epochs for VGG-16 and ResNet-20, and 100 epochs for LSTM-PTB with a varying number of workers. We measure the top-1 convergence accuracy for FNN-3, VGG-16 and ResNet-20, and the perplexity score for LSTM.

Figure 3 shows the convergence performance with 8 workers. The performance with 2, 4 and 16 workers are included in the Appendix. These results show that, for all the cases, A2SGD achieves the closest top-1 accuracy to dense SGD within the same number of epochs, and outperforms the other algorithms in terms of convergence accuracy. Top-K performs the best overall among the rest of algorithms. A2SGD achieves 97.82%, 87.82%, 88.80% top-1 accuracy, and 135.53 perplexity for FNN-3, ResNet20, VGG16 and LSTM, respectively. In addition, A2SGD achieves 2.5% and 1.3% better top-1 accuracies than Top-K for ResNet20 and VGG16, respectively. Furthermore, Top-K, Gaussian-K, and QSGD all exhibit a varying amount of accuracy drops with more workers.

4.3 Gradient Synchronization Complexities and Scaling Efficiency

As discussed in $\S 3$, our A2SGD algorithm is designed to improve gradient synchronization with reduced communication traffic without costly computation to process the gradients. To gain an insight on its impact to computation and communication in gradient synchronization, we have characterized the asymptotic computation complexity and the amount of communication traffic (# bits) per worker for A2SGD, in comparison with dense SGD, QSGD, Top-K and Gaussian-K. In data-parallel distributed SGD, each worker hosts a full copy of the model and the gradients after each training iteration. We assume a model with n parameters, therefore n gradients as well.

Communication Complexity. In terms of communication traffic, it is evident that dense SGD has to transfer all gradients from each worker, i.e., 32n bits. A2SGD transfers two means, i.e., 64 bits. Top-K and Gaussian-K both transfer k gradients, i.e., 32k bits. Alistarh et al. [21] reported that QSGD transfers 2.8n+32 bits. Thus A2SGD is the only algorithm that achieves $\mathcal{O}(1)$ communication complexity per worker, which can greatly increase the communication efficiency in the gradient

synchronization of large DNN models. The amount of communication traffic for these algorithms is shown by Column 3 of Table 2.

Computation Complexity. Dense SGD does not process local gradients, and has a computation complexity of $\mathcal{O}(1)$. All other algorithms store a local residual or error vector from the transferred gradients, with a computation complexity of $\mathcal{O}(n)$. A2SGD has an overall computation complexity of $\mathcal{O}(n)$ because, for each model, it traverses all gradients to compute two separate averages, which is still $\mathcal{O}(n)$. Gaussian-K has a computation complexity of $\mathcal{O}(n)$ in its formulation of the gaussian estimation model as stated in [25]. In addition, it has an additional overhead to estimate the threshold based on its gaussian model.

In the Python implementation without GPU support [42], QSGD computes the second norm (a complexity of $\mathcal{O}(n)$) and then applies quantization for each gradient. Thus its total computation complexity is $\mathcal{O}(n^2)$. A max heap-based implementation of TopK has an overall computation complexity of $\mathcal{O}(n+klogn)$, where n is the complexity of constructing the max heap and klogn for selecting the largest k elements. The computation comparison is shown by Column 2 of Table 2. Note that GPU based implementations can have higher complexity to facilitate the need of GPU parallelization [48, 49]. For both QSGD and Top-K, the cost to maintain a local error vector is one order lower, and does not change the overall computation complexity.

Our analysis of these algorithms in terms of their asymptotic computation complexity confirms the superb overall efficiency of A2SGD. While achieving $\mathcal{O}(1)$ communication complexity per worker, it maintains the minimal asymptotic computation complexity of $\mathcal{O}(n)$, without the overhead for threshold estimation like Gaussian-K.

Table 2: Com	parison of	Gradient	Synchro	nization	Comr	olexities a	nd Scaling	Efficiency
			J					

Algorithm	Computation Complexity	Communication (# bits)	Scaling Efficiency (8 Workers) (FNN/VGG/ResNet/LSTM)
Dense SGD	$\mathcal{O}(1)$	32n	(1.83 / 2.34 / 2.52 / 2.34×)
QSGD	$\mathcal{O}(n^2)$	2.8n + 32	(1.73 / 0.66 / 2.34 / 0.26×)
Top-K	$\mathcal{O}(n + klogn)$	32k	(1.76 / 2.40 / 1.92 / 1.50×)
Gaussian-K	$\mathcal{O}(n)$	32k	(1.79 / 2.97 / 2.40 / 6.58×)
A2SGD	$\mathcal{O}(n)$	64	(1.80 / 3.06 / 2.50 / 6.37×)

Furthermore, we have evaluated the scaling efficiency of A2SGD in comparison to the other algorithms. We measure the throughput of each algorithm as the number of images processed per second. Then the scaling efficiency is calculated as the normalized throughput with an increasing number of workers. Since there is no gradient synchronization for only one worker, we use the throughput of dense SGD with two workers for normalization in order to demonstrate synchronization costs. Specifically, it is calculated as $Scaling\ Efficiency = (t_8/t_2^D)$, where t_2^D is the throughput (average images processed per iteration) of dense SGD with 2 workers, and t_8 is the overall throughput with 8 workers for any specific algorithm. A higher throughput reflects the better efficiency in processing images. As shown by the last column in Table 2, A2SGD and Gaussian-K have better scaling efficiency than the other three algorithms. The reason Gaussian-K performs comparably to A2SGD is because of its Allgather implementation of gradient exchange as will be discussed in §4.4.

4.4 Execution Time

Given our understanding on the complexity of A2SGD and its scaling efficiency, we further evaluate the benefit of A2SGD to the execution time of DNN training models. We first measure the average execution time per iteration for all algorithms. Figure 4 shows the comparison across four different models. For the smaller models FNN-3 and ResNet-20, A2SGD and Gaussian-K perform comparably to dense SGD, and slightly better than QSGD and Top-K. These two models have a smaller number of parameters, which lead to an immaterial difference on the 100-Gbps high-bandwidth network. The longer execution time per iteration of QSGD and Top-K is due to their higher computation costs.

For the bigger models VGG-16 and LSTM-PTB, A2SGD and Gaussian-K deliver much faster execution time per iteration, compared to dense SGD, Top-K and QSGD. For the biggest model, Gaussian-K achieves better execution time per iteration than A2SGD. We could not attribute this

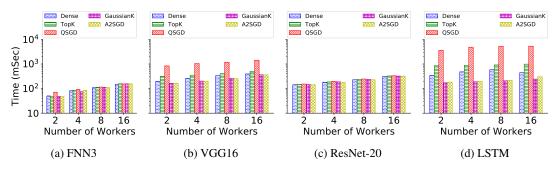


Figure 4: Comparison of Average Iteration Time

difference to the comparisons listed in Table 2. By examining the implementation of Gaussian-K, we realize that this is because Gaussian-K uses Allgather for exchanging gradients, which is faster than Allreduce adopted by A2SGD on 100-Gbps high-bandwidth networks [50, 46]. Furthermore, the execution per iteration is always the longest for QSGD. Compared to dense SGD, this is because the overhead from its high computation dominates over the benefit of communication reduction on the 100-Gbps high-bandwidth network. Moreover, all algorithms exhibit longer execution time per iteration with an increasing number of workers. This is because of the collective nature of gradient synchronization, i.e., more communication time for synchronization across more workers.

We also evaluate the benefit of A2SGD to the total execution time with an increasing number of workers. Figure 5 shows the comparison across all algorithms for four different models. Despite the increasing execution time per iteration, all algorithms deliver faster total execution time with more workers, a manifestation on the strength of data parallel distributed SGD algorithms. Again, for FNN-3 and ResNet-20, A2SGD and Gaussian-K perform comparably to dense SGD, and slightly better than QSGD and Top-K, for the same reasons as previously stated. For the bigger models VGG-16 and LSTM-PTB, A2SGD and Gaussian-K again achieve better performance than dense SGD, Top-K and QSGD. Gaussian-K is slightly faster than A2SGD for its Allgather implementation. QSGD suffers from its high computation overhead compared to dense SGD, and its high communication costs compared to the other models.

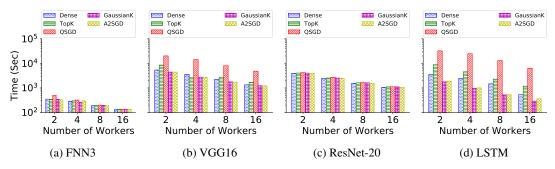


Figure 5: Comparison of Total Execution Time

Discussion. These evaluation results demonstrate that, while achieving $\mathcal{O}(1)$ communication complexity, A2SGD delivers the best overall performance in terms of convergence accuracy, scaling efficiency and execution time. Our comparative analysis also identifies an opportunity for A2SGD. While we have adopted the Allreduce implementation for gradient exchange in our initial implementation of A2SGD, it can be expanded with an Allgather-based alternative like Gaussian-K. We plan to pursue this optimization and update all our evaluation results. Also note that, compared to Gaussian-K, A2SGD does not have to go through an initial estimation of the threshold, and its computation cost is slightly lower.

5 Conclusion

In this paper, we have examined the scalability challenge of gradient synchronization in distributed SGD and proposed a two-level gradient averaging algorithm A2SGD for distributed workers to exchange only two averages and achieve $\mathcal{O}(1)$ communication complexity per worker. We have

theoretically analyzed the convergence of A2SGD. Our experimental results have confirmed the convergence of A2SGD and demonstrated that A2SGD achieves an overall improvement compared to other sparsification and quantization algorithms [25, 27, 21]. For all these algorithms, we also have systematically analyzed their computation and communication complexities during gradient synchronization and pointed out that A2SGD outperforms the others asymptotically.

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A Appendix

A.1 Proof of Theorem 1 on the Convergence of A2SGD

Theorem 1. When the learning system is updated as follows:

$$w_{t+1} = w_t - \eta_t(g_t + \nabla \mu_t), \tag{10}$$

then, it **converges almost surely** toward minimum w^* , *i.e.* $\mathbb{P}(\lim_{t \to +\infty} w_t = w^*) = 1$.

Proof:

$$h_{t+1} - h_t = -2\eta_t (w - w^*)^T (g_t + \nabla \mu_t) + \eta_t^2 (g_t + \nabla \mu_t)$$
(11)

Taking conditional expectation of previous equation:

$$\mathbb{E}\{h_{t+1} - h_t | \mathcal{D}_t\} = -2\eta_t(w_t - w^*)\mathbb{E}\{g_t + \nabla \mu_t | \mathcal{D}_t\} + \eta_t^2 \mathbb{E}\{g_t + \nabla \mu_t | \mathcal{D}_t\}$$
(12)

This expression can be further simplified using the following condition: $\nabla_w C(w_t) = (g_t + \nabla \mu_t)$

$$\mathbb{E}\{h_{t+1} - h_t | \mathcal{D}_t\} = -2\eta_t(w_t - w^*)\nabla_w C(w_t) + \eta_t^2 \mathbb{E}\{\|g_t + \nabla \mu_t\|^2 | \mathcal{D}_t\}$$
(13)

$$\implies \mathbb{E}\{h_{t+1} - h_t | \mathcal{D}_t\} + 2\eta_t(w_t - w^*) \nabla_w C(w_t) = \eta_t^2 \mathbb{E}\{\|g_t + \nabla \mu_t\|^2 | \mathcal{D}_t\}$$
 (14)

From Assumption 3., we can further have:

$$\mathbb{E}\{h_{t+1} - h_t | \mathcal{D}_t\} + 2\eta_t(w_t - w^*)\nabla_w C(w_t) \le \mathbb{A}\eta_t^2 + \mathbb{B}\eta_t^2 \parallel w - w^* \parallel^2 = \mathbb{A}\eta_t^2 + \mathbb{B}\eta_t^2 h_t$$
 (15)

$$\mathbb{E}\{h_{t+1} - (1 + \eta_t^2 \mathbb{B})h_t | \mathcal{D}_t\} \le -2\eta_t (w_t - w^*)^T \nabla_w C(w_t) + \eta_t^2 \mathbb{A}$$
(16)

Eq. 16 satisfies the condition of Lemma 1, which proves Theorem 10.

A.2 Convergence Accuracy with 2, 4 and 16 Workers

Note: All QSGD results are evaluated using quantization level 4. Threshold for TopK and GaussianK is 0.001d in all the experiments.

Figure 6 shows the convergence performance with 2 workers.

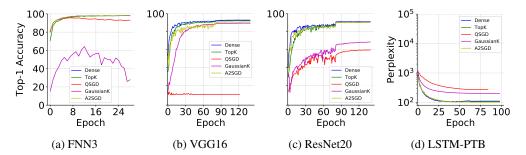


Figure 6: Comparison of Convergence Accuracy with 2 Workers

Figure 7 shows the convergence performance with 4 workers.

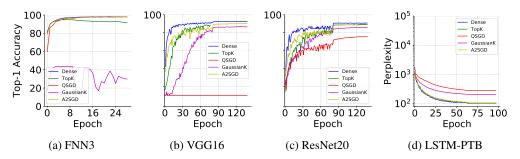


Figure 7: Comparison of Convergence Accuracy with 4 Workers

Figure 8 shows the convergence performance with 16 workers.

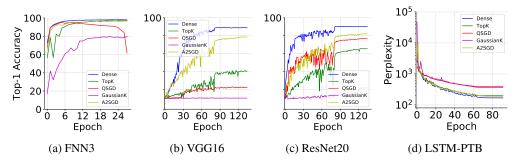


Figure 8: Comparison of Convergence Accuracy with 16 Workers