

# Residential Energy Management Using a Novel Interval Optimization Method

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**Abstract**—In this paper, a new interval optimization method is proposed to manage the uncertainty of stochastic variables to the problem of Residential Energy Management (REM). This new method is called Stochastic Predicted Bands (SPB) and it considers the uncertainty of decision making variables without knowledge of the Probability Density Function (PDF). The modeling of uncertainty is done by bands based on the prediction of stochastic variables. Besides, an auxiliary parameter is defined to provide flexibility to the decision-maker to be optimistic or conservative. Hence, applying the *optimistic coefficient* to the SPB method results in the enhancement of its performance. This new method is called Modified Stochastic Predicted Bands (MSPB). The simulation results of the test system show the performance of the proposed model in solving energy management problems via SPB method.

**Index Terms**—Residential energy management, interval optimization, decision-making under uncertainty, stochastic predicted bands, power scheduling.

## I. INTRODUCTION

Residential buildings with linked devices via communications channels are called smart homes [1]. They are known as *prosumers*, and have an important role in the optimization of electrical energy scheduling [1]. The Residential Energy Management System (REMS) is necessary for achieving an economic improvement through automation technologies. However, there are challenges in the REMSs consisting of inaccurate forecasts of energy generation and demand patterns, and heavy computational burden [2]-[8].

Various research papers have investigated the optimal scheduling of home energy, and different algorithms and methods have been presented. For example, in [3], the bi-level day-ahead REM program it has been shown that the system operator optimizes the centralized multi-objective problem based on fuzzy decision making. In [4], a novel decomposition approach has been used in an independent REM method. In [5], the REM problem has been solved by modeling the controllable loads and the loads that depend on the weather conditions. In the proposed method of [6], Demand Response (DR) program has been applied automatically to control the appliances under uncertainty of outdoor temperature and electricity price. In [7], the probabilistic optimization method has been used to solve the REM problem. In [8], the decentralized approach has been proven to manage the energy and operate

the PV power output. An energy service modeling method has been described in [9]. Particle swarm optimization (PSO) has been used to solve the optimization problem in [9]. As observed in the literature review, different methods have been utilized in the REMSs to decrease the computational burden of the REM problem. Numerous advantages can be achieved by decreasing the computational burden of the algorithms employed in REMSs such as reduction of the REMS energy consumption and obtaining enough time to concentrate on the forecasting methods with high accuracy [10].

In this paper, the uncertainty of decision making variables are modeled based on new interval optimization method. This method is called SPB that models the uncertainty of the variables in the REMS.

The rest of this paper is organized as follows. Section II introduces the proposed interval optimization framework of the REM problem. Then, problem formulation is described in Section III and the simulation results of the case study are illustrated in Section IV. Finally, the findings of the paper are concluded in Section V.

## II. PROPOSED INTERVAL OPTIMIZATION METHOD

In this section, we introduce the proposed method for modeling stochastic variables in the decision-making problem. There are different methods to model uncertainty based on its type in the problems that the interested readers are referred to [11]-[13]. There are similarities between the proposed method of this paper and the Stochastic Optimization (SO) methods. However, in this approach, presenting the uncertainty is not done by stating the scenarios. Knowing the PDF of decision-making variables is one of the necessities in most of the stochastic scenario-based methods [14]. It is clear that PDFs of stochastic variables are not always available. Besides, SO models are a large computational burden to the systems. Hence, our proposed method considers the uncertainty of the decision-making variables, taking into account the drawbacks of the SO method.

### A. Stochastic Predicted Bands (SPB) Method

In this section, the SPB method is defined to model the uncertainty. It consists of four steps which are described below:

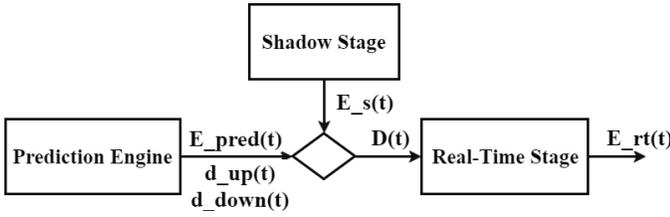


Fig. 1: Simple flowchart of SPB method.

1) *Step 1*: This model consists of two stages and it is not a bi-level optimization problem. The first stage is called the shadow stage because it does not actually executed. Also, the uncertainty of variables is not considered. The variables in this stage are also called shadow variables. The second stage is called real-time stage where the uncertainty of variables is considered, and the associated variables are called real-time variables. The shadow variables play an important role in converging the real-time variables to their optimum decisions when their uncertainties converge to zero. Hence, the shadow variables should be determined in the first step.

2) *Step 2*: In this method, the uncertainty of variables is considered based on their predicted amounts. Hence, short-term forecasting of variables is done in the second step. Besides,  $\sigma_{up}$  and  $\sigma_{down}$  are parameters that are defined to state the amounts of upper and lower variances of the predicted variable in comparison with its actual amount, respectively.

3) *Step 3*: In this step, the difference between the shadow amount of variables,  $E_t^S$ , and their predicted amount in each time,  $E_t^{pred}$  is determined. Also, a simple flowchart of the SPB method is illustrated in Fig. 1.

$$D_t = E_t^S - E_t^{pred} \quad (1)$$

4) *Step 4*: According to the state of  $D_t$ , the real-time decision-making variables,  $E_t^{rt}$ , are limited to the max and min bands which are outlined below:

- If  $D_t$  is positive, it means that the scheduling amount is more than the predicted amount. Hence, the real-time amount should be greater than the predicted amount to converge to the amount of scheduling variable.
- If  $D_t$  is negative, the predicted amount of the variable is more than the scheduling one. Hence, as the real-time variable likes to converge to the amount of its scheduling variable, the real-time variable will be limited to the predicted amount as its maximum band. Therefore, the minimum limitation of the real-time variable will be based on the upper variance of its prediction because the variables predicted amount is more than its scheduling amount.

(2) is defined to clarify the above explanations:

$$\begin{cases} E_t^{pred} \leq E_t^{rt} \leq E_t^{pred} + \sigma_{down} & D_t \geq 0 \\ E_t^{pred} - \sigma_{up} \leq E_t^{rt} \leq E_t^{pred} & D_t \leq 0 \end{cases} \quad (2)$$

For instance, it is supposed that the amount of shadow variable is determined to be equal to 10 in  $t=4$ . Also, the

predictor system forecasts that the amount of that stochastic variable equals 11 in  $t=4$ , while the upper and lower variances are considered to be 0.5 and 0.3, respectively. Hence,  $D_t$  will be negative in this case, and the real-time variable should be limited to the bands as following:

$$\begin{aligned} D_{t=4} &= 10 - 11 = -1 \\ 11 - 0.5 &\leq E_t^{rt} \leq 11 \end{aligned}$$

### B. Modified Stochastic Predicted Bands (MSPB) Method

One of the drawbacks of the SPB method is that the uncertainty of the stochastic variables cannot be modeled completely based on the predicted bands. Also, the variables tend to converge to the maximum and minimum bands based on their amounts in the shadow stage. This way, results of the decision-making variables are completely optimistic because they always adapt to the bands to optimize the objective function of the problem. Hence, the stochastic variables stick only to the maximum or minimum bands to optimize the problem.

In this section, an auxiliary parameter is defined as a slack parameter in order to give freedom to the decision-maker to apply its knowledge regarding the stochastic behavior of the uncertain variable. This parameter is called an *optimistic coefficient*,  $\alpha$ , and its amount can be between 0 and 1. This method is called MSPB. (3) is the modified version of the (2), if the MSPB method is utilized in the problem.

$$\begin{cases} E_t^{pred} \alpha + (E_t^{pred} - \sigma_{up})(1 - \alpha) \leq E_t^{rt} \\ \leq (E_t^{pred} + \sigma_{down}) \alpha + E_t^{pred} (1 - \alpha) & D_t \geq 0 \\ (E_t^{pred} - \sigma_{up}) \alpha + E_t^{pred} (1 - \alpha) \leq E_t^{rt} \\ \leq E_t^{pred} \alpha + (E_t^{pred} + \sigma_{down})(1 - \alpha) & D_t \leq 0 \end{cases} \quad (3)$$

## III. PROPOSED RESIDENTIAL ENERGY MANAGEMENT SYSTEM

In this section, MSPB is used to model uncertainty in the REMS. However, the SPB and MSPB as novel interval optimization methods can be used in different scale of power system problems that face uncertainty, and there is a lack of full information.

### A. System Objective

In this section, a model of power scheduling in a building is presented. The objective function is to maximize the revenue of energy services provided in a residential energy management system. As seen in (4), the objective function includes four parts. The first part represents the revenue from selling the energy produced to the electricity market. The total cost of energy consumption is presented in the second term. The value of energy which is not served is stated in the third term. Finally, the spillage costs of non-dispatchable energies are presented in the last term. In this model, the cost of reactive power as a major element of ancillary service has been ignored, so the interested reader is referred to [15]-[16]. The proposed objective function consists of two stages: the shadow the real-time stages.

$$OF = EC \quad (4)$$

$$EC = \sum_{t=1}^{N_t} (\lambda_{pvb} P_{pvb,o_t}^S + \lambda_w P_{w,o_t}^S - \lambda_{net} P_{net_t}^S + \lambda_{pvb} P_{pvb,o_t}^{rt} + \lambda_w P_{w,o_t}^{rt} + \lambda_{ev} P_{ev,o_t}^{rt} - \lambda_{net} P_{net_t}^{rt} - (VOLL_{sh} L_{sh_t}^{shed} + VOLL_{swh} L_{swh_t}^{shed} + VOLL_{pp} L_{pp_t}^{shed} + VOLL_{mrs} L_{mrs_t}^{shed}) - (V_w^S S_{w_t} + V_{pvb}^S S_{pvb_t}))$$

### B. First Stage

The uncertainty of decision-making variables is not considered in this stage.

$$P_{net_t}^S + P_{pvb,i_t}^S + P_{w,i_t}^S = L_{sh_t}^S + L_{swh_t}^S + L_{pp_t}^S + L_{mrs_t}^S \quad (5)$$

$$-f_{max} \leq P_{net_t}^S - (P_{pvb,o_t}^S + P_{w,o_t}^S) \leq f_{max} \quad (6)$$

(5) establishes the power balance equation of the devices in the smart home. Besides, (6) represents the power flow limitation through the distribution line which ended to the building. Moreover, there are some limitations corresponding to all appliances. Only maximum and minimum limitations of energy produced/consumed by each device are defined in this stage because the uncertainty is not considered in the shadow stage. Also, components of the REMS that cause flexibility are not modeled. Hence, the constraints of the energy storage systems and load shedding are not defined in the first stage. These flexibility is needed when system faces the uncertainty, so all flexible agents are modeled in the second stage.

$$P_{pvb_t}^S = P_{pvb,i_t}^S + P_{pvb,o_t}^S \quad (7)$$

$$P_{w_t}^S = P_{w,i_t}^S + P_{w,o_t}^S \quad (8)$$

$$P_{pvb_t}^{min} \leq P_{pvb_t}^S \leq P_{pvb_t}^{max} \quad (9)$$

$$P_{w_t}^{min} \leq P_{w_t}^S \leq P_{w_t}^{max} \quad (10)$$

$$L_{sh_t}^{min} \leq L_{sh_t}^S \leq L_{sh_t}^{max} \quad (11)$$

$$L_{swh_t}^{min} \leq L_{swh_t}^S \leq L_{swh_t}^{max} \quad (12)$$

$$L_{pp_t}^{min} \leq L_{pp_t}^S \leq L_{pp_t}^{max} \quad (13)$$

$$L_{mrs_t}^{min} \leq L_{mrs_t}^S \leq L_{mrs_t}^{max} \quad (14)$$

### C. Second Stage

In this stage, the uncertainties of decision-making variables are considered. In this paper, only uncertainty of the wind and PV power generation is considered, and uncertainty of the outdoor temperature and the must-run services are ignored for simplicity. Hence, the amounts of these variables are determined based on the outputs of the first stage and the uncertainty in the real-time operation. The power balance equation in the real-time is represented in (15). Besides, the

power flow limitation through the distribution line in the real-time is described in (16).

$$P_{net_t}^{rt} + P_{pvb,i_t}^{rt} + P_{w,i_t}^{rt} + P_{ev,i_t}^{rt} = L_{sh_t}^{rt} + L_{swh_t}^{rt} + L_{pp_t}^{rt} + L_{mrs_t}^{rt} - (L_{sh_t}^{shed} + L_{swh_t}^{shed} + L_{pp_t}^{shed} + L_{mrs_t}^{shed}) \quad (15)$$

$$-f_{max} \leq P_{net_t}^{rt} - (P_{pvb,o_t}^{rt} + P_{w,o_t}^{rt} + P_{ev,o_t}^{rt}) \leq f_{max} \quad (16)$$

1) *PV-Battery System*: The power output of the PV-battery system in the real-time,  $P_{pvb_t}^{rt}$ , is obtained based on (17). According to (17),  $P_{pv_t}^{rt}$  is the power output of the PV panels in the real-time,  $P_{b_t}^{rt}$  is the storage power of the battery in the real-time and  $S_{pvb_t}$  is the spillage power of the PV-battery system. The state of charge balance equation is defined based on (20). As seen in (20),  $C_i$  is the initial state of charge of the battery. (22) shows that the total power output of the PV-battery system equals its power output which was consumed in the building and the amount of power generation that is sold to the power market.

$$P_{pvb_t}^{rt} = P_{pv_t}^{rt} - P_{b_t}^{rt} - \omega_t - S_{pvb_t} \quad (17)$$

$$P_{pv_t}^{min} \leq P_{pv_t}^{rt} \leq P_{pv_t}^{max} \quad (18)$$

$$P_{b_t}^{min} - C_{t-1} \leq P_{b_t}^{rt} \leq P_{b_t}^{max} - C_{t-1}, t \geq 2 \quad (19)$$

$$C_t = C_{t-1} + \omega_t, t \geq 2 \quad (20)$$

$$C_t = C_i, t = 1$$

$$\omega_t^{min} \leq \omega_t \leq \omega_t^{max} \quad (21)$$

$$P_{pvb_t}^{rt} = P_{pvb,i_t}^{rt} + P_{pvb,o_t}^{rt} \quad (22)$$

2) *Electric Vehicle (EV)*: EV plays as an electrical storage system than can be used economically based on the charge and discharge strategies in the REM problem. There are different factors that should be considered to model the effect of the use of EV in the REM problem. These factors are mobility patterns and battery characteristics of the EV. The power generation of the EV is represented in (23) and (30). (24) represents the state of charge balance equation in the EV, and  $C_i^{ev}$  is the initial state of charge in the EV. (29) enforces power limitations of the energy storage system in the EV.

$$P_{ev_t}^{rt} = -P_{ev,b_t}^{rt} - \omega_t^c + \omega_t^d \quad (23)$$

$$C_t^{ev} = C_{t-1}^{ev} + \omega_t^c \eta_{G2V} - \omega_t^d / \eta_{V2G} - \omega_t^m / \eta_{V2T}, t \geq 2 \quad (24)$$

$$C_t^{ev} = C_i^{ev}, t = 1$$

$$P_{ev,d_t}^{min} \eta_{ev} (1 - u_t^{ev}) \leq \omega_t^d \leq P_{ev,d_t}^{max} \eta_{ev} (1 - u_t^{ev}) \quad (25)$$

$$P_{ev,c_t}^{min} \eta_{ev} u_t^{ev} \leq \omega_t^c \leq P_{ev,c_t}^{max} \eta_{ev} u_t^{ev} \quad (26)$$

$$0 \leq \omega_t^d \leq (C_t^{ev} - P_{ev,d_t}^{min}) \eta_{ev} \quad (27)$$

$$0 \leq \omega_t^c \leq (P_{ev,c_t}^{max} - C_t^{ev}) \eta_{ev} \quad (28)$$

$$P_{ev,d_t}^{max} - C_{t-1}^{ev} \leq P_{ev,b_t}^{rt} \leq P_{ev,c_t}^{max} - C_{t-1}^{ev}, t \geq 2 \quad (29)$$

$$P_{ev_t}^{rt} = P_{ev,i_t}^{rt} + P_{ev,o_t}^{rt} \quad (30)$$

3) *Wind System*: The power output of the wind micro-turbine is calculated according to (31). In (31),  $P_{w_t}^{rt}$  is the power output of the wind system,  $P_{w,p_t}^{rt}$  is the potential

power output of the wind micro-turbine based on the real-time weather conditions, and  $S_{w_t}$  is the spillage power of the wind system.

$$P_{w_t}^{rt} = P_{w,p_t}^{rt} - S_{w_t} \quad (31)$$

$$P_{w_t}^{min} \leq P_{w,p_t}^{rt} \leq P_{w_t}^{max} \quad (32)$$

$$P_{w_t}^{rt} = P_{w,i_t}^{rt} + P_{w,o_t}^{rt} \quad (33)$$

4) *Space Heater*: The space heater is responsible for maintaining the indoor temperature at the desired temperature. There is a differential equation between the indoor temperature and the power consumed by the space heater device. (34) represents the performance of the space heater based on the relation between the indoor temperature and the electrical load of the space heater. As seen in (34),  $\theta_0$  is the initial indoor temperature and it has been proposed that this amount is equal to the desired temperature.

$$\theta_{in_t+1} = \theta_{in_t} e^{-1/RC} + L_{sh_t}^{rt} R(1 - e^{-1/RC}) + \theta_{out_t}^{pred}(1 - e^{-1/RC}), t \geq 2 \quad (34)$$

$$\theta_{in_t} = \theta_0 = \theta_{des}, t = 1$$

$$-1 \leq \theta_{in_t} - \theta_{des} \leq 1 \quad (35)$$

$$L_{sh_t}^{min} \leq L_{sh_t}^{rt} \leq L_{sh_t}^{max} \quad (36)$$

5) *Storage Water Heater*: Storage water heater is responsible for storing the heat in the water tank via residents. The max and min limitations of the storage water heater's power and energy consumed are expressed in (37) and (38), respectively.

$$L_{swh_t}^{min} \leq L_{swh_t}^{rt} \leq L_{swh_t}^{max} \quad (37)$$

$$U_{swh_t}^{min} \leq \sum_{t=1}^{N_t} L_{swh_t}^{rt} \leq U_{swh_t}^{max} \quad (38)$$

6) *Pool Pump*: Running hours of the pool pump should not be more than  $T_{ON}$  hours in a day. (39) represents the max and min limitations of the pool pump power consumed in each hour.

$$L_{pp_t}^{min} z_t \leq L_{pp_t}^{rt} \leq L_{pp_t}^{max} z_t \quad (39)$$

$$\sum_{t=1}^{N_t} z_t \leq T_{ON} \quad (40)$$

7) *Must-run Services*: Must-run services include electrical loads that should be provided quickly such as lighting, entertainment, etc. (41) makes the assumption that there is no uncertainty in the prediction of the electrical loads of the must-run services.

$$L_{mrs_t}^{rt} = L_{mrs_t}^{pred} \quad (41)$$

8) *Spillage Limits*: The spillage amount of the wind and the PV-battery systems are expressed in (42) and (43), respectively.

$$0 \leq S_{w_t} \leq P_{w,p_t}^{rt} \quad (42)$$

$$0 \leq S_{pv_t}^{rt} \leq P_{pv_t}^{rt} \quad (43)$$

9) *Load Shedding Limits*: Load shedding is the amount of the electrical load which is not served. (44)-(47) enforce the load shedding constraints of each electrical load.

$$0 \leq L_{sh_t}^{shed} \leq L_{sh_t}^{rt} \quad (44)$$

$$0 \leq L_{swh_t}^{shed} \leq L_{swh_t}^{rt} \quad (45)$$

$$0 \leq L_{pp_t}^{shed} \leq L_{pp_t}^{rt} \quad (46)$$

$$0 \leq L_{mrs_t}^{shed} \leq L_{mrs_t}^{rt} \quad (47)$$

10) *Integration with the Modified Stochastic Predicted Bands(MSPB) Method*: In this model, SPB and MSPB methods are defined to model the uncertainty of variables in the REM problem. In this paper, the uncertainty of the wind and PV power generation are considered based on (48)-(51). Besides, outdoor temperature is considered as a deterministic variable.

$$D_{w_t} = P_{w_t}^S - P_{w_t}^{pred} \quad (48)$$

$$D_{pv_t} = P_{pv_t}^S - P_{pv_t}^{pred} \quad (49)$$

$$\begin{cases} P_{w_t}^{pred} \alpha_w + (P_{w_t}^{pred} - \sigma_w^{up})(1 - \alpha_w) \leq P_{w_t}^{rt} \\ \leq (P_{w_t}^{pred} + \sigma_w^{down}) \alpha_w + P_{w_t}^{pred}(1 - \alpha_w) & D_{w_t} \geq 0 \\ (P_{w_t}^{pred} - \sigma_w^{up}) \alpha_w + P_{w_t}^{pred}(1 - \alpha_w) \leq P_{w_t}^{rt} \\ \leq P_{w_t}^{pred} \alpha_w + (P_{w_t}^{pred} + \sigma_w^{down})(1 - \alpha_w) & D_{w_t} \leq 0 \end{cases} \quad (50)$$

$$\begin{cases} P_{pv_t}^{pred} \alpha_{pv} + (E_t^{pred} - \sigma_{pv}^{up})(1 - \alpha_{pv}) \leq P_{pv_t}^{rt} \\ \leq (P_{pv_t}^{pred} + \sigma_{pv}^{down}) \alpha_{pv} + P_{pv_t}^{pred}(1 - \alpha_{pv}) & D_{pv_t} \geq 0 \\ (P_{pv_t}^{pred} - \sigma_{pv}^{up}) \alpha_{pv} + P_{pv_t}^{pred}(1 - \alpha_{pv}) \leq P_{pv_t}^{rt} \\ \leq P_{pv_t}^{pred} \alpha_{pv} + (P_{pv_t}^{pred} + \sigma_{pv}^{down})(1 - \alpha_{pv}) & D_{pv_t} \leq 0 \end{cases} \quad (51)$$

## IV. CASE STUDY

### A. Energy Service system in a Smart Home

To assess the performance of the proposed REM model, the physical system from [9] is applied. However, some extension of the system parameters are made to the system. The case study is described in Fig. 2. The maximum energy produced by the PV system is 2-kW. The battery can store between 0.48 and 2.4 kWh, and the maximum charging/discharging rates are 400 W. Besides, charging/discharging efficiencies are 90%. The maximum energy produced by the wind micro-turbine is 6-kW. The EV can store between 1.77 and 5.9 kWh, and the maximum charging/ discharging rates are 3 kW. Charging/discharging efficiencies are 90%. Also, the EV is considered to be out of home between 6 AM and 5 PM. The maximum heating power equals 2 kW to maintain the temperature of the house within  $\pm 1$  of the desired temperature (23°C). The thermal resistance of the building shell is equal to 18°C/kW, and C equals 0.525 kWh/°C . The energy capacity of the storage water heater is 10.46 kWh (180 L) which has 2 kW heating element. The rated power of the pool pump is 1.1 kW, and it can run for maximum of 6 hours during the day. The performance of the proposed REM model is assessed in three cases. The program implemented is solved in GAMS 23.7 [17]. Table I presents the predicted data of stochastic variables.

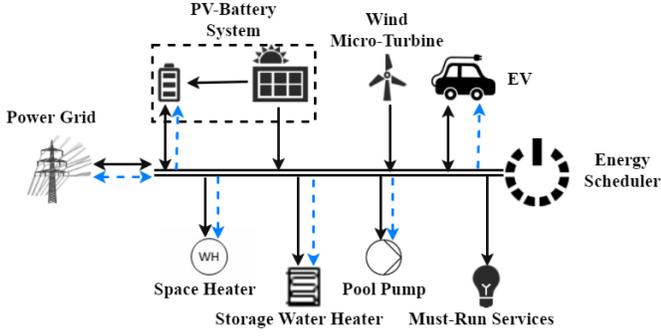


Fig. 2: An extended sample smart home in [9].

TABLE I: Predicted Data of Uncertain Variables

t	$p_{pv,t}^{pred}$	$\sigma_{pv}^{up}$	$\sigma_{pv}^{down}$	$p_{w,t}^{pred}$	$\sigma_w^{up}$	$\sigma_w^{down}$	$q_{out,t}^{pred}$	$I_{mrs,t}^{pred}$
1	0	0.03	0.01	4	0.28	0.19	5.5	0.3
2	0	0.03	0.01	3.7	0.28	0.19	5.5	0.3
3	0	0.03	0.01	3.6	0.28	0.19	5.2	0.3
4	0	0.03	0.01	3.3	0.28	0.19	5.2	0.3
5	0	0.03	0.01	3.4	0.28	0.19	4.8	0.4
6	0	0.03	0.01	3	0.28	0.19	5.5	0.6
7	0.25	0.03	0.01	2.4	0.28	0.19	6.5	0.8
8	0.75	0.03	0.01	1.8	0.28	0.19	7.5	0.8
9	1.25	0.03	0.01	2	0.28	0.19	9.8	0.7
10	1.75	0.03	0.01	1.5	0.28	0.19	10	0.55
11	1.9	0.03	0.01	1	0.28	0.19	11	0.5
12	1.9	0.03	0.01	0.8	0.28	0.19	12	0.5
13	1.9	0.03	0.01	0.7	0.28	0.19	12	0.5
14	1.75	0.03	0.01	0.6	0.28	0.19	12	0.5
15	1.25	0.03	0.01	1.3	0.28	0.19	11	0.6
16	0.75	0.03	0.01	1.7	0.28	0.19	10	0.8
17	0.25	0.03	0.01	2.1	0.28	0.19	9	1.5
18	0	0.03	0.01	2.9	0.28	0.19	8.5	1.8
19	0	0.03	0.01	3.7	0.28	0.19	8	1.7
20	0	0.03	0.01	3.5	0.28	0.19	7.5	1.1
21	0	0.03	0.01	4	0.28	0.19	7	0.9
22	0	0.03	0.01	5	0.28	0.19	6.5	0.7
23	0	0.03	0.01	5.7	0.28	0.19	6.2	0.6
24	0	0.03	0.01	5.9	0.28	0.19	6	0.4

TABLE II: PRICE DATA OF THE SYSTEM

Time (hour)	Price (\$/MW)			
	$\lambda_{pvb}$	$\lambda_{ev}$	$\lambda_w$	$\lambda_{net}$
23-7	2.2	1	2.2	0.0814
8-14	2.2	1	2.2	0.1408
15-20	2.2	1	2.2	0.3564
21-22	2.2	1	2.2	0.1408

TABLE III: VOLL AND SPILLAGE COSTS

Time (hour)	VOLL (\$/MW)				Spillage Cost (\$/MW)	
	SH	SWH	PP	MRS	PVB	Wind
22-7	1	1	-0.5	2.2	4	6
8-21	1	1	0.25	2.2	4	6

Table II presents the price data of the system. Moreover, the Value of Loss Loads (VOLL), and the spillage costs of the wind and the PV-battery power generation are presented in Table III. Note that  $\alpha_w$  and  $\alpha_{pv}$  to equal 1 in case 1, so the SPB approach considers the uncertainty in case 1.

TABLE IV: Impact of uncertainty of unit generations on EC

EC (\$)	No uncer. of wind and PV	Uncer. of wind	Uncer. of PV
	652.683	665.087	660.969

## B. Simulation Results

1) *Impact of Uncertainty*: This section presents the impact of uncertainty of wind and PV power output on EC which is presented in Table IV. However, the outdoor temperature and must-run services are considered as deterministic variables for simplicity. Table IV states that the uncertainty of the wind and PV power output increases the amount of EC. Per the SPB method, upper/lower variance of the predicted variables will equal zero. Therefore, the decision-making variables can be more/less than the predicted amount of these variables based on the upper/lower variance of the prediction when there is uncertainty in the prediction. As shown, the uncertainty of the wind increases the amount of EC in comparison with the uncertainty of PV because the penetration of wind power is more than the penetration of PV power output in this case study.

2) *Impact of Optimistic Coefficient*: In this section, the MSPB method is used to apply the uncertainty of wind and PV power generation. In this section, only the impact of  $\alpha_w$  is assessed. It is considered that  $\alpha_{pv}$  equals 1. As seen in Fig. 3, increasing the amount of  $\alpha_w$  has the EC and wind energy output increase. However, this increment is not uniform.

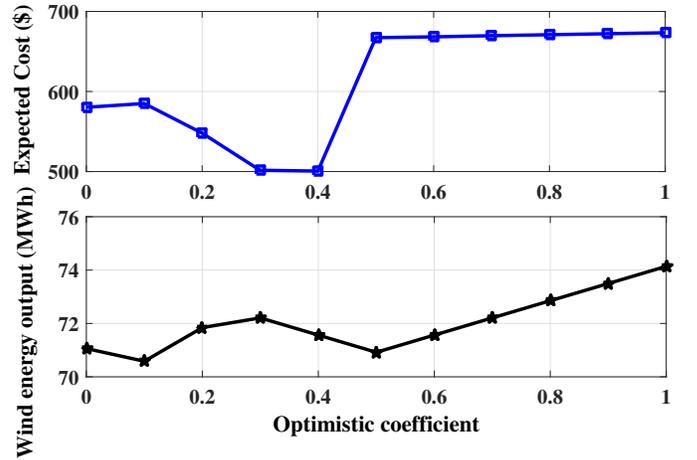


Fig. 3: Impact of OC on wind energy output and system expected cost

3) *Impact of Prediction Accuracy*: The impact of wind power prediction accuracy is evaluated in three scenarios based on its *optimistic coefficient*. Fig. 4 shows the influence of the Prediction Accuracy Coefficient (PAC) on the EC and wind energy output. In this case, it is assumed that upper prediction accuracy is equal to 15% and the lower prediction accuracy is equal to 10% when PAC equals 1. Additionally, upper prediction accuracy equals 10% and lower prediction accuracy is equal to 6.67% when the PAC equals 0.67, and

upper prediction accuracy equals 22.5% and down prediction accuracy equals 15% when the PAC equals 1.5. As seen in Fig. 4, increasing the amount of wind power PAC can have a positive effect on the EC in all scenarios. However, the increasing of PAC does not get the wind energy output to increase in all scenarios. Therefore, the pessimistic impact of when it equals zero is the main reason why increasing the PAC has the wind energy output decrease. It is noticeable that the simulation results of the system are more realistic when  $\alpha_w$  equals zero because increasing prediction error has negative effect on the system energy output. Hence, this point is seen when  $\alpha_w$  equals zero.

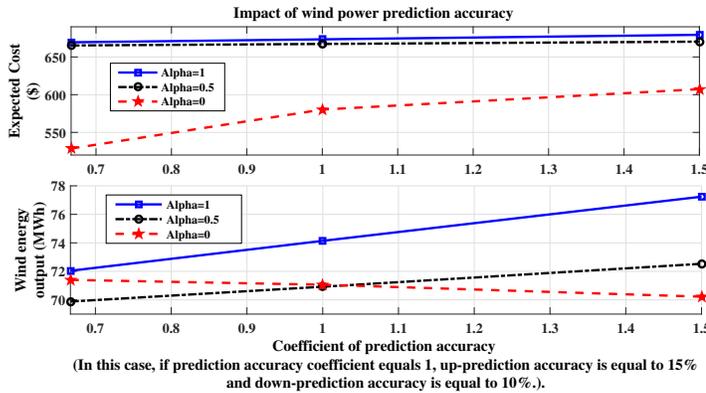


Fig. 4: Impact of wind power prediction accuracy on wind energy output and system expected cost.

## V. CONCLUSIONS

In this paper, Stochastic Predicted Bands (SPB) method has been defined as a novel interval method and utilized to model the uncertainty of the decision-making variables in the REM problem. The bands based on prediction of the stochastic variables have been used to model the uncertainty of these variables. Also, Optimistic Coefficient (OC) has been defined for the first time in this paper to enhance the performance of the SPB method. Therefore, this new interval approach has been called Modified Stochastic Predicted Bands (MSPB) method. OC is a slack parameter for the decision-maker system. Hence, the flexibility, due to the OC, performs robust and interval optimization methods only in the MSPB method.

The performance of the proposed model is evaluated based on the influences of the uncertainty of wind and PV power generation. Additionally, the performance of MSPB is assessed in this paper. This is done by analyzing the effects of the optimistic and prediction accuracy coefficients on the system simulation results. According to the simulation results, increasing the amount of the OC can make the optimistic impacts on the system outputs and increase the system EC and the power generation output of the uncertain energy resources.

Lastly, the proposed algorithm considers the uncertainty in the REM problem, it has many advantages such as reducing the computational burden of the system. However, this method has some drawbacks too. For example, the performance of decision-maker and predictor systems are independent. While,

the predictor system can be updated based on the outputs of the decision-maker system. The mentioned topic requires further research which will be considered in our future works.

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