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# Impact of Energy Consumption in a Production Inventory Model with Price- and Carbon Emission-Sensitive Demand

Hong-Nguyen Nguyen<sup>1,2,\*</sup>, Matthieu Godichaud<sup>1</sup>, Lionel Amodeo<sup>1</sup>

**Abstract**—As companies strive to reduce energy consumption and minimize their carbon footprint, there is a growing trend among customers to choose eco-friendly products. In order to address these concerns, the present study puts forth a new economic production quantity (EPQ) model that takes into account various factors such as energy usage, carbon emissions, and market demand in relation to product price and environmental impact. This is achieved through an analysis of the working phases of the manufacturing machine. By maximizing the overall profitability of the system, optimal decisions can be made regarding cycle time, production rate, demand rate, and machine states during non-production phases (standby or powered off). A Mixed integer nonlinear programming (MINLP) problem is proposed and analyzed. A case study demonstrates that meeting customer expectations for sustainable products can lead to lower profits for businesses, but it also results in reduced energy consumption and environmental emissions. The sensitivity analysis demonstrates that market demand, price sensitivity coefficient, and unit production cost have a more substantial impact on profit compared to the other parameters.

## I. INTRODUCTION

In recent years, high energy prices and fierce competition in the global supply chain make the cost of operating and purchasing manufacturing systems a major challenge for companies. The efficient use of energy in various industrial processes, including production, warehousing, and transportation, is a significant concern in the context of reducing CO<sub>2</sub> emissions and mitigating climate change. In order to tackle this challenge, scholars have endeavored to integrate these factors into inventory management models with the aim of minimizing both energy consumption and emissions-related expenses. In particular, adjusting the production rate has emerged as a popular method for suppliers to reduce inventory costs [1]. Several inventory models have been proposed, extending the demand assumption of the basic inventory model in different directions, including deterministic and stochastic models [2], [3]. Additionally, some researchers have considered the impact of customer preferences for environmentally friendly products on demand by integrating environmental factors into the demand function and assessing its influence on firms' inventory policies [4], [5]. Overall, these approaches can help businesses achieve more sustainable and environmentally responsible supply chain management practices. The current literature lacks research that combines

inventory modeling with sustainability considerations and price-dependent demand and carbon emissions. To fill this gap, the paper aims to make specific contributions in this area. Our main contributions can be listed as follows:

(i) Novel inventory model for Economic Production Quantity (EPQ) that takes into account the energy consumed during production. In this model, the costs associated with CO<sub>2</sub> emissions resulting from energy consumption will be factored into a carbon tax, which may impact the demand from environmentally conscious customers.

(ii) Maximize the profits of the enterprise by making optimal decisions related to production rate, demand rate, cycle time, and machine status during non-production phases.

The paper is structured as follows: Section II provides a comprehensive review of the inventory model with a focus on sustainable aspects and demand sensitivity assumptions. Section III outlines the problem studied and details the mathematical modeling employed. In Section IV, the MINLP is analyzed and simplified. Section V presents a case study to illustrate the model and highlights managerial insights. Additionally, sensitivity analysis is performed to further assess the effectiveness of the proposed model.

## II. LITERATURE REVIEW

The relevant studies for this paper can be categorized into two streams: (1) Inventory models that consider sustainable development factors, including energy consumption and greenhouse gas emissions, and (2) Sustainable inventory models that incorporate dependent demand.

### A. Inventory model with sustainable aspects

Harris's initial inventory model [6], known as the economic order quantity (EOQ) model, was later expanded to the EPQ model for manufacturing enterprises. As supply chains have evolved, inventory models have become more complex and updated to reflect real-world situations. This includes considerations for energy efficiency and GHG reduction, which are now government-mandated for sustainable development. As a result, there has been a growing number of studies on this topic. Jaber et al. [7] propose a two-tier supply chain model that incorporates the carbon emission trading mechanism and emissions from the production process. Zanoni et al. [8] focus on enhancing energy efficiency in the production process by adjusting the production rate with a two-machine production model. Bazan et al. [9] study the single-vendor, single-buyer model with the Vendor-Managed Inventory with Consignment Stock (VMI-CS) agreement policy, taking into account the multi-level emission-taxing

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scheme. Marchi et al. [10] propose an EPQ model that considers the effect of worker training on the production process as well as energy efficiency. Then, the work is extended by integrating inventory modeling with VMI-CS into the supply chain and assessing its impact on GHG emissions [11]. Nezami and Heydar [12] integrate energy price variations into the EPQ model. Fichtinger et al. [13] analyze the impact of warehouse operations on emissions through inventory modeling. Tiwari et al. [14] propose a two-tier supply chain model that considers integrated deteriorating and imperfect quality items while taking carbon emissions into account. Nguyen et al. [15] examine the effects of using different types of Specific Energy Consumption (SEC) functions in EPQ models that consider energy consumption in production. These notable studies highlight the critical need and development of inventory modeling studies that integrate sustainability aspects in the present era.

### B. Sustainable inventory model with dependent demand

Several researchers have contributed to bringing basic inventory models closer to reality. One approach is to change the input assumption of demand rate as a known constant. A number of inventory models with price-dependent demand have been proposed [2], [3], [16]–[18]. With the shift in consumer spending towards environmental concerns, several authors have studied the integration of demand that is sensitive to the level of emissions of the product. Hovelague and Bironneau [5] integrate the EOQ model with the demand that depends on both price and emissions. In this model, both carbon tax and carbon trading mechanisms are analyzed to maximize corporate profits. Later, Ruidas et al. [19] integrate price-sensitive demand into the EPQ model, which is applied to an imperfect production system. The objective of this paper is to establish an inventory model that incorporates interval numbers for various carbon parameters and considers multiple carbon emission regulatory policies, including simple tax, cap and purchase, cap and reward, and permitted cap policies. De-la-Cruz-Márquez et al. [20] introduce an inventory model that accounts for carbon emissions and shortages while considering the price-sensitive demand for growing items (such as farmyard animals). The model determines the optimal policy for the selling price, order quantity, and backordering quantity to maximize the expected total profit per unit of time.

The previous literature reviews indicate a growing trend in research on sustainable inventory models. However, the combination of demand that is dependent on price and emissions during the production process has not been extensively studied. This paper aims to address this gap by expanding the EPQ inventory model. The model considers energy consumption in the production process in detail by examining the different working stages of the machine. Emissions are calculated based on the energy consumed under the carbon tax mechanism. As customers increasingly prefer environmentally friendly products, the model incorporates demand that depends on both market prices and carbon emissions from the production process. The MINLP

problem is analyzed and simplified for numerical analysis, and optimal decisions are made to maximize company profits regarding demand rate, production rate, cycle time, and machine working mode. The model is validated through case studies and sensitivity analysis, which provide useful managerial insights. Table I provides an overview of the literature review.

TABLE I  
SUMMARY OF LITERATURE REVIEWS

Refs.	Inventory model	Type of Dep. demand	Optimal machine mode	Sustainable aspects
[7]	EPQ	—	—	CO <sub>2</sub>
[8]	EPQ	—	✓	Energy
[9]	EPQ	—	—	Energy, CO <sub>2</sub>
[10]	EPQ	—	—	Energy
[12]	EPQ	—	—	Energy
[14]	EPQ	—	—	Energy, CO <sub>2</sub>
[15]	EPQ	—	✓	Energy
[16]	EOQ	- Stock level - Price	—	—
[2]	EPQ	- Price	—	—
[3]	EPQ	- Price - Quality	—	CO <sub>2</sub>
[18]	EOQ	- Price	—	—
[5]	EOQ	- Price - Carbon emission	—	CO <sub>2</sub>
[19]	EPQ	- Price	—	CO <sub>2</sub>
[20]	EPQ	- Price - Shortages - Carbon emission	—	CO <sub>2</sub>
[4]	EOQ	- Price - Carbon emission	—	CO <sub>2</sub>
<b>This study</b>	<b>EPQ</b>	<b>- Price - Carbon emission</b>	<b>✓</b>	<b>Energy, CO<sub>2</sub></b>

## III. MODEL DEVELOPMENT

### A. Problem statement

This study analyzes a single-product, single-machine production system's energy consumption during production and non-production time. The machine is turned off or kept on standby after production time and activated before the next cycle to optimize energy consumption, as illustrated in Fig. 1. Demand is affected by price and customer sensitivity to carbon emissions. The model also integrates carbon tax policies to assess inventory management's effectiveness in managing carbon footprint. The production rate can be controlled and optimized to maximize overall average profit, including production rate, demand rate, and non-production machine state.

The following notations are used to model the problem:

#### Parameters:

$S$ : setup cost (€/setup)

$H$ : inventory holding cost (€/unit.h)

$W$ : idle power of the machine (kW)

$K$ : energy consumption of the machine to produce one unit (kWh/unit)

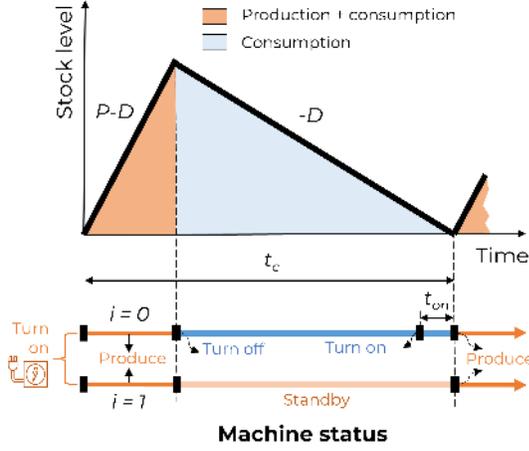


Fig. 1. Inventory level and the feasible machine modes during non-production time

$\alpha$ : maximal market demand (unit/h)  
 $\beta$ : price sensitivity coefficient  
 $\delta$ : carbon emission sensitivity coefficient  
 $E_e$ : energy cost (€/kWh)  
 $E_t$ : electricity standard emission (tonCO<sub>2</sub>/kWh)  
 $C_{tx}$ : carbon tax (€/ton)  
 $F$ : multiplication factor for the power required during the setup of a machine

$t_{on}$ : setup time of the machine on one cycle (h)  
 $C$ : unit production cost (€/unit)

**Dependent variables:**

$V$ : price for on unit of product (€)  
 $M_i$ : total carbon emissions for mode  $i$  (tonCO<sub>2</sub>/h)  
 $E_i$ : power consumption of the machine for mode  $i$  (kW)  
 $Pr_i$ : average profit for mode  $i$  (€/h)

**Decision variables:**

$P$ : production rate (unit/h)  
 $D$ : demand rate (unit/h)  
 $t_c$ : cycle time (h)  
 $i$ : the binary variable presents for machine mode during the non-production time.  $i$  is equal to 0 if the machine is shut down, and 1 if the machine is kept in standby

The model considers the following assumptions:

- The demand rate is smaller than the production rate ( $P > D > 0$ );
- The production rate is controllable and limited in a specific interval  $[P_{min}, P_{max}]$ ;
- Shortages are not allowed;
- The lead time is zero.

**B. Mathematical modeling**

Following the same assumption made in Hovelague et al. [5], the demand in this study is not constant but depends on both market price and customer concern regarding the product's emission level. The demand is expressed as follows:

$$D = \alpha - \beta V - \delta M \quad (1)$$

Where the average total carbon emission  $M$  is solely associated with the energy consumption during the production process. From (1), the price can be presented by the demand as (2)

$$V = \frac{\alpha - D - \delta M}{\beta} \quad (2)$$

Nguyen et al. [15] proposed the concept of consumption power, which is the combined energy consumption during both the production and non-production phases of the machine (expressed in equations (3) and (4)). The machine's two modes during the non-production time are represented by an index ( $i$ ) with a value of either 0 or 1.

$$E_0(t_c, P, D) = \left(\frac{W}{P} + K\right) D + \frac{FWt_{on}}{t_c} \quad (3)$$

$$E_1(D) = \left(\frac{W}{P} + K\right) D + \left(1 - \frac{D}{P}\right) W = W + KD \quad (4)$$

From the energy consumption components in (3) and (4) for each case of  $i$ , the total carbon emission for each case can be calculated by the function:  $M_i = E_i(t_c, P, D)E_t$ .

The average profit functions are defined as

$$Pr_i(D, P, t_c) = \frac{\alpha - D - \delta E_t E_i(t_c, P, D)}{\beta} D - CD - \left[\frac{S}{t_c} + \frac{HDt_c}{2} \left(1 - \frac{D}{P}\right)\right] - E_i(t_c, P, D)(E_e + C_{tx}E_t) \quad (5)$$

where the components of the equation, in order, represent sales revenue, production cost, traditional inventory cost, energy cost and emissions cost.

**IV. RESOLUTION ANALYSIS**

The problem now can be summarized as follows:

**Maximize**  $\{Pr_0(D, P, t_c); Pr_1(D, P, t_c)\}$

**s.t.**

$P_{min} \leq P \leq P_{max}$   
 $t_{on} < \left(1 - \frac{D}{P}\right) t_c$  for the case:  $i = 0$  [15].

The objective average profit function for each instance of machine mode in the non-production period is analyzed. The case that generates a higher profit will be selected. The average profit function can be divided into two cases of  $i$ .

**A. Case  $i = 0$  (the machine is turned off-on)**

The average profit function for this case is

$$Pr_0(D, P, t_c) = \frac{\alpha - D - \delta E_t \left[\left(\frac{W}{P} + K\right) D + \frac{FWt_{on}}{t_c}\right]}{\beta} D - CD - \left[\frac{S}{t_c} + \frac{HDt_c}{2} \left(1 - \frac{D}{P}\right)\right] - \left[\left(\frac{W}{P} + K\right) D + \frac{FWt_{on}}{t_c}\right] (E_e + C_{tx}E_t) \quad (6)$$

The second partial derivatives of  $Pr_0$  with respect to  $t_c$  as present in (7) is smaller than zero with all  $t_c$  larger than

zero. Therefore,  $Pr_0$  is concave with respect to  $t_c$  for any given value of  $P$  and  $D$ .

$$\frac{\partial^2 Pr_0}{\partial t_c^2} = -\frac{2S + 2FWt_{on} \left( E_e + C_{tx}E_t + \frac{DE_t\delta}{\beta} \right)}{t_c^3} \quad (7)$$

By set the first partial derivative to zero, we can present the value of  $t_c$  and  $Pr_0$  by  $D$  and  $P$  as

$$t_{c,0}^*(P, D) = \sqrt{\frac{S + FWt_{on} \left( E_e + C_{tx}E_t + \frac{DE_t\delta}{\beta} \right)}{\frac{DH}{2} \left( 1 - \frac{D}{P} \right)}} \quad (8)$$

$$Pr_0(t_{c,0}^*(P, D), P, D) = \frac{\alpha - D - DE_t\delta \left( \frac{W}{P} + K \right)}{\beta} D - CD - 2\sqrt{\left[ S + FWt_{on} \left( E_e + C_{tx}E_t + \frac{DE_t\delta}{\beta} \right) \right] \frac{DH}{2} \left( 1 - \frac{D}{P} \right)} - D \left( \frac{W}{P} + K \right) (E_e + C_{tx}E_t) \quad (9)$$

Now the profit function has been simplified by removing the variable  $t_c$ . Although the convexity of (9) cannot be mathematically verified, the optimal solutions that maximize  $Pr_0(t_{c,0}^*(P, D), P, D)$  can only be obtained numerically with specific sets of input parameters.

### B. Case $i = 1$ (the machine is kept in standby mode)

In this case, the average profit function can be expressed as

$$Pr_1(D, P, t_c) = \frac{\alpha - D - \delta E_t(W + KD)}{\beta} D - CD - \left[ \frac{S}{t_c} + \frac{HDt_c}{2} \left( 1 - \frac{D}{P} \right) \right] - (W + KD)(E_e + C_{tx}E_t) \quad (10)$$

The first partial derivative of  $Pr_1$  with respect to  $P$  is

$$\frac{\partial Pr_1}{\partial P} = -\frac{D^2 H t_c}{2P^2} < 0, \forall P > 0 \quad (11)$$

Therefore,  $P_1^* = P_{min}$ .

The second partial derivative of  $Pr_1$  with respect to  $t_c$  as shown in (12) demonstrates that  $Pr_1$  is concave with this variable. Hence,  $t_{c,1}^*$  and  $Pr_1$  now can be presented by the last variable  $D$  as (13) and (14).

$$\frac{\partial^2 Pr_1}{\partial t_c^2} = -\frac{2S}{t_c^2} < 0, \forall t_c > 0 \quad (12)$$

$$t_{c,1}^*(D) = \sqrt{\frac{S}{\frac{DH}{2} \left( 1 - \frac{D}{P_{min}} \right)}} \quad (13)$$

$$Pr_1(t_{c,1}^*(D), P_{min}, D) = \frac{\alpha - DE_t\delta(W + KD)}{\beta} D - CD - 2\sqrt{\frac{SDH}{2} \left( 1 - \frac{D}{P_{min}} \right)} - (W + KD)(E_e + C_{tx}E_t) \quad (14)$$

The optimal solutions that maximize  $Pr_1(t_{c,1}^*(D), P_{min}, D)$  in (14) can also be numerically achieved.

## V. NUMERICAL ANALYSIS

This section presents case studies that illustrate the properties of the model. The model is analyzed under various scenarios, considering the impact of emission-dependent demand. The study concludes with a sensitivity analysis that evaluates the role of the parameters in the optimal decision-making process.

### A. Case studies

The input data used in the study was sourced and adjusted from previous works [8] to fit the model. The related sensitive demand parameters and CO<sub>2</sub> emission are taken from [5] and [21]. The data used for the numerical analysis are summarized in Table II.

TABLE II  
INPUT PARAMETERS FOR THE NUMERICAL ANALYSIS

$S$	100 €/setup	$t_{on}$	0.01 h
$H$	0.05 €/(unit.h)	$W$	100 kW
$E_t$	5e-4 ton/kWh	$F$	2
$C_{tx}$	120 €/ton	$K$	10 kWh/unit
$\delta$	1	$E_e$	0.2 €/kWh
$\alpha$	25 unit/h	$P$	[150,300] unit/h
$\beta$	0.08	$C$	100 €/unit

Table III displays the maximum average profit ( $Pr_i^*$ ) and decision variables ( $t_{c,i}^*$ ,  $P_i^*$ ,  $D_i^*$ ,  $W_{p,i}^*$ ) values for each scenario of  $i$ . The optimal decision of switching off/on the machine during the non-production period is evident (in bold). In this case, the optimal production rate of  $P_{max}$  allows for reduced production time and thus, greater energy savings during the non-active period. This is reflected in the average cost of energy ( $C_{e,0}^*$ ) and CO<sub>2</sub> emissions ( $C_{CO_2,0}^*$ ). Notably, emission reduction provides price and demand advantages over the  $i = 1$  scenario.

The difference between the two profit functions is calculated as follows:

$$\Delta = Pr_1 - Pr_0 = \left[ \frac{FWt_{on}}{t_c} - W \left( 1 - \frac{D}{P} \right) \right] (E_e + C_{tx}E_t + \frac{\delta E_t D}{\beta}) \quad (15)$$

The parameter  $\Delta$ 's sign is dependent on the values of two input parameters,  $F$  and  $t_{on}$ . When the value of these two parameters is high enough, the cost of machine setup after a shutdown becomes significantly high. This phenomenon is observed in various industries, such as industrial printing presses, semiconductor manufacturing equipment, and CNC machines, among others. To obtain the optimal machine mode for  $i = 1$ , we aim to increase the values of these two parameters for verification purposes. The results of this effort are also presented in Table III.

The analysis reveals that modifying the values of  $F$  and  $t_{on}$  has no effect on the optimal results of the case  $i = 1$ . Elevating these two parameters leads to higher energy consumption costs, carbon emissions costs (increased 2 times), and lower profit (-4.83%). Additionally, the optimal value of  $t_{c,0}^*$  must increase to reduce their impact on the objective function, resulting in higher inventory costs compared to

TABLE III  
OPTIMAL SOLUTIONS WITH DIFFERENT VALUES OF  $F$  AND  $t_{on}$

$i$	$t_{c,i}^*$	$P_i^*$	$D_i^*$	$W_{p,i}^*$	$Pr_i^*$	$C_{e,i}^*$	$C_{CO_2,i}^*$
$t_{on} = 0.01; F = 2$							
<b>0</b>	<b>22.30</b>	<b>300.00</b>	<b>8.33</b>	<b>207.86</b>	<b>866.88</b>	<b>17.23</b>	<b>5.17</b>
1	22.57	150.00	8.31	207.49	836.70	36.62	10.99
$t_{on} = 10; F = 10$							
0	126.86	300.00	8.21	208.85	825.01	32.73	9.82
<b>1</b>	<b>22.57</b>	<b>150.00</b>	<b>8.31</b>	<b>207.49</b>	<b>836.70</b>	<b>36.62</b>	<b>10.99</b>

the previous case study. Consequently, the study suggests that keeping the machine in standby mode provides a more significant economic benefit.

### B. Effects of emission consideration on the demand

To further analyze the properties of the model, we compare the current model with a simpler version of itself that assumes the rate of demand depends only on market prices and not on production emissions, using the function:  $D = \alpha - \beta V$ . Table IV displays the optimal outcomes of the proposed model (optimal case is in bold). It is evident that the economic benefits of this scenario where customers were assumed to be indifferent toward the emissions produced during the product manufacturing process, are higher than the studying scenario in both cases of  $i$ . Where the percentage value results from comparing to the corresponding case of the studying model as presented in Table III. This is validated by the positive difference in the average returns between the two scenarios, as depicted in (16), for all positive values of  $P$ ,  $D$ , and  $t_c$ .

$$\Delta' = Pr_{non,i} - Pr_i = \frac{E_i \delta E_t}{\beta} D \quad (16)$$

Where  $Pr_{non,i}$  stands for the average profit of the comparing scenario (demand is not dependent on the emission). Although this scenario may achieve a greater advantage in terms of demand ratio and market price in the short term, it comes at the expense of higher emissions and energy consumption costs. These costs are likely to pose a significant challenge in light of changing market trends, such as the growing environmental consciousness among consumers, the current upward trend in energy prices, and the introduction of policies aimed at reducing energy consumption and promoting sustainability. Moreover, governmental incentives and penalties for companies based on their carbon footprint (such as carbon trading and carbon caps) are becoming increasingly prevalent.

### C. Sensitivity analysis

The model's sensitivity analysis was conducted by varying each input parameter (as presented in Table II) from  $-25\%$  to  $+25\%$  with a step size of  $5\%$  to investigate its impact on the maximum average total return, as depicted in Figure 2-4. Here, the maximal profits are archived for the case

TABLE IV  
OPTIMAL SOLUTIONS WITH DIFFERENT VALUES OF  $F$  AND  $t_{on}$  FOR THE INDEPENDENT EMISSION DEMAND MODEL

$i$	$t_{c,i}^*$	$P_i^*$	$D_i^*$	$W_{p,i}^*$	$Pr_i^*$	$C_{e,i}^*$	$C_{CO_2,i}^*$
$t_{on} = 0.01; F = 2$							
<b>0</b>	<b>18.41</b>	<b>300.00</b>	<b>12.38</b>	<b>157.80</b>	<b>1908.76</b>	<b>25.60</b>	<b>7.68</b>
					120.19%	48.57%	48.57%
1	18.77	150.00	12.38	157.75	1884.10	44.76	13.43
					125.18%	22.23%	22.23%
$t_{on} = 10; F = 10$							
0	97.77	150.00	12.31	158.66	1863.30	46.71	14.01
					125.85%	42.70%	42.70%
<b>1</b>	<b>18.77</b>	<b>150.00</b>	<b>12.38</b>	<b>157.75</b>	<b>1884.10</b>	<b>44.76</b>	<b>13.43</b>
					125.18%	22.23%	22.23%

$i = 0$ . The results show that most of the parameters have an inverse relationship with the profit, except for variable  $\alpha$ . Among them, parameters maximal market demand ( $\alpha$ ), price sensitivity coefficient ( $\beta$ ), and unit production cost ( $C$ ), have a significant influence on the profit change (varied up to 90%), while the other parameters have a relatively minor effect (varied under 1%). However, given the recent surge in energy prices, the introduction of the carbon tax, and the growing environmental awareness among customers, managers must rely on the model to make informed decisions in the future. Additionally, integrating bonus/penalty mechanisms or trading for carbon emissions in the model can emphasize the importance of sustainability factors in production.

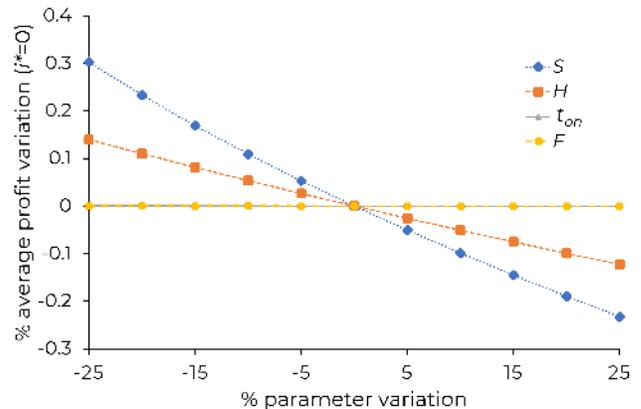


Fig. 2. Percentage change in total cost when varying a parameter ( $S, H, t_{on}, F$ ).

## VI. CONCLUSIONS

This study proposes a novel inventory production model that integrates considerations of energy consumption and emissions in the production process while incorporating demand based on market prices and customers' preferences for green products. Energy consumption is analyzed specifically by examining the working stages of the machine. Mathematical modeling is developed by separating the problem

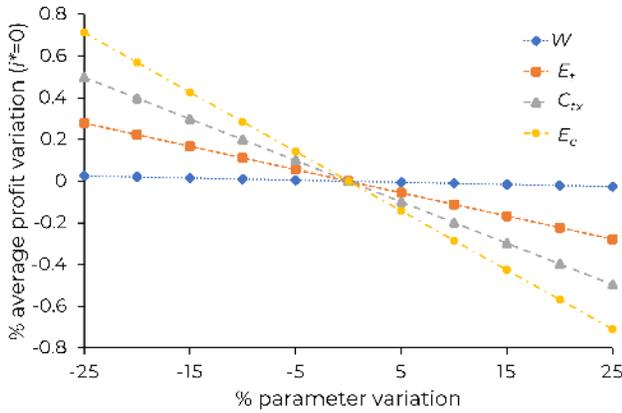


Fig. 3. Percentage change in total cost when varying a parameter ( $\alpha, \beta, \delta, K, C$ ).

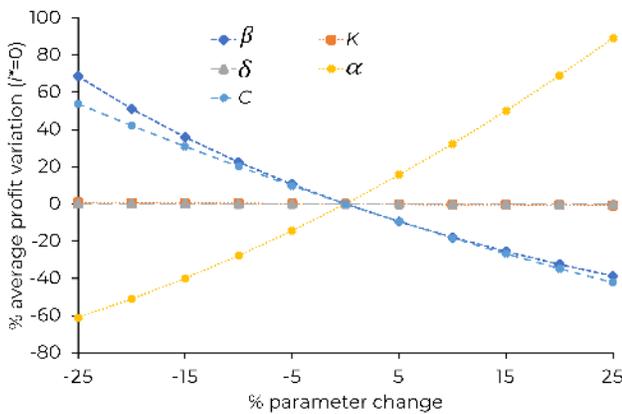


Fig. 4. Percentage change in total cost when varying a parameter ( $W, C_{tx}, E_t, E_c$ ).

into two cases with respect to machine states during the non-production period. The MINLP problem is analyzed to simplify finding the maximum profit and decision variables related to the demand rate, production rate, cycle time, and optimal machine working mode variables during non-production phases. Case studies demonstrate that although considering customer awareness of emissions may lead to immediate disadvantages for manufacturers, it is a necessary consideration in the long run for production to deal with increasing energy prices, government mechanisms related to carbon emissions, and shifting consumer consciousness.

Future research may explore emissions from storage and transportation in the supply chain or incorporate additional carbon emission mechanisms into the model, such as carbon trading and carbon caps.

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