A Distributed Algorithm for Overlapped Community Detection in Large-Scale Networks

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Abstract—**Overlapped community detection in social networks has become an important research area with the increasing popularity and complexity of the networks. Most of the existing solutions are either centralized or parallel algorithms, which are computationally intensive - require complete knowledge of the entire networks. But it isn't easy to collect entire network data because the size of the actual networks may be prohibitively large. This may be a result of either privacy concerns (users of a social network may be unwilling to reveal their social links) or technological impediments (implementation of an efficient web crawler). Performing in-network computation solves both problems utilizing the computational capability of the individual nodes of the network. Simultaneously, nodes communicate and share data with their neighbours via message passing, which may go a long way toward mitigating individual nodes' privacy concerns in the network.**

All the aforementioned concerns motivated us to design a decentralized or distributed technique to detect overlapped communities in a large-scale network. It is desirable because this technique does not offer a single point of failure, and the system as a whole can continue to function even when many of the nodes fail. To overcome the disadvantages of the existing solutions, in this paper, we address the overlapped community detection problem for large-scale networks. We present an efficient distributed algorithm, named DOCD, to identify the overlapped communities in the network. The efficiency of DOCD algorithm is verified with extensive simulation study on both synthetic and real networks data such as, `Dolphin`, `Zachary karate club`, `Football club`, and `Facebook ego` networks. We show that DOCD algorithm is capable of keeping the asymptotically same results with the existing classical centralized algorithms [1]–[5] in terms of community modularity and the number of identified communities. The DOCD algorithm can also efficiently identify the overlapped nodes and overlapped communities with a small number of rounds of communication and computation.

Index Terms—**Overlapped Community, Community Detection, Social Networks, Large-Scale Networks, Distributed Algorithms**

I. INTRODUCTION

Large-scale networks, such as social networks (e.g., Facebook, Twitter, Linkedin, ResearchGate, Instagram, etc.), consist of a large number of connected individuals or users; typically, the number of nodes is millions, and the number of links between the users is billions in such a social network. The size of such networks is growing with an increasing number of nodes at an enormous rate day by day. Thus, in such networks, the demand for analysis and characterization is increasing rapidly. This analysis provides an immense knowledge for understanding the features of the network entities and other related characteristics. For instance, the connectivity between users in the social network represents their friendship relation or neighbourhood relation or both. They can belong to a group, but analysis can establish that those group of people is from the same school or college or maybe from the same religion, which in turn refers to a *community*. To identify such *communities* is one of the techniques by which we can analyze and characterize the networks. Mathematically, a network is considered a graph where nodes represent vertices, and links between nodes represent edges of the graph. *Community detection* in networks is a process of partitioning the underlying network graph into subgraphs (i.e., communities), which are internally densely and externally sparsely connected. This is a fundamental analysis technique that confers the modular composition of a network.

Community detection has been used in a broad range of applications, such as to find research communities in DBLP databases, to identify functional groups of a particular virus strains for vaccine development, and to classify content on social media sites. Prior knowledge of communities helps to understand the processes like rumor spreading or epidemic spreading in a network. For example, there is a very high chance of spreading some infectious diseases very fast within a community where community members are physically staying together or very near to each other (e.g., housing complex, town, city, village, etc.) or sharing common places (e.g., market, shopping mall, school, college, rail station, etc.). Therefore, appropriate preventive measures can be applied before spreading, or essential help can be provided to the affected communities. Similarly, communities allow us to create a large-scale map of a network, making it easier to study and analyze.

Moreover, in social networks, a person belongs to more than one social groups such as family, friends, colleagues, where each group can be treated as an individual community. Thus, a person can simultaneously associate with as many communities as he wishes. Therefore, if a node belongs to more than one community in the network, it is an *overlapped node*. When those overlapped nodes formed a community, which is termed as *overlapped community*. Such overlapped communities are frequently visible in social networks like Facebook, Twitter, etc. Finding overlapped communities refers to the overlapped community detection problem in networks.

Designing practical algorithms for overlapped community detection in network graphs is an important and challenging problem. However, centralized or parallel algorithms are computationally intensive and require complete knowledge of the networks, which is not feasible for large-scale networks. Therefore, an efficient, distributed algorithm to find overlapped communities in large-scale networks is needed. This paper proposes an efficient, distributed algorithm to identify overlapped communities in large-scale networks using local information and message passing.

Our Contributions: This paper studies distributed overlapped community detection problem in large-scale networks and makes the following contributions.

1) We propose a distributed algorithm (DOCD) that can efficiently identify the overlapped communities in large-scale networks. To the best of our knowledge, this is the first distributed algorithm to identify an overlapped community.
2) The number of nodes in the networks is not an input of the algorithm. It is scalable and robust with respect to the number of nodes in the networks.
3) The time and message complexities of the algorithm are $O(n^2 m)$ and $O(\mathcal{D})$, respectively, where m, n, and $\mathcal{D}$ are the number of nodes, edges, and diameter of the underlying network graph.
4) We report on the performance of our algorithm through simulation. The efficiency of the DOCD algorithm is verified with extensive simulation study on both synthetic and real networks data such as, Dolphin, Zachary karate club, Football club, and Facebook ego networks.
5) We show that the DOCD algorithm (with local information) is capable of keeping the asymptotically same results with the existing classical centralized algorithms [1]–[5] (which need the information of entire networks) in terms of community modularity, number of identified communities, overlapped nodes, and communities.

The rest of the paper is organized as follows. Section II presents brief literature review. In Section III, we present the preliminaries and formulate the problem. Section IV presents the overlapped community detection algorithm. The detailed analysis of the message and time complexity is presented in Section V. Section VI, evaluates the performance of the proposed method and finally, Section VII, concludes the paper.

II. RELATED WORKS

Several community detection techniques are reported in the literature, which is mostly centralized and parallel solutions. The community modularity proposed by Girvan & Newman [1] is the widely used [2], [6], [7] metric to measure connectivity among community member, i.e., the structure of a community in an underlying network graph. Community merging between any two communities leads to the maximum modularity gain, and this technique has been used in a CNM algorithm [6]. The CNM algorithm is a hierarchical agglomeration algorithm for detecting the community structure. Further,

the CNM algorithm has been used in [2], [8]. Several other measurement techniques are proposed [3], [9] to detect the quality of the community structure. For instance, the paper [9] presents an algorithm that detects disjoint communities in a network using another community modularity metric called WCC, which uses *triangular* structures in the community. Ghoshal [3] defines another *modularity* metric to show the improvement of their algorithm compare to the technique proposed in [1], [6]. The authors in [10] proposed an iterative search algorithm for community detection that uses a community description model evaluates the quality of a partition, where the partition is done based on external-link separation among the communities and internal-link compactness within communities.

Brandes et al. [11] proved that identifying communities with maximum modularity is NP-hard, even for the restricted version with a bound of two on the number of clusters, i.e., communities, and established a lower bound on the approximation factor. Therefore, heuristics are used in practice to allow the processing of large inputs. However, even such heuristics could take a very long time or run out of memory on modern days' computer. Hence, parallel or distributed solutions are very much essential to reduce computation time. An MPI based parallel heuristics for community detection has been proposed in [7]. They developed a parallel version of the Louvain method [2] to reduce the time complexity. However, their proposed parallel method requires repetitive tasks such as graph coloring, meta node creation in each iteration. The authors in [12] proposed parallel version of the label propagation method [13], Louvain method [2], called as PLM. Next, they extended this PLM method to PLMR method. Finally, the authors combined both the PLM and PLMR algorithms to present a two-phase approach called as EPP. A divisive spectral method has been presented in [14], where the authors first used a sparsification operation followed by a repeated bisection spectral algorithm to find the community structures.

However, all the above works are based on finding the disjoint communities. Another important direction of the identification of communities is whether the communities are overlapped community or not. This problem has been addressed in [15], and the authors proposed a centralized algorithm that finds the overlapped communities. Furthermore, Said et al. [16] used a genetic algorithm to find the overlapped communities in social and complex networks. The authors in [5], proposed a two-step genetic algorithm to find the overlapped communities. They first encountered the disjoint communities, and from the disjoint communities, the overlapped communities have been identified using the community modularity as the optimization function. Reihanian et al. [17] proposed a generic framework to find the overlapping communities in social networks, where paper focused on rating-based social networks. The members within a community have the same topics of interest. The strengths of the relationships between the members are based on the rate of their viewpoints' unity, where the strengths of connections of intra-communities are

much more than those of inter-communities. All the solutions have been proposed so far for addressing the community detection problem in large-scale networks. However, many of them are centralized or parallel with expensive procedures either requires complete knowledge of the networks or computationally intensive. Using local information, designing practical distributed algorithms for community detection is an important and challenging problem in large-scale networks.

III. PRELIMINARIES

A. Basic Definitions

Network graph: A network graph is denoted as $G(V, E)$, which shows interconnections between a set of entities $V = \{v_1, v_2, \ldots, v_n\}$. Each entity $v_i \in V$ is represented by a node or vertex. Connections between nodes are represented through links or edges set E. Let $n = |V|$ and $m = |E|$ be the number of vertices and number of edges in G, respectively. $G(V, E)$ is a simple undirected graph.

Community in a Graph $G(V, E)$**:** We denote a set $C = \{C_1, C_2, \ldots, C_k\}$ which consists of k number of communities, where each community $C_i \in C$, $1 \leq i \leq k$, consists of set of nodes of V, i.e., $C_i \subseteq V$. The size of a community C_i is presented by $|C_i| = \lambda$, where $C_i = \{v_1, v_2, \ldots, v_\lambda\}$ and $V = C_1 \cup C_2 \cup \ldots \cup C_k$.

A node may belong to one or more communities. If a node v belongs to at least two communities such as C_i and C_j, i.e., $v \in C_i \cap C_j$ then v is treated as an overlapped node.

Overlapped node: A node is said to be an overlapped node if it belongs to at least two communities.

Overlapped community: A community is said to be an overlapped community if it consists of at least one overlapped node.

Cluster coefficient: The *cluster coefficient* [16] of a node v is denoted as: $CC_v = \frac{2 \times \bar{\mu}}{\delta(\delta-1)}$, where $\bar{\mu}$ is the total number of links between neighbours and δ is the total number of neighbours of v.

Node Modularity (NM): The *node modularity* of a node v is defined as:

$$NM_v = \frac{(2 \times \mu)}{\delta(\delta - 1)}, \tag{1}$$

where μ is the 'total number of links between neighbours within its own community of v' and δ is the total number of neighbours of v.

Overlapped Node Modularity (ONM): The *Overlapped Node Modularity* is defined as:

$$ONM_v = \frac{(2 \times \mu')}{\delta(\delta - 1)}, \tag{2}$$

where μ' is the 'total number of links between neighbours belong to communities of overlapped node v'.

For example, let v be an overlapped node belongs to two different communities C_i and C_j. Hence, all the links of its neighbours that belong to C_i and C_j are only be considered for computing ONM_v of v. The value of NM_v and ONM_v is the same when v is not an overlapped node.

Community Modularity: Community modularity measures community structures. It quantifies how a node within a community is strongly connected with other nodes in the network. We use this metric to measure the connectivity between nodes and identifying communities. The *community modularity* of a community $C_i \in C$ is defined as: $CM_{C_i} = \frac{\sum_{j=1}^{\lambda} NM_j}{\lambda}$.

Overall Community Modularity: The *overall community modularity* of the set of communities C is defined as: $CM_C = \frac{\sum_{i=1}^{k} CM_{C_i}}{k}$.

B. Distributed Computing Model

We consider a synchronized communication network which consists of n nodes. Each node has a unique id of an $O(\log n)$ bits. Initially, each node knows its own id and the ids of its neighbours in the network. We use *CONGEST* model [18], [19] of distributed computing, wherein each communication round, every node may send an $O(\log n)$-bit message to each of its neighbours. In this synchronous system, the computation of every node proceeds in rounds. In each round, every node of the network sends messages, and its neighbours receive these messages in the same round.

We consider the distributed network model as a network graph $G = (V, E)$, where nodes represent processors (or computing entities), and the edges represent communication links among the processes. Nodes communicate through the edges in synchronous rounds. At the beginning, each node knows following additional information:

- $\Gamma(v)$: list of neighbours of a node v and adjacent edges of each neighbour;
- $\gamma(v)$: a list of communities where neighbours of v belongs to. Each entry in this list is represented by a 3-tuple: $\{c\_id, c\_size, CM\}$, where c_id is the community ID, c_size is the number of node in the community and CM is the *community modularity*;
- $join = True/False$: if a node joins a community, it sets $join = True$, otherwise $False$;
- $head = True/False$: if a node becomes community head, it sets $head = True$, otherwise $False$;
- $parent = True/False$: if a node becomes parent, it sets $parent = True$, otherwise $False$;
- $CL(v)$: represents a list of communities where v belongs to at the same time. Each entry in the list $CL(v)$ is represented as a 4-tuple: $\{c\_id, c\_size, p\_id, CM\}$, where c_id and c_size are defined above, p_id is parent ID or NULL, and CM is the *community modularity*, respectively.
- $\mathcal{Z}(v)$: a set of overlapped communities $\{\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_s\}$ identified by v.

C. Problem Definition

Let $G(V, E)$ be an undirected unweighted underlying network graph, where V is a set of vertices or nodes, and E is a set of edges. Our target is to partition V into $\{C_1, C_2, \ldots, C_k\}$ which consists of k number of communities such that $V = C_1 \cup C_2 \cup \ldots \cup C_k$.

The problem is to identify the overlapped communities $\mathcal{Z} = \{\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_s\}$ such that a node $v \in \mathcal{Z}_t$ if $v \in \cap_{i \in I_t} C_i$ where $v \in V$, $t \leq s$ and $I_t \subseteq \{1, 2, \ldots, k\}$ with $|I_t| \geq 2$.

IV. DOCD ALGORITHM

In this section we explain *Distributed Overlapped Community Detection* (DOCD) algorithm to find the overlapped communities in a network graph $G(V, E)$.

A. Outline of the DOCD algorithm

The DOCD algorithm executes in two phases: Phase-I (Community selection and formation) and Phase-II (Community reorganization). In the first step of Phase-I, community heads are selected, and each community head forms a new community. In the next step, each non-community head selects one or more communities and join them. This community selection process continues until all the nodes join at least one community. Phase-I terminates after all nodes join their respective communities. In Phase-II, all non-community heads join one or more new communities or leave from its selected communities. Next, the community heads do the same process to leave their community or join a new community. In the case of joining a new community refers to the community merging process. Finally, after the community merging process, each node identifies the overlapped communities. A flow diagram of the DOCD algorithm is illustrated in Fig. 1.
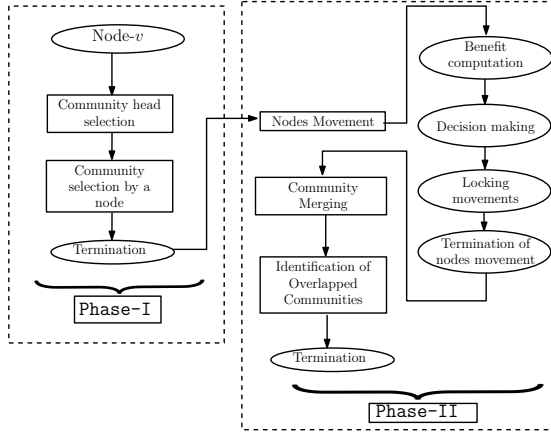

Fig. 1. Outline of the DOCD algorithm

A detailed description of the DOCD algorithm is given below.

B. Phase-I: Community Selection and Formation

In this phase, following messages are to be exchanged between the nodes.

- CC_msg$\langle CC_v \rangle$:
  - Send: Node v sends this message to the neighbour after computing the cluster coefficient CC_v.
  - Receive: On receiving these messages from all of its neighbour u, node v compares its CC_v with CC_u.
- Join_Com$\langle v, c\_id, p\_id \rangle$:

- Send: On joining a community a node v sends this message with its community id c_id, and parent id p_id.
- Receive: On receiving this message, node v updates its $\gamma(v)$.

- Complete$\langle v, NM_v, c\_size \rangle$:
  - Send: After all neighbours of v select their communities, v sends this massage to its parent.
  - Receive: On receiving this message, v checks its $\gamma(v)$ to verify that all of its neighbours select their communities or not.

The detail description of Phase-I is described below.

a) **Community head selection:** Each node v computes cluster coefficient CC_v using $\Gamma(v)$. It exchange CC_msg$\langle CC_v \rangle$ with its all neighbours. It finds the maximum among all cluster coefficients received from its neighbours (excluding its own) and stores in CC_{max}. If its own cluster coefficient, $CC_v > CC_{max}$ then v elects itself as a community head. In case of multiple maximum, id can be used to select the community head. The community head v updates the following information:

- The community head inserts a 4-tuple: $\{c\_id, c\_size, p\_id, CM\}$ with $c\_id = v$, $c\_size = 1$, $p\_id = NULL$, and $CM = CC_v$ in its $CL(v)$.
- It sets $head = True$, $parent = False$ and $join = True$.

Finally, v sends a Join_Com$\langle v, c\_id, p\_id \rangle$ message to its neighbours. Each community head maintains a list of all nodes belongs to its community.

b) **Community selection by a node:** On receiving Join_Com$\langle v, c\_id, p\_id \rangle$ message, node u first includes the 3-tuple: $(c\_id, -1, -1)$ in $\gamma(u)$. If it receives multiple Join_Com messages, then it selects the community which consists of maximum number of its neighbours. u selects multiple communities in case of a tie - having multiple maximum neighbours. In this case, u becomes an *overlapped* node and it will have more than one parents.

For each selected community, node u inserts an individual entry of a 4-tuple: $\{c\_id, c\_size, p\_id, CM\}$ with $c\_id = v$, $c\_size = 1$, $p\_id = v$, and $CM = CC_u$ in its $CL(u)$. It sets $head = False$, $parent = False$ and $join = True$. Finally, it sends Join_Com$\langle u, c\_id, p\_id \rangle$ message. When a node v (already joined a community) receives Join_Com$\langle u, c\_id, p\_id \rangle$ message from node u, then node v checks whether it becomes a parent of u or not. Node v checks the p_id value, if $p\_id == v$ then it updates $parent = True$.

c) **Termination of Phase-I:** If a node v is a not a parent (i.e., $parent = False$), then it checks its $\gamma(v)$ to ensure whether all of its neighbours joined their respective communities or not. If joined, it computes the node modularity NM_v by eqn 1. If v is an overlapped node, it computes NM_v for the individual communities. It sets $c\_size = 1$ and sends Complete$\langle v, NM_v, c\_size \rangle$ message to its parent then it locally terminates Phase-I.

Now, if a node v is a parent (i.e., $parent = True$), it waits until it receives all the `Complete` messages from its children. If the parent node v is an an overlapped node and received all the `Complete` messages from its children then for each community, it does the following tasks.

- Node v computes NM_v and $NM_{avg} = \frac{NM_v + \sum_{i=1}^{l} NM_i}{l+1}$, where NM_i is obtained from the `Complete` message of neighbour i, l is the total number of its children in the same community.
- v updates $c\_size = 1 + \sum_{i=1}^{l} c\_size_i$ and sends `Complete`$\langle v, NM_{avg}, c\_size \rangle$ message to its parent.
- If v is a community head (i.e., $p\_id = NULL$), it computes NM_v and $CM = \frac{NM_v + \sum_{i=1}^{l'} NM_i}{l'+1}$, where NM_i is obtained from received `Complete` message of neighbour i, and l' is the total number of its children. It updates $c\_size = 1 + \sum_{i=1}^{l'} c\_size_i$.

Finally, v locally terminates `Phase-I`. When each community head receives `Complete` message from its neighbours, it starts the execution of `Phase-II` procedure, which is described below.

C. `Phase-II`: Community Reorganization

This phase is divided into two sub-phases: `Nodes movement` and `Community Merging` which are discussed below. Here we introduce the following messages those are to be transmitted during the execution of the process.

- `Movement`$\langle v, CM, c\_size \rangle$:
  - `Send`: Each community head v sends this message to its community members for a possible movement from one community to another.
  - `Receive`: on receiving the message, each member computes the benefits for its current community from which it moves or joins other communities where its neighbours belong to.
- `ONM_msg`$\langle ONM_v \rangle$:
  - `Send`: Node v sends this message to the neighbour after computing the overlapped node modularity ONM_v.
  - `Receive`: On receiving these messages from all of its neighbour u, node v compares its ONM_v with ONM_u.
- `Decision`$\langle v, NM_v, benefit, leave \rangle$:
  - `Send`: When a node v decides to join or leave a community, it sends this message. A variable $leave = True/False$, is used to know whether it is leaving or not from a community.
  - `Receive`: On receiving this message, the community head decides whether its members are allowed to leave or not.
- `Leave_Accepted` $\langle v, CM, c\_size \rangle$:
  - `Send`: When a community head u decides to allow node v to leave from its community, it sends this message.

- `Receive`: On receiving this message, a node v leaves a community.
- `Join_Req`$\langle v, NM_v \rangle$:
  - `Send`: v sends the message when it decides to join neighbour community.
  - `Receive`: On receiving this message, a node v sends this message to community head. When the community head receives this message, it updates CM and c_size.

1) Nodes Movement: Each community head starts execution of `Nodes movement` by sending a `Movement` message to the members of its community. On receiving this message, each member v forwards this message. If c_id of the message is same with its own community then it updates CM and c_size in $CL(v)$. If it is not same, then the message came from its neighbour with different communities, then it updates CM and c_size in $\gamma(v)$. Next, it does the following tasks.

a) Benefit computation: Node v computes *benefit* to include itself for each of the communities in $\gamma(v)$ and stores the maximum positive benefits with the community id in a list BL_{nbr}. Similarly, it computes *benefits* to exclude itself for each of its communities in $CL(v)$. It selects the maximum positive benefit and stores in a list BL_{self}. In case of multiple maximum, it keeps all maximum benefits associate with each of the community ids in BL_{self} or in BL_{nbr} as per the aforementioned explanation. The detail description of benefit computation is given below:

Let C_i and C_j be two communities with size l and l' respectively. Let $u \in C_i$ and $v \in C_j$ and u and v are neighbours of each other. u computes *benefit* of community modularity for the community $C_i \setminus \{u\}$ (excluding u from C_i) and $C_j \cup \{u\}$ (including u into C_j), respectively. If u leaves C_i, the community modularity of C_i may increase or decrease. Similarly, if u joins C_j, that may increase or decrease the community modularity of C_j. The community modularity of C_i and C_j are CM_{C_i} and CM_{C_j}. u computes the *benefit* of C_i after excluding u from C_i:
$benefit_{(C_i \setminus \{u\})} = CM_{C_i}^{(-u)} - CM_{C_i}$ where $CM_{C_i}^{(-u)} = \frac{(CM_{C_i} \times l) - NM_u}{l-1}$. Similarly, u computes the *benefit* of C_j after including u into C_j: $benefit_{(C_j \cup \{u\})} = CM_{C_j}^{(+u)} - CM_{C_j}$ where $CM_{C_j}^{(+u)} = \frac{(CM_{C_j} \times l') + NM_u}{l'+1}$.

b) Decision making: Movement of v is decided based on following conditions:

- if BL_{self} and BL_{nbr} both are empty then v maintains the status quo with current communities.
- if BL_{self} is non-empty and BL_{nbr} is empty then v maintains the status quo with current communities.
- if BL_{self} is empty and BL_{nbr} is non-empty then v joins the neighbour communities along with maintains the status quo with current communities.
- if BL_{self} and BL_{nbr} both are non-empty then v decides to leave from its current communities and joins neighbour community of positive benefit.

c) **Locking movements:** Suppose a node u computes the benefit for its neighbour v's community and decides to join that community. Similarly, node v computes benefits for its neighbour w's community and decides to join that community. If u and v both move to their intended communities, then the benefit computation by node u is incorrect because of the movement of v. To eradicate this problem, we use a locking strategy by which among the neighbouring nodes, only one node is allowed to move. To execute it, u and v both compute ONM_u and ONM_v by eqn 2 and exchange it via the `ONM_msg` message. In general, on the comparison, if ONM_u is the minimum [1] among all other competitors (e.g., v), then node u is allowed to move. Other competitors (e.g., v) lock their movement for this round. If $ONM_u = ONM_v$ for all v, then the id of the nodes can be used to break the tie and give priority to one node. If a node has at least one child having degree one in the community then it also locks itself for the movement. A node u having minimum ONM_u, sends a `Decision`$\langle v, NM_v, benefit, leave \rangle$ message to its community head, where $benefit = benefit_{(C_i \setminus \{u\})}$ and $leave = True$, respectively.

d) **Termination of nodes movement:** On receiving `Decision` message from the members, each community head v makes a list ML of the members who want to leave the community (i.e., whose $leave = True$). v arranges the list ML in decreasing order based on the values of $benefit$. v recomputes the community modularity CM' excluding first member u of the list ML. If $CM' \geq CM$ then community head v allows u to leave the community, where CM is the community modularity including u. Then the community head v, removes entry of u from ML, decreases the c_size by one, updates CM by CM' and sends `Leave_Accepted` $\langle u, CM, c\_size \rangle$ message to all of the community members. The community head v repeats the above process unless the list ML is exhausted. On receiving a `Leave_Accepted` $\langle u, CM, c\_size \rangle$ message, a member forwards the message to its neighbour within the community and updates CM and c_size. When u receives this message, it removes the entry of 4-tuple associated with the community from $CL(u)$. When all the children of u receive this message, they select a neighbour with minimum id as a new parent from the same community and update CM and c_size accordingly. All the neighbours of u update their γ.

When a node u (belonging to a community C_i) wants to join a new community C_j of a neighbour w, it inserts an entry: $\{c\_id, c\_size, p\_id, CM\}$ with $c\_id = j$, $c\_size =$ size of C_j, $p\_id = w$, and $CM = CM$ of C_j in its $CL(u)$. Next, u sends `Join_Req`$\langle u, NM_u \rangle$ message to the neighbour w. If w is not the community head then it forwards this message to its community head. On receiving this message the community head updates CM and c_size. A non-head node u terminates the `Node movement` round once it decides the final decision of its movement: staying in the same community

or joining in new communities or leaving from the current community and joining new communities. The community head proceeds for the next round `community merging` once the decision of `Node movement` in its own community is over.

2) ***Community Merging***: The community heads are the candidate for executing community merging procedures. This is similar to the aforementioned `Node movement` procedure. In this process, each community head v computes the benefits and stores in BL_{nbr} and BL_{self}. Next, if its BL_{self} and BL_{nbr} both are non-empty then v decides to merge with the neighbouring community of the positive benefits. If node v wants to join a new community C_j of community head u, then it sends a `Merge_Req`$\langle v, c\_id, ONM_v \rangle$ message to u, where $c\_id = j$ for C_j. If a node v wanted to join C_j, but receives a `Merge_Req` message from u, in that case, lower id node allows other to join by sending a `Confirm` $\langle v, c_v, t, CM \rangle$ message. Meanwhile, if v receives a `Merge_Req` message from w, then it ignores the message. On the other case, if node v does not want to join any community and receives a `Merge_Req` message from node w, then it sends `Confirm`$\langle v, c\_id, c\_size, CM \rangle$ message to node w. When node v joins community of u after receiving the `Confirm`$\langle u, c\_id, c\_size, CM \rangle$, it updates CM and c_size and sends `Update_Com` $\langle v, c\_id, c\_size, CM \rangle$ message to all the nodes in the merge community. Each member after receiving this message, updates its $CL(v), \gamma(v), CM, c\_size$, respectively. This process terminates when no community heads move from one community to another.

3) **Identification of Overlapped Communities::** When a node v joins a community, it keeps this community information in its $CL(v)$. If $CL(v)$ consists of multiple entries, then node v is an overlapped node. During the whole execution of the `Phase-II` procedure, each v updates its $\gamma(v)$. Node v can easily get its neighbours community information from its $\gamma(v)$. Node v checks its $\gamma(v)$ and extracts the neighbour ids of the same communities, and thus, it can identify the overlapped community $\mathcal{Z}(v) = \{\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_s\}$ locally. The outline of `DOCD` is presented in Algorithm 1.

V. COMPLEXITY ANALYSIS

In the first round of `Phase-I`, the community heads are selected based on cluster coefficients and ids. In the second round, one-hop neighbours of all the community head select their communities. Similarly, two-hop neighbours decide their communities in the third round, and thus the execution of `Phase-I` proceeds hop by hop in the network. If the network diameter is $\mathcal{D}$, then we require $(\mathcal{D}+1)$ such rounds to terminate the whole process of `Phase-I`. Hence, the time complexity of the `Phase-I` is $O(\mathcal{D})$. In the whole execution of `Phase-I`, each node v sends one `CC_msg`, one `Join_Com`, and one `Complete` message. In this phase per edge requires constant number of messages which is at most six to be traveled during the execution of whole `Phase-I` procedure. Hence message complexity is $O(m)$.

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[1]Minimum value of ONM_u signifies that most of the neighbours of u are belonging to different communities.

Algorithm 1: DOCD

> **Input:** Node v: $\Gamma(v)$
> **Output:** $\mathcal{Z}(v)$
> // v **Executes Phase-I**
> - Community head selection;
> - Community selection by a node;
> - Termination of Phase-I;
>
> // **Executes Phase-II**
> **if** v *is a non-head member* **then**
>   **if** *receives* Movement *message* **then**
>   node v executes the following task in sequence:
>   - Benefit Computation;
>   - Decision Making;
>   - Locking Movements;
>   - Termination Nodes Movement.
> **else**
>   // v is a community head
>   send Movement message to the members within the community;
>   **if** *receives all* Decision *messages from all community members* **then**
>   Executes Community Merging;
>
> v identifies overlapped communities $\mathcal{Z}(v)$;

In Phase-II, each community head initially sends the Movement message to its community neighbours. The neighbour of the community heads forward this message to its community neighbours and so on. Hence, total $\mathcal{D}$ rounds is required for forwarding the Movement message in the worst case. Each member of a community computes its benefits and makes decisions for its movement. It needs another round of message exchange to know whether it can move or not. In the same round, it sends Decision message to the community head and a Join_Req message to its neighbours. Hence, it takes $2\mathcal{D}$ rounds of message exchange. In the next round, each community head sends a Leave_Accepted message for allowing a member to move. Similarly, this message is forwarded by each of the members that take $\mathcal{D}$ rounds. Hence, Phase-II: Nodes movement takes $O(\mathcal{D})$ rounds. In Phase-II: Community merging, each community head computes its benefits. It needs another round to know whether it can merge with the neighbouring community or not. In the next round, it may send or receive a Confirm message and a Update_Com message. So, Phase-II: Community merging takes $O(\mathcal{D})$ rounds. Hence, time complexity of the Phase-II is $O(\mathcal{D})$. In the whole process of Phase-II, each community head sends one Movement message. Hence, it becomes total $|C|$ Movement messages. Next, the community head may generate total (c_size-1) Leave_Accepted messages sent to be among the community members. Since, there are $|C|$ communities, then in the worst case total $|C|$ Merge_Req messages and $|C|$ Confirm messages may generate during the whole process. Finally, total c_size number of Update_Com messages may exchange for updating the community. Similarly, each node in the network, who decides for movement, exchanges a ONM_msg and a Decision message. Hence, total $2n$ messages are to be generated. Finally, a member can join $|C|-1$ communities, thus n nodes may generate at most $n \cdot (|C|-1)$ Join_Req messages. Hence, total message complexity is $O(|C|+c\_size+2n+n \cdot (|C|-1)) = O(n \cdot |C|) \approx O(n^2)$, where maximum possible value

of $|C|$ is n. In worst case each message can pass through every edge in the network, hence message complexity of DOCD algorithm is $O(n^2m)$ and time complexity is $O(\mathcal{D})$.

VI. PERFORMANCE EVALUATION

Extensive simulation studies have been done to evaluate the performance of our proposed DOCD algorithm. In
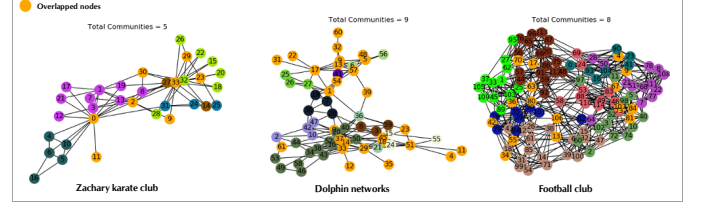

Fig. 2. Run time snapshot of real world networks (benchmark datasets)

our simulation study, we use real world networks (benchmark datasets) like, Dolphin, Zachary karate club, Football club, and Facebook networks, respectively. The algorithm is implemented in Intel Xeon 2.6GHz, 16 cores, 64 GB RAM, machine using Python-3. A runtime snapshot of the DOCD algorithm is shown in Fig. 2 for Dolphin, Zachary karate club and Football club networks.

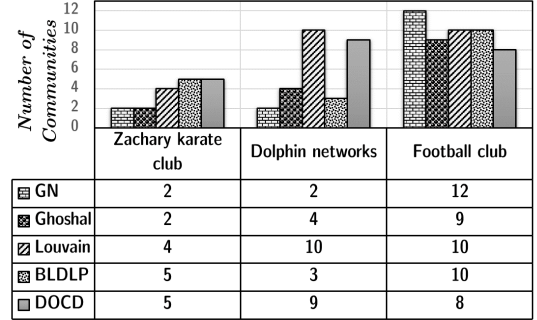|	Zachary karate club	Dolphin networks	Football club
GN	2	2	12
Ghoshal	2	4	9
Louvain	4	10	10
BLDLP	5	3	10
DOCD	5	9	8

Fig. 3. Comparison with number of communities on different networks

The different and same colored nodes in Fig. 2 show different and same community, respectively. All the orange-colored nodes refer to the overlapped nodes.

Fig. 3 shows number of identified communities on Dolphin, Zachary karate club and Football club networks, respectively. We have compared our results with GN [1], Louvain [2], Ghoshal [3], and BLDLP [4] methods.

We observe that DOCD identifies almost the same number of communities as a result obtained by Louvain [2] for all three benchmark datasets, whereas for Football club network, the result of DOCD is almost similar with the outcome of [3]. Hence, DOCD, the algorithm performs almost comparable results compared to the centralized algorithms in terms of identifying number communities with existing works.

Fig. 4 shows community modularity values of different algorithms for Dolphin, Zachary karate club and
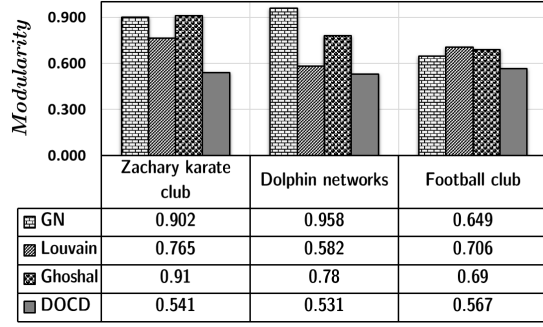
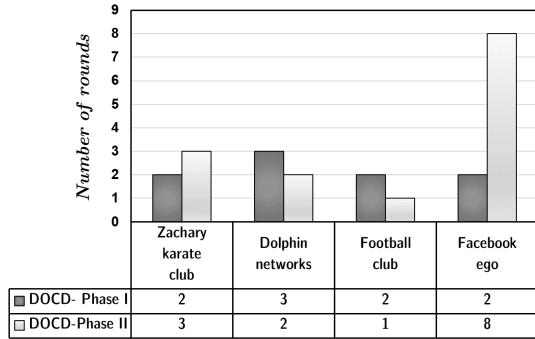Fig. 4. Comparison with community modularity on different networks

	Zachary karate club	Dolphin networks	Football club
▦ GN	0.902	0.958	0.649
▨ Louvain	0.765	0.582	0.706
▩ Ghoshal	0.91	0.78	0.69
▦ DOCD	0.541	0.531	0.567

Fig. 5. Number of rounds needed to find overlapped communities for different networks

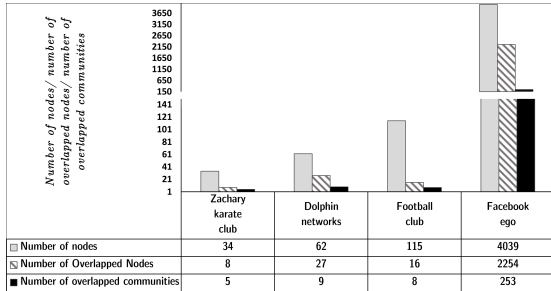|	Zachary karate club	Dolphin networks	Football club	Facebook ego
▦ DOCD- Phase I	2	3	2	2
□ DOCD-Phase II	3	2	1	8

Fig. 6. Number of nodes, number of overlapped nodes and number of overlapped communities in different networks

	Zachary karate club	Dolphin networks	Football club	Facebook ego
□ Number of nodes	34	62	115	4039
▨ Number of Overlapped Nodes	8	27	16	2254
■ Number of overlapped communities	5	9	8	253

Football club networks. Compare to others, the community modularity value of DOCD algorithms is less. The reason for the same is that DOCD finds overlapped communities where community modularity is computed using overlapped nodes, whereas [1], [2], and [3] compute disjoint communities.

Fig. 5 shows the number of rounds required to find overlapped communities in Zachary karate club, Dolphin and Football club networks, respectively using DOCD algorithm. We show the results for each phase individually. Fig. 6 shows the variation of the actual number of nodes vs. number of overlapped nodes and overlapped communities identified from the Zachary karate club, Dolphin and Football club networks, respectively.

Table I shows the comparison study with the method [5], for

Data set	No. of communities		Community modularity		No. of overlapped nodes	
	Meena [5]	DOCD	Meena [5]	DOCD	Meena [5]	DOCD
Karate	4	5	0.4198	0.541	5	8
Dolphin	5	9	0.5285	0.531	Unknown	27
Football	7	8	0.5851	0.567	Unknown	16

TABLE I
COMPARISON WITH [5] WITH RESPECT TO THE NUMBER OF COMMUNITIES, COMMUNITY MODULARITY AND NUMBER OF OVERLAPPED NODES.

the number of identified communities, community modularity, and the number of overlapped nodes, respectively. It is interesting to observe that the number of identified communities are always greater than the result reported in [5]. The comparison of the number of overlapped nodes is insignificant because the number of overlapped nodes directly affects the community modularity value. Thus, our DOCD algorithm results better community modularity as compared to [5]

Method	Zachary Karate club		Dolphin networks		Football club		Algorithm	Community
	$|C|$	CM	$|C|$	CM	$|C|$	CM		
GN [1]	2	0.90	2	0.95	12	0.64	centralized	Disjoint
Louvain [2]	4	0.91	10	0.78	10	0.69	Centralized	Disjoint
Ghoshal [3]	2	0.76	4	0.58	9	0.70	Centralized	Disjoint
Meena [5]	4	0.41	5	0.52	7	0.58	Centralized	Overlapped
DOCD	5	0.54	9	0.53	8	0.56	Distributed	Overlapped

TABLE II
SHOWING COMPARISON SUMMARY WITH THE NUMBER OF COMMUNITIES $|C|$ AND COMMUNITY MODULARITY CM

The DOCD algorithm identifies overlapped communities in a large-scale networks and it produces comparative results with the existing centralized algorithms [1]–[3], which are shown in Table II. Moreover, DOCD algorithm is capable of identifying the overlapped communities within a small number of rounds of communications, as shown in Fig. 5.

VII. CONCLUSION

This paper has proposed a distributed algorithm DOCD, to solve the overlapped community detection problem in large-scale networks. The total number of nodes in the networks is not an input of the algorithm. It is scalable and robust with respect to the number of nodes in the networks. The time and message complexities of the algorithm are $O(n^2 m)$ and $O(\mathcal{D})$, respectively, where m, n, and $\mathcal{D}$ are the number of nodes, edges, and diameter of the network graph. To show the performance of the algorithm, we have done an extensive simulation study with benchmark data sets. We showed that our distributed algorithm keeps the asymptotically same numbers of communities and community modularity with existing centralized algorithms. The proposed solution can also work for a dynamic network by recomputing community modularity for a neighborhood change, but complexity will be high. So, as future work, one can design an efficient distributed algorithm for dynamic networks.

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