Shape from planar curves: a linear escape from Flatland

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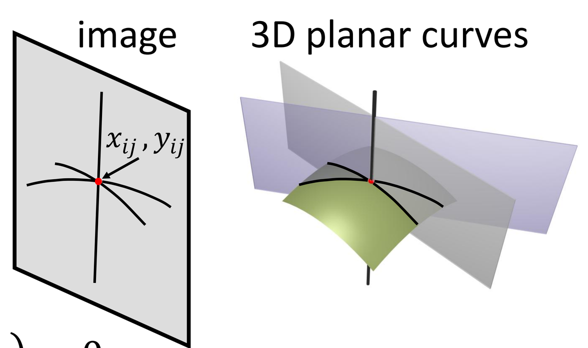
Introduction

We revisit the problem of recovering 3D shape from a single photo containing planar curves. Previous work [1,2,3] did not explicitly explore the space of flat solutions and its orthogonal complement

- For **intersecting planar curves**, we study the space of solutions and derive a stable linear method
- For **parallel planar curves**, we demonstrate special cases where the shape is similarly solved
- Our work **unifies relevant literature** on shape from contour, single view modeling and structured light

Intersecting curves



Consider planar curves

$$z_i(x,y) = a_i x + b_i y + d_i$$

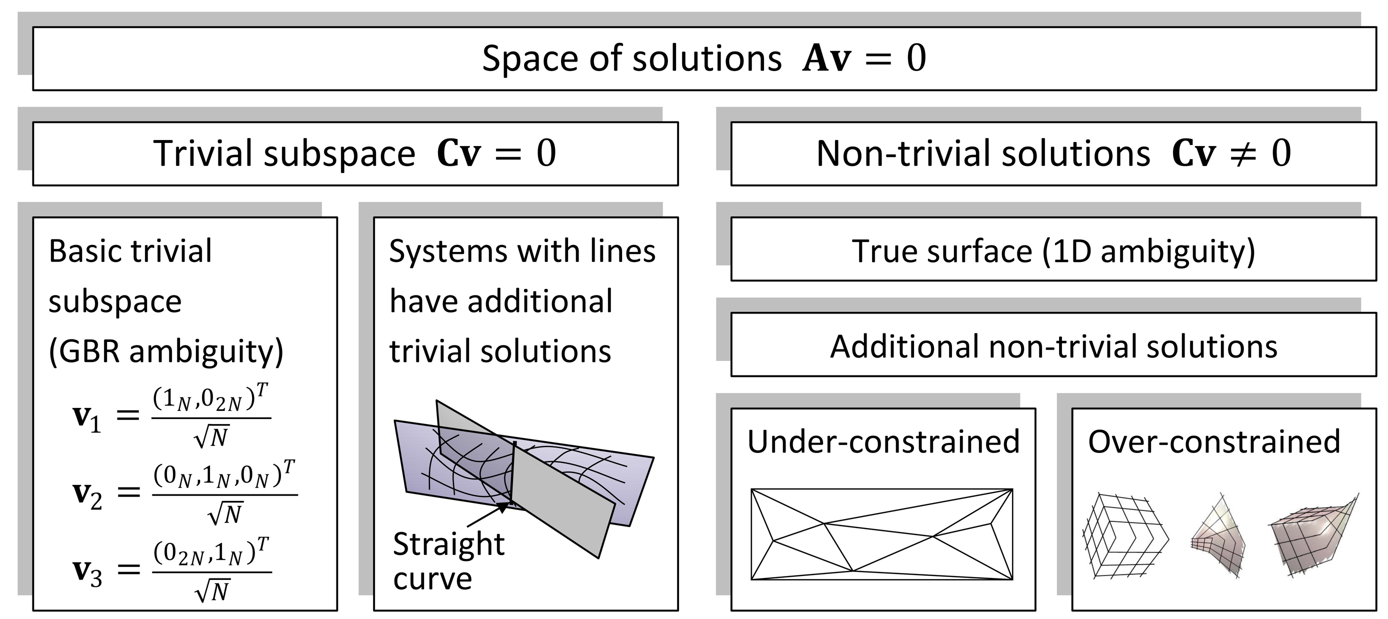
At intersection points

$$z_i(x_{ij}, y_{ij}) - z_j(x_{ij}, y_{ij}) =$$
$$(a_i - a_j)x_{ij} + (b_i - b_j)y_{ij} + (d_i - d_j) = 0$$

The homogenous linear system in vector form

$$\mathbf{Av} = 0$$
$$\mathbf{v} = (a_1, \ldots, a_N, b_1, \ldots, b_N, d_1, \ldots, d_N)^T$$

Let $\mathbf{C}$ be a matrix so that $\|\mathbf{Cv}\|^2$ is the linear regression residue of fitting a plane to a sample of 3D points on the curves



Ambiguity resolution: pick a vector orthogonal to the trivial subspace

- For a large number of random planes, the relative magnitude of the trivial component is likely to be small $\frac{1}{\sqrt{N}}\left\|\left(\sum a_i, \sum b_i\right)\right\| / \|(a_1, \ldots, a_N, b_1, \ldots, b_N)\|$

Simple linear method

$$\underset{\mathbf{v}}{\operatorname{argmin}} \|\mathbf{Av}\| \text{ s.t. } \|\mathbf{v}\| = 1, \mathbf{v} \perp \operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$$

- Breaks in presence of straight lines
- May collapse to a nearly-flat solution

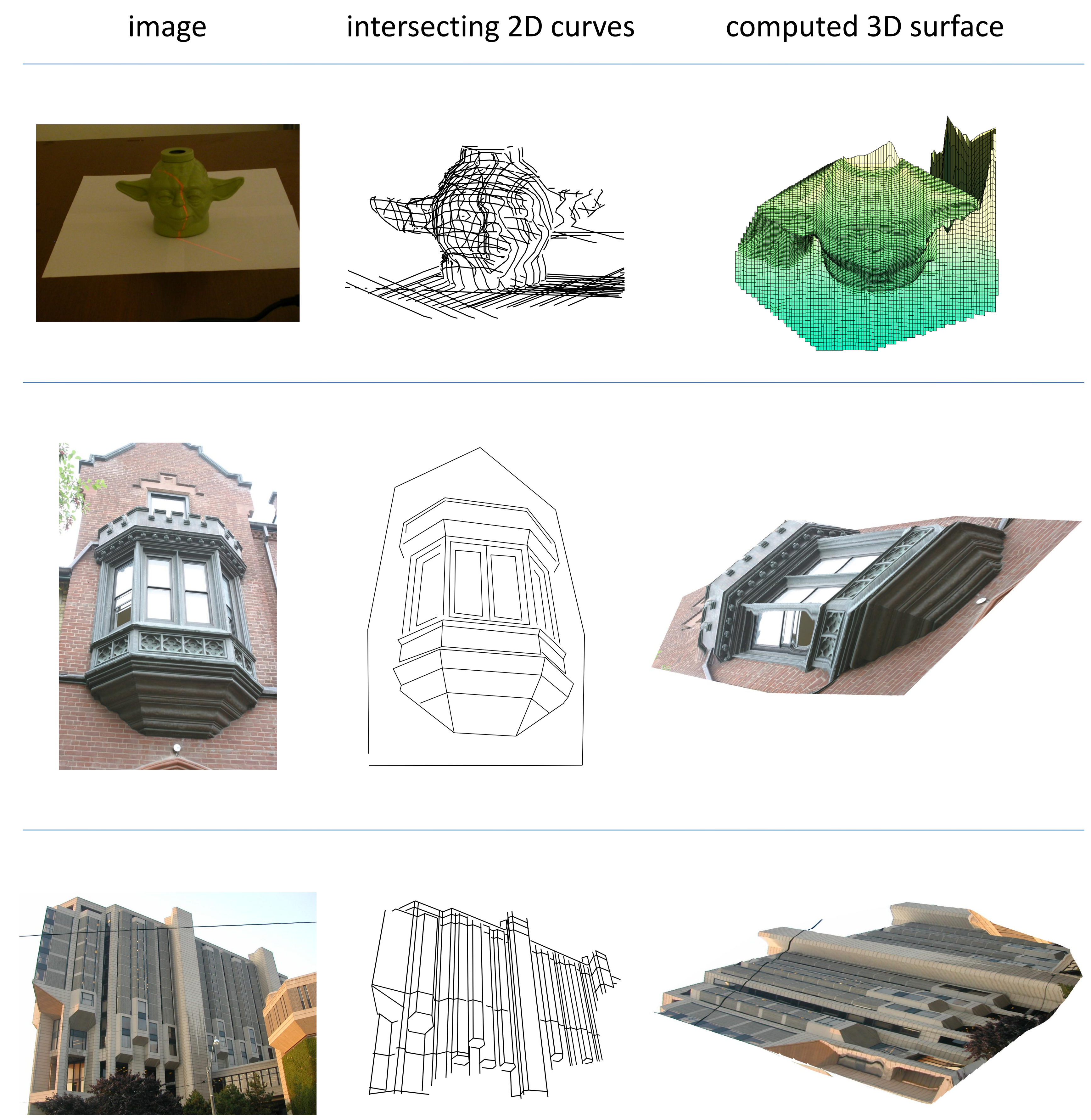
Our method

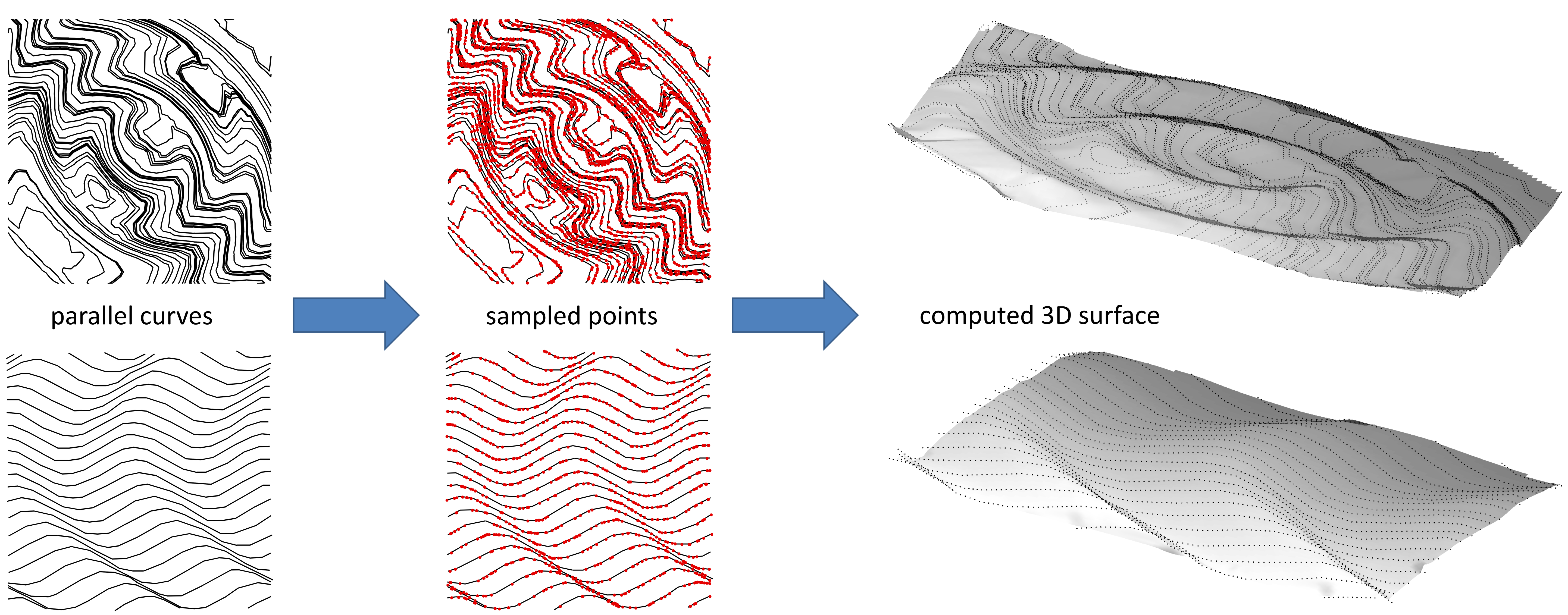
$$\underset{\mathbf{v}}{\operatorname{argmin}} \|\mathbf{Av}\| \text{ s.t. } \|\mathbf{Cv}\| = 1, \ \mathbf{v} \perp Null(\mathbf{C})$$

$\mathbf{v} = \mathbf{C}^{+}\mathbf{w}$, where $\mathbf{w}$ is the last right singular vector of $\mathbf{AC}^{+}$

Similar linear formulation for perspective projection

- Singular values are independent of the focal length



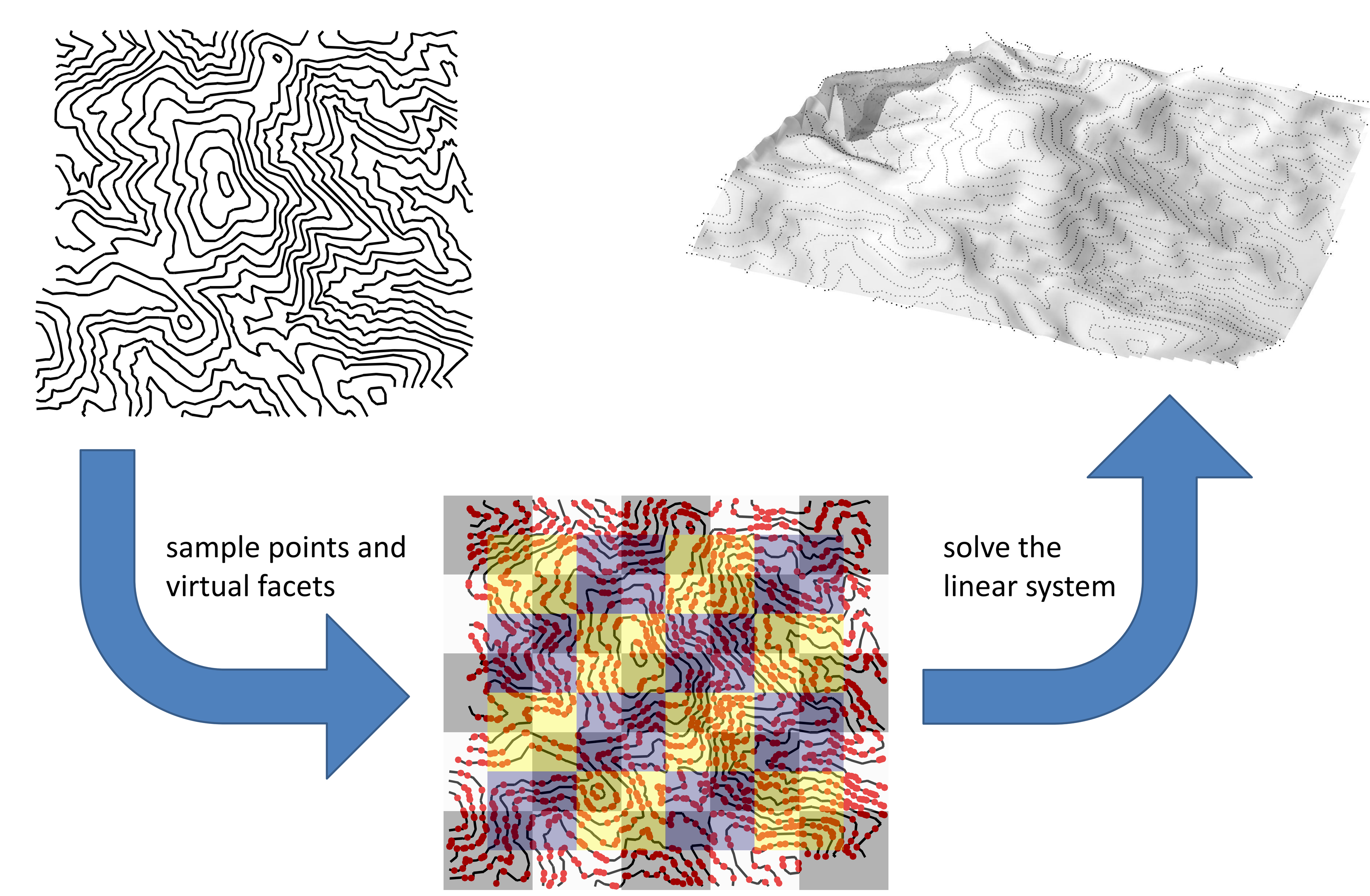


Parallel curves

- Humans can perceive 3D from parallel planar cross sections
- Do not assume fixed gaps between the planes. Not standard shape from texture
- We demonstrate reconstructions in special cases

Global planarity

- For curves arranged around a principle plane, the shape might be a singular vector of $\begin{bmatrix}\mathbf{R}\\ \mathbf{C}\end{bmatrix}$, $\|\mathbf{Rv}\|^2 = \lambda(Var(a_i) + Var(b_i))$



Local planarity

- Reduce to the intersecting case by assuming virtual planar facets

References

[1] K. Sugihara. *Machine Interpretation of Line Drawings*. MIT press, 1986

[2] C. Rothwell and J. Stern. Understanding the shape properties of trihedral polyhedra. Technical Report 2661, INRIA, 1995

[3] J.-Y. Bouguet, M. Weber, and P. Perona. What do planar shadows tell us about scene geometry? In *Proc. CVPR'99, pages 514–520, 1999*