SHIFT15M: Fashion-specific dataset for set-to-set matching with several distribution shifts

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Figure 1: t-SNE [130] visualization for the SHIFT15M.

Abstract

This paper addresses the problem of set-to-set matching, which involves matching two different sets of items based on some criteria, especially in the case of high-dimensional items like images. Although neural networks have been applied to solve this problem, most machine learning-based approaches assume that the training and test data follow the same distribution, which is not always true in real-world scenarios. To address this limitation, we introduce SHIFT15M, a dataset that can be used to evaluate set-to-set matching models when the distribution of data changes between training and testing. We conduct benchmark experiments that demonstrate the performance drop of naive methods due to distribution shift. Additionally, we provide software to handle the SHIFT15M dataset in a simple manner, with the URL for the software to be made available after publication of this manuscript. We believe proposed SHIFT15M dataset provide a valuable resource for evaluating set-to-set matching models under the distribution shift.

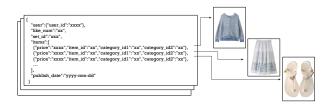


Figure 2: Overview of SHIFT15M dataset.

from shift15m.datasets import NumLikesRegression
dataset = NumLikesRegression(root="./data", download=True)
(x_tr, y_tr), (x_te, y_te) = dataset.load_dataset(target_shift=True)

Figure 3: Minimum sample code using SHIFT15M data loader.

1. Introduction

One of the key problems for fashion data analysis is setto-set matching [110, 7, 3]. For example, we can consider a task that measures the degree of completion of an outfit by matching sets of clothing items (i.e., for two sets A ={hat, shirt, skirt} and B = {jacket, shoes}, the matching score of A and B corresponds to the goodness of the outfit $A \cup B$). To solve this, we need to investigate neural networks that handle sets [119, 73, 155, 60, 125, 136, 159, 137]. We summarize neural networks that deal with sets in Section 4.

Another common phenomenon in the domain of fashion is a trend change. These phenomena are observed at various scales, ranging from annual trend changes such as fashionable colors to seasonal trend changes such as summer to winter clothing. In the field of machine learning, such an assumption can be defined as a distribution shift (or dataset shift) [102, 88, 121, 117, 48, 117, 148, 87]. We assume that training examples $\{(x_i^{tr}, y_i^{tr})\}_{i=1}^{n_{tr}}$ are independently and identically distributed (i.i.d.) according to some fixed but unknown distribution $p_{tr}(x, y)$, which can be decomposed into the marginal distribution and the conditional probability distribution, i.e., $p_{tr}(\boldsymbol{x}, y) = p_{tr}(\boldsymbol{x})p_{tr}(y|\boldsymbol{x})$. We also denote the test examples by $\{(\boldsymbol{x}_i^{te}, y_i^{te})\}_{i=1}^{n_{te}}$ drawn from a test distribution $p_{te}(\boldsymbol{x}, y) = p_{te}(\boldsymbol{x})p_{te}(y|\boldsymbol{x})$.

Definition 1.1. (Covariate shift [118]) We consider that the two distributions $p_{tr}(x, y)$ and $p_{te}(x, y)$ satisfy the covariate shift assumption if the following conditions hold:

$$p_{tr}(\boldsymbol{x}) \neq p_{te}(\boldsymbol{x}), \quad p(y|\boldsymbol{x}) = p_{tr}(y|\boldsymbol{x}) = p_{te}(y|\boldsymbol{x}).$$

Definition 1.2. (Target shift [156]) We consider that the two distributions $p_{tr}(x, y)$ and $p_{te}(x, y)$ satisfy the target shift assumption if the following conditions hold:

$$p_{tr}(y) \neq p_{te}(y), \quad p(\boldsymbol{x}|y) = p_{tr}(\boldsymbol{x}|y) = p_{te}(\boldsymbol{x}|y).$$

Definition 1.3. (General distribution shift [121]) Let $\mathcal{Z} \subset \{\mathcal{Z}, \mathcal{Y}\}$ ve a set of immutable variables whose marginal distribution should remain fixed, $\mathcal{W} \subset \{\mathcal{X}, \mathcal{Y}\} \setminus \mathcal{Z}$ be a set of mutable variables whose distribution can be shifted, and $\mathcal{V} = \{\mathcal{X}, \mathcal{Y}\} \setminus \{\mathcal{W} \cup \mathcal{Z}\}$ be the remaining dependent variables. This partition of the variables defines a factorization of p_{tr} into

$$p_{tr}(\boldsymbol{v}|\boldsymbol{w},\boldsymbol{z})p_{tr}(\boldsymbol{w}|\boldsymbol{z})p_{tr}(\boldsymbol{z}), \qquad (1)$$

where $z \in Z$, $w \in W$ and $v \in V$. We consider that the two distributions $p_{tr}(x, y)$ and $p_{te}(x, y)$ satisfy the general dataset shift assumption if the following hold:

$$p_{tr}(\boldsymbol{w}|\boldsymbol{z}) \neq p_{te}(\boldsymbol{w}|\boldsymbol{z}). \tag{2}$$

Notably, this formulation generalizes other dataset shifts. For example, if we let $\mathcal{Z} = \emptyset$ and $\mathcal{W} = \mathcal{X}$, then this corresponds to a covariate shift.

To address these problems, we provide SHIFT15M, a realworld dataset that can handle the above two problem settings, that is, the set-to-set matching dataset with distribution shift. Our SHIFT15M dataset is built on data accumulated over the past 10 years in our fashion SNS (see Figure 1). In this SNS, users could post combinations of their clothing items and other users could bookmark them as favorites. The data accumulated by this service, which has been in operation for a decade from 2010 to 2020, is very useful for dealing with distribution shifts in the fashion sector. Figure 2 shows an overview of the SHIFT15M dataset. Each column is a set of posted fashion items, with information such as the user who posted, the date of publication, and the price of each item. We hope that our SHIFT dataset will encourage research on set-to-set matching tasks under the distribution shift.

1.1. Contribution

Our contributions are summarized as follows:

 We propose SHIFT15M, a fashion-specific dataset that can properly evaluate models for set-to-set matching under the distribution shift assumptions. SHIFT15M also enables the performance evaluation of the model under various magnitudes of dataset shifts by switching the magnitude. Figure 4 shows several sample images from the SHIFT15M dataset. In Section 2, we introduce overall statistics on the SHIFT15M dataset.

- We provide open-source software to handle the SHIFT15M dataset in a very simple way. Figure 3 shows the minimum sample code of our software;
- We propose first-step benchmark methods for set-toset matching under distribution shift, numerical experiments show the usefulness of these methods. Section 3 presents the proposed benchmark methods and the results of comparative experiments.

2. Statistics on the SHIFT15M dataset

In this section, we present some statistics for our SHIFT15M dataset. First, Table 1 shows the overview of statistics on the SHIFT15M dataset. Since our fashion SNS was launched in 2010, the number of users and posts gradually increased from 2010, reaching a peak around $2014\sim2015$, and slowly decreasing until 2020, the year the service was terminated. Also, the number of items in a set tends to increase over the years, indicating that users tend to construct outfits with more and more items.

The top panel of Figure 5 shows the trend of price for items included in the SHIFT15M dataset. It can be seen that the fashion items posted by users are becoming more expensive every year. The bottom panel of Figure 5 shows the trend of the number of likes for posted sets.

Figure 6 plots the trend of the number of posted sets by year. This figure shows that our fashion SNS, the source of the SHIFT15M dataset, was most active around $2014 \sim 2015$.

Also, each item from the SHIFT15M dataset has two categories specifying the types of the item. Figure 11 show the distributions of the number of items belonging to each category. The figures show that there are categories in which the number of items changes from year to year and categories in which the number of items does not change much throughout the entire period.

Finally, we confirm the covariate shift of the image features included in SHIFT15M. If covariate shift assumption 1.1 holds, we should be able to construct a classifier $f : x \mapsto y = \{0, 1\}$, where x is the image feature of the item and y is the binary classification output for two years. Figure 7 shows the experimental results. The results show that classification between distant years (e.g., acc. of 2010 vs. 2020 is 0.85) is easier, while classification between close years (e.g., acc. of 2010 vs. 2011 is 0.62) is more difficult, indicating a gradual shift in image features. Figure 8 also shows the experimental results of item categorization when the training and test data were generated from different years.



Figure 4: Several sample images from SHIFT15M dataset. See Appendix A for more sample items.

Table 1: Statistics on the SHIFT15M dataset.

Property	Total	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
#sets	2,555,147	1,423	4,813	131,611	466,583	730,443	617,844	299,502	137,510	92,944	59,412	13,062
#items	15,218,721	8,327	29,140	756,532	2,644,564	4,305,802	3,731,864	1,853,647	855,036	576,022	373,549	84,238
mean set size	6.03	5.85	6.05	5.74	5.66	5.89	6.04	6.18	6.21	6.19	6.28	6.44
median set size	6.00	6.00	6.00	5.00	5.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
mean #likes	26.98	0.94	2.00	15.74	16.84	23.24	37.37	35.67	32.41	24.89	21.34	16.01
median #likes	9.00	0.00	1.00	8.00	6.00	6.00	13.00	18.00	23.00	19.00	17.00	12.00
#unique users	193,574	289	571	16,922	52,283	80,290	49,441	18,854	7,511	4,442	2,739	853

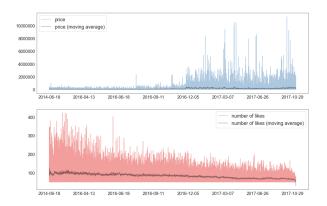


Figure 5: Top panel: the trend of price for items included in SHIFT15M. Bottom panel: the trend of the number of likes for posted sets.

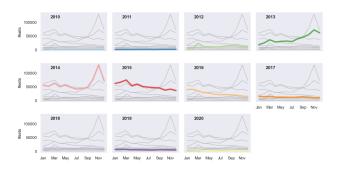


Figure 6: Trends of the number of posted sets by year.

This figure shows that the closer the years of the training and test data are, the higher the classification accuracy.

As we will see in the following sections, density ratio

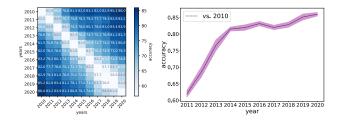


Figure 7: Covariate shift of image features.

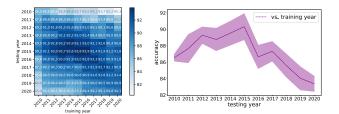


Figure 8: Category classification results under the covariate shift.

 $p_{te}(\boldsymbol{x})/p_{tr}(\boldsymbol{x})$ is essential in distribution shift adaptation. Let p(train) and p(test) be the probability that some data are generated from train and test distributions, respectively. If we assume that p(train) = p(test) = 0.5, we can estimate density ratio as $p(\text{test}|\boldsymbol{x})/p(\text{train}|\boldsymbol{x})$ via Bayes' theorem:

$$\frac{p_{te}(\boldsymbol{x})}{p_{tr}(\boldsymbol{x})} = \frac{p(\boldsymbol{x}|\text{test})}{p(\boldsymbol{x}|\text{train})} = \frac{p(\text{train})}{p(\text{test})} \frac{p(\text{test}|\boldsymbol{x})}{p(\text{train}|\boldsymbol{x})} = \frac{p(\text{test}|\boldsymbol{x})}{p(\text{train}|\boldsymbol{x})}$$

Conversely, the Bayes optimal classifier $g^*(\boldsymbol{x})$ can be written

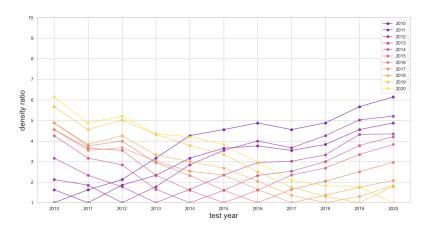


Figure 9: Density ratio estimation by using binary classifiers.

as a function of the density ratio
$$r(\boldsymbol{x}) = rac{p_{te}(\boldsymbol{x})}{p_{tr}(\boldsymbol{x})}$$

$$r(\boldsymbol{x}) = \frac{p_{te}(\boldsymbol{x})}{p_{tr}(\boldsymbol{x})} = \frac{g^{*}(\boldsymbol{x})}{1 - g^{*}(\boldsymbol{x})}, \ g^{*}(\boldsymbol{x}) = \frac{p_{te}(\boldsymbol{x})}{p_{tr}(\boldsymbol{x}) + p_{tr}(\boldsymbol{x})}$$

Figure 9 shows the density ratio estimation by using the above binary classifiers. These density ratios induce the importance weighted set-to-set matching algorithm in the following section. See Appendix E for more details.

3. Benchmarks

In this section, we introduce several numerical experiments on the SHIFT15M dataset.

3.1. Importance weighted set-to-set matching

As the benchmark strategy for the distribution shift adaptation on the set-to-set matching, we propose importance weighted set-to-set matching which is based on IWERM.

Definition 3.1. (Importance weighted ERM [118]) Importance Weighted Empirical Risk Minimization (IWERM) uses the density ratio $p_{te}(\mathbf{x})/p_{tr}(\mathbf{x})$ as the weighting function:

$$\hat{h} = \operatorname*{arg\,min}_{h \in \mathcal{H}} \frac{1}{n_{tr}} \sum_{i=1}^{n_{tr}} \frac{p_{te}(\boldsymbol{x}_i^{tr})}{p_{tr}(\boldsymbol{x}_i^{tr})} \ell(h(\boldsymbol{x}_i^{tr}), y_i^{tr}).$$
(3)

Adopting the density ratio as the weighting function, as in Definition 3.1, leads to the following statistically important property.

Theorem 3.1. (Consistency of IWERM[118]) If we set $w(x) = p_{te}(x)/p_{tr}(x)$ as the weighting function, the empirical error computed by the weighted ERM is a consistent estimator of the expected error in the test distribution.

Using the above ideas, we propose a novel covariate shift adaptation method for set-to-set matching. Let $\mathcal{L}(\mathcal{V}, \mathcal{W}, f)$

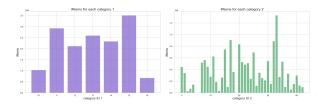


Figure 10: Overall distribution of the number of items in each category.

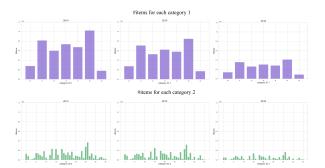


Figure 11: Yearly distribution of the number of items in each category.

be the *K*-pair-set loss [110] function for the set matching, which is defined as follows:

$$\mathcal{L}(\mathcal{V}, \mathcal{W}, f) = -\frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{K} \delta_{ij} \log \frac{\exp(f(\mathcal{V}_i, \mathcal{W}_j))}{\sum_{k=1}^{K} \exp(f(\mathcal{V}_i, \mathcal{W}_k))},$$

where δ is Kronecker's delta, and we can modify $\mathcal{L}(\mathcal{V}, \mathcal{W}, f)$ as follows:

$$\mathcal{L}_w(\mathcal{V}, \mathcal{W}, f) = -\frac{1}{K} \sum_{i=1}^K \sum_{j=1}^K \delta_{ij} \Gamma_{i,j}^p \log \frac{\Gamma_{i,j}^f}{\sum_{k=1}^K \Gamma_{i,k}^f}, \quad (4)$$

where $\Gamma^p_{i,j} = e^{p(test|\mathcal{V}_i \cup \mathcal{W}_j)}$ and $\Gamma^f_{i,j} = e^{f(\mathcal{V}_i, \mathcal{W}_j)}$. This

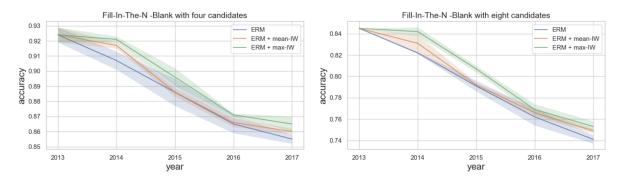


Figure 12: Plots for Fill-In-The-N-Blank experiments.

Table 2: Experimental results of the Fill-In-The-N-Blank with four candidates. Evaluation metrics are the accuracy [%]

Models	2013	2014	2015	2016	2017
ERM [110]	$0.924(\pm 0.005)$	$0.907(\pm 0.006)$	$0.886(\pm 0.009)$	$0.865(\pm 0.006)$	$0.855(\pm 0.003)$
ERM + mean-IW	$0.924(\pm 0.005)$	$0.917(\pm 0.002)$	$0.886(\pm 0.003)$	$0.866(\pm 0.003)$	$0.860(\pm 0.002)$
ERM + max-IW	$0.924(\pm 0.005)$	${\bf 0.921} (\pm 0.002)$	$0.896(\pm 0.006)$	${\bf 0.871} (\pm 0.001)$	$\bf 0.865 (\pm 0.005)$

modification can be regarded as a weighting based on the probability that the pair is included in the test set.

Here, we propose two weighting strategies:

$$\begin{aligned} \max\text{-IW}: \ p(test|\mathcal{V}_i \cup \mathcal{W}_j) &= \max_{\boldsymbol{x} \in \mathcal{V}_i \cup \mathcal{W}_j} w(\boldsymbol{x}), \\ \text{mean-IW}: \ p(test|\mathcal{V}_i \cup \mathcal{W}_j) &= \frac{1}{|\mathcal{V}_i \cup \mathcal{W}_j|} \sum_{\boldsymbol{x} \in \mathcal{V}_i \cup \mathcal{W}_j} w(\boldsymbol{x}), \end{aligned}$$

where w(x) is the weighting function. Next, we approximate w(x) by using unlabeled data from both p_{tr} and p_{te} . In IWERM, the squared error can be decomposed as follows:

$$\mathbb{E}_{p_{te}}\left[\Delta^{2}\right] = \mathbb{E}_{p_{tr}}\left[w(\boldsymbol{x})\Delta^{2}\right]$$
$$= \mathbb{E}_{p_{tr}}\left[\hat{w}(\boldsymbol{x})\Delta^{2}\right] + \mathbb{E}_{p_{tr}}\left[(w(\boldsymbol{x}) - \hat{w}(\boldsymbol{x}))\Delta^{2}\right],$$

where $\Delta^2 = ||f(\boldsymbol{x}) - y||^2$ and $\hat{w}(\boldsymbol{x})$ is the approximator of the weighting function $w(\boldsymbol{x})$. The second term is bounded as

$$\mathbb{E}_{p_{tr}} \left[(w(\boldsymbol{x}) - \hat{w}(\boldsymbol{x})) \Delta^2 \right] \\
\leq \frac{1}{2} \left(\mathbb{E}_{p_{tr}} \left[\Delta^2 \right] + \mathbb{E}_{p_{tr}} \left[(w(\boldsymbol{x}) - \hat{w}(\boldsymbol{x}))^2 \right] \right). \quad (5)$$

Let s be the indicator of the distributions, where s = 1 corresponds to the train distribution and s = 0 corresponds to the test distribution, and we assume that p(s) = 0.5. Then, we also assume that

$$p(\boldsymbol{x}|s) = \begin{cases} p_{tr}(\boldsymbol{x}) & (s=1), \\ p_{te}(\boldsymbol{x}) & (s=0). \end{cases}$$
(6)

Then, we have $w(\boldsymbol{x}) = \frac{p(\boldsymbol{x}|s=0)}{p(\boldsymbol{x}|s=1)}$. Let $g(\boldsymbol{x})$ be the optimal source discriminator which identifies whether \boldsymbol{x} is generated p_{tr} or p_{te} . Then, we can write as $g(\boldsymbol{x}) = p(s = 1|\boldsymbol{x}) = \frac{1}{1+w(\boldsymbol{x})}$. Suppose that the density ratio $p_{te}(\boldsymbol{x})/p_{tr}(\boldsymbol{x})$ is bounded by $\beta > 0$, we have $\frac{1}{1+\beta} \leq g(\boldsymbol{x}) \leq 1$ for all \boldsymbol{x} . From the unlabeled data generated from p_{tr} and p_{te} , we can learn the estimator \hat{g} of g. Then, we can write the weight estimation term as

$$\begin{split} & \mathbb{E}_{p_{tr}} \Big[(w(\boldsymbol{x}) - \hat{w}(\boldsymbol{x}))^2 \Big] = \mathbb{E}_{p_{tr}} \left[\left(\frac{g(\boldsymbol{x}) - \hat{g}(\boldsymbol{x})}{g(\boldsymbol{x})\hat{g}(\boldsymbol{x})} \right)^2 \right] \\ & \leq (1 + \beta)^4 \mathbb{E}_{p_{tr}} \Big[(g(\boldsymbol{x}) - \hat{g}(\boldsymbol{x}))^2 \Big] \\ & = (1 + \beta)^4 \mathbb{E}_{p_{te}} \Bigg[(g(\boldsymbol{x}) - \hat{g}(\boldsymbol{x}))^2 \frac{p_{tr}(\boldsymbol{x})}{p_{te}(\boldsymbol{x})} \Bigg] \\ & \leq 2(1 + \beta)^4 \mathbb{E}_{p_{te}} \Big[(g(\boldsymbol{x}) - \hat{g}(\boldsymbol{x}))^2 \Big] \\ & = 2(1 + \beta)^4 \Bigg\{ \mathbb{E}_{p_{te}} \Big[(s - g(\boldsymbol{x}))^2 \Big] - \mathbb{E}_{p_{te}} \Big[(g(\boldsymbol{x}) - \hat{g}(\boldsymbol{x})) \Big] \Bigg\}. \end{split}$$

This indicates that the weighting function is approximated by the function $g(\mathbf{x})$.

3.2. Experimental results on set-to-set matching problem under the covariate shift assumption

We introduce benchmark results for a set-to-set matching under the covariate shift. The model architecture is the same as the previous work [110], which is based on the architecture of Transformer [134, 73, 91]. Our task can be considered an extended version of a standard task, Fill-In-The-

Table 3: Experimental results of the Fill-In-The-N-Blank with eight candidates. Evaluation metrics are the accuracy [%]

Models	2013	2014	2015	2016	2017
ERM [110] ERM + mean-IW ERM + max-IW	$0.845(\pm 0.000)$ $0.845(\pm 0.000)$ $0.845(\pm 0.000)$	$\begin{array}{c} 0.822(\pm 0.001)\\ 0.831(\pm 0.008)\\ 0.842(\pm 0.004)\end{array}$	$0.791(\pm 0.005)$ $0.792(\pm 0.002)$ $0.807(\pm 0.003)$	$\begin{array}{c} 0.762 (\pm 0.008) \\ 0.766 (\pm 0.004) \\ 0.769 (\pm 0.005) \end{array}$	$\begin{array}{c} 0.741(\pm 0.004) \\ 0.749(\pm 0.002) \\ 0.753(\pm 0.005) \end{array}$

Blank [21], which requires us to select an item that best extends an outfit from among four candidates. Because selecting a set corresponds to filling multiple blanks, we consider the set matching problem as Fill-In-The-N-Blank [110]. To construct the correct pair of sets to be matched, we randomly halve the given outfit \mathcal{O} into two non-empty proper subsets \mathcal{V} and \mathcal{W} , as follows: $\mathcal{O} \to {\mathcal{V}, \mathcal{W}}$, where $\mathcal{V} \cap \mathcal{W} = \emptyset$. Tables 2, 3 and Figure 12 show the experimental results of the Fill-In-The-N-Blank with four and eight candidates. In these experiments, data from 2013 are used as training data, and data from 2013~2017 are used as test data. ERM refers to empirical risk minimization [131, 132, 17], which assumes that $p_{tr}(\boldsymbol{x}) = p_{te}(\boldsymbol{x})$. From these results, we can see that the covariate shift adaptive set-to-set matching methods can achieve better performances than the ordinal ERM. This means that for set-to-set matching on the SHIFT15M dataset, we need to apply some distribution shift adaptation methods.

Table 4 and 5 show the experimental results for the various models. The models used in the experiments are the same as [110]. To quote,

- Cross Attention and Cross Affinity [110]: Set-to-Set matching models which proposed by [110], with the attention-based and affinity-based functions, respectively.
- Set Transformer [73]: Set Transformer which introduced by applying a self-attention based Transformer to a set of data. Set Transformer is trained through supervised or unsupervised learning and transforms a set of data into a vector representation to recognize set features. By using Set Transformer f_{ST} , we perform the extension by calculating the matching score between the two sets \mathcal{V} and \mathcal{W} via the inner product $f_{ST}(\mathcal{V})^{\top}f_{ST}(\mathcal{W})$, sharing the weights between the two f_{ST} .
- We consider a union of two sets as a set-input for the extension of BERT [27] and omit the individual token embedding. We use the segment embedding to designate items of X and Y. We use three variants: BERT_{BASE} is the same model as described in [27]; BERT_{BASE-AP} uses the average pooling in the last layer; and BERT_{SMALL} is a four-layered version of BERT_{BASE} with eight heads, and the hidden size is 512.

- GNN [21]: We combine two sets as one input for the extension of GNN [21]. Because this model is not presented to train in an end-to-end with the feature extractor, we do not finetune the CNN in fashion set matching, where pre-trained CNNs are used, but train it in an end-to-end manner for the group re-id task. Note that we omit the context provided from the external graphs in the evaluation stage to apply this model in the same scenarios of our tasks. We set the training epoch to 256 in the group re-id to enhance the training results of the GNN.
- HAP2S [154]: A conventional CNN trained by Hard-Aware Point-to-Set loss.

See Appendix B.2 for the additional figures.

3.3. Additional experimental results on regression problem under the target shift assumption

Additionally, we present benchmark results for a regression problem with the target shift. Although SHIFT15M is a dataset for set-to-set matching, it can also be used for simple regression problems by setting the input to image features and the output to attributes associated with the items.

The target variable is the number of likes that each instance possesses, and the input variables are the user ID and the prices of the items. In this experiment, we evaluate the robustness of the model for different shift magnitudes, and the magnitudes of the shift are measured by the Wasserstein distance. We use the simple linear regression as the ordinal ERM, and we compare this with two other covariate shift adaptation methods, IWERM and AIWERM.

Definition 3.2. (Adaptive importance weighted ERM [118]) AIWERM uses $(p_{te}(\boldsymbol{x})/p_{tr}(\boldsymbol{x}))^{\alpha}$ for $\alpha \in [0,1]$ as the weighting function:

$$\hat{h} = \operatorname*{arg\,min}_{h \in \mathcal{H}} \frac{1}{n_{tr}} \sum_{i=1}^{n_{tr}} \left(\frac{p_{te}(\boldsymbol{x}_i^{tr})}{p_{tr}(\boldsymbol{x}_i^{tr})} \right)^{\alpha} \ell(h(\boldsymbol{x}_i^{tr}), y_i^{tr}).$$
(7)

Definition 3.3. (Relative importance weighted ERM [146]) RIWERM uses $p_{te}(\boldsymbol{x})/((1-\alpha)p_{tr}(\boldsymbol{x}) + \alpha p_{te}(\boldsymbol{x}))$ for $\alpha \in [0, 1]$ as the weighting function:

$$\hat{h} = \operatorname*{arg\,min}_{h \in \mathcal{H}} \frac{1}{n_{tr}} \sum_{i=1}^{n_{tr}} \frac{p_{te}(\boldsymbol{x}_i^{tr})}{m_{\alpha}(\boldsymbol{x}_i^{tr})} \ell(h(\boldsymbol{x}_i^{tr}), y_i^{tr}), \quad (8)$$

Models 2013 2014 2015 2016 2017 Set Transformer [73] $0.791(\pm 0.036)$ $0.743(\pm 0.041)$ $0.710(\pm 0.047)$ $0.698(\pm 0.050)$ $0.675(\pm 0.051)$ Set Transformer + mean-IW $0.791(\pm 0.036)$ $0.755(\pm 0.037)$ $0.732(\pm 0.041)$ $0.710(\pm 0.048)$ $0.696(\pm 0.049)$ Set Transformer + max-IW $0.760(\pm 0.037)$ $0.700(\pm 0.045)$ $0.791(\pm 0.036)$ $0.731(\pm 0.038)$ $0.714(\pm 0.045)$ BERT_{SMALL} [27] $0.898(\pm 0.008)$ $0.882(\pm 0.005)$ $0.860(\pm 0.005)$ $0.842(\pm 0.007)$ $0.830(\pm 0.011)$ BERT_{SMALL} + mean-IW $0.893(\pm 0.003)$ $0.839(\pm 0.008)$ $0.898(\pm 0.008)$ $0.866(\pm 0.005)$ $0.844(\pm 0.007)$ BERT_{SMALL} + max-IW $0.898(\pm 0.008)$ $0.895(\pm 0.002)$ $0.878(\pm 0.003)$ $0.859(\pm 0.002)$ $0.851(\pm 0.004)$ BERT_{BASE} [27] $0.880(\pm 0.011)$ $0.875(\pm 0.009)$ $0.844(\pm 0.004)$ $0.827(\pm 0.007)$ $0.817(\pm 0.010)$ BERT_{BASE} + mean-IW $0.880(\pm 0.011)$ $0.878(\pm 0.010)$ $0.853(\pm 0.007)$ $0.840(\pm 0.013)$ $0.822(\pm 0.004)$ $0.870(\pm 0.012)$ BERT_{BASE} + max-IW $0.880(\pm 0.011)$ $0.877(\pm 0.003)$ $0.852(\pm 0.003)$ $0.830(\pm 0.003)$ $0.860(\pm 0.007)$ $0.869(\pm 0.008)$ $0.825(\pm 0.013)$ $0.802(\pm 0.010)$ $0.785(\pm 0.009)$ BERT_{BASE-AP} [27] BERT_{BASE-AP} + mean-IW $0.869(\pm 0.008)$ $0.864(\pm 0.009)$ $0.837(\pm 0.009)$ $0.829(\pm 0.010)$ $0.797(\pm 0.012)$ BERT_{BASE-AP} + max-IW $0.869(\pm 0.008)$ $0.866(\pm 0.005)$ $0.844(\pm 0.004)$ $0.833(\pm 0.008)$ $0.810(\pm 0.002)$ GNN [21] $0.370(\pm 0.032)$ $0.361(\pm 0.035)$ $0.355(\pm 0.035)$ $0.351(\pm 0.035)$ $0.349(\pm 0.033)$ $0.357(\pm 0.033)$ GNN + mean-IW $0.370(\pm 0.032)$ $0.366(\pm 0.033)$ $0.360(\pm 0.032)$ $0.354(\pm 0.033)$ $0.362(\pm 0.030)$ $0.359(\pm 0.030)$ GNN + max-IW $0.370(\pm 0.032)$ $0.367(\pm 0.032)$ $0.362(\pm 0.031)$ HAP2S [154] $0.433(\pm 0.041)$ $0.420(\pm 0.044)$ $0.409(\pm 0.053)$ $0.385(\pm 0.053)$ $0.370(\pm 0.062)$ HAP2S + mean-IW $0.433(\pm 0.041)$ $0.428(\pm 0.042)$ $0.415(\pm 0.045)$ $0.408(\pm 0.050)$ $0.395(\pm 0.057)$ $0.413(\pm 0.047)$ HAP2S + max-IW $0.433(\pm 0.041)$ $0.427(\pm 0.042)$ $0.418(\pm 0.042)$ $0.400(\pm 0.055)$ $0.836(\pm 0.014)$ Cross Attention [110] $0.920(\pm 0.007)$ $0.892(\pm 0.009)$ $0.867(\pm 0.012)$ $0.844(\pm 0.012)$ Cross Attention + mean-IW $0.920(\pm 0.007)$ $0.901(\pm 0.008)$ $0.873(\pm 0.012)$ $0.850(\pm 0.013)$ $0.843(\pm 0.014)$ Cross Attention + max-IW $0.877(\pm 0.010)$ $0.920(\pm 0.007)$ $0.913(\pm 0.009)$ $0.849(\pm 0.011)$ $0.841(\pm 0.013)$ Cross Affinity [110] $0.924(\pm 0.005)$ $0.907(\pm 0.006)$ $0.886(\pm 0.009)$ $0.865(\pm 0.006)$ $0.855(\pm 0.003)$ Cross Affinity + mean-IW $0.924(\pm 0.005)$ $0.917(\pm 0.002)$ $0.886(\pm 0.003)$ $0.866(\pm 0.003)$ $0.860(\pm 0.002)$ Cross Affinity + max-IW $0.896(\pm 0.006)$ $0.871(\pm 0.001)$ $0.865(\pm 0.005)$ $0.924(\pm 0.005)$ $0.921(\pm 0.002)$

Table 4: Experimental results of the Fill-In-The-N-Blank with four candidates with several models. Evaluation metrics are the accuracy [%].

Table 5: Experimental results of the Fill-In-The-N-Blank with eight candidates with several models. Evaluation metrics are the accuracy [%]

Models	2013	2014	2015	2016	2017
Set Transformer [73]	$0.711(\pm 0.042)$	$0.690(\pm 0.040)$	$0.663(\pm 0.037)$	$0.637(\pm 0.044)$	$0.605(\pm 0.041)$
Set Transformer + mean-IW	$0.711(\pm 0.042)$	$0.702(\pm 0.038)$	$0.685(\pm 0.043)$	$0.662(\pm 0.040)$	$0.630(\pm 0.040)$
Set Transformer + max-IW	$0.711(\pm 0.042)$	$0.708(\pm 0.037)$	$0.699(\pm 0.044)$	$0.675(\pm 0.040)$	$0.641 (\pm 0.045)$
BERT _{SMALL} [27]	$0.824(\pm 0.015)$	$0.799(\pm 0.023)$	$0.772(\pm 0.025)$	$0.740(\pm 0.039)$	$0.716(\pm 0.039)$
BERT _{SMALL} + mean-IW	$0.824(\pm 0.015)$	$0.815 (\pm 0.016)$	$0.787(\pm 0.023)$	$0.759(\pm 0.031)$	$0.720(\pm 0.030)$
BERT _{SMALL} + max-IW	$0.824(\pm 0.015)$	$0.813(\pm 0.015)$	$0.805(\pm 0.020)$	$0.782(\pm 0.035)$	$0.732(\pm 0.030)$
BERT _{BASE} [27]	$0.810(\pm 0.017)$	$0.780(\pm 0.027)$	$0.764(\pm 0.035)$	$0.733(\pm 0.043)$	$0.700(\pm 0.044)$
BERT _{BASE} + mean-IW	$0.810(\pm 0.017)$	$0.799(\pm 0.020)$	$0.778(\pm 0.028)$	$0.750(\pm 0.022)$	$0.714(\pm 0.033)$
$BERT_{BASE} + max-IW$	$0.810(\pm 0.017)$	$0.805(\pm 0.023)$	$0.794 (\pm 0.021)$	$0.764 (\pm 0.020)$	$0.738(\pm 0.029)$
BERT _{BASE-AP} [27]	$0.801(\pm 0.012)$	$0.765(\pm 0.028)$	$0.741(\pm 0.030)$	$0.719(\pm 0.042)$	$0.694(\pm 0.048)$
$BERT_{BASE-AP}$ + mean-IW	$0.801(\pm 0.012)$	$0.788(\pm 0.025)$	$0.763(\pm 0.026)$	$0.734(\pm 0.026)$	$0.709(\pm 0.030)$
$BERT_{BASE-AP} + max-IW$	$0.801(\pm 0.012)$	$0.795(\pm 0.025)$	$0.777(\pm 0.020)$	$0.748(\pm 0.028)$	$0.722(\pm 0.021)$
GNN [21]	$0.346(\pm 0.030)$	$0.329(\pm 0.033)$	$0.319(\pm 0.036)$	$0.301(\pm 0.045)$	$0.287(\pm 0.050)$
GNN + mean-IW	$0.346(\pm 0.030)$	$0.337(\pm 0.031)$	$0.325(\pm 0.040)$	$0.310(\pm 0.040)$	$0.299(\pm 0.044)$
GNN + max-IW	$0.346(\pm 0.030)$	$0.342(\pm 0.035)$	$0.338(\pm 0.036)$	$0.317(\pm 0.038)$	$0.306(\pm 0.038)$
HAP2S [154]	$0.382(\pm 0.044)$	$0.367(\pm 0.046)$	$0.340(\pm 0.048)$	$0.325(\pm 0.045)$	$0.310(\pm 0.053)$
HAP2S + mean-IW	$0.382(\pm 0.044)$	$0.371(\pm 0.040)$	$0.343(\pm 0.048)$	$0.331(\pm 0.046)$	$0.312(\pm 0.051)$
HAP2S + max-IW	$0.382(\pm 0.044)$	$0.380(\pm 0.043)$	$0.359(\pm 0.046)$	$0.342(\pm 0.046)$	$0.324(\pm 0.048)$
Cross Attention [110]	$0.839(\pm 0.002)$	$0.808(\pm 0.005)$	$0.772(\pm 0.005)$	$0.740(\pm 0.009)$	$0.717(\pm 0.010)$
Cross Attention + mean-IW	$0.839(\pm 0.002)$	$0.810(\pm 0.009)$	$0.788(\pm 0.007)$	$0.752(\pm 0.006)$	$0.720(\pm 0.008)$
Cross Attention + max-IW	$0.839(\pm 0.002)$	$0.836(\pm 0.003)$	$0.800(\pm 0.005)$	$0.783(\pm 0.007)$	$0.743(\pm 0.003)$
Cross Affinity [110]	$0.845(\pm 0.000)$	$0.822(\pm 0.001)$	$0.791(\pm 0.005)$	$0.762(\pm 0.008)$	$0.741(\pm 0.004)$
Cross Affinity + mean-IW	$0.845(\pm 0.000)$	$0.831(\pm 0.008)$	$0.792(\pm 0.002)$	$0.766(\pm 0.004)$	$0.749(\pm 0.002)$
Cross Affinity+ max-IW	$0.845(\pm 0.000)$	$0.842(\pm 0.004)$	$0.807(\pm 0.003)$	$0.769(\pm 0.005)$	$0.753(\pm 0.005)$

Table 6: Experimental results for the regression problem with distribution shift adaptation. Evaluation metrics are the MSE.

Models	W = 0	W = 10	W = 20	W = 30	W = 40	W = 50
ERM	$9.36(\pm 0.02)$	$10.44(\pm 0.04)$	$17.10(\pm 0.06)$	$28.80(\pm 0.05)$	$39.56(\pm 0.05)$	$48.84(\pm 0.05)$
IWERM (optimal)	$9.36(\pm 0.02)$	$25.67(\pm 0.12)$	$32.58(\pm 0.12)$	$26.83(\pm 0.11)$	$20.19(\pm 0.10)$	$14.52(\pm 0.10)$
RIWERM ($\alpha = 0.25$)	$9.36(\pm 0.02)$	$9.34(\pm 0.04)$	$9.53(\pm 0.03)$	$11.37(\pm 0.04)$	$14.89(\pm 0.09)$	$17.00(\pm 0.14)$
RIWERM ($\alpha = 0.50$)	$9.36(\pm 0.02)$	$9.73(\pm 0.04)$	$9.57(\pm 0.03)$	$9.69(\pm 0.04)$	$12.37(\pm 0.10)$	$14.68(\pm 0.15)$
RIWERM ($\alpha = 0.75$)	$9.36(\pm 0.02)$	$11.59(\pm 0.05)$	$11.35(\pm 0.05)$	$9.39(\pm 0.03)$	$10.46(\pm 0.09)$	$12.60(\pm 0.14)$

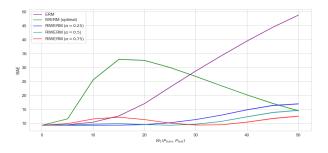


Figure 13: Experimental results of the regression problem for the number of likes

where
$$m_{\alpha}(\boldsymbol{x}_{i}^{tr}) = (1 - \alpha)p_{tr}(\boldsymbol{x}_{i}^{tr}) + \alpha p_{te}(\boldsymbol{x}_{i}^{tr})$$
.

Note that there are other variants of IWERM, as in [65].

Figure 13 and Table 6 show the experimental results of the regression task. These results show that the performance of ERM decreases with increasing shift magnitude, while IWERM and its variants allow the robustness to target shifts.

4. Related works and conclusion

There are many studies for handling set data with neural networks. DeepSets [155] proposes a model that satisfies the key concepts of permutation invariant and permutation equivariant for approximating functions that deal with sets.

Definition 4.1 (Permutation invariant). A set function f is said to be permutation invariant if $f(\mathcal{V}, \mathcal{W}) = f(\pi_w \mathcal{W}, \pi_v \mathcal{V})$ for permutations π_w and π_v .

Definition 4.2 (Permutation equivariance). A set function f is said to be permutation equivariant if $f(\pi_w W, \pi_v V) = \pi_w f(W, V)$ for permutations π_w and π_v . Note that f is permutation invariant for permutations within V.

Theorem 4.1 (Sum-decomposable[155]). A function f on set \mathcal{X} from countable particle space is invariant if and only if there exists a decomposition,

$$f(\mathcal{X}) =
ho \left(\sum_{\boldsymbol{x} \in \mathcal{X}} \phi(\boldsymbol{x})\right), \quad f =
ho \circ \sum \circ \phi,$$

with appropriate functions ϕ and ρ .

It is pointed out that the necessary and sufficient of sumdecomposable is guaranteed only for countable sets [136].

Theorem 4.2 ([136]). A continuous function f on finite sets \mathcal{X} , $|\mathcal{X}| < p$, is invariant if and only if it is sum-decomposable via \mathbb{R}^p .

That is, for an arbitrary continuous function f, the image space of ϕ has to have at least dimension p, which is both necessary and sufficient. For more details, see Appendix C.

SetTransformer [73] is an attention-based neural network module that allows us to handle sets as inputs. SetVAE [63], an extension of VAE to set data, has also been proposed.

Several distribution shift datasets exist for general classification and regression tasks where the input is a vector. WILDS [68] is the collection of benchmark datasets [10, 8, 126, 52, 24, 16, 20, 153, 93, 85] under the distribution shift, including histopathological images, satellite images or sequence of source code tokens. PACS [74] and Office-Home [135] adopt the image style to differentiate distributions, and VLCS [31] takes data collected independently from four sources as environments. Also, DomainNet [160] extends PACS to a far larger scale.

There are also a number of studies that evaluate robustness to distribution shifts by introducing artificial distribution shifts, such as noise corruptions [42, 49, 90, 108, 145], spatial transformations [30, 32], ImageNet [26] variants (e.g. ImageNet-A [50], ImageNet-C [49], ImageNet-R [48]), and adversarial examples [14, 123, 72, 45, 143, 18]. However, a recent study [124] has indicated that there is no correlation between the robustness of such artificial distribution shifts and the robustness of natural distribution shifts.

We believe that our SHIFT15M is a very useful dataset for evaluating the still underdeveloped task of set-to-set matching under natural distribution shifts. See Appendix D and F for more related literature including domain adaptation, out-of-domain generalization, concept drift adaptation, or other fashion datasets. We also provide the datasheet for the SHIFT15M in Appendix H, which is based on [41].

4.1. Future works

- More ablation studies: more model architectures and parameter influences need to be investigated.
- Experiments on model calibration: model calibration and distribution shift are known to be closely re-

lated [138, 95, 70]. We expect that observing the metrics for evaluating model calibration in experiments on the SHIFT15M will provide meaningful insights.

• Additional API development: currently, our API is based on PyTorch [97]. In the future, we would like to expand the APIs for other machine learning libraries (e.g., TensorFlow [1] or Keras [19]).

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A. Sample items from the SHIFT15M

Figure 15,16,17,18 show the additional sample items from the SHIFT15M dataset.

In addition, we provide the year-wise visualization for the SHIFT15M dataset with t-SNE in Figure 19,20,21 like as Figure 1. t-SNE is a dimensionality reduction technique that is often used for visualizing high-dimensional data in a lower-dimensional space. In this case, the data being visualized is likely a collection of fashion-related features, such as color, texture, and style, that have been extracted from the dataset. These figures are likely to show the distribution of fashion-related features across different years or time periods, which can help to reveal trends and patterns in the fashion industry over time.

B. Details and additional figures for the numerical experiments

Here, we describe the details for numerical experiments and introduce additional figures.

B.1. Details for numerical experiments

Training Settings We use a stochastic gradient descent method [106, 61] with a learning rate of 0.005, a momentum of 0.5, and a weight decay of 0.00004. We train both the CNN and set-matching model in an end-to-end manner. In each iteration, we randomly swap pairs of sets and items in each set, and randomly flip images horizontally, to learn all the methods stably.

Preparing Set Pairs To construct the correct pair of sets to be matched, we randomly halve the given outfit \mathcal{O} into two non-empty proper subsets \mathcal{X} and \mathcal{Y} as follows: $\mathcal{O} \to {\mathcal{X}, \mathcal{Y}}$, where $\mathcal{X} \cap \mathcal{Y} = \emptyset$. Here, we extend this setting to include more general situations. We select Q outfits $\mathcal{O}^{(1)}, \ldots, \mathcal{O}^{(Q)}$ randomly and split the respective outfits in half $\mathcal{O}^{(q)} \to {\mathcal{X}^{(q)}, \mathcal{Y}^{(q)}}$, where $q \in {1, \ldots, Q}$. We regard the two sets ${\mathcal{X}^{(1)}, \ldots, \mathcal{X}^{(Q)}}$ and ${\mathcal{Y}^{(1)}, \ldots, \mathcal{Y}^{(Q)}}$ as the correct pair, which consists of Q fashion styles. In the training phase, we set Q = 4. Figure 14 shows the overview of set-to-set matching problem.

B.2. Additional figures for the numerical experiments

Figure 22,23,24,25,26 show the additional figures for the Fill-In-The-N Blank experiments. These figures are based on Table 4 and 5.

Figure 27,28 and 29 shows the PCA [35] dimensionality reduction for items in each year. The color of each point corresponds to the item category.



Figure 14: This figure is a citation of Figure 1 from Y.Saito et al. [110]. Set-to-set matching aims to answer a fundamental question: which candidate set is more compatible with the reference set than others? In this process, we match the reference set with each candidate set and select the best pair based on some criteria.

C. Details for the sum-decomposable set function

From Definition 4.1, we say that (ρ, ϕ) is a sumdecomposition of f. Given sum-decomposition (ρ, ϕ) , we write $\Phi(\mathcal{X}) = \sum_{\boldsymbol{x} \in \mathcal{X}} \phi(\boldsymbol{x})$ and $f = \rho \circ \Phi$. We may also refer to the function $\rho \circ \Phi$ as a sum-decomposition.

Definition C.1. Let (ρ, ϕ) be a sum-decomposition. Write *Z* for the domain of ρ and the codomain of ϕ . We refer to *Z* as the latent space of the sum-decomposition (ρ, ϕ) .

Definition C.2. Given a space Z, we say that f is sumdecomposable via Z if f has a sum-decomposition whose latent space is Z.

Definition C.3. We say that f is continuously sumdecomposable when there exists a sum-decomposition (ρ, ϕ) of f such that both ρ and ϕ are continuous. (ρ, ϕ) is then a continuous sum-decomposition of f.

We give a brief reproduction of the statements and proof of two key theorems from [155].

Theorem C.1. Let $f : 2^{\mathcal{U}} \to \mathbb{R}$ where \mathcal{U} is countable. Then f is sum-decomposable via \mathbb{R} .

Proof. Since \mathcal{U} is countable, each $x \in \mathcal{U}$ can be mapped to a unique element in \mathbb{N} by a bijective function $c : \mathcal{U} \to \mathbb{N}$. If we can choose ϕ so that Φ is invertible, then we can set $\rho = f \circ \Phi^{-1}$, giving

$$f = \rho \circ \Phi, \tag{9}$$

so f is sum-decomposable via \mathbb{R} .

Considering the mapping $\phi(\boldsymbol{x}) = 4^{-c(\boldsymbol{x})}$, each set $\mathcal{X} \subset \mathcal{U}$ corresponds to a unique real number $r \coloneqq \Phi(\mathcal{X})$. The

number r can be decoded to the set \mathcal{X} and the element $c^{-1}(n) \in \mathcal{U}$ belongs to \mathcal{X} if and only if the *n*-th digit of r is 1. This decoding procedure shows that Φ is invertible, and the conclusion follows.

Theorem C.2. Let $M \in \mathbb{N}$, and let $f : [0,1]^M \to \mathbb{R}$ be a continuous permutation-invariant function. Then, f is continuously sum-decomposable via \mathbb{R}^{M+1} .

Theorem C.3. Deep Sets can represent any continuous permutation-invariant function function of M elements if the dimesion of the latent space of the model is at least M + 1.

Recent work [137] argue that the guarantee of sumdecomposability via \mathbb{R} given by Theorem C.1 cannot hold in practice, and prove that the guarantee of sumdecomposability via \mathbb{R}^{M+1} is essentially the bestpossible.

Theorem C.4. Let $M, N \in \mathbb{N}$, with M > N. Then there exist continuous permutation-invariant functions $f : \mathbb{R}^M \to \mathbb{R}$ which are not continuously sum-decomposable via \mathbb{R}^N .

Theorem C.5. Let $M \in \mathbb{N}$, and let $f : \mathbb{R}^M \to \mathbb{R}$ be a continuous permutation-invariant function. Then, f is continuously sum-decomposable via \mathbb{R}^M .

Theorem C.6. Denote the set of subsets [0,1] containing at most M elements by $[0,1]^{\leq M}$. Let $f:[0,1]^{\leq M} \to \mathbb{R}$ be continuous and permutation-invariant. Then, f is continuously sum-decomposable via \mathbb{R}^M .

Let M be a positive integer, $U \subset \mathbb{R}^M$ be compact, and $f: U \to \mathbb{R}$.

Definition C.4. Let $\epsilon > 0$. (ϕ, ρ) is a within- ϵ sumdecomposition of f if $|f(\mathcal{U}) - \rho(\Phi(\mathcal{U}))| < \epsilon$ for every $\mathcal{U} \in U$.

Definition C.5. A sequence $(\phi, \rho)_k := \{(\phi_k, \rho_k) : k \in \mathbb{N}\}$ is an approximate sum-decomposition of f if, for any $\epsilon > 0$, there is some $K \in \mathbb{N}$ such that (ϕ_K, ρ_K) is a within- ϵ sum-decomposition of f. We also require that (ϕ_k, ρ_k) is a sequence of ever-closer approximations to f. The existence of an approximate sum-decomposition of f guarantees that f can be approximated arbitrarily closely by sumdecomposition.

Theorem C.7. Let $M, N \in \mathbb{N}$ with M > N, and recall that $I_M = [-1, 1]^M \subset \mathbb{R}^M$. Then there exists a continuous permutation-invariant function $f : I_M \to \mathbb{R}$ which has no continuous approximate sum-decomposition via \mathbb{R}^N .

D. Generalization bounds under the distribution shift

The SHIFT15M dataset is a valuable resource for evaluating the performance of machine learning models in settings where the underlying distribution of the data may shift over time. One important aspect of this dataset is that it allows researchers to calculate the distance between the distributions in the train and test splits. This distance measure can provide valuable insight into how much the distribution has shifted between the two sets of data, which in turn can help researchers to better understand the behavior of their models under distributional shifts.

Furthermore, researchers have proposed several generalization bounds that are specific to distribution shifts and that depend on the distance between distributions. These bounds provide a way to quantify the relationship between the performance of a model and the extent of the distributional shift, which is a crucial factor in evaluating the robustness of a model. By considering these generalization bounds when analyzing experimental results on the SHIFT15M dataset, researchers can gain a more precise understanding of how their models perform under distribution shifts and how to improve them. Overall, the SHIFT15M dataset and the associated generalization bounds represent important tools for evaluating and improving the robustness of machine learning models. Here, we introduce several known generalization bounds under the distribution shift [105].

Definition D.1 (Total variation distance). Denote by \mathcal{B} be the set of measurable subsets under two probability distributions p_1 and p_2 . Then, the total variation distance between p_1 and p_2 is defined as

$$d_{TV}(p_1, p_2) = 2 \sup_{B \in \mathcal{B}} |p_1(B) - p_2(B)|.$$
(10)

Let

$$\mathcal{R}_{\mathfrak{D}}^{\ell}(h) = \mathbb{E}_{(\boldsymbol{x}, y) \sim \mathfrak{D}}\left[\ell(\boldsymbol{x}, y)\right].$$
(11)

Theorem D.1 ([12]). Given two domains \mathfrak{S} and \mathfrak{T} , and a hypothesis class \mathcal{H} over $\mathcal{X} \times \mathcal{Y}$, the following holds for any $h \in \mathcal{H}$.

$$\begin{aligned} &\mathcal{R}^{\ell_{01}}_{\mathfrak{T}}(h) \leq \mathcal{R}^{\ell_{01}}_{\mathfrak{S}}(h) + d_{TV}(\mathfrak{S}_{\mathcal{X}},\mathfrak{T}_{\mathcal{X}}) \\ &+ \min\left\{ \mathbb{E}_{\boldsymbol{x} \sim \mathfrak{S}_{\mathcal{X}}}\left[J(\boldsymbol{x}; \mathfrak{S}_{\mathcal{X}}, \mathfrak{T}_{\mathcal{X}}) \right], \mathbb{E}_{\boldsymbol{x} \sim \mathfrak{T}_{\mathcal{X}}}\left[J(\boldsymbol{x}; \mathfrak{S}_{\mathcal{X}}, \mathfrak{T}_{\mathcal{X}}) \right] \right\}, \end{aligned}$$

where $J(x; \mathfrak{S}_{\mathcal{X}}, \mathfrak{T}_{\mathcal{X}}) = |f_{\mathfrak{S}}(x) - f_{\mathfrak{T}}(x)|$, and $f_{\mathfrak{S}}(x)$ and $f_{\mathfrak{T}}(x)$ are source and target true labeling functions associated with \mathfrak{S} and \mathfrak{T} , respectively.

Definition D.2. Given marginal distributions of two domains $\mathfrak{S}_{\mathcal{X}}$ and $\mathfrak{T}_{\mathcal{X}}$ over the input space \mathcal{X} , let \mathcal{H} be a hypothesis class, and denote $\mathcal{H}\Delta\mathcal{H}$ the symmetric difference hypothesis space defined as

$$h \in \mathcal{H} \Delta \mathcal{H} \iff h(\boldsymbol{x}) = g(\boldsymbol{x}) \oplus g'(\boldsymbol{x})$$
 (12)

for some $(g,g') \in \mathcal{H}^2$, where \oplus stands for XOR operation. Let I(h) denote the set for which $h \in \mathcal{H}\Delta\mathcal{H}$ is the characteristic function, that is $\boldsymbol{x} \in I(h) \Leftrightarrow g(\boldsymbol{x}) = 1$. The $\mathcal{H}\Delta\mathcal{H}$ -divergence between $\mathfrak{S}_{\mathcal{X}}$ and $\mathfrak{T}_{\mathcal{X}}$ is defined as:

$$d_{\mathcal{H}\Delta\mathcal{H}}(\mathfrak{S}_{\mathcal{X}},\mathfrak{T}_{\mathcal{X}}) = 2 \sup_{h \in \mathcal{H}\Delta\mathcal{H}} \left| \Pr_{\mathfrak{S}_{\mathcal{S}}}(I(h)) - \Pr_{\mathfrak{T}_{\mathcal{X}}}(I(h)) \right|.$$
(13)

Lemma D.2. Let \mathcal{H} be a hypothesis space of VC dimension $VC(\mathcal{H})$. If S_u , T_u are unlabeled samples of size m each, drawn independently from $\mathfrak{S}_{\mathcal{X}}$ and $\mathfrak{T}_{\mathcal{X}}$ respectively, then for any $\delta \in (0, 1)$ with probability at least $1 - \delta$ over the random choice of the samples we have

$$d_{\mathcal{H}\Delta\mathcal{H}}(\mathfrak{S}_{\mathcal{X}},\mathfrak{T}_{\mathcal{X}}) \leq \hat{d}_{\mathcal{H}\Delta\mathcal{H}}(\mathfrak{S}_{u},\mathfrak{T}_{u}) + 4\sqrt{\frac{2VC(\mathcal{H})\ln 2m + \ln\frac{2}{\delta}}{m}},$$

where $\hat{d}_{\mathcal{H}\Delta\mathcal{H}}(\mathfrak{S}_u,\mathfrak{T}_u)$ is the empirical $\mathcal{H}\Delta\mathcal{H}$ -divergence estimated on \mathfrak{S}_u and \mathfrak{T}_u .

Lemma D.3 ([11]). Let \mathcal{H} be a hypothesis space. Then, for two unlabeled samples S_u , T_u of size m we have

$$d_{\mathcal{H}\Delta\mathcal{H}}(S_u, T_u) = 2\left\{1 - \min_{h \in \mathcal{H}\Delta\mathcal{H}} \left(\frac{1}{m} \sum_{\boldsymbol{x} \in H_0} 1_{\boldsymbol{x} \in S_u} + \frac{1}{m} \sum_{\boldsymbol{x} \in H_1} 1_{\boldsymbol{x} \in T_u}\right)\right\},\$$

where $H_0 = \{ x : h(x) = 0 \}$ and $H_1 = \{ x : h(x) = 1 \}.$

Lemma D.4 ([11]). Let \mathfrak{S} and \mathfrak{T} be two domains on $\mathcal{X} \times \mathcal{Y}$. For any pair of hypothesis $(h, h') \in \mathcal{H} \Delta \mathcal{H}^2$, we have

$$\left|\mathcal{R}^{\ell_{01}}_{\mathfrak{T}}(h,h') - \mathcal{R}^{\ell_{01}}_{\mathfrak{S}}(h,h')\right| \le \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(\mathfrak{S}_{\mathcal{X}},\mathfrak{T}_{\mathcal{X}}), \quad (14)$$

where

$$\mathcal{R}^{\ell}_{\mathfrak{D}}(h,h') = \mathbb{E}_{\boldsymbol{x} \sim \mathfrak{D}_{\mathcal{X}}}\left[\ell(h(\boldsymbol{x}),h'(\boldsymbol{x}))\right].$$
(15)

Theorem D.5 ([11]). Let \mathcal{H} be a hypothesis space of VC dimension $VC(\mathcal{H})$. If S_u and T_u are unlabeled samples of size m' each, drawn independently from $\mathfrak{S}_{\mathcal{X}}$ and $\mathfrak{T}_{\mathcal{X}}$, respectively, then for any $\delta \in (0, 1)$ with probability at least $1 - \delta$ over the random choice of the samples, we have that for all $h \in \mathcal{H}$

$$\mathcal{R}_{\mathfrak{T}}^{\ell_{01}}(h) \leq \mathcal{R}_{\mathfrak{S}}^{\ell_{01}}(h) + \frac{1}{2}\hat{d}_{\mathcal{H}\Delta\mathcal{H}}(S_u, T_u) + 4\sqrt{\frac{2VC(\mathcal{H})\ln 2m' + \ln\frac{2}{\delta}}{m'}} + \lambda$$

where λ is the combined error of the ideal hypothesis h^* .

The trade-off between source risk, divergence and the capability to adapt for distribution shift is a crucial phenomenon that plays a significant role in experiments related to distribution shift adaptation using SHIFT15M. This tradeoff refers to the balance that needs to be maintained between the risk of the source, the extent of divergence between the source and target domains, and the ability of a model to adapt to distribution shift. Therefore, it is essential to evaluate the performance of models while keeping this trade-off in mind. By doing so, researchers can ensure that the models they develop are not only accurate but also robust and adaptable to changes in the underlying distribution of the data. Ultimately, this can lead to better real-world performance of machine learning models and improve their utility in various applications.

Since our numerical experiments use the Wasserstein distance W_p^p as the distance between distributions, we also introduce generalization bounds based on this distance:

$$W^p_p(\mathfrak{S}_{\mathcal{X}},\mathfrak{T}_{\mathcal{X}}) \coloneqq \inf_{\gamma \in \Pi(\mathfrak{S}_{\mathcal{X}},\mathfrak{T}_{\mathcal{X}})} \int_{\mathcal{X} \times \mathcal{X}} c(\boldsymbol{x}, \boldsymbol{x}')^p d\gamma(\boldsymbol{x}, \boldsymbol{x}'),$$

where $c : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_+$ is a cost function for transporting one unit of mass x to x', and $p \in [1, +\infty]$.

Lemma D.6 ([104]). Let $\mathfrak{S}_{\mathcal{X}}, \mathfrak{T}_{\mathcal{X}} \in \mathcal{P}(\mathcal{X})$ be two probability measures on \mathbb{R}^d . Assume that the cost function $c(\boldsymbol{x}, \boldsymbol{x}') = \|\phi(\boldsymbol{x}) - \phi(\boldsymbol{x}')\|_{\mathfrak{H}_{k_\ell}}$, where \mathfrak{H} is an RKHS equipped with kernel $k_\ell : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ induced by $\phi : \mathcal{X} \to \mathfrak{H}_{k_\ell}$ and $k_\ell(\boldsymbol{x}, \boldsymbol{x}') = \langle \phi(\boldsymbol{x}), \phi(\boldsymbol{x}') \rangle_{\mathfrak{H}_{k_\ell}}$. Assume further that the loss function $\ell_{h,f} : \boldsymbol{x} \mapsto \ell(h(\boldsymbol{x}), f(\boldsymbol{x}))$ is convex, symmetric and bounded and obeys the triangular equality and has the parametric form $|h(\boldsymbol{x}) - f(\boldsymbol{x})|^q$ for some q > 0. Assume also that kernel k_ℓ in the RKHS \mathfrak{H}_{k_ℓ} is square-root integrable with respect to both $\mathfrak{S}_{\mathcal{X}}, \mathfrak{T}_{\mathcal{X}}$ for all $\mathfrak{S}_{\mathcal{X}}, \mathfrak{T}_{\mathcal{X}} \in \mathcal{P}(\mathcal{X})$ where \mathcal{X} is separable and $0 \leq k_\ell(\boldsymbol{x}, \boldsymbol{x}') \leq K, \forall \boldsymbol{x}, \boldsymbol{x}' \in \mathcal{X}$ if $\|\ell\|_{\mathfrak{H}_{k_\ell}} \leq 1$, then the following holds:

$$\mathcal{R}_{\mathfrak{T}}^{\ell_q}(h,h') \le \mathcal{R}_{\mathfrak{S}}^{\ell_q}(h,h') + W_1(\mathfrak{S}_{\mathcal{X}},\mathfrak{T}_{\mathcal{X}}), \quad (16)$$

for all $(h, h') \in \mathfrak{H}^2_{k_\ell}$.

This lemma allows us to relate the source and target errors by using Wasserstein distance. Also, we have

$$\begin{aligned} \|\phi(\boldsymbol{x}) - \phi(\boldsymbol{x}')\|_{\mathfrak{H}} &= \sqrt{\langle \phi(\boldsymbol{x}) - \phi(\boldsymbol{x}'), \phi(\boldsymbol{x}) - \phi(\boldsymbol{x}') \rangle_{\mathfrak{H}}} \\ &= \sqrt{k(\boldsymbol{x}, \boldsymbol{x}) - 2k(\boldsymbol{x}, \boldsymbol{x}') + k(\boldsymbol{x}', \boldsymbol{x}')}. \end{aligned}$$

Theorem D.7 ([15]). Let μ be a probability measure in \mathbb{R}^d so that for some $\alpha > 0$, $\in_{\mathbb{R}^d} \exp\{\alpha \| \boldsymbol{x} \|^2\} d\mu < \infty$, and $\hat{\mu} = \frac{1}{N} \sum_{i=1}^N \delta_{\boldsymbol{x}_i}$ be its associated empirical measure defined on a sample of independent variables $\{\boldsymbol{x}_i\}_{i=1}^N$ drawn from μ . Then for any d' > d and $\zeta' < \sqrt{2}$ there exists some constant N_0 depending on d' and some square exponential moment of μ such that, for any $\epsilon > 0$ and $N \ge N_0 \max(\epsilon^{-(d'+2)}, 1)$,

$$\Pr\left\{W_1(\mu, \mu') > \epsilon\right\} \le \exp\left\{-\frac{\zeta'}{2}N\epsilon^2\right\}$$
(17)

where d' and ζ' can be calculated explicitly.

Theorem D.8. Under the assumption of Lemma D.6, let S_u and T_u be two samples of size N_S and N_T drawn i.i.d. from $\mathfrak{S}_{\mathcal{X}}$ and $\mathfrak{T}_{\mathcal{X}}$, respectively. Let $\mathfrak{S}_{\mathcal{X}} = \frac{1}{N_S} \sum_{i=1}^{N_S} \delta_{\boldsymbol{x}_i^S}$ and $\mathfrak{T}_{\mathcal{X}} = \frac{1}{N_T} \sum_{i=1}^{N_T} \delta_{\boldsymbol{x}_i^T}$ be the associated empirical measures. Then for any d' > d and $\zeta' < \sqrt{2}$, there exists some

constant N_0 depending on d' such that for any $\delta > 0$ and $\min(N_S, N_T) \ge N_0 \max(\delta^{-(d'+2)}, 1)$ with probability at least $1 - \delta$ for all h, we have

$$\begin{aligned} \mathcal{R}_{\mathfrak{T}}^{\ell_q}(h) &\leq \mathcal{R}_{\mathfrak{S}}^{\ell_q}(h) + W_1(\hat{\mathfrak{S}}_{\mathcal{X}}, \hat{\mathfrak{T}}_{\mathcal{X}}) \\ &+ \sqrt{\frac{2\ln\frac{1}{\delta}}{\zeta'}} \left(\sqrt{\frac{1}{N_S}} + \sqrt{\frac{1}{N_T}}\right) + \lambda_1 \end{aligned}$$

where λ is the combined error of the ideal hypothesis h^* that minimizes the combined error of $\mathcal{R}^{\ell_q}_{\mathfrak{S}}(h) + \mathcal{R}^{\ell_q}_{\mathfrak{T}}(h)$.

E. Density ratio estimation for the importance weighted set-to-set matching

In the experiments, we proposed the importance-weighted set-to-set matching as the baseline method. Recall that, under the covariate shift assumption, we have

$$\begin{split} \mathbb{E}_{tr} \left[\frac{p_{te}(\boldsymbol{x})}{p_{tr}(\boldsymbol{x})} \ell(h(\boldsymbol{x}), y) \right] &= \int \frac{p_{te}(\boldsymbol{x})}{p_{tr}(\boldsymbol{x})} \ell(h(\boldsymbol{x}), y) p_{tr}(\boldsymbol{x}, y) d\boldsymbol{x} dy \\ &= \int \ell(h(\boldsymbol{x}), y) p_{te}(\boldsymbol{x}, y) d\boldsymbol{x} dy \\ &= \mathbb{E}_{te} \left[\ell(h(\boldsymbol{x}), y) \right]. \end{split}$$

Density ratio estimation is a technique used in machine learning to quantify the difference between the distributions of two datasets. The goal of density ratio estimation is to estimate the density ratio $r(\boldsymbol{x})$, which is defined as the ratio of the probability density functions of the test distribution $p_{te}(\boldsymbol{x})$ and the training distribution $p_{tr}(\boldsymbol{x})$ as $r(\boldsymbol{x}) = p_{te}(\boldsymbol{x})/p_{tr}(\boldsymbol{x})$.

The density ratio is a powerful tool because it allows us to compare the distributions of the two datasets and measure the extent of the distribution shift. In particular, if we can estimate the density ratio accurately, we can obtain a consistent estimator under the distribution shift. This means that we can use this estimator to accurately predict the performance of our model on the test set, even if the distribution of the test set is different from that of the training set.

To estimate the density ratio, we used a probabilistic classifier. However, there are several other strategies that exist for estimating density ratios. Each of these methods has its own strengths and weaknesses, and the choice of method will depend on the specific application and the characteristics of the data.

The main idea of moment matching is, to match the moments of $\hat{p}_{te}(\boldsymbol{x}) = \hat{r}(\boldsymbol{x})p_{tr}(\boldsymbol{x})$ and $p_{te}(\boldsymbol{x})$. For example, matching the mean is

$$\int \boldsymbol{x}\hat{r}(\boldsymbol{x})p_{tr}(\boldsymbol{x})d\boldsymbol{x} = \int \boldsymbol{x}p_{te}(\boldsymbol{x})d\boldsymbol{x}.$$
 (18)

However, matching a finite number of moments does not necessarily yield the true density ratio even asymptotically. Kernel mean matching [53, 46] allows that All moments are efficiently matched in Gaussian RKHS \mathfrak{H} :

$$\min_{\hat{r}\in\mathfrak{H}}\left\|\int K(\boldsymbol{x},\cdot)\hat{r}(\boldsymbol{x})p_{tr}(\boldsymbol{x})d\boldsymbol{x}-\int K(\boldsymbol{x},\cdot)p_{tr}(\boldsymbol{x})d\boldsymbol{x}\right\|_{\mathfrak{H}}^{2}.$$

KLIEP [92, 122] minimize KL-divergence from $p_{te}(x)$ to $\hat{p}_{te}(x) = \hat{r}(x)p_{tr}(x)$ as

$$\min_{\hat{r}} D_{KL}[p_{te}(\boldsymbol{x}) \| \hat{p}_{te}] = \min_{\hat{r}} \int p_{te}(\boldsymbol{x}) \frac{p_{te}(\boldsymbol{x})}{\hat{r}(\boldsymbol{x}) p_{tr}(\boldsymbol{x})} d\boldsymbol{x}.$$

Least-Squares Importance Fitting (LSIF) [57] minimize squared loss:

$$\min_{\hat{r}} \int \left(\hat{r}(\boldsymbol{x}) - r(\boldsymbol{x})\right)^2 p_{tr}(\boldsymbol{x}) d\boldsymbol{x}.$$

F. Other related literature

Here we present the rest of the relevant literature.

F.1. Concept drift

In addition to covariate shift and target shift, the following concept drift [128, 39, 82] is also well known.

Definition F.1 (Concept drift). We consider that the two distributions $p_{tr}(x, y)$ and $p_{te}(x, y)$ satisfy the concept drift assumption if the following conditions hold:

$$p_{tr}(\boldsymbol{x}|\boldsymbol{y}) \neq p_{te}(\boldsymbol{x}|\boldsymbol{y}),$$
$$p_{tr}(\boldsymbol{y}|\boldsymbol{x}) \neq p_{tr}(\boldsymbol{y}|\boldsymbol{x}).$$

Unlike covariate shift and target shift, concept drift assumes that the conditional probabilities between the two distributions are different. This means that a model that is well-specified in the training distribution will be missspecified in the test distribution, making it the most difficult problem setup. The strategies for addressing concept drift can be broadly categorized as follows

- concept drift detection;
- · concept drift understanding;
- concept drift adaptation.

Concept drift detection refers to the strategies that characterize and quantify concept drift via identifying change points or change time intervals [9]. Many algorithms focus on tracking changes in the online error rate of base classifiers, such as Drift Detection Method (DDM) [37], LLDD [36], EDDM [6], HDDM [34] or FW-DDM [79]. Other strategies based on data distribution [84, 120, 83] or multiple hypothesis test [2, 139, 158].

Concept drift understanding refers to retrieving concept drift information about "When" (the time at which the concept drift occurs and how long the drift lasts) [115, 84], "How" (the severity/degree of concept drift) [94], and "Where" (the drift regions of concept drift) [78].

The main approaches for concept drift adaptation are training new models for global drift [5, 86], model ensemble for recurring drift [44, 69, 29], or adjusting existing models for regional drift [54, 38, 147].

F.2. Domain adaptation

Domain adaptation [23, 22, 40, 111] is often used in a similar context to distribution shift adaptation. It is often referred to as visual domain adaptation [33, 55, 140, 98], especially in the field of computer vision. This concept is often referred to indistinguishably from covariate shift. Depending on the availability of the source and target domain data, the domain adaptation problem can be defined in many different ways.

- **supervised domain adaptation**: In supervised domain adaptation [89, 129], labeled data is available in both the source and target domains. The model is trained on the labeled data from the source domain and then adapted to the target domain using the labeled data from the target domain.
- semi-supervised domain adaptation: In semisupervised domain adaptation [71, 109, 28, 151], a small amount of labeled data is available in the target domain in addition to the labeled data in the source domain. The model is trained on the labeled data from both domains and then adapted to the target domain using both the labeled and unlabeled data from the target domain.
- **unsupervised domain adaptation**: In unsupervised domain adaptation [40, 58, 77, 113, 81], labeled data is only available in the source domain. The model is trained on the labeled data from the source domain and then adapted to the target domain using only the unlabeled data from the target domain.
- multi-source domain adaptation: Unlike traditional domain adaptation, which involves adapting from a single source domain, multi-source domain adaptation [99, 149, 161] deals with situations where there are multiple source domains with different but related feature distributions.

F.3. Out-of-distribution generalization

Another similar concept is out-of-distribution generalization [4, 48, 76, 117, 152]. The main difference with distribution shift problem is that in the out-of-distribution generalization problem setting we do not have any access to the test data during training. For example, under the covariate shift hypothesis, we often assume that we are allowed to access unlabeled test data. Major approaches for out-of-distribution generalization are disentangled representation learning [13, 51, 62], causal representation learning [150, 112], domain generalization [162, 75, 163, 74], invariant learning [100, 107, 47], stable learning [116, 157] and distributionally robust optimization [103, 25, 43].

F.4. Other related topics

Active learning Active learning [114, 101, 66] is a subfield of machine learning that focuses on developing algorithms that can automatically select the most informative data to be labeled by an expert or a human annotator. The goal of active learning is to achieve high accuracy models with minimal labeled data. Active learning can help mitigate the effects of distribution shift by actively selecting the most informative data points to be labeled by an expert or a human annotator. By doing so, the active learning algorithm can ensure that the labeled data used to train the model is representative of the data that the model will encounter in the real world.

Continual learning Continual learning [96], also known as lifelong learning or incremental learning, is a subfield of machine learning that deals with the problem of learning from a continuous stream of data over time, without forgetting previously learned knowledge. One of the key challenges in continual learning is avoiding catastrophic forgetting [67, 59], which occurs when a model overwrites previously learned knowledge with new information. Both catastrophic forgetting and distribution shift can lead to poor model performance and may require the model to be retrained or updated to address these issues. Continual learning algorithms are designed to address these challenges by enabling models to learn from a continuous stream of data over time, without forgetting previously learned knowledge and without being affected by distribution shift.

Transfer learning Transfer learning [141, 127, 164] is a machine learning technique that involves leveraging knowledge learned from one task to improve performance on a different, but related task. In transfer learning, a model is first trained on a source task, which provides a foundation of knowledge and skills. Then, the model is fine-tuned on a target task, which is related to the source task but may have different characteristics. Transfer learning is motivated by the fact that many tasks in machine learning share common features and patterns. By leveraging knowledge from a related task, a model can learn to recognize patterns and features more effectively, even if the target task has different characteristics.

F.5. Fashion datasets

Several fashion datasets have been introduced in the research community to facilitate the development and evaluation of fashion-related machine learning models. These datasets contain various types of fashion-related data, including images, textual descriptions, and attribute labels.

Fashion MNIST [144] Fashion MNIST is a publicly available dataset of Zalando's article images that is widely used for research and development in computer vision and machine learning. The dataset consists of a training set of 60,000 examples and a test set of 10,000 examples, each of which is a 28×28 grayscale image. The images in the dataset are associated with labels from 10 different classes.

DeepFashion [80] DeepFashion is a dataset containing around 800K diverse fashion images with their rich annotations (46 categories, 1,000 descriptive attributes, bounding boxes and landmark information) ranging from well-posed product images to real-world-like consumer photos.

Fashionpedia [56] Fashionpedia consists of user uploaded 48K street-fashion photos collected from free license websites such as Unsplash, Kaboompics etc. These photos contain people wearing variety of clothes and accessories captured in different background, weather and camera conditions.

Polyvore [133] Polyvore is a crowd-sourced dataset containing outfits or sets of fashion items that complement each other. It consists of manually labeled 68K outfits that are split into 53K, 10K and 5K into training, validation and testing sets, respectively

Polyvore-disjoint [133] Polyvore-disjoint is a subset of the Polyvore dataset created by removing outfits that have common items between training, validation and testing sets. The dataset is challenging compared to Polyvore dataset and consists of 32K outfits. During inference, we use only product images patches and do not use the metadata associated with the products.

Fashion IQ [142] The Fashion IQ dataset is a collection of images and associated metadata designed to facilitate research on natural language-based interactive image retrieval in the fashion domain. It is the fashion dataset that includes human-written relative captions that have been annotated for similar pairs of images, as well as real-world product descriptions and attribute labels as side information. The dataset was created to help researchers develop new methods for retrieving fashion images using natural language queries.

G. Open questions

Learning guarantees for set-to-set matching under the distribution shift As introduced in Appendix D, there are

a number of studies on generalized error analysis under distribution shifts for ordinal classification and regression problems. However, theoretical analysis of set matching under distributional shifts remains unexplored. Moreover, even under the i.i.d. assumption, there is little research on theoretical analysis of set matching [64].

Correlation of performance on SHIFT15M dataset with performance on other different datasets As introduced in Section 4, there are many datasets for classification and regression under distribution shifts. Since SHIFT15M provides data loaders for ordinary classification and regression as well as set matching, it is useful to examine the correlation between performance on these tasks and performance on other data sets.



Figure 15: Additional sample items from 2014.



Figure 16: Additional sample items from 2015.



Figure 17: Additional sample items from 2016.



Figure 18: Additional sample items from 2017.



Figure 19: t-SNE visualization for the SHIFT15M (2015).

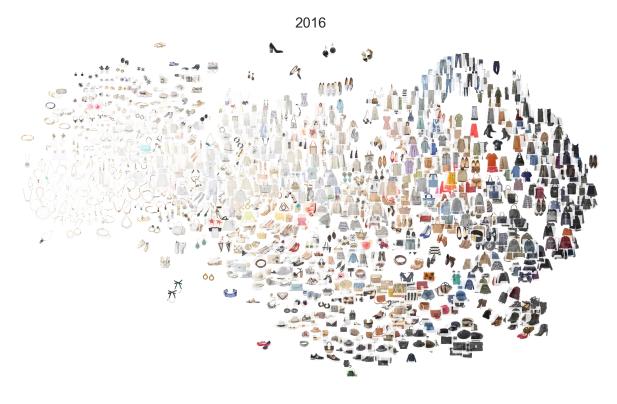


Figure 20: t-SNE visualization for the SHIFT15M (2016).

2015



Figure 21: t-SNE visualization for the SHIFT15M (2017).

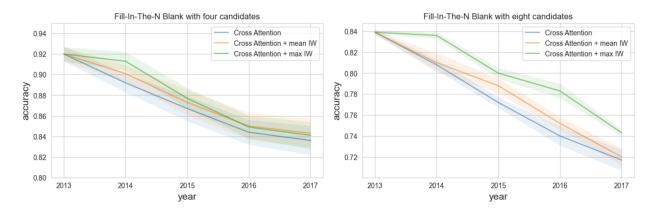


Figure 22: Plots for Fill-In-The-N-Blank experiments with Cross Attention.

2017

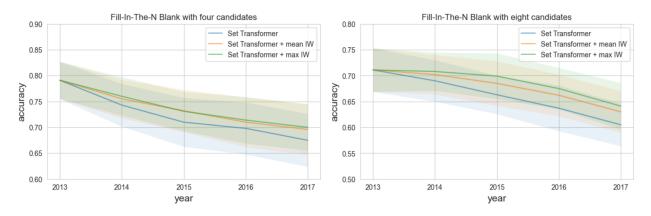


Figure 23: Plots for Fill-In-The-N-Blank experiments with Set Transformer.

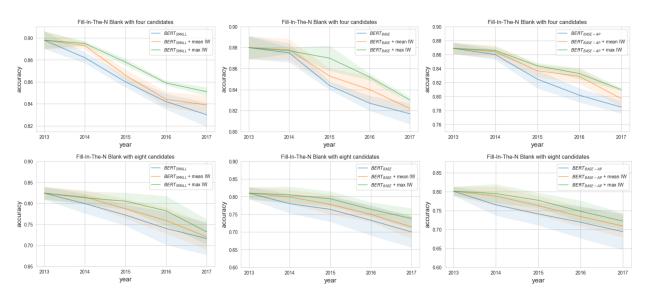


Figure 24: Plots for Fill-In-The-N-Blank experiments with BERT.

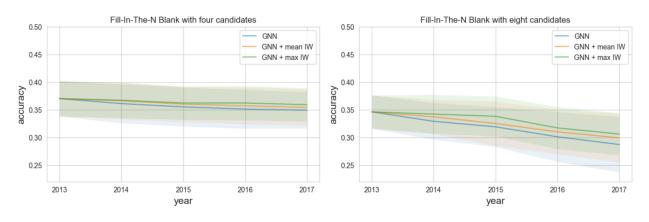


Figure 25: Plots for Fill-In-The-N-Blank experiments with GNN.

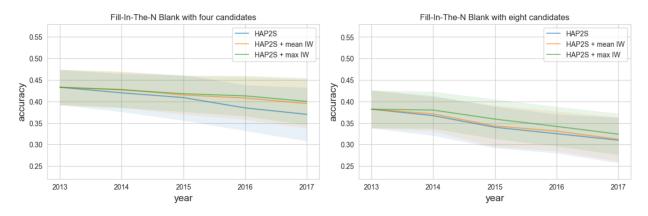


Figure 26: Plots for Fill-In-The-N-Blank experiments with HAP2S.

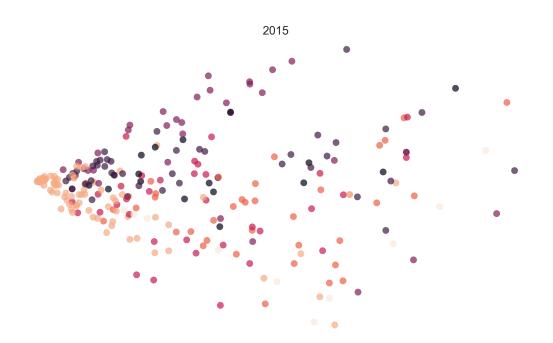


Figure 27: PCA for the SHIFT15M (2015). The color of each point corresponds to the item category.

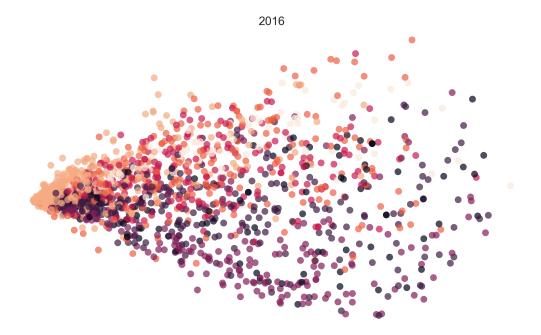


Figure 28: PCA for the SHIFT15M (2016). The color of each point corresponds to the item category.

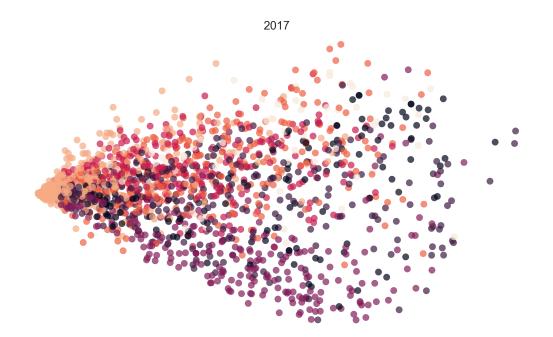


Figure 29: PCA for the SHIFT15M (2017). The color of each point corresponds to the item category.

H. Datasheet for SHIFT15M

In accordance with [41], we provide the datasheet for SHIFT15M. This datasheet offers an overview of crucial information about the dataset that can aid users in making informed decisions regarding its use. It encompasses a comprehensive range of details, including but not limited to the dataset's creation purpose, sources of data, and data processing methods. Overall, the datasheet provides a vital resource for those interested in utilizing the SHIFT15M dataset, as it offers a transparent and comprehensive account of the dataset's characteristics and construction.

Motivation The purpose of this section is to emphasize the importance of transparency and clarity in the process of dataset creation, particularly with regards to the motivations behind the creation of the dataset and any potential conflicts of interest that may arise. Dataset creators are encouraged to clearly articulate their reasons for creating the dataset, including the research questions or goals that the dataset is intended to address.

Composition Dataset creators are advised to review these questions thoroughly prior to commencing any data collection, and provide responses after data collection has concluded. The primary purpose of the questions in this section is to equip dataset consumers with the information necessary to make informed decisions about utilizing the dataset for their specific needs. Additionally, some of the questions have been tailored to obtain information about adherence to the General Data Protection Regulation (GDPR) of the European Union or similar regulatory frameworks in other jurisdictions. Notably, questions specific to datasets involving individuals have been consolidated towards the end of the section. It is recommended a broad interpretation of what constitutes a dataset relating to people. For instance, any dataset containing text that was produced by individuals can be considered to relate to people.

Collection Process To ensure potential issues are identified, dataset creators are advised to review the questions in this section before initiating data collection and then furnish responses upon completion of collection, similar to the previous section. Apart from the objectives set out in the preceding section, the questions in this section are aimed at extracting information that could assist researchers and practitioners in producing alternative datasets with comparable attributes. Similarly, inquiries exclusive to datasets pertaining to individuals are categorized towards the end of this section.

Preprocessing/cleaning/labeling Before proceeding with any preprocessing, cleaning, or labeling, it is recommended

that dataset creators review the questions presented in this section and subsequently provide responses upon completing these tasks. The purpose of the questions in this section is to equip dataset consumers with the requisite information to evaluate whether the "raw" data has been processed in a manner that aligns with their intended use.

Uses The questions in this section serve to prompt dataset creators to consider the appropriate and inappropriate uses of their dataset. By explicitly specifying such tasks, dataset creators can assist dataset consumers in making informed decisions, minimizing potential hazards or adverse outcomes.

Distribution Dataset creators should provide answers to these questions prior to distributing the dataset either internally within the entity on behalf of which the dataset was created or externally to third parties.

Maintenance As with the questions in the previous section, dataset creators should provide answers to these questions prior to distributing the dataset. The questions in this section are intended to encourage dataset creators to plan for dataset maintenance and communicate this plan to dataset consumers.

Motivation

For what purpose was the dataset created? Was there a specific task in mind? Was there a specific gap that needed to be filled? Please provide a description.

This paper addresses the problem of set-to-set matching, which involves matching two different sets of items based on some criteria, especially in the case of high-dimensional items like images. Although neural networks have been applied to solve this problem, most machine learning-based approaches assume that the training and test data follow the same distribution, which is not always true in real-world scenarios. To address this limitation, we introduce SHIFT15M, a dataset that can be used to evaluate set-to-set matching models when the distribution of data changes between training and testing. We conduct benchmark experiments that demonstrate the performance drop of naive methods due to distribution shift. Additionally, we provide software to handle the SHIFT15M dataset in a simple manner, with the URL for the software to be made available after publication of this manuscript. We believe proposed SHIFT15M dataset provide a valuable resource for evaluating set-to-set matching models under the distribution shift.

Who created this dataset (e.g., which team, research group) and on behalf of which entity (e.g., company, institution, organization)? Anonymized until after the paper is accepted.

Who funded the creation of the dataset? If there is an associated grant, please provide the name of the grantor and the grant name and number. Not applicable.

Any other comments?

Composition

What do the instances that comprise the dataset represent (e.g., documents, photos, people, countries)? Are there multiple types of instances (e.g., movies, users, and ratings; people and interactions between them; nodes and edges)? Please provide a description.

The SHIFT15M dataset is an extensive collection of outfits that were previously shared on a fashion-oriented social networking service. The service is no longer available, but the dataset continues to be a valuable resource for researchers studying set-to-set matching problems, particularly in the context of fashion. The dataset contains a vast array of information about each outfit, including details about the user who posted it and some meta-information. Each record in the dataset comprises five different fields, making it easy to organize and analyze the data.

- set_id: An ID that identifies the outfit that was posted.
- items: Provides information about the items that comprise the posted outfit and consists of 4 subfields.
 - item_id: An ID that identifies an item.
 - category_id1: An ID indicating the item category (e.g., outerwear, tops, ...).
 - category_id2: An ID indicating the item subcategory (e.g., T-shirts, blouses, ...).
 - price: Price of the item. user: Provides information about the user who posted the outfit and consists of 2 subfields. An ID that identifies the user who posted the outfit. A list of brands that users have voted for as their favorites. The number is an ID that identifies the brand.

- like_num: the number of times this outfit has been favorited by other users.
- publish_date: The date the outfit was posted.

How many instances are there in total (of each type, if appropriate)?

The dataset consists of 15,218,721 item images and 2,555,147 outfits which created by users of our fashion SNS.

Does the dataset contain all possible instances or is it a sample (not necessarily random) of instances from a larger set? If the dataset is a sample, then what is the larger set? Is the sample representative of the larger set (e.g., geographic coverage)? If so, please describe how this representativeness was validated/verified. If it is not representative of the larger set, please describe why not (e.g., to cover a more diverse range of instances, because instances were withheld or unavailable).

The fashion-oriented social networking service from which we collected outfits for the SHIFT15M dataset was a rich source of data that allowed us to obtain insights into the fashion trends and preferences of millions of users. With approximately 2 million users, the website was a bustling hub of activity where fashion enthusiasts could share their outfit ideas, provide feedback to others, and explore the latest styles and trends.

The vast majority of users on the website were women in their 20s and 30s, representing a demographic that is known for their fashion-forward mindset and interest in new trends. This demographic was particularly valuable for our research, as it allowed us to obtain a large amount of data on outfits that were representative of current fashion trends.

Our collection period spanned over a decade, starting on January 1st, 2010 and ending on April 6th, 2020. During this time, we meticulously gathered outfits consisting of multiple items, each of which was carefully categorized into a specific category. Our focus was on outfits that contained four or more items from the main categories, including outerwear, tops, bottoms, shoes, bags, hats, and accessories. This selection criterion was chosen with the aim of creating a dataset that would be useful for set-to-set matching tasks involving fashion items.

What data does each instance consist of? "Raw" data (e.g., unprocessed text or images) or features? In either case, please provide a description.

Each item consists of 4096-dimensional features extracted via the VGG16 model trained using the ILSVRC2012 dataset.

Is there a label or target associated with each instance? If so, please provide a description.

Indeed, each instance in the SHIFT15M dataset contains a wealth of information that can be leveraged for various tasks. Along with the outfit items, each instance also includes several numerical values such as the category ID and number of likes. These values provide additional context that can be used to train models for various set-to-set matching problems.

One of the strengths of the SHIFT15M dataset is its versatility. By choosing one of these numerical values as the target variable, researchers can easily switch between several tasks, each with its own unique set of challenges and opportunities. For example, if the focus is on predicting outfit popularity, the number of likes can be used as the target variable. On the other hand, if the goal is to perform set-to-set matching between outfits, the number of likes can be used as the target variable.

Is any information missing from individual instances? If so, please provide a description, explaining why this information is missing (e.g., because

it was unavailable). This does not include intentionally removed information, but might include, e.g., redacted text.

It is important to note that the SHIFT15M dataset only includes items that belong to the main categories, such as outerwear, tops, bottoms, shoes, bags, hats, and accessories. This means that items outside of these categories, such as underwear or background images for collage, are missing from the dataset.

While this selection criterion may seem limiting, it was chosen to ensure that the dataset is focused on items that are most commonly used in fashionrelated set-to-set matching tasks. By excluding items outside of the main categories, we were able to curate a dataset that is more manageable and less noisy, while still providing a diverse range of fashion items for analysis.

Are relationships between individual instances made explicit (e.g., users' movie ratings, social network links)? If so, please describe how these relationships are made explicit.

Each instance is assigned the ID of the user who submitted the outfit.

Are there recommended data splits (e.g., training, development/validation, testing)? If so, please provide a description of these splits, explaining the rationale behind them.

SHIFT15M is a valuable dataset that simulates various types of dataset shifts that are commonly observed in real-world applications. The collected data spans a decade, from 2010 to 2020, and encompasses various shifts that arise due to factors such as changes in user behavior, fashion trends, and cultural preferences. To make it easy for researchers to evaluate their models on the SHIFT15M dataset, we have developed software that allows them to experiment with different types and sizes of shifts. The software automates the train/val/test splitting process, making it easier for researchers to evaluate the performance of their models under various shift scenarios. With this software, researchers can simulate shifts that arise due to various factors and assess their models' robustness to such shifts. By doing so, they can gain insights into how their models perform in real-world settings where data distributions are constantly changing.

Are there any errors, sources of noise, or redundancies in the dataset? If so, please provide a description. No.

Is the dataset self-contained, or does it link to or otherwise rely on external resources (e.g., websites, tweets, other datasets)? If it links to or relies on external resources, a) are there guarantees that they will exist, and remain constant, over time; b) are there official archival versions of the complete dataset (i.e., including the external resources as they existed at the time the dataset was created); c) are there any restrictions (e.g., licenses, fees) associated with any of the external resources that might apply to a future user? Please provide descriptions of all external resources and any restrictions associated with them, as well as links or other access points, as appropriate. The dataset is self-contained.

Does the dataset contain data that might be considered confidential (e.g., data that is protected by legal privilege or by doctor-patient confidentiality, data that includes the content of individuals non-public communications)? If so, please provide a description. Does the dataset contain data that, if viewed directly, might be offensive, insulting, threatening, or might otherwise cause anxiety? If so, please describe why.

No.

Does the dataset relate to people? If not, you may skip the remaining questions in this section.

Yes. Each instance is a combination of outfits created by an individual and preferred by that individual.

Does the dataset identify any subpopulations (e.g., by age, gender)? If so, please describe how these subpopulations are identified and provide a description of their respective distributions within the dataset. No.

Is it possible to identify individuals (i.e., one or more natural persons), either directly or indirectly (i.e., in combination with other data) from the dataset? If so, please describe how.

It is impossible to identify individuals from the dataset.

Does the dataset contain data that might be considered sensitive in any way (e.g., data that reveals racial or ethnic origins, sexual orientations, religious beliefs, political opinions or union memberships, or locations; financial or health data; biometric or genetic data; forms of government identification, such as social security numbers; criminal history)? If so, please provide a description.

No.

Any other comments?

Collection Process

How was the data associated with each instance acquired? Was the data directly observable (e.g., raw text, movie ratings), reported by subjects (e.g., survey responses), or indirectly inferred/derived from other data (e.g., part-of-speech tags, model-based guesses for age or language)? If data was reported by subjects or indirectly inferred/derived from other data, was the data validated/verified? If so, please describe how.

Except for the item attributes, the data was generated by users. Item attributes (category and price) were collected from e-commerce sites that sell the item. All data was viewable on the website.

What mechanisms or procedures were used to collect the data (e.g., hardware apparatus or sensor, manual human curation, software program, software API)? How were these mechanisms or procedures validated?

The fashion-oriented social networking service that we collected the SHIFT15M dataset from provided its users with an easy-to-use outfit editor that allowed them to create and publish their outfits on the platform. This editor featured a wide range of clothing items, accessories, and other fashion-related items that users could choose from to create their outfits. Once a user had selected the items they wanted to include in their outfit, the editor registered this selection as a new outfit on the platform.

To ensure that this outfit creation function was tested appropriately, we followed general software development procedures. This involved conducting thorough testing to ensure that the editor functioned as intended, with no bugs or glitches that could affect the accuracy or reliability of the data collected. By following this rigorous testing process, we were able to gather a high-quality dataset that accurately reflects the fashion choices made by users on the social networking service during the ten-year collection period.

If the dataset is a sample from a larger set, what was the sampling strategy (e.g., deterministic, probabilistic with specific sampling probabilities)?

We collected a complete dataset without sampling to create our dataset, except for data deleted by the user.

Who was involved in the data collection process (e.g., students, crowdworkers, contractors) and how were they compensated (e.g., how much were crowdworkers paid)?

Anonymized until after the paper is accepted.

Over what timeframe was the data collected? Does this timeframe match the creation timeframe of the data associated with the instances (e.g., recent crawl of old news articles)? If not, please describe the time-frame in which the data associated with the instances was created.

The dataset was collected in the period of 2010 2020. Each outfit includes a timestamp that describes when the outfit created.

Were any ethical review processes conducted (e.g., by an institutional review board)? If so, please provide a description of these review processes, including the outcomes, as well as a link or other access point to any supporting documentation.

No.

Does the dataset relate to people? If not, you may skip the remaining questions in this section.

Yes. Each instance is a combination of outfits created by an individual and preferred by that individual.

Did you collect the data from the individuals in question directly, or obtain it via third parties or other sources (e.g., websites)?

Collected directly through the website.

Were the individuals in question notified about the data collection? If so, please describe (or show with screenshots or other information) how notice was provided, and provide a link or other access point to, or otherwise reproduce, the exact language of the notification itself. Notified in the Terms of Service.

Noulled in the Terms of Service.

Did the individuals in question consent to the collection and use of their data? If so, please describe (or show with screenshots or other information) how consent was requested and provided, and provide a link or other access point to, or otherwise reproduce, the exact language to which the individuals consented.

The use of the service was deemed as consent.

If consent was obtained, were the consenting individuals provided with a mechanism to revoke their consent in the future or for certain uses? If so, please provide a description, as well as a link or other access point to the mechanism (if appropriate).

It is possible to contact the company that provided the service.

Has an analysis of the potential impact of the dataset and its use on data subjects (e.g., a data protection impact analysis) been conducted? If so,

please provide a description of this analysis, including the outcomes, as well as a link or other access point to any supporting documentation. No, there had been no potential impact analysis conducted.

Any other comments?

Preprocessing/cleaning/labeling

Was any preprocessing/cleaning/labeling of the data done (e.g., discretization or bucketing, tokenization, part-of-speech tagging, SIFT feature extraction, removal of instances, processing of missing values)? If so, please provide a description. If not, you may skip the remainder of the questions in this section.

We extracted the CNN features from images and treated them as input data in our image-based tasks. As a result, our dataset contains the features but does not include raw photos, making them anonymized. The CNN we used is an official pre-trained VGG16, and we adopted the outputs of the 'fc6' layer before applying ReLU as the feature. We exclude the outfits that contain less than four items. Other than that, we did not remove any instances in creating our dataset. However, we excluded some data in each independent task. In detail, please refer to each task description.

Was the "raw" data saved in addition to the preprocessed/cleaned/labeled data (e.g., to support unanticipated future uses)? If so, please provide a link or other access point to the "raw" data. No.

Is the software used to preprocess/clean/label the instances available? If so, please provide a link or other access point.

All software are provided on the SHIFT15M repository.

Any other comments?

Uses

Has the dataset been used for any tasks already? If so, please provide a description.

Benchmarks using this dataset and the specified evaluation protocol are listed in GitHub page.

Is there a repository that links to any or all papers or systems that use the dataset? If so, please provide a link or other access point.

All benchmarks that use this dataset will be available at GitHub page.

What (other) tasks could the dataset be used for?

Here, we list candidate tasks for which SHIFT15M can be applied as follows:

- set-to-set matching;
- regression (e.g., number of likes or sum of prices);
- classification (e.g., category ids or publish years).

Is there anything about the composition of the dataset or the way it was collected and preprocessed/cleaned/labeled that might impact future uses? For example, is there anything that a future user might need to know to avoid uses that could result in unfair treatment of individuals or groups (e.g., stereotyping, quality of service issues) or other undesirable harms (e.g., financial harms, legal risks) If so, please provide a description. Is there anything a future user could do to mitigate these undesirable harms? No.

Are there tasks for which the dataset should not be used? If so, please provide a description.

This dataset is distributed in a way that excluding raw images and anonymizing the users/brands. Therefore, it requires the dataset users not to reconstruct raw images from the image features or restore the anonymized parts in a future task.

Any other comments?

Distribution

Will the dataset be distributed to third parties outside of the entity (e.g., company, institution, organization) on behalf of which the dataset was created? If so, please provide a description.

Yes. The dataset will be distributed to third parties based on the licence.

How will the dataset will be distributed (e.g., tarball on website, API, GitHub) Does the dataset have a digital object identifier (DOI)?

The dataset will be distributed via a website or the links indicated in our Github repository. We will add DOI for the SHIFT15M dataset.

When will the dataset be distributed?

The dataset will be first released in August 2021.

Will the dataset be distributed under a copyright or other intellectual property (IP) license, and/or under applicable terms of use (ToU)? If so, please describe this license and/or ToU, and provide a link or other access point to, or otherwise reproduce, any relevant licensing terms or ToU, as well as any fees associated with these restrictions.

The SHIFT15M dataset will be made available for distribution under the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0) license. This means that users are free to share and adapt the dataset, as long as they provide attribution and do not use it for commercial purposes. For more information about the license, please refer to the following link: https://creativecommons.org/licenses/ by-nc/4.0/. This license ensures that the dataset can be used by the academic community for research purposes, and that any derivative works or publications based on the dataset will be properly attributed.

Have any third parties imposed IP-based or other restrictions on the data associated with the instances? If so, please describe these restrictions, and provide a link or other access point to, or otherwise reproduce, any relevant licensing terms, as well as any fees associated with these restrictions.

There are no fees or restrictions.

Do any export controls or other regulatory restrictions apply to the dataset or to individual instances? If so, please describe these restrictions, and provide a link or other access point to, or otherwise reproduce, any supporting documentation.

Unknown.

Any other comments?

Maintenance

Who will be supporting/hosting/maintaining the dataset?

Anonymized until after the paper is accepted.

How can the owner/curator/manager of the dataset be contacted (e.g., email address)?

All changes to the dataset will be announced through the GitHub Releases.

Is there an erratum? If so, please provide a link or other access point.

To ensure the accuracy and transparency of the SHIFT15M dataset, any changes made to the dataset will be immediately announced through the GitHub Releases page. This will include updates to the dataset's documentation, modifications to the dataset's format or metadata, or any other changes that may impact the dataset's use. Additionally, any errors or issues found in the dataset will be listed in the "Errata" section of the SHIFT15M repository. This will allow users to stay informed of any updates or issues related to the dataset and ensure that they are working with the most accurate and up-to-date version of the data.

Will the dataset be updated (e.g., to correct labeling errors, add new instances, delete instances)? If so, please describe how often, by whom, and how updates will be communicated to users (e.g., mailing list, GitHub)? All changes to the dataset will be announced through the GitHub Releases.

If the dataset relates to people, are there applicable limits on the retention of the data associated with the instances (e.g., were individuals in question told that their data would be retained for a fixed period of time and then deleted)? If so, please describe these limits and explain how they will be enforced.

No.

Will older versions of the dataset continue to be supported/hosted/maintained? If so, please describe how. If not, please describe how its obsolescence will be communicated to users.

They will continue to be supported with all information on SHIFT15M repository. We also provide the contribution guides for software that supports the dataset.

If others want to extend/augment/build on/contribute to the dataset, is there a mechanism for them to do so? If so, please provide a description. Will these contributions be validated/verified? If so, please describe how. If not, why not? Is there a process for communicating/distributing these contributions to other users? If so, please provide a description.

Others may do so and should contact the original authors about incorporating fixes/extensions.

Any other comments?