Data Driven Condition Monitoring of Wind Power Plants Using Cluster Analysis

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***Abstract*—Along with the rapid growth of the wind energy sector, reducing the wind energy costs caused by unplanned downtimes and maintenances has aroused great concern of researchers. Condition monitoring system (CMS) is widely used for detecting anomalies of wind power plants (WPPs) so as to reduce the downtimes and optimize the maintenance plan. However, current solutions to condition monitoring of WPPs focus mostly on detecting a particular anomaly on a single component or a subsystem. Optimizing the maintenance plan of whole wind power plant requires a solution to system level condition monitoring of WPPs.**

This paper gives a procedure for system level condition monitoring of WPPs using data driven method, that provides an overall picture of the system statuses. Firstly, cluster analysis is utilized to automatically learn the normal behavior model of WPPs from the observations. Two clustering algorithms are explored to choose a suitable one for modeling the WPPs. The presented anomaly detection algorithm uses the learned model as reference to detect the system anomalies. The effectiveness of this approach is evaluated with real world data.

I. Introduction

According to a wind market statistic by the GWEC (Global Wind Energy Council) [1], the global wind power capacity grew continuously for the last 17 years. In 2014, the global wind industry had a 44% rise of annual installations and the worldwide total installed capacity accumulated to 369553 megawatt at the end of 2014, 31% of which is installed in China. In Europe, renewable energy from WPPs covers up to 11% of the energy demand [2]. Wind power has become the fastest growing renewable energy source. To enhance its competence against the other energy sources, it is desired to reduce the cost of generating wind power.

WPPs are the most expensive equipment in the wind power industry. Even the inspection and maintenance of WPPs are very costly. Such costs for 750kW turbines might come to about 25%-30% of the wind power generating costs [3]. Furthermore, the availability to generate power is also reduced due to the downtime during the maintenance of WPPs. In a case study, Nilsson [4] denotes an unscheduled downtime with 1000 € per man-hour, with costs of up to 300000 € for replacements.

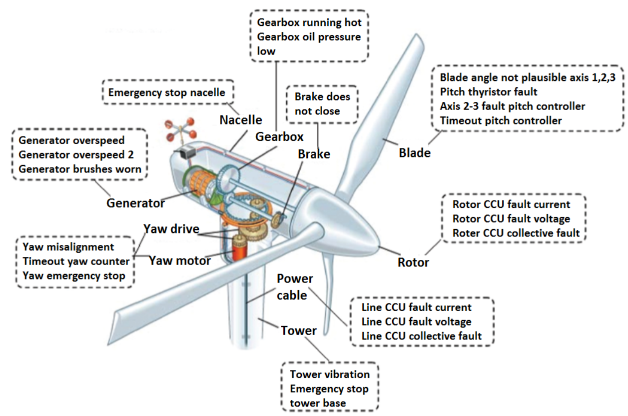

Fig. 1. status description in relation with WPPs components [5]

To avoid these unscheduled downtimes and minimize the inspection and maintenance costs, CMS is widely utilized for continuous monitoring the statuses of critical components in WPPs (Fig. 1). The core task of a CMS is anomaly detection. By means of CMS, the anomalies in WPPs can be detected early and the maintenance can be thus planed more efficiently [4]. Fausto et al. [6] reviewed the classic techniques and methods used for condition monitoring of WPPs, such as vibration analysis, oil analysis, signal processing methods; Through continuous monitoring of WPPs, a huge amount of data are collected which represents the statuses of WPPs against the time and can be analyzed using artificial intelligent (AI) algorithms to detect the system anomalies. A survey of AI based methods for condition monitoring of WPPs, such as Support Vector Machines (SVM), k-nearest neighbor (k-NN), are presented in [7]. Because of the complexity of WPPs, the analysis of the relationships between the anomalies and the observed data in system level is extremely difficult [8]. Either the classic methods in [6] or the AI methods in [7] just focus on detecting a particular anomaly on a single component or a subsystem (component level) in WPPs and rely on specific type of sensor data. The CMSs based on such methods have very limited impact on the efficient planning of

WPPs maintenance due to the lack of an overall picture of the whole system.

Therefore, system-level anomaly detection in WPPs is highly desired. For this purpose, the following challenges are tracked in this work:

- How can the status of WPPs be modeled in system level?
- How can a new status be detected as a system anomaly without assumptions on available types of sensors?

To address these challenges, a data driven approach for system level condition monitoring of WPPs is presented in this paper. Using cluster analysis, the system normal behaviors can be learned automatically from system observations without understanding of the complex relations between anomalies and data. This learned model shows the conditions of the whole WPPs. Additionally, this approach does not depend on any particular structure of WPPs. Thus, it is universally applicable for condition monitoring of WPPs. Cluster analysis do not heavily rely on a-priori knowledge about the system to be modeled. Hence, the influence of the harsh environment can also be modeled through analysis of the environment data in relation with the WPPs data. This learned model can be used as reference model for further anomaly detection task.

The paper is organized as follows. First, the state of the art for data driven condition monitoring of WPPs is resumed in Section II; Then, the proposed solution addressing the forementioned challenges is presented in Section III. Furthermore, experimental results for the proposed approach using real world data are detailed in Section IV. Finally, a conclusion of this paper can be found in Section V.

II. Related Work

According to the solution idea of this work, the studies on condition monitoring of WPPs using cluster analysis are investigated. Two well-known clustering algorithms, DBSCAN and spectral clustering, are utilized to handle the complex correlations in the WPPs data.

As stated in [9], the models used for anomaly detection of complex systems should be learned automatically and data driven approaches to learning such models should be moved into the research focus. A wide range of data-driven algorithms that deal with modeling the system behavior for anomaly detection and diagnosis are available in the literature. Dai and Gao [10] have reviewed various data-driven algorithms in perspective of fault detection and diagnosis.

As one of the classic density based clustering method, DBSCAN shows its advantages over the statistical method on anomaly detection in temperature data [11]. DBSCAN is resistant to noise and can recognize patterns of arbitrary shapes. Thang and Kim developed a new way to use DBSCAN with different parameters for different clusters in the field anomaly detection of network traffic [12].

Compared to the traditional approaches to clustering (e.g. k-means, single linkage), spectral clustering is very simple to implement and can be solved efficiently by standard linear algebra methods [13]. Piero and Enrico proposed a spectral clustering based method for fault diagnosis where fuzzy logic is used to measure the similarity and the fuzzy C-Means is used for clustering the data [14]. Siddharth et al. [15] presented an application of spectral clustering in network intrusion detection.

Due to the high complexity of WPPs and its harsh working environment, the modeling of WPPs on system level is very challenging. Most data-driven solutions to WPPs condition monitoring concentrate on the errors of one particular component (in component level) [16]. These methods are designed to detect specific faults (e.g. fault in gearbox, generator).

The application of such methods is available in different studies. In [17], a shock pulse method is adapted for bearing monitoring. A multi-agent system is developed in [18] for condition monitoring of the wind turbine gearbox and oil temperature. In [19], the ultrasonic and radiographic techniques are used for non-destructive testing of the WPPs blades. Using these methods can prevent the WPPs breakdowns caused by the particular faults. For enhancing the availability and the reliability of the whole WPPs, a method for monitoring the WPPs on system-level is required.

In this work, DBSCAN and spectral clustering are used for condition monitoring of WPPs. This approach is aimed to model WPPs on system-level in order to perform automatic anomaly detection. To the best of our knowledge, no application of either DBSCAN or spectral clustering in condition monitoring of WPPs exists.

III. Solution

Since it is unrealistic to get a large number of fault cases with corresponding annotation in WPPs, the main idea of the presented solution is to automatically learn a model of normal system behaviors from the observations using cluster analysis. Clustering is primarily an unsupervised machine learning method. In our case, clustering is performed on the system normal behavior data. Thus, it is used in semi-supervised manner.

At first, the observed data set of the system normal behaviors is preprocessed using Principal Component Analysis (PCA) [20]. After that, clustering algorithms are applied on these preprocessed data set to generate a model in system level. In the final step, the learned system model is used for automatic anomaly detection.

A. Step 1: Data Preprocessing

WPPs include many components and sensors. Therefore, the observed data of WPPs are high-dimensional. The analysis of these high-dimensional data sets is time consuming. Besides, DBSCAN is not suitable to cluster high dimensional data because density is more difficult to define in high dimensional space. Therefore, a method to reduce dimensionality should be applied to the data before cluster analysis. To enhance the performance of the analysis, PCA is utilized to reduce the dimensionality. PCA is based on the assumption, that most information about the data to be analyzed is located in a linear lower dimensional subspace of the original data. Therefore, PCA can reduce the dimensionality of data with limited losses of information.

B. Step 2: Clustering based modeling

The goal of cluster analysis is to partition data points into different groups. Similarity of points is defined by a minimal intra-cluster distance, whereas different clusters aim for a maximum inter-cluster distance. Thus, cluster analysis can be utilized to find the pattern of a system direct using the multi-dimensional data without explicit descriptions about the system features. Thus, it can be performed on unlabeled data set for modeling complex systems with seasonal components, e.g. WPPs.

In the presented solution, a system model for anomaly detection should characterize the normal system behaviors and can be used to identify unusual behaviors. For most complex systems, the normal behaviors might consist of multiple modes that depend on different factors, e.g. work environments, operations of the systems. When the cluster analysis is performed on a data set representing the normal behaviors of a system, multiple clusters can be recognized. Each cluster represents a particular status of the system. Then such multiple clusters can be used as the normal behavior model of a system for anomaly detection.

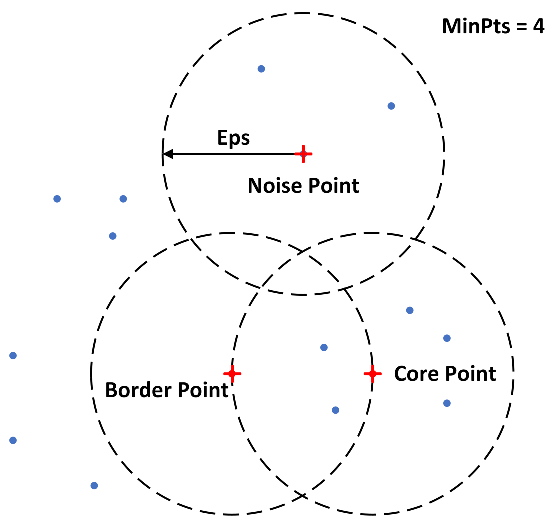

Fig. 2. Core, border and noise point in DBSCAN [21]

In DBSCAN, the density for a particular point is defined as the number of neighbor points within a specified radius of that point [22]. Two user-defined parameters are required: Eps - the radius; $MinPts$ - the minimal number of neighbors in the Eps. DBSCAN uses such center-based density to classify the data points as (see Figure 2):

- **core point**: number of neighbors in $Eps \geq MinPts$;
- **border point**: it is not a core point, but is the neighbor of minimal one core point;
- **noise point**: neither a core nor a border point.

Two core points that are within Eps of each other are defined as density-reachable core points. DBSCAN partitions the data into clusters by iteratively labeling the data points and collecting density-reachable core points into the same cluster. As result, DBSCAN delivers several clusters in which noise points are also collected in one cluster.

The idea of spectral clustering is to represent the data in form of a similarity graph $G(V, E)$ where each vertex $v_i \in V$ presents a data point in the dataset (see Figure 3). Each edge $e_{ij} \in E$ between two vertices v_i and v_j carries a non-negative weight w_{ij} (similarity between the two points). According to w_{ij}, the connectedness between two vertices in $G(V, E)$ can be defined. Then, the clustering problem can be handled as graph partition [23] using spectral graph theory [24]. G is divided into smaller components, such that the vertices within the small components have high connection and there are few connections between the small components. These small components correspond to the clusters in the results of spectral clustering and can be used as normal behavior model for anomaly detection.

Compared to the traditional approaches to clustering (e.g. k-means, DBSCAN), spectral clustering use the connectivity in similarity graph instead of geometrical proximity in the original data space to partition data points. Therefore, it does not make strong assumptions on the statistics of data set. Although the data points are not geometrically separated with each other very well, spectral clustering can still deliver better results. Moreover, it can be solved efficiently by standard linear algebra methods [13]. Another advantage of spectral clustering is the ability to handle the high dimensional data using spectral analysis [25].

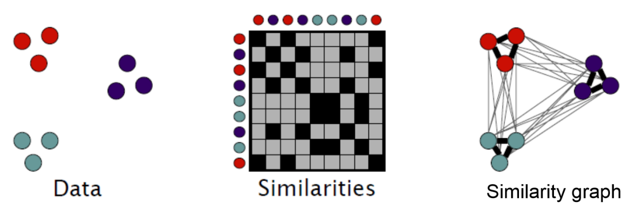

Fig. 3. Similarity graph construction [26]

C. Step 3: Anomaly Detection

Within the presented solution, anomaly detection is based on the assumption denoted by Chandola et al. [27]:

"normal data instances lie close to their closest cluster centroid, while anomalies are far away from their closest cluster centroid."

Accordingly, a metric is needed to calculate the deviation of an actual observation from the learned model. Different distance metrics, such as Euclidean distance [28], Mahalanobis distance [29] or Manhattan distance [30], are available. Mahalanobis distance is selected here for the calculation of deviation. Compare with Euclidean distance which assumes that all the features of the data are isotropic, Mahalanobis distance takes the correlations between features into consideration by using covariance matrix and mean of data set. Thus, it is a very useful approach to distance measurement of multi-dimensional data when attributes are correlated with each other.

DBSCAN generated clusters provide a discrimination of core and border data points. Only core points are used to measure the distance between an observation and the core points. Spectral clustering computes clusters in a dimensionally reduced space but gives no further information about core or border points. Measuring the distance between such clusters is achieved with the cluster centers in the presented solution.

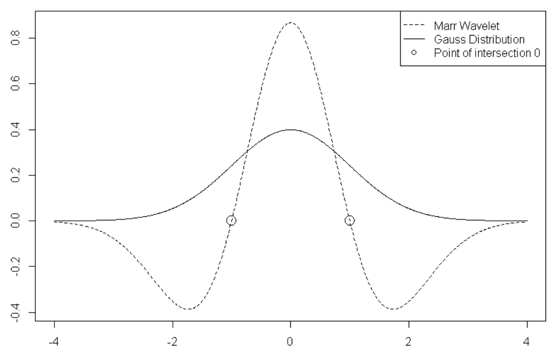

Fig. 4. Characteristics of Gaussian distribution in comparison to Marr Wavelet (dashed). Spots are marked where the Marr Wavelet reach zero

Algorithm 1 Anomaly Detection

Input: **Centers** ▷ centers of the clusters
Input: **O** ▷ input observation
Output: **Boolean** ▷ anomaly or not

procedure ANOMALY_DETECTION(**Centers**, **O**)
  $\mathbf{O_{PCA}}$ = mapToPCASpace(**O**)
  $\psi = \max\{marr(distance(\mathbf{O_{PCA}}, \mathbf{Centers}))\}$
  if ($\psi < 0$) **then**
    anomaly: **TRUE**
  else
    anomaly: **FALSE**
  end if
  return (**anomaly**)
end procedure

Absolute distance measuring is missing a threshold to decide whether an observation is an anomaly or not. Even though utilizing a Gaussian density function to provide an indicator for classification, a threshold is still needed. In this project, a Marr wavelet function is used to decide whether a new observation is close enough from the clusters or not. Instead of a Gaussian distribution, the characteristic form of a Marr wavelet [31] allows a classification where the threshold can be set to zero, see Figure 4.

The distance measuring used in this work is computed as follows.

Let $X_{cluster} = [x_1, \cdots, x_l]$ be the center (a vector with l dimensions) of a cluster and $O_{new} = [o_1, \cdots, o_l]$ a new observation with same dimension as $X_{cluster}$. Then the distribution function to measure if a new observation is part of the cluster or not is formed as:

$$\psi(X_{cluster}, O_{new}) = \frac{2}{\sqrt{3\sigma}\pi^{\frac{1}{4}}} \cdot (1 - \frac{k^2}{\sigma^2}) \cdot \exp(-\frac{k^2}{2\sigma^2})$$

Where k is the Mahalanobis distance between the new observation and the center of a cluster, which is defined as:

$$k = \sqrt{(O_{new} - \mu)^T S^{-1} (O_{new} - \mu)}$$

μ is the mean of $X_{cluster}$ and S is the covariance matrix of $X_{cluster}$.

Algorithm 1 illustrates the presented anomaly detection approach. At first, a new observation (O) is mapped to the same space as the learned model using the transformation matrix from the PCA (see III-A). Then the mapped observation (O_{PCA}) is compared with the centers of the clusters. The value of ψ is used as indicator for anomaly detection. In case that

$$\psi < 0,$$

it is an anomaly. Otherwise, it is a normal status.

IV. Results

The data used in the evaluation is collected over a duration of 4 years from 11 real WPPs in Germany with 10 minutes resolution. The dataset consists of 12 variables which describe the work environment (e.g. wind speed, air temperature) and the status of WPPs (e.g. power capacity, rotation speed of generator, voltage of the transformer).

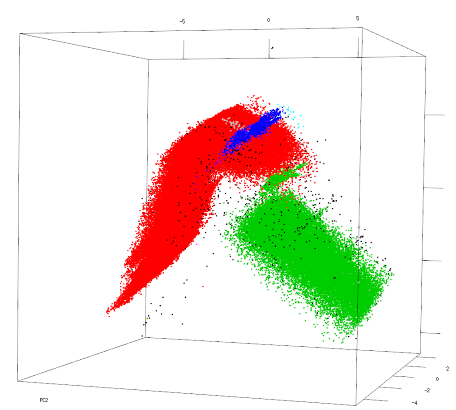

Fig. 5. Results of DBSCAN plotted in the space of raw data

TABLE I
EVALUATION RESULTS OF WIND POWER STATION DATA.

	True Pos.	True Neg.	False Pos.	False Neg.	Bal. Acc.	F1-Measure
DBSCAN	1812	6827	186	2719	68.66%	55.50
Spectral Clustering	3832	6328	685	699	87.40%	84.71

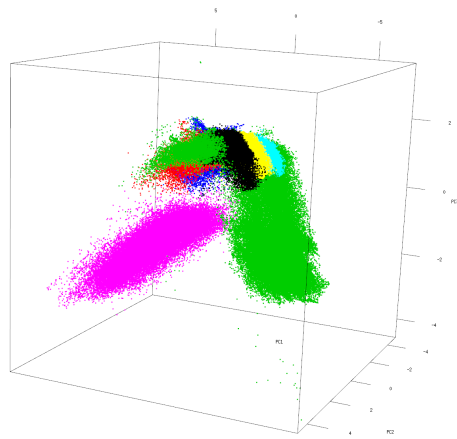

Fig. 6. Results of spectral clustering plotted in the space of raw data

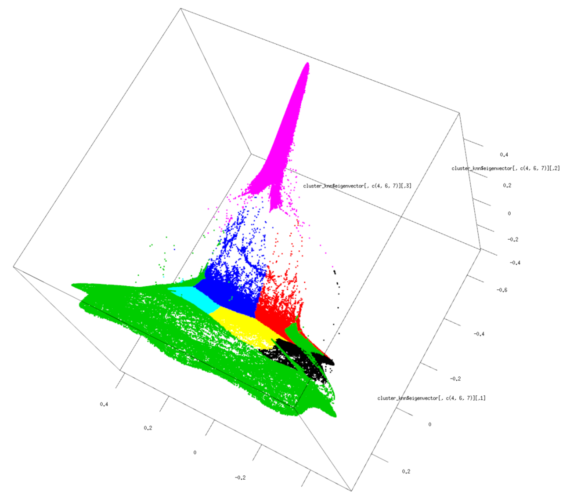

Fig. 7. Results of spectral clustering plotted in the space of eigenvectors

For evaluation, a training data set of 232749 observations of normal behaviors was used to build the normal behavior model of WPPs. The evaluation data set of 11544 observations contains 4531 reported failures and 7013 observations of normal behaviors.

Before clustering, the training data and the evaluation data were projected into a new data space with 6 dimensions using PCA. Then, DBSCAN and spectral clustering were performed on the preprocessed data set separately. 5 clusters were generated by DBSCAN (see Figure 5), while 7 clusters were generated by spectral clustering (Figure 6).

After clustering, the stated method of anomaly detection was evaluated with the evaluation data set. Table I shows the confusion matrix [21] as a result of the evaluation. Here, true negative denotes a correct prediction of normal state and true positive means a correct classification of a failure. F1-Measure accesses the accuracy of classification accounting both precision and recall [32], which concentrates on one class (usually positive) and gives a comprehensive view of the performance on classification of this class. In our case, F1-Measure is appropriate for analyzing the system's performance in anomaly detection.

DBSCAN still has trouble with high-dimensional data, even though the PCA has been already performed to reduce the data dimensionality. As shown in Figure 5, DBSCAN has generated 2 large clusters. The cluster in red color has concave boundary. Using the core points of this cluster to detect the anomaly leads to a very high rate of false negative.

On the contrary, spectral clustering can handle high-dimensional data, although the attributes of the data have complex correlations with each other. The data are clustered in the eigenvector space of the laplacian matrix that represents the connectivity in similarity graph (Figure 7) [13]. The data set with concave boundary has been divided into 6 smaller clusters. The accuracy of spectral clustering based anomaly detection is much better than the DBSCAN based method. On the whole, spectral clustering outperforms DBSCAN by modeling of the WPPs. Therefore, spectral clustering is chosen to generate the normal behavior model of WPPs in our solution.

V. CONCLUSION

In this work, a data driven approach for system-level condition monitoring of WPPs was presented. Two clustering algorithms, DBSCAN and spectral clustering, are explored for system level modeling the normal behaviors of WPPs. An anomaly detection method is developed and evaluated with real WPPs data, which use Mahalanobis distance and Marr wavelet as metric of the similarities between the new observations and the learned normal behavior model. No specific type of sensor data is given in this method. Therefore, this method can be utilized for any type of WPPs. Spectral clustering has shown its advantage over DBSCAN on modeling the WPPs. The anomaly detection based on the model learned with spectral clustering reaches a F1-Measure of 84.71% and a ballanced accuracy of 87.40%.

The clustering based anomaly detection can operate automatically and is easy to be adapted to different complex systems. But the accuracy of anomaly detection using cluster center is highly dependent on the used clustering algorithm. Spectral clustering is appropriate for modeling WPPs.

In the future work, the learned model will be trained and evaluated using data from more WPPs with different working environment to improve the performance of the presented method. Beyond the task of anomaly detection, diagnosis of the root cause of an anomaly is also a sensible functionality of a CM system. The presented solution will be extended by a root cause analysis. Such an extension can support maintenance personal to trace the detected anomaly.

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