

Multiple-Source and Multiple-Destination Charge Migration in Hybrid Electrical Energy Storage Systems*

Yanzhi Wang, Qing Xie, and
Massoud Pedram

University of Southern California
Los Angeles, CA, USA 90089
{yanzhiwa, xqing, pedram}@usc.edu

Younghyun Kim and Naehyuck
Chang

Seoul National University
Seoul, Korea 151-744
{yhkim, naehyuck}@elpl.snu.ac.kr

Massimo Poncino

Politecnico di Torino
Torino, Italy 10129
massimo.poncino@polito.it

Abstract— Hybrid electrical energy storage (HEES) systems consist of multiple banks of heterogeneous electrical energy storage (EES) elements that are connected to each other through the Charge Transfer Interconnect. A HEES system is capable of providing an electrical energy storage means with very high performance by taking advantage of the strengths (while hiding the weaknesses) of individual EES elements used in the system. Charge migration is an operation by which electrical energy is transferred from a group of source EES elements to a group of destination EES elements. It is a necessary process to improve the HEES system’s storage efficiency and its responsiveness to load demand changes. This paper is the first to formally describe a more general charge migration problem, involving multiple sources and multiple destinations. The multiple-source, multiple-destination charge migration optimization problem is formulated as a nonlinear programming (NLP) problem where the goal is to deliver a fixed amount of energy to the destination banks while maximizing the overall charge migration efficiency and not depleting the available energy resource of the source banks by more than a given percentage. The constraints for the optimization problem are the energy conservation relation and charging current constraints to ensure that charge migration will meet a given deadline. The formulation correctly accounts for the efficiency of chargers, the rate capacity effect of batteries, self-discharge currents and internal resistances of EES elements, as well as the terminal voltage variation of EES elements as a function of their state of charges (SoC’s). An efficient algorithm to find a near-optimal migration control policy by effectively solving the above NLP optimization problem as a series of quasi-convex programming problems is presented. Experimental results show significant gain in migration efficiency up to 35%.

Keywords—hybrid electrical energy storage system; charge management; charge migration

I. INTRODUCTION

Electrical energy storage (EES) systems are deployed to increase power availability, reliability and efficiency, mitigate the supply-demand mismatch, and regulate the peak-power demand [1, 2, 3]. Unfortunately, none of the existing types of EES elements can fulfill all the desirable performance characteristics of an ideal electrical storage means e.g., high power and energy densities, low cost/weight per unit capacity, high cycle efficiency, long cycle life, and low environmental effects. A *hybrid* EES (HEES) system, as introduced in [4, 5, 6], is an EES system comprising of two or more heterogeneous EES elements, connected to each other through the Charge Transfer Interconnect (CTI). The HEES system can exploit the strengths of each type of EES element while hiding its weaknesses, so as to achieve a combination of performance metrics

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that is superior to those of any of its individual EES components. For the HEES system to be useful in practice, it is essential to efficiently implement three charge management operations in the HEES system: *charge allocation*, *charge replacement*, and *charge migration* [6].

Along with charge allocation and replacement, charge migration, which moves charge from a group of source EES elements to a group of destination EES elements, is a crucial operation in a HEES system. Charge migration will ensure the availability and responsiveness of some best-suited EES elements, which have desired characteristics with respect to the load demand or source input in terms of self-leakage and (or) output power rating, to service future load demands or to receive energy from power sources [6].

The simple case of single-source and single-destination (SSSD) charge migration operation has been investigated in [6]. However, the more general multiple-source and multiple-destination (MSMD) charge migration, which is not supported in [6], may be often needed in a large-scale HEES system. This paper is the first to formally describe the MSMD charge migration problem and optimization. Compared to performing multiple SSSD charge migration operations sequentially, performing a single MSMD charge migration results in lower charge transfer time and higher migration efficiency.

We first introduce a generalized HEES system architecture and the corresponding electrical circuit models for chargers and EES element arrays. Next, we formulate the MSMD charge migration optimization problem as the problem of delivering a fixed amount of energy to the destination banks and targeting at the maximization of the overall charge migration efficiency defined as the ratio of the total energy received by the destination EES element arrays to the total energy drawn from the source EES element arrays. The goal of the MSMD charge migration optimization solution is to provide the optimal voltage level for the CTI, the amount of charging currents among the destination EES banks, and the amount of discharging currents among the source EES banks.

The MSMD charge migration problem is formulated as a nonlinear programming problem (NLP), which is generally hard to solve efficiently. The more complicating factor here, however, is the fact that the SoC of an EES array at time instance t is a function of the charging or discharging currents before that time. Therefore, a one-shot solution of the NLP can result in a solution that is useless in practice. To address this latter issue, we propose solving the MSMD charge migration problem over short intervals while updating the SoC’s of various EES arrays in between consecutive intervals. More precisely, we divide the total charge migration time (also called a relative deadline in this paper) into a fixed number of timing intervals. The boundary point between two consecutive intervals is called a *decision epoch*. At each decision epoch, we solve a *spontaneous MSMD* (sMSMD) optimization problem in which the SoC and OCV levels for all EES arrays are known. Assume that the relative deadline T_d is divided into S timing intervals, i.e., $T_d = S \cdot \Delta t$. At decision epoch $t = i \cdot \Delta t$, the remaining amount of energy that must still be delivered to the destination banks is equally divided among the remaining time intervals. The result is called an energy quantum. The goal of the sMSMD optimization problem is to

maximize the charge transfer efficiency while delivering at least this quantum of energy to the destination banks. The optimal solution to the sMSMD optimization problem will not only specify the values of the CTI voltage and charge/discharge current of various EES banks, but also determine the optimal amount of energy (which is still no less than the required quantum) that can be transferred in one timing interval. The action is taken, the SoC and OCV levels of all EES arrays are updated and the process of setting up and solving the sMSMD optimization problem at the next decision epoch is repeated. The process continues until the full amount of required energy is transferred to the destination banks. The sMSMD optimization problem at each decision epoch can be solved in an iterative manner, where in each iteration we solve a quasi-convex optimization problem in polynomial time. Details are provided below.

II. HEES SYSTEMS

A. HEES System Architecture

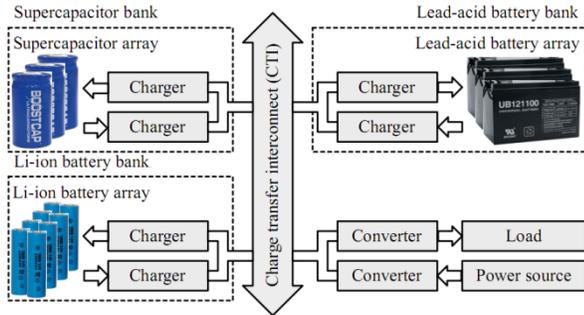


Figure 1: Architecture of the proposed HEES system.

Figure 1 illustrates a conceptual block diagram of the proposed HEES system architecture. The system is comprised of multiple heterogeneous EES banks, connecting to each other through the CTI. Each EES bank consists of an EES (element) array and two chargers, enabling charge transferring into and out of the EES element arrays through the CTI regardless of their different SoC's, terminal voltages and power ratings. Each EES array consists of multiple homogeneous EES elements with the same SoC and other characteristics, organized in an appropriately constructed two-dimensional array using series and/or parallel connections. As mentioned before, no single type of EES element can fulfill all the desirable performance requirements such as high power and energy densities, low cost/weight per unit capacity, high cycle efficiency, and long cycle life in energy storage systems. Therefore, heterogeneous EES banks can be used in a complementary manner in the HEES system to exploit the strengths of each type of EES element while hiding their shortcomings.

B. Bank Model

1) Charger

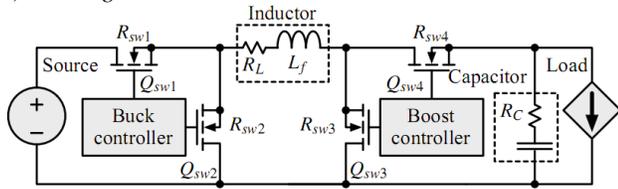


Figure 2: Buck-boost converter architecture.

A charger is a PWM (pulse width modulation) buck-boost switching converter which regulates its output charging current into a desired value according to our proposed algorithm, with model shown in Figure 2. The input voltage, input current, output voltage and output current of the charger are denoted by V_{in} , I_{in} , V_{out} , and I_{out} , respectively. We use P_{conv} to denote the power loss of the charger, which includes the conduction loss, the switching loss and the controller loss [7], and we have:

$$P_{conv} = V_{in} \cdot I_{in} - V_{out} \cdot I_{out}. \quad (1)$$

Based on the relation between V_{in} and V_{out} , the charger has two working modes: the buck mode ($V_{in} > V_{out}$) and otherwise the boost mode. In the buck mode, the converter power loss P_{conv} is given by

$$P_{conv} = I_{out}^2 \cdot (R_L + D \cdot R_{sw1} + (1-D) \cdot R_{sw2} + R_{sw4}) + \frac{(\Delta I)^2}{12} (R_L + D \cdot R_{sw1} + (1-D) \cdot R_{sw2} + R_{sw4} + R_C) + V_{in} \cdot f_s \cdot (Q_{sw1} + Q_{sw2}) + V_{in} \cdot I_{controller}, \quad (2)$$

where $D = V_{out}/V_{in}$ is the PWM duty ratio and $\Delta I = V_{out} \cdot (1-D)/(L_f \cdot f_s)$ is the maximum current ripple; f_s is the switching frequency; $I_{controller}$ is the current flowing into the controller; R_L and R_C are the equivalent series resistances of the inductor L and the capacitor C , respectively; R_{swi} and Q_{swi} are the turn-on resistance and gate charge of the i -th MOSFET switch in Figure 2, respectively.

In the boost mode, the power loss P_{conv} is given by

$$P_{conv} = \left(\frac{I_{out}}{1-D} \right)^2 \cdot (R_L + D \cdot R_{sw3} + (1-D) \cdot R_{sw4} + R_{sw1} + D \cdot (1-D) \cdot R_C) + \frac{(\Delta I)^2}{12} (R_L + D \cdot R_{sw3} + (1-D) \cdot R_{sw4} + R_{sw1} + (1-D) \cdot R_C) + V_{out} \cdot f_s \cdot (Q_{sw3} + Q_{sw4}) + V_{in} \cdot I_{controller}, \quad (3)$$

where $D = 1 - V_{in}/V_{out}$ and $\Delta I = V_{in} \cdot D/(L_f \cdot f_s)$ in this case.

2) EES Element Array

This paper introduces the MSMD charge migration problem with two representative EES elements to deliver the main concepts and avoid diversity: Li-ion battery and supercapacitor. We use an electronic equivalent circuit model in [8] for the Li-ion battery model, which is suitable for developing the mathematical formulation. More specifically, the relation between the battery array open circuit voltage $V_{array}^{OC}(t)$ and closed circuit voltage $V_{array}^{CC}(t)$ is given by

$$V_{array}^{CC}(t) = V_{array}^{OC}(t) - V_{tl}(t) - V_{ts}(t) - I_{array}(t) \cdot R_s, \quad (4)$$

where $V_{tl}(t)$ and $V_{ts}(t)$ are the voltage drops across the internal capacitances, $I_{array}(t)$ is the array current and R_s is the internal series resistance. Similar relationship can be applied to the supercapacitor array in which the internal resistance is negligible.

The open circuit terminal voltage (OCV) value $V_{array}^{OC}(t)$ of a supercapacitor or a battery element array is a function of its SoC value at time t , $SoC(t)$, defined as the ratio between the charge stored in an element array and the total charge when the array is fully charged. For supercapacitor element arrays, the OCV value $V_{array}^{OC}(t)$ is a linear function of the SoC value. For battery element arrays, the OCV-SoC relation is given as follows:

$$V_{array}^{OC}(t) = b_1 \cdot e^{b_2 \cdot SoC(t)} + b_3 \cdot SoC^3(t) + b_4 \cdot SoC^2(t) + b_5 \cdot SoC(t) + b_6, \quad (5)$$

where those b_i are empirically determined parameters [9].

A primary disadvantage of the supercapacitor array is its high self-discharge rate. A supercapacitor may lose more than 20% of its stored energy per day even if no load is connected to it [4]. The supercapacitor array power loss due to self-discharge is given by

$$P_{sd}(t) = V_{array}^{OC}(t) \cdot I_{sd}(t) = C_{cap} (V_{array}^{OC}(t))^2 / \tau, \quad (6)$$

where $I_{sd}(t)$ is the self-discharge current, C_{cap} is the capacitance of the supercapacitor array and τ is the self-discharge time constant. On the other hand, the battery element arrays have negligible self-discharge effect within the time range of charge migration.

The rate capacity effect of batteries explains that the charging and discharging efficiencies decrease with the increasing of charging and discharging currents, respectively. More precisely, the Peukert's Law [10] describes that the charging and discharging efficiencies of a battery element array as functions of the charging current I_c and discharging current I_d , respectively, are given by

$$\eta_{rate,c}(I_c) = k_c / (I_c)^{\alpha_c}, \quad \eta_{rate,d}(I_d) = k_d / (I_d)^{\alpha_d}, \quad (7)$$

where k_c , α_c , k_d , and α_d are constants known *a priori*. In contrast, the rate capacity effect of supercapacitor is negligible, i.e., their charging and discharging efficiencies equal to one.

III. MSMD CHARGE MIGRATION: ARCHITECTURE AND PROBLEM FORMULATION

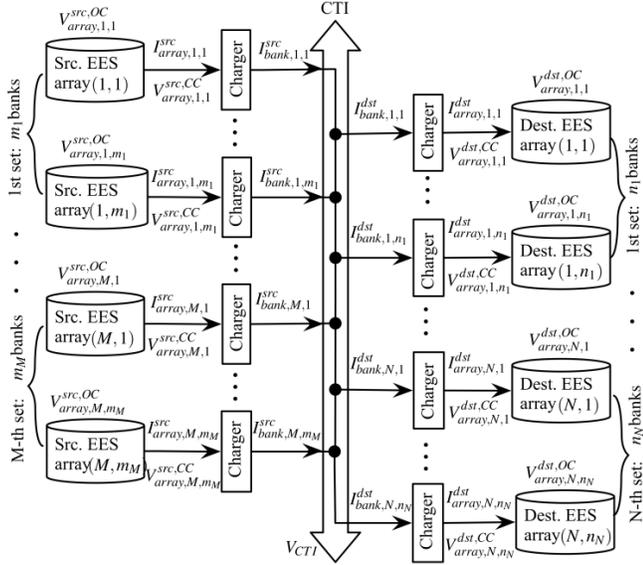


Figure 3: MSMD charge migration system architecture.

Figure 3 presents the conceptual architecture for the MSMD charge migration in a HEES system. For given groups of source EES banks and destination EES banks, the proposed near-optimal MSMD charge migration algorithm shall effectively determine the CTI voltage level, discharging currents from source EES banks and charging currents for destination EES banks. The goal of the MSMD charge migration algorithm is to maximize the charge migration efficiency during the migration process, considering distinct properties of EES element arrays and efficiencies of chargers. The near-optimal CTI voltage, discharging currents and charging currents may vary over time due to the changes of SoC's of the EES element arrays. As shown in Figure 3, we have M ($M \geq 1$) sets of source EES banks, each consisting of m_i , $i \in \{1, 2, \dots, M\}$ EES banks, on the left of the CTI, and N ($N \geq 1$) sets of destination EES banks, each consisting of n_j , $j \in \{1, 2, \dots, N\}$ EES banks, on the right of the CTI, where each bank consists of an EES element array, a *discharging control charger* and a *charging control charger* that connect the EES array to CTI. The charging control chargers in source EES banks and discharging control chargers in destination EES banks are turned off in the case of MSMD charge migration process and thus are removed from the schematic for simplicity. At time instance t , $V_{array,i,k}^{src,OC}(t)$ and $V_{array,i,k}^{src,CC}(t)$ ($i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$) denote the open circuit terminal voltage (OCV) and closed circuit terminal voltage (CCV) of the k -th EES array in the i -th set of source EES banks, respectively, and $V_{array,j,l}^{dst,OC}(t)$ and $V_{array,j,l}^{dst,CC}(t)$ ($j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$) denote the OCV and CCV of the l -th EES array in the j -th set of destination EES banks, respectively. These two voltage values are generally not equal to each other due to the internal resistance of the EES array. Their relation is described in Section II-B. The source and destination EES arrays have self discharge with the power rate of $P_{sd,i,k}^{src}(t)$ and $P_{sd,j,l}^{dst}(t)$, respectively. The input and output currents of the discharging control charger connecting the k -th EES array in the i -th set of source EES banks to the CTI are denoted by the *array discharging current*, $I_{array,i,k}^{src}(t)$, and *bank discharging current*, $I_{bank,i,k}^{src}(t)$, respectively. The power loss of the charger is denoted by $P_{conv,i,k}^{src}(t)$ and is a function of its input and output voltages and

currents, as shown in Section II-B. Similarly, we define the *array charging current* $I_{array,j,l}^{dst}(t)$, *bank charging current* $I_{bank,j,l}^{dst}(t)$, and the power loss $P_{conv,j,l}^{dst}(t)$ of the charging control charger connecting the l -th EES array in the j -th set of destination EES banks to the CTI.

In an MSMD charge migration process, the system controller supervises the bank discharging currents $I_{bank,i,k}^{src}(t)$ of all the source EES banks, the bank charging currents $I_{bank,j,l}^{dst}(t)$ of all the destination EES banks, as well as the CTI voltage $V_{CTI}(t)$ at time instance t . All other voltage and current values are either given (e.g., EES array OCVs) or are *associated variables*. The MSMD migration problem is constrained by the energy conservation law. As illustrated in Figure 3, the power flowing into each destination EES bank is used to charge the corresponding EES element array and drive the corresponding charging control charger, i.e., at time t , we have:

$$V_{CTI}(t) \cdot I_{bank,j,l}^{dst}(t) = P_{conv,j,l}^{dst}(t) + V_{array,j,l}^{dst,CC}(t) \cdot I_{array,j,l}^{dst}(t) \quad (8)$$

where V_{in} , I_{in} , V_{out} , and I_{out} of the charger connecting CTI and the l -th EES array in the j -th set of destination EES banks are $V_{CTI}(t)$, $I_{bank,j,l}^{dst}(t)$, $V_{array,j,l}^{dst,CC}(t)$, and $I_{array,j,l}^{dst}(t)$, respectively. We further denote the *charge increasing speed* of the l -th EES array in the j -th set of destination EES banks by $I_{array,j,l}^{dst,inc}(t)$, when neglecting the *self-discharge effect*. Obviously, $I_{array,j,l}^{dst,inc}(t) \leq I_{array,j,l}^{dst}(t)$ and is a concave and monotonically increasing function of $I_{array,j,l}^{dst}(t)$, denoted by $I_{array,j,l}^{dst,inc}(t) = f_{inc,j,l}(I_{array,j,l}^{dst}(t))$, and is given by

$$I_{array,j,l}^{dst,inc}(t) = f_{inc,j,l}(I_{array,j,l}^{dst}(t)) \quad (9)$$

$$= I_{array,j,l}^{dst}(t) \cdot \eta_{rate,c,j,l}^{dst}(I_{array,j,l}^{dst}(t)).$$

Moreover, each bank discharging current comes from source EES array through the corresponding discharging control charger, i.e.,

$$V_{CTI}(t) \cdot I_{bank,i,k}^{src}(t) = V_{array,i,k}^{src,CC}(t) I_{array,i,k}^{src}(t) - P_{conv,i,k}^{src}(t) \quad (10)$$

where V_{in} , I_{in} , V_{out} , and I_{out} of the charger connecting the k -th EES array in the i -th set of source EES banks to CTI are $V_{array,i,k}^{src,CC}(t)$, $I_{array,i,k}^{src}(t)$, $V_{CTI}(t)$, and $I_{bank,i,k}^{src}(t)$, respectively. We further denote the *charge decreasing speed* of the k -th EES array in the i -th set of source EES banks when neglecting the *self-discharge effect* by $I_{array,i,k}^{src,dec}(t) \cdot I_{array,i,k}^{src}(t) \geq I_{array,i,k}^{src}(t)$ and is a convex and monotonically increasing function of $I_{array,i,k}^{src}(t)$, denoted by $I_{array,i,k}^{src,dec}(t) = f_{dec,i,k}(I_{array,i,k}^{src}(t))$, and is given by

$$I_{array,i,k}^{src,dec}(t) = f_{dec,i,k}(I_{array,i,k}^{src}(t)) \quad (11)$$

$$= I_{array,i,k}^{src}(t) / \eta_{rate,d,i,k}^{src}(I_{array,i,k}^{src}(t)).$$

Furthermore, the total current flowing into the CTI equals to the total current flowing out of the CTI, i.e.,

$$\sum_{i=1}^M \sum_{k=1}^{m_i} I_{bank,i,k}^{src}(t) = \sum_{j=1}^N \sum_{l=1}^{n_j} I_{bank,j,l}^{dst}(t). \quad (12)$$

The initial OCVs of all the EES element arrays can be derived based on the initial EES array SoC's, using the OCV-SoC relation (5) for battery arrays or the linear function for supercapacitor arrays. Suppose that the migration process starts at time $T_0 = 0$, we formulate the MSMD charge migration problem as transferring E_1^{dst} , E_2^{dst} , ..., E_N^{dst} amounts of energy to the 1st, 2nd, ..., N -th set of destination EES banks, respectively, within certain time limit T_d , such that the energy loss in the 1st, 2nd, ..., M -th set of source EES banks are no more than E_1^{src} , E_2^{src} , ..., E_M^{src} , respectively. We formally describe the MSMD charge migration problem as follows:

Given: $V_{array,i,k}^{src,OC}(t)|_{t=0}$ for $i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$ and $V_{array,j,l}^{dst,OC}(t)|_{t=0}$ for $j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$, amounts of energy to be migrated E_j^{dst} for $j \in \{1, 2, \dots, N\}$, energy loss constraints E_i^{src} for $i \in \{1, 2, \dots, M\}$, and relative deadline T_d .

Find: the optimal $V_{CTI}(t)$, $I_{bank,i,k}^{src}(t)$ ($i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$), and $I_{bank,j,l}^{dst}(t)$ ($j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$), for $t \in [0, T_d]$.

Maximize: the charge migration efficiency η_{MSMD} , defined as:

$$\frac{\int_0^{T_d} \sum_{j=1}^N \sum_{l=1}^{n_j} (V_{array,j,l}^{dst,OC} \cdot I_{array,j,l}^{dst,inc}(t) - P_{sd,j,l}^{dst}(t)) dt}{\int_0^{T_d} \sum_{i=1}^M \sum_{k=1}^{m_i} (V_{array,i,k}^{src,OC} \cdot I_{array,i,k}^{src,dec}(t) + P_{sd,i,k}^{src}(t)) dt} \quad (13)$$

Subject to:

i) *Energy Conservation:* (8) - (12) are satisfied.

ii) *Calculating SoC:* SoC values at time $t \in [0, T_d]$ are given by

$$SoC_{i,k}^{src}(t) = SoC_{i,k}^{src}(0) - \frac{\int_0^t (I_{array,i,k}^{src,dec}(\tau) + I_{sd,i,k}^{src}(\tau)) d\tau}{C_{i,k}^{src,full}}, \quad (14)$$

$$SoC_{j,l}^{dst}(t) = SoC_{j,l}^{dst}(0) + \frac{\int_0^t (I_{array,j,l}^{dst,inc}(\tau) - I_{sd,j,l}^{dst}(\tau)) d\tau}{C_{j,l}^{dst,full}}, \quad (15)$$

for $i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$ and $j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$. In (14) and (15), $C_{i,k}^{src,full}$ and $C_{j,l}^{dst,full}$ denote the full charge of the k -th EES array in the i -th set of source EES banks, and the l -th EES array in the j -th set of destination EES banks, respectively. The self-discharge currents $I_{sd,i,k}^{src}(t)$ and $I_{sd,j,l}^{dst}(t)$ can be calculated using (6), and the SoC values calculated in (14), (15) should be within the range of $[0, 1]$.

iii) *Achieving Specified Energy Delivery to Destination Banks:* the amount of energy shall be migrated into the j -th ($j \in \{1, 2, \dots, N\}$) set of destination EES banks equals to E_j^{dst} , i.e.,

$$\int_0^{T_d} \sum_{l=1}^{n_j} V_{array,j,l}^{dst,OC} \cdot (I_{array,j,l}^{dst,inc}(t) - I_{sd,j,l}^{dst}(t)) dt = E_j^{dst} \quad (16)$$

iv) *Bounding Energy Depletion of the Source Banks:* the amount of energy loss in the i -th ($i \in \{1, 2, \dots, M\}$) set of source EES banks is no more than E_i^{src} , i.e.,

$$\int_0^{T_d} \sum_{k=1}^{m_i} V_{array,i,k}^{src,OC} \cdot (I_{array,i,k}^{src,dec}(t) + I_{sd,i,k}^{src}(t)) dt \leq E_i^{src} \quad (17)$$

v) *Calculating OCV and CCV from SoC:* the OCV-SoC relation for battery (5) and supercapacitor (linear function), and the OCV-CCV relation (4).

vi) *Non-negativity of Charging/Discharging Currents:* the output currents of discharging control and charging control chargers, i.e., the bank discharging currents and array charging currents, are no less than zero at any time $t \in [0, T_d]$, i.e.,

$$I_{bank,i,k}^{src}(t) \geq 0, \quad I_{array,j,l}^{dst}(t) \geq 0, \quad (18)$$

for each $i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$, and $j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$.

IV. OPTIMIZATION METHOD

In this section we first discuss the spontaneous multiple-source, multiple-destination (sMSMD) charge migration problem targeting at maximizing the spontaneous migration efficiency, and propose an efficient solution for the sMSMD charge migration problem. After that we consider the original MSMD charge migration problem, and provide a solution in a discrete time space by performing the near-optimal sMSMD optimization at each decision epoch with lower bounds on the amount of delivered energy to destination banks and upper bound on the amount of energy depleted from the source banks.

A. Spontaneous Charge Migration Optimization

In the sMSMD charge migration problem formulation, we omit the time index t for simplicity in writing. In this problem, we have EES array OCVs $V_{array,i,k}^{src,OC}$ for $i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$ and $V_{array,j,l}^{dst,OC}$ for $j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$, derived from EES array SoC values. The system control variables are the CTI voltage

V_{CTI} , bank discharging currents $I_{bank,i,k}^{src}$ ($i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$) and bank charging currents $I_{bank,j,l}^{dst}$ ($j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$). However, in the sMSMD charge migration optimization problem formulation we use the *charge increasing speeds* $I_{array,j,l}^{dst,inc}$ ($j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$) when neglecting self-discharge of destination EES arrays as optimization variables, instead of the bank charging currents, to make the optimization problem easier to solve, which is equivalent since the bank charging currents can be calculated and used for system control as long as the sMSMD charge migration problem has been solved. The objective function to be maximized is (derived from (13)):

$$\eta_{sMSMD} = \frac{\sum_{j=1}^N \sum_{l=1}^{n_j} (V_{array,j,l}^{dst,OC} \cdot I_{array,j,l}^{dst,inc} - P_{sd,j,l}^{dst})}{\sum_{i=1}^M \sum_{k=1}^{m_i} (V_{array,i,k}^{src,OC} \cdot I_{array,i,k}^{src,dec} + P_{sd,i,k}^{src})}. \quad (19)$$

We further derive the constraints from (8) - (12) and (18) by omitting time index t . Moreover, there exists additional constraint come from (14) and (15) that the bank charging currents $I_{bank,j,l}^{dst} = 0$ (or equivalently, $I_{array,j,l}^{dst,inc} = 0$) if $SoC_{j,l}^{dst} = 100\%$, and the bank discharging current $I_{bank,i,k}^{src} = 0$ if $SoC_{i,k}^{src} = 0$. It can be observed that the sMSMD migration optimization problem is a non-linear and non-convex optimization problem, and therefore effective heuristics have to be developed to find the near-optimal solution.

We first consider a *quasi-convex* version of sMSMD (qsMSMD) charge migration problem to find the optimal $I_{bank,i,k}^{src}$ ($i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$) and $I_{array,j,l}^{dst,inc}$ ($j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$) values aiming at maximization of spontaneous charge migration efficiency η_{sMSMD} , under the assumption that the CTI voltage V_{CTI} and the source and destination EES bank CCVs are given. In the qsMSMD optimization problem, the power loss of each discharging control charger $P_{conv,i,k}^{src}$ is a quadratic function of $I_{bank,i,k}^{src}$ according to (2), (3), and therefore the array discharging current $I_{array,i,k}^{src}$ becomes a convex function of $I_{bank,i,k}^{src}$ according to (10). In addition, due to the fact that $I_{array,i,k}^{src,dec}$ is a convex and monotonically increasing function of $I_{array,i,k}^{src}$, $I_{array,i,k}^{src,dec}$ becomes a convex function of $I_{bank,i,k}^{src}$ as well due to the rules of convexity of composite functions [11]. Therefore the nominator of (19) is a linear function of optimization variables $I_{bank,i,k}^{src}$ ($i \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, m_i\}$) and $I_{array,j,l}^{dst,inc}$ ($j \in \{1, 2, \dots, N\}, l \in \{1, 2, \dots, n_j\}$), while the denominator of (19) is a convex function of those optimization variables (self-discharge power terms $P_{sd,j,l}^{dst}$ and $P_{sd,i,k}^{src}$ can be viewed as constants.) Hence the spontaneous charge migration efficiency η_{sMSMD} to be maximized is a quasi-concave function of optimization variables in the qsMSMD optimization problem. Note that constraints (8) - (11) are already integrated into the objective function, and constraint (18) will become convex inequality constraint if we use $I_{array,j,l}^{dst,inc} \geq 0$ instead of $I_{array,j,l}^{dst} \geq 0$ in the constraint (they are equivalent.) Moreover, constraint (12) will become convex inequality constraint if we modify it into the following form:

$$\sum_{i=1}^M \sum_{k=1}^{m_i} I_{bank,i,k}^{src}(t) \geq \sum_{j=1}^N \sum_{l=1}^{n_j} I_{bank,j,l}^{dst}(t), \quad (20)$$

which is also intuitive. Hence, the qsMSMD charge migration optimization problem becomes a quasi-convex programming problem, and its optimal solution can be calculated in the following way: since the feasibility problem $\eta_{sMSMD} \geq \beta$ for $\forall \beta \geq 0$ in the qsMSMD charge migration problem setup (i.e., CTI voltage, EES bank CCVs are given) is a convex optimization problem and can be solved in polynomial time [11], we could use bisection method to effectively derive the maximum feasible β value efficiency η_{sMSMD} could achieve, which will be the optimal spontaneous charge migration efficiency of the qsMSMD charge migration problem. The optimal variable values can be determined accordingly.

We propose the following two heuristics to determine V_{CTI} , $V_{array,i,k}^{src,CC}$ ($i \in \{1, 2, \dots, M\}$, $k \in \{1, 2, \dots, m_i\}$), and $V_{array,j,l}^{dst,CC}$ ($j \in \{1, 2, \dots, N\}$, $l \in \{1, 2, \dots, n_j\}$) values, which are assumed given in solving the qsMSMD charge migration optimization problem.

Heuristic A: Assume that the optimal spontaneous migration efficiency η_{sMSMD} is a quasi-concave function with respect to V_{CTI} . We propose to solve a qsMSMD migration problem with a fixed V_{CTI} value, and use the ternary search algorithm to effectively search feasible region of V_{CTI} to get the optimal V_{CTI} value. Simulation results validate the assumption of quasi-concavity and prove the efficiency of ternary search for finding the best-suited V_{CTI} value.

Heuristic B: We start from the initial values $V_{array,i,k}^{src,CC} = V_{array,i,k}^{src,OC}$ and $V_{array,j,l}^{dst,CC} = V_{array,j,l}^{dst,OC}$, and find the final $V_{array,i,k}^{src,CC}$ and $V_{array,j,l}^{dst,CC}$ values iteratively. In each iteration, we solve a qsMSMD migration problem with $V_{array,i,k}^{src,CC}$ and $V_{array,j,l}^{dst,CC}$ values assumed to be fixed and update such values using (4) at the end of the iteration.

We finally obtain the near-optimal solution to the sMSMD charge migration optimization problem by iteratively solving the qsMSMD migration problem. The complete algorithm is given in Algorithm 1.

Algorithm 1: Near-Optimal sMSMD Charge Migration Problem Solver

Input: $V_{array,i,k}^{src,OC}$ ($i \in \{1, 2, \dots, M\}$, $k \in \{1, 2, \dots, m_i\}$), $V_{array,j,l}^{dst,OC}$ ($j \in \{1, 2, \dots, N\}$, $l \in \{1, 2, \dots, n_j\}$), V_{CTI} range $[V_{CTI}^{min}, V_{CTI}^{max}]$, threshold V_e , value $0 < \alpha < 0.5$.

Output: Near-optimal sets $\{V_{bank,i,k}^{src,opt}\}$ and $\{V_{array,j,l}^{dst,inc,opt}\}$, and CTI voltage V_{CTI}^{opt} .

Repeat

For each $V_{CTI} \in \{(1 - \alpha) \cdot V_{CTI}^{min} + \alpha \cdot V_{CTI}^{max}, (1 - \alpha) \cdot V_{CTI}^{max} + \alpha \cdot V_{CTI}^{min}\}$;

 Initialize $V_{array,i,k}^{src,CC} \leftarrow V_{array,i,k}^{src,OC}$ ($i \in \{1, 2, \dots, M\}$, $k \in \{1, 2, \dots, m_i\}$),
 $V_{array,j,l}^{dst,CC} \leftarrow V_{array,j,l}^{dst,OC}$ ($j \in \{1, 2, \dots, N\}$, $l \in \{1, 2, \dots, n_j\}$);

Repeat

- 1) Update $\{V_{bank,i,k}^{src}\}$ and $\{V_{array,j,l}^{dst,inc}\}$ by solving qsMSMD migration problem with input values V_{CTI} , $\{V_{array,i,k}^{src,CC}\}$ and $\{V_{array,j,l}^{dst,CC}\}$;
- 2) Calculate the optimal spontaneous migration efficiency from the above qsMSMD optimization, denoted by $\eta_{sMSMD}(V_{CTI})$;
- 3) Update $\{V_{array,i,k}^{src,CC}\}$ and $\{V_{array,j,l}^{dst,CC}\}$ using (4);

Until both $\{V_{array,i,k}^{src,CC}\}$ and $\{V_{array,j,l}^{dst,CC}\}$ converge

End

If $\eta_{sMSMD}((1 - \alpha)V_{CTI}^{min} + \alpha V_{CTI}^{max}) > \eta_{sMSMD}((1 - \alpha)V_{CTI}^{max} + \alpha V_{CTI}^{min})$;

$V_{CTI}^{max} \leftarrow (1 - \alpha) \cdot V_{CTI}^{max} + \alpha \cdot V_{CTI}^{min}$;

Else: $V_{CTI}^{min} \leftarrow (1 - \alpha) \cdot V_{CTI}^{min} + \alpha \cdot V_{CTI}^{max}$;

Until $V_{CTI}^{max} - V_{CTI}^{min} < V_e$

Return $\{V_{bank,i,k}^{src,opt}\} \leftarrow \{V_{bank,i,k}^{src}\}$, $\{V_{array,j,l}^{dst,inc,opt}\} \leftarrow \{V_{array,j,l}^{dst,inc}\}$, $V_{CTI}^{opt} \leftarrow V_{CTI}$

B. Global MSMD Charge Migration Optimization Algorithm

In the global MSMD charge migration optimization algorithm which may last for a few minutes or hours, we have to consider not only the charge migration efficiency, but also how to satisfy the *migration energy constraint* (16) and the *energy loss constraint* (17). Therefore, we propose to solve the global MSMD charge migration problem in a discrete time space, by solving a series of sMSMD charge migration optimization problems one at each decision epoch, with additional constraints so that it can be guaranteed that constraints (16) and (17) shall be satisfied.

Note that if the migration controller makes greedy decisions so that for some specific set of destination EES banks (e.g., the j -th set, for instance), it is possible that E_j^{dst} (the target) amount of energy has been migrated into such set of banks at an early stage, while at that time the migration energy constraints of other sets of EES banks have not been satisfied. This may result in efficiency degradation since the j -th set of EES banks will be turned off without being charged any more after that time. Therefore in our proposed heuristic for global charge migration, *all sets of EES banks finish charging at the same*

time, i.e., we know that $E_1^{dst}, E_2^{dst}, \dots, E_N^{dst}$ amounts of energy shall be transferred into the 1st, 2nd, ..., N -th set of destination EES banks, respectively, and these transfers shall finish at the same time, and we denote this finishing time by T_e (of course $T_e \leq T_d$ and T_e is not known until charge migration finishes.) To achieve this goal, we incorporate the following constraint to the sMSMD charge migration optimization at each decision epoch (assume at time $t \in [0, T_e]$):

$$\frac{\sum_{l=1}^{n_j} (V_{array,j,l}^{dst,OC}(t) \cdot I_{array,j,l}^{dst,inc}(t) - P_{sd,j,l}^{dst}(t))}{\sum_{l=1}^{n_{j'}} (V_{array,j',l}^{dst,OC}(t) \cdot I_{array,j',l}^{dst,inc}(t) - P_{sd,j',l}^{dst}(t))} = \frac{E_j^{dst}}{E_{j'}^{dst}} \quad (21)$$

for each pair $(j, j') \in \{1, 2, \dots, N\} \times \{1, 2, \dots, N\}$ and $j \neq j'$. This implies that for any set (the j -th set, for instance) of destination EES banks, the energy accumulating rate is proportional to its target migration value E_j^{dst} at any time t during charge migration process.

Furthermore, we shall make sure that charge migration will finish within the time limit T_d . To achieve this goal, we use $E_{rem}^{dst}(t)$ for $t \in [0, T_e]$ to denote the remaining energy to be migrated into the destination EES banks at time t , which can be calculated as follows:

$$E_{rem}^{dst}(t) = \sum_{j=1}^N E_j^{dst} - \int_0^t \sum_{j=1}^N \sum_{l=1}^{n_j} (V_{array,j,l}^{dst,OC}(\tau) \cdot I_{array,j,l}^{dst,inc}(\tau) - P_{sd,j,l}^{dst}(\tau)) d\tau. \quad (22)$$

We add the following constraint to the sMSMD charge migration optimization performed at each decision epoch $t \in [0, T_e]$ so that charge migration can be guaranteed to finish before deadline T_d :

$$\sum_{j=1}^N \sum_{l=1}^{n_j} (V_{array,j,l}^{dst,OC}(t) \cdot I_{array,j,l}^{dst,inc}(t) - P_{sd,j,l}^{dst}(t)) \geq \frac{E_{rem}^{dst}(t)}{T_d - t}. \quad (23)$$

Next we shall discuss about the fairness issue, i.e., how constraint (17) shall be satisfied in the global MSMD charge migration solution. For each set of source EES banks (the i -th set, for instance), we use $E_{i,drawn}^{src}(t)$ to denote the energy drawn from that set of source EES banks from the beginning of the migration process to time t , i.e.,

$$E_{i,drawn}^{src}(t) = \int_0^t \sum_{k=1}^{m_i} (V_{array,i,k}^{src,OC}(\tau) I_{array,i,k}^{src,dec}(\tau) + P_{sd,i,k}^{src}(\tau)) d\tau \quad (24)$$

Then the following constraint on the i -th set of source EES banks ($i \in \{1, 2, \dots, M\}$) can be added to the sMSMD charge migration optimization performed at each decision epoch $t \in [0, T_e]$ so that the source energy loss constraint (17) can be satisfied:

$$\frac{\sum_{k=1}^{m_i} (V_{array,i,k}^{src,OC}(t) \cdot I_{array,i,k}^{src,dec}(t) + P_{sd,i,k}^{src}(t))}{\sum_{j=1}^N \sum_{l=1}^{n_j} (V_{array,j,l}^{dst,OC}(t) \cdot I_{array,j,l}^{dst,inc}(t) - P_{sd,j,l}^{dst}(t))} \leq \frac{E_i^{src} - E_{i,drawn}^{src}(t)}{E_{rem}^{dst}(t)}. \quad (25)$$

Since constraint (21) is affine equality constraint, and constraints (23) and (25) are convex inequality constraints, the sMSMD charge migration optimization with addition constraints (21), (23) and (25) can be solved with no additional time complexity as the original sMSMD charge migration optimization problem.

V. EXPERIMENTAL RESULTS

In this section, we compare our proposed near-optimal solution of the MSMD charge migration problem, with various baseline systems with constant bank discharging/charging currents. Experiments are carried out with different deadline T_d values. In fact, the baseline systems unaware of detailed models of EES arrays and power converters cannot guarantee to meet both the deadline constraint (16) and the energy loss constraint (17). Therefore for the need of comparison we make the assumption that baseline systems are aware of such detailed models and can meet all the related constraints. The CTI voltage $V_{CTI}(t)$ of the baseline systems are set to be constant

throughout the charge migration process. Moreover, the *overall energy migration rate*, defined as the increasing speed of the amount of energy migrated into all the destination EES banks, are set to be constant (are equal to $\sum_{j=1}^N E_j^{dst}/T_d$) for the baseline setups, such that the charge migration process will finish *just before the deadline*. Furthermore, the bank charging currents among destination EES banks in the baseline systems are properly set such that the energy accumulating speed of each set of destination EES banks (the j -th, for instance) is proportional to its target migration value E_j^{dst} , and therefore constraint (16) can be satisfied. Within each set of destination EES banks, the bank charging currents of different banks are set to be the same. Also, the bank discharging currents among source EES banks in the baseline systems are again properly set such that the energy decreasing speed of each set of source EES banks (the i -th, for instance) is proportional to its energy loss constraint value E_i^{src} , and therefore constraint (17) can be satisfied. Finally, within each set of source EES banks, the bank discharging currents of different banks are set to be the same.

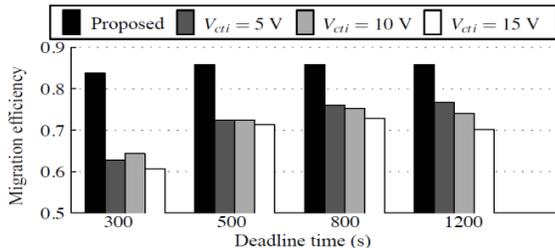


Figure 4: Comparison of charge migration efficiencies on charge migration process from supercapacitor banks to battery banks, with different T_d values.

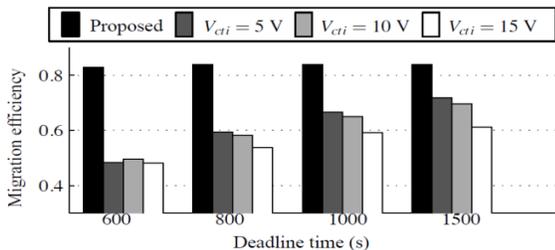


Figure 5: Comparison of charge migration efficiencies on charge migration process from battery banks to supercapacitor banks, with different T_d values.

In the first test case we compare our near-optimal MSMD charge migration policy with three baseline setups with constant CTI voltage 5 V, 10 V, 15 V, respectively, on the charge migration process from one set of source EES banks, consisting of four supercapacitor banks with array OCVs 13.5 V, 13.5 V, 11.0 V, 11.0 V, respectively, to one set of destination EES banks, consisting of four battery banks with array OCVs 3.0 V, 3.0 V, 12.0 V, 12.0 V, respectively. The total energy transferred into destination banks during the charge migration process equals to 50% of the total energy stored in all the four source supercapacitor banks. The comparison results between the proposed method and baseline setups are illustrated in Figure 4. As shown in this figure, four sets of experiments are carried out with relative deadline (T_d) values 300 s, 500 s, 800 s, and 1200 s, respectively.

In the second test case we compare our near-optimal MSMD charge migration policy with three baseline setups with constant CTI voltage 5 V, 10 V, 15 V, respectively, on the charge migration process from one set of source EES banks, consisting of four battery banks with array OCVs 4.0 V, 4.0 V, 12.0 V, 12.0 V, respectively, to two sets of destination EES banks, both consisting of two supercapacitor banks with array OCVs 2.25 V and 3.0 V, respectively. The total energy transferred into destination supercapacitor banks during the migration process equals to 30% of the total energy required to fully charge all the four destination supercapacitor banks. Besides, the 1st and the 2nd sets of destination

supercapacitor banks are supposed to receive 70% and 30% of the total migrated energy, respectively. The comparison results are illustrated in Figure 5. As shown in this figure, four sets of experiments have been carried out with relative deadline (T_d) values 600 s, 800 s, 1000 s, and 1500 s, respectively.

As can be seen from Figure 4 and Figure 5, the proposed near-optimal solution for MSMD charge migration problem consistently outperforms baseline systems under the same deadline constraint (the same T_d value), with migration efficiency enhancement compared to baseline systems ranging from 9.1 % to 35 %. In the case when the deadline is very tight, the proposed near-optimal solution, although forced to operate with overall energy migration rate restricted by the deadline value, outperforms baseline systems due to the freedom of selecting and adjusting the optimal CTI voltage, the discharging currents among various banks in each set of source EES banks, and the charging currents among various banks in each set of destination EES banks. In the case when the deadline is relatively loose, the proposed near-optimal solution has additional degree of freedom of choosing the optimal overall energy migration rate. Therefore, in the latter case the MSMD charge migration process may finish before the deadline time, and the charge migration efficiency will converge to a maximum value along with the increase of relative deadline value T_d .

VI. CONCLUSION

This paper pioneers a multiple-source and multiple-destination (MSMD) charge migration problem for hybrid electrical energy storage (HEES) systems that allows multiple simultaneous charge migration operations. MSMD migration requires to derive the optimal shared voltage of the charge transfer interconnect (CTI) as well as the charging or discharging current of each EES bank. We formulate the MSMD charge migration problem as a non-linear and non-convex programming problem and present an efficient approach to find a near-optimal solution. We achieve spontaneous MSMD (sMSMD) charge migration efficiency optimization at each decision epoch of the MSMD charge migration process in an iterative manner, and in each iteration step we solve a quasi-convex optimization problem with polynomial time complexity. Experimental results demonstrate significant charge migration efficiency improvement up to 35% against the baseline charge migration methods.

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