A codebook generation algorithm for document image compression

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Abstract

Pattern-matching based document compression systems rely on finding a small set of patterns that can be used to represent all of the ink in the document. Finding an optimal set of patterns is NP-hard; previous compression schemes have resorted to heuristics. We extend the cross-entropy approach, used previously for measuring pattern similarity, to this problem. Using this approach we reduce the problem to the fixed-cost k-median problem, for which we present a new algorithm with a good provable performance guarantee. We test our new algorithm in place of the previous heuristics (First Fit, with and without generalized Lloyd's (k-means) postprocessing steps). The new algorithm generates a better codebook, resulting in an overall improvement in compression performance of almost 17%.

1 INTRODUCTION

For scanned text, pattern matching based compression achieves the best compression ratios currently known: roughly four times that of 2D-FAX coding [11]. Pattern matching based compression has been studied by several research groups [1, 6, 9, 11]. This kind of compression involves the following steps:

Extract from the image of the scanned document a sequence of *glyphs*. Each glyph typically represents one connected blob of ink occuring somewhere in the document, represented as a positioned bitmap.

Partition the set of glyphs into equivalence classes of glyphs with similar bitmaps; for each class, compute a representative *pattern* — a bitmap representing the bitmaps of all glyphs in the class. This set of canonical patterns is the *codebook*.

Transmit the codebook and, for each glyph in the sequence of glyphs representing the document, the glyph's position and the index of its representative pattern.

For *lossy* compression, the process stops here. For *lossless* compression, for each glyph, also transmit the difference between the glyph and its representative, so that the glyph's bitmap can be exactly reconstructed.

For effective compression, the transmission step requires additional compression techniques to economically represent the codebook, indices, and position information. For this work, the codebook is compressed using Moffatt's two-level context-based method (following [11]). The glyph indices are coded using PPMC. The glyph positions are coded using structure-based position coding developed in [15].

Several recent papers have focussed on how to measure the *distance* between pairs of bitmaps [7, 14] via cross-entropy measures. These measures approximate the number of bits needed to encode one bitmap given the other under various models. In this paper we use the measure proposed in [14].

The main contribution of this paper is a new method for the *partitioning* step. Recent works have used relatively ad-hoc heuristics for this step, with the most succesful bearing a resemblance to Lloyd's algorithm for vector quantizer design [4].

The conceptual basis of the new method is an extension of a particular cross-entropy model, proposed and approximated in [14], for measuring distances between bitmaps. Here is a summary of this model. Each bitmap is assumed to have been generated by scanning an ideal character (a blob of ink) at some random offset. Thus, any ideal character induces a probability distribution on the bitmaps. Conversely, any bitmap induces a probability distribution on the ideal characters — the uniform distribution on those ideal characters consistent with the bitmap. The *patterns* in this model are these distributions. Each such pattern has a succint representation as a bitmap. The *distance* between a pattern and a glyph is the conditional entropy of the glyph given the distribution on ideal characters associated with the pattern.

To extend this model to the partitioning problem, we make the useful but perhaps unwarranted assumption that the a-priori distribution of patterns is uniform. With this assumption, given the set of glyphs in the document, each set of possible patterns has a conditional probability. We call the set of patterns with the maximum conditional probability the *minimum-entropy* codebook. We define the *cost* of a pattern to be the number of bits needed to represent it, the *cost* of a codebook to be the sum of the costs of its patterns, and the *distortion* of a codebook to be the number of bits needed to represent all the glyphs in the document given the codebook. The minimum-entropy codebook is the one with minimum cost plus distortion.

The combinatorial problem of finding this codebook reduces to a well-studied problem called the fixed-cost k-median problem, for which we propose a new approximation algorithm with a good provable performance guarantee. We use this algorithm to find a good codebook. We compare our partition algorithm, which we will call Greedy k-median (GKM), with First Fit, First Fit followed by k-means post-processing steps, and two plausible versions of GKM combined with Lloyd's k-means algorithm. By simply substituting GKM for First Fit in the partitioning step, we reduce the size of the compressed documents in our test suite by 17%.

2 PREVIOUS WORK

Previous systems[11] used a simple codebook generation algorithm which we call the First Fit algorithm. This algorithm works as follows:

FIRST-FIT(S)
1. Let the set of equivalence classes $P \leftarrow \emptyset$
2. *for each* glyph in the document $c \in S$
3. *do* find a $p \in P$ such that $distance(p, c) < T$
(the distance between p and c is defined as the distance between the first element of p and c.)
4. *if* there is such a p, add c to p, *continue* with next c
5. *else* $P \leftarrow P \cup \{\{c\}\}$
6. return P

The First Fit algorithm runs in $O(mn)$ time (m is the number of equivalence classes, n is the number of glyphs). The algorithm is fast, but does not have a good performance guarantee relative to the optimal partition. In the worst case, the number of equivalence classes is n times worse than the optimal partition. For example, consider the glyph list $L = \{a, b, c, d\}$ and matching pairs $\{a, b\}, \{b, c\}, \{b, d\}$. Notice that a does not match directly with c and d. The First Fit algorithm partitions a and b into an equivalence class. Because c and d do not match to a, which is the first element of the partition $\{a, b\}$, two partitions are created for c and d. So the final partition is $\{\{a, b\}, \{c\}, \{d\}\}$. If we change the order of the list to $\{b, a, c, d\}$, however, the algorithm finds the partition $\{\{b, a, c, d\}\}$.

Several compression systems, including MGTIC [13] and CDIS [16], combine a bitmap averaging method with the First-Fit algorithm to form a multi-pass pattern classification method. For each equivalence class, all of the bitmaps in the class are averaged and the result is thresholded to obtain a new pattern. The glyphs are then repartitioned, with each glyph being assigned to the first matching pattern. Patterns with no matched glyphs are deleted. This process is iterated until little or no improvement results.

We also test a variant of this algorithm which reassigns each glyph to its *best* matching pattern. This algorithm is a specialization of the Generalized Lloyd (k-means) algorithm for vector quantizer design, because the repartitioned equvalence classes satisfy the nearest-neighbor condition [4]. We call this algorithm "*modified* k-means" because the averaging and thresholding steps do not necessarily produce an optimal pattern for each equivalance class. This can degrade the performance of the algorithm.

3 THE GREEDY K-MEDIAN ALGORITHM

The *weighted fixed-cost k-median* problem is the following. Let $G = (V, E)$ be a directed or undirected graph with non-negative edge weights $d : E \rightarrow R$ and non-negative vertex weights $c : V \rightarrow R$. For any subset S of vertices, define the *cost* of S to be $c(S) = \sum_{w \in S} c(w)$; define the *distortion* of S to be $d(S) = \sum_{v \in V} d(v, S)$ where

$d(v, S) = \min_{w \in S} d(v, w)$. The problem is to find a set minimizing the cost plus the distortion.

Our problem of finding a minimum-entropy codebook reduces to this problem in a directed graph with a vertex v for each glyph in the document. The cost of v is the cost of the glyph's pattern; the cost of edge (u, v) is the distance between the glyph of u and the pattern of v's glyph. Any subset of vertices in the graph then represents a codebook of corresponding cost and distortion. Because of the properties of glyphs and their patterns, the weights in this graph satisfy the usual triangle inequality, as well as $c(v) \leq c(u) + d(u, v)$ for all u, v.

3.1 Previous work.

Hochbaum [5] gives a polynomial-time algorithm that finds a set for which the cost plus the distortion is at most $1 + \ln n$ times the minimum possible. Roughly, Hochbaum observes that the problem reduces to a traditional weighted set-cover problem with exponentially many sets. She then argues it suffices to consider only quadratically many of these sets and obtains a cubic-time algorithm by adapting the weighted greedy set cover algorithm of Chvatal [2].

For the variant in which the desired cost k is specified, and the problem is to minimize the resulting distortion, Lin and Vitter [8] give a polynomial-time approximation algorithm with the following performance guarantee: given $\epsilon > 0$, the algorithm returns a set of cost at most $(1 + 1/\epsilon)(1 + \ln(|V|))k$ with distortion at most $1 + \epsilon$ times the minimum distortion for a set of cost k. They present a related algorithm for the fixed-cost problem that returns a set with cost plus distortion bounded by

$$(1 + \epsilon)d(\mathrm{OPT}) + (1 + 1/\epsilon)(1 + \ln(|V|))c(\mathrm{OPT}),$$

where OPT is the optimal set. These algorithms each solve a linear program and then round the solution using the set-cover algorithm of Chvatal.

3.2 A New Greedy Algorithm

Our new algorithm is somewhat simpler to implement than either of the aforementioned algorithms and provides comparable performance guarantee in graphs with weights satisfying the triangle inequality. The algorithm is based on the greedy algorithm for minimizing a linear function subject to a submodular constraint [10] (a generalization of the greedy set-cover algorithm).

Define the *capped distortion* of a set S to be

$$\delta(S) = \sum_{v \in V} \min(d(v, S), c(v)).$$

The algorithm is the following.

GREEDY-K-MEDIAN($G = (V, E), c, w$)
1. $S' \leftarrow \emptyset$
2. *do* $S \leftarrow S'$
3. choose $v \in V$ that maximizes $(\delta(S) - \delta(S \cup \{v\}))/c(v)$

4. $S' \leftarrow S \cup \{v\}$
5. *while* $c(S') + \delta(S') < c(S) + \delta(S)$
6. *return* S

The loop executes linearly many times and each iteration takes quadratic time. Thus, GREEDY-K-MEDIAN runs in cubic time.

3.3 Analysis.

Fix a graph G with vertex weights c and edge weights w. Recall that $c(S)$ and $d(S)$ denote the cost and distortion, respectively, of a set S. Let OPT denote the set of vertices such that $c(\mathrm{OPT}) + d(\mathrm{OPT})$ is minimal.

The main point of the analysis is the following.

Claim: *GREEDY-K-MEDIAN maintains the invariant*

$$\frac{\delta(S) - d(\mathrm{OPT})}{\delta(\emptyset) - d(\mathrm{OPT})} \leq \exp(-c(S)/c(\mathrm{OPT})).$$

Proof: At the beginning of any iteration, adding to S all of the vertices in OPT would result in a set with $\delta(S) = d(\mathrm{OPT})$. Thus, since δ is supermodular, some vertex w in OPT can be added such that

$$\frac{\delta(S) - \delta(S \cup \{w\})}{c(w)} \geq \frac{\delta(S) - d(\mathrm{OPT})}{c(\mathrm{OPT})}.$$

Together with the choice of v (and a little algebra) it follows that

$$\delta(S \cup \{v\}) - d(\mathrm{OPT}) \leq (\delta(S) - d(\mathrm{OPT}))(1 - c(v)/c(\mathrm{OPT})).$$

The result follows inductively, using $1 - c(v)/c(\mathrm{OPT}) \leq \exp(-c(v)/c(\mathrm{OPT}))$. With a little more work (details are in the full paper), one can show:

Corollary: *The set S returned by GREEDY-K-MEDIAN has cost plus distortion bounded by*

$$d(\mathrm{OPT}) + \left(1 + \ln \frac{\delta(\emptyset)}{c(\mathrm{OPT})}\right) \times c(\mathrm{OPT}).$$

If the weights satisfy the triangle inequality, it is not hard to see that $c(\mathrm{OPT}) + d(\mathrm{OPT}) \geq \max_v c(v) \geq \delta(\emptyset)/|V|$. Thus, the previous corollary and a little algebra imply:

Corollary: *Given a graph with edge and vertex weights satisfying the triangle inequality, the set S returned by GREEDY-K-MEDIAN has cost plus distortion bounded by*

$$2 \times d(\mathrm{OPT}) + (1 + \ln |V|) \times c(\mathrm{OPT}).$$

In such graphs, if the cost of the optimal solution is much smaller than the distortion (as one might expect in a problem corresponding to a reasonably long document), these performance guarantees are fairly strong. In particular, when the ratio between the two is about $\ln |V|$, the performance guarantee is constant; when the ratio grows larger, the performance guarantee tends to 1 (this follows from the first corollary).

Test document	Codebook size			Decrease in codebook size	
	F. F.	K-means	GKM	F.F. – K-means	F.F. – GKM
CCITT1	177	157	162	11.3%	8.5%
CCITT4	164	163	224	0.6%	-36.6%
BROOKS1	408	386	281	5.4%	31.1%
BROOKS2	365	339	287	7.1%	21.4%
BROOKS3	332	309	247	6.9%	25.6%
BROOKS4	371	343	290	7.5%	21.8%
LAMBDA1	614	583	313	5.0%	49.0%
LAMBDA2	214	180	120	15.9%	43.9%
LAMBDA3	713	677	331	5.0%	53.6%
LAMBDA4	648	596	368	8.0%	43.2%
Average				7.3%	26.2%

Table 1: *Codebook size for the First Fit algorithm with and without modified k-means optimization and the GKM algorithm. With the exception of CCITT4, GKM dominates First Fit. On average, GKM has a 26% smaller codebook than First Fit, and a 19% smaller codebook than First Fit with modified k-means.*

3.4 GKM with k-means improvement

We next combine GKM with the k-means algorithm as follows:

GKM WITH MODIFIED K-MEANS(G,c,d)
0. define $\text{rate}(S, v) = (\delta(S) - \delta(S \cup \{v\}))/c(v)$
1. $S' \leftarrow \emptyset$
2. *do* $S \leftarrow S'$
3. choose $v' \in V$ that maximizes $\text{rate}(S, v')$
4. *do* $v \leftarrow v'$
5. $v' \leftarrow$ the centroid of $\{w \in V | d(w, v) = \min(d(w, S), c(w))\}$
6. *while* $\text{rate}(S, v') > \text{rate}(S, v) + \epsilon$
7. $S' \leftarrow S \cup \{v\}$
8. *while* $c(S') + \delta(S') < c(S) + \delta(S)$
9. return S

The centroid in step 5 is (some estimation of) the *optimal* center for the set of vertices (glyphs) assigned to the vertex (pattern) v. The algorithm considers it as a possible alternative to v. Similarly, we can use the modified k-means algorithm as a postprocessing step:

GKM FOLLOWED BY MODIFIED K-MEANS
1. Compute the initial equivalence classes by GREEDY-K-MEDIAN.
2. Average the glyphs in each equivalence class and creating an ideal pattern for each class.
3. Reclassify the glyphs by matching each glyph with its closest ideal pattern. Delete empty classes.
4. Go to step 2 while the number of equivalence classes is decreased by at least some constant k.

Test document	Codebook size		Decrease in codebook size
	GKM	GKM k-means	GKM–GKM k-means
CCITT1	162	142	12.3%
CCITT4	224	138	38.4%
BROOKS1	281	275	2.1%
BROOKS2	287	226	21.3%
BROOKS3	247	194	21.5%
BROOKS4	290	219	24.5%
LAMBDA1	313	303	3.2%
LAMBDA2	120	110	8.3%
LAMBDA3	331	314	5.1%
LAMBDA4	368	366	0.5%
Average			13.7%

Table 2: *Codebook size for GKM and GKM followed by modified k-means. GKM followed by modified k-means results in smaller number of pattern for all the test documents. The average improvement is 13.7%.*

4 EXPERIMENTAL RESULTS

We have implemented the GKM algorithm with and without modified k-means post-processing. $cost(c)$ is estimated as the area of each glyph. We compute $cost(c|p)$ using the spatial sampling error based cross entropy measure mentioned above [14]. We also implemented the First Fit algorithm with multi-pass optimization as used in the MGTIC system [11]. We have tested these configurations on ten high quality scanned document pages with over 20,000 glyphs. Two of the pages, CCITT1 and CCITT4, are from the CCITT test image set, scanned at 200 dpi. The other eight pages, the BROOKS series and the LAMBDA series, are from the MIT AI lab's technical document collection and are scanned at 300 dpi. We measured the performance of each codebook generation algorithm according to its use in document image compression. For a fair comparison, we adjusted pattern matching thresholds in each case so that no characters were misclassified.

Table 1 compares the sizes of the codebook produced by the First Fit codebook generation algorithm with and without the multi-pass optimization, and our GKM algorithm. In each case we used the same underlying pattern matching algorithm, EPM [14]. We use the first match version of the modified k-means optimization (used in MGTIC [11]) because it produces slightly better results and runs faster than the best match version. Compared with the First Fit algorithm, GKM reduces the codebook size by an average of 26%, a remarkable improvement. Notice that for CCITT4, the number of patterns grows. In this case, GKM shifts the costs of coding one glyph given another glyph to the costs of patterns while minimizing the total cost. The modified k-means optimization reduces the codebook size by an average of 7.2%, a smaller gain than GKM.

Table 2 compares the sizes of the codebook produced by the plain GKM algorithm and the GKM followed by modified k-means algorithm. GKM followed by modified k-means is better than plain GKM. GKM with k-means (not shown) performed slightly worse than plain GKM. The mixed results of the k-means algorithm may be due to

Test document	Lossy compression ratio		Lossless compression ratio		Size of output GKM/F.F.	
	F.F.	GKM	F.F.	GKM	(lossy)	(lossless)
CCITT1	93.7:1	95.7:1	34.5:1	35.4:1	98.2%	97.5%
CCITT4	54.9:1	44.5:1	11.2:1	10.7:1	123.4%	104.7%
BROOKS1	87.2:1	105.4:1	42.8:1	47.5:1	82.7%	90.1%
BROOKS2	75.2:1	83.8:1	21.8:1	23.1:1	89.7%	94.4%
BROOKS3	80.3:1	90.9:1	24.4:1	23.1:1	88.3%	105.6%
BROOKS4	72.5:1	80.1:1	20.8:1	22.0:1	90.5%	94.5%
LAMBDA1	54.9:1	89.8:1	30.5:1	39.3:1	61.1%	77.6%
LAMBDA2	145.0:1	236.7:1	79.4:1	101.9:1	61.6%	77.9%
LAMBDA3	48.0:1	68.0:1	13.6:1	20.6:1	70.5%	66.0%
LAMBDA4	47.9:1	75.5:1	24.0:1	29.7:1	63.4%	80.8%
Average					82.9%	88.9%

Table 3: *Overall compression ratios for plain GKM and First Fit. GKM improves compression significantly. The average lossy output of GKM is about 83% of First Fit's output.*

thresholding effects.

Table 3 compares overall compression ratios achieved using GKM and First Fit. We used the same lossy glyph position coding scheme [15] and Entropy-based Pattern Matching (EPM) [14] to compress the test images. With the exception of CCITT4, GKM dominated First Fit. The use of GKM reduced the size of the compressed documents by an average of 17%.

We also compared reconstructed images resulting from plain First Fit and GKM. As you can see in Figure 1, GKM usually picks a better quality bitmap as the pattern, resulting in better reconstructed images.

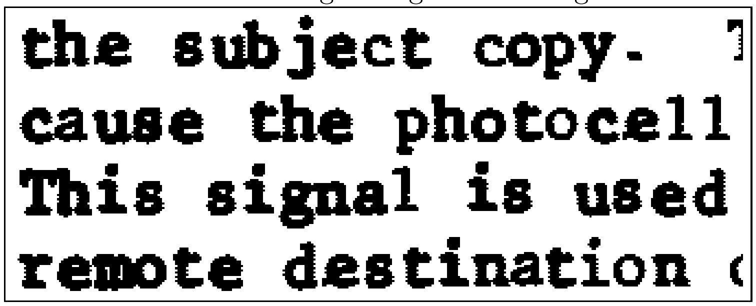
5 DISCUSSION

GKM performed surprising well compared to the popular First Fit algorithm in our experiments. We had expected to improve compression by a few percent, but we were unprepared for the 17% improvement in overall compression performance that we saw. Clearly the glyph clustering algorithm is more important than previously realized.

K-means postprocessing steps helped First Fit, but did not help GKM, presumably because GKM does a good job on its own. We had believed at first that K-means steps could not degrade a codebook, because of the performance guarantees associated with this algorithm. However, after seeing codebooks degraded, we realized that the distortion measure that K-means is guaranteed not to degrade is not ours, that our thresholded (non-linear!) distance measures do not fit into the K-means framework, and that K-means minimizes distortion, not library size. Use this "universal" algorithm with caution.

GKM runs in cubic time, taking several minutes to partition a single dense page of text on a moderately aged DEC Alpha workstation. This is far too long, although since only encoding is slow, this extra time could be tolerable for some applications.

Reconstructed image using First Fit algorithm



Reconstructed image using GKM

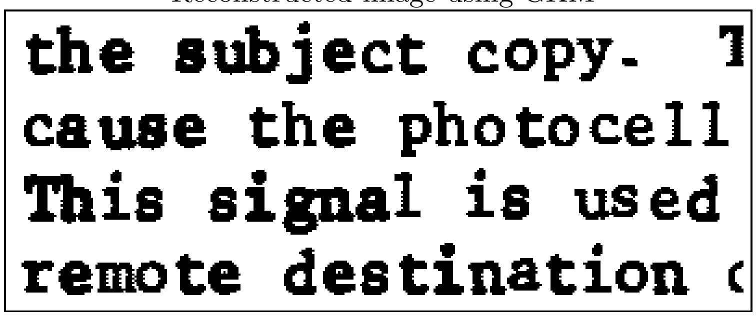


Figure 1: *A randomly selected zoomed portion of CCITT1 reconstructed by First Fit and GKM. It is clear that First Fit and GKM pick different glyphs as the representitive patterns to reconstruct the original image. In most cases, GKM picks a better quality bitmap.*

We expect investigations into efficient data structures for GKM to be fruitful.

A shortcoming in our model that we would like to address (at a later date) is the fact that we assume that all patterns are equally likely: there is no explicit pattern indexing cost in the minimization. If we could include estimates of these costs in the minimization, then patterns which are likely in some context would be favored over patterns which are unlikely in some context. Since text entropies are small compared to glyph entropies, this would be a small effect, but we might see these costs breaking ties between two patterns which match a glyph almost equally well, as often occurs with “l” and “1” and occasionally “a” and “s”. In essence we would like to develop a minimum entropy model of the document which encourages consistent spelling by taking into account the syntactic structure of the document.

References

[1] R. N. Ascher and Nagy, “A means for achieving a high degree of compaction on scan-digitized printed text,” IEEE Transactions on Computers, c-23(11):1174-1179, 1974.

[2] V. Chvatal, “A greedy heuristic for the set-covering problem”' Mathematics of Operations Research, Vol. 4, Number 3, p 233-235, 1979.

[3] T. Cormen, C. Leiserson, and R. Rivest, *Introduction to Algorithms*, pp. 974-978, The MIT Press, Cambridge, 1991.

[4] Allen Gersho and Robert Gray, *Vector Quantization and Signal Compression*, pp. 362-369, Kluwer Academic Publishers,Boston/Dordrecht/London, 1992

[5] Dorit S. Hochbaum, "Heuristics for the fixed cost median problem," Mathematical Programming 22 (1982) pp. 148-162.

[6] M. J. Holt and C.S.Xydeas, "Recent developments in image data compression for digital facsimile," ICL Technical Journal, pp. 123-146, 1986.

[7] S. Inglis and I.H.Witten, "Compression-based template matching," Proc. IEEE Data Compression Conference, pp.106-115, IEEE Computer Society Press, Los Alamitos, CA, 1994.

[8] J-H Line and J.S. Vitter, "Approximations with miminum packing constraint violation," Proc. 24th ACM Symp. on Theory of Computing, p 771-782, 1992.

[9] K. M. Mohiuddin, "Lossless binary image compression based on pattern matching," Proc. International Conference on Computers, System and Signal Processing, pp.447-451, 1984.

[10] G. Nemhauser and L. Wolsey, *Integer and Combinatorial Optimization*, pp.709-712, John Wiley & Sons, 1988.

[11] I. H. Witten, T.C. Bell, H. Emberson, and S. Inglis, "Textual image compression: two-stage lossy/lossless encoding of textual images," Proceedings of the IEEE, v. 86, No. 6, pp. 878-888, 1994.

[12] I. H. Witten, Alistair Moffat, and T. C. Bell, *Managing Gigabytes: compressing and indexing documents and images*, Van Nostrand Reinhold, New York, 1994.

[13] I. H. Witten, Alistair Moffat, T. Bell, et al. Software kit for mg. Available via anonymous ftp from munnari.oz.au in the directory /pub/mg.

[14] Qin Zhang and John M. Danskin, "Entropy-based pattern matching for document image compression," Proceedings of the International Conference on Image Processing 1996, pp 192-199, Lausanne, Switzerland, 16-19 September 1996.

[15] Qin Zhang and John M. Danskin, "A pattern-based lossy compression scheme for document images," Electronic Publishing. Vol. 8 (2&3), pp 235-246 June & September 1995.

[16] Qin Zhang and John M. Danskin, "Bitmap Reconstruction for Document Image Compression," to appear in SPIE's International Symposium on Voice, Video, and Data Communications, 18-22 November 1996, Boston, MA.