Incorporating Control Performance Tuning into Economic Model Predictive Control

Olumuyiwa I. Olanrewaju and Jan M. Maciejowski

Abstract— **Economic model predictive control (eMPC), where an economic objective is used directly as the objective function of the control system, has gained much popularity in recent literature. However, with a purely economic objective, the control designer has no influence over the control performance of the process. In this paper, we propose a means of tuning the objective function in order to give some level of control performance. Also, the stability proof for eMPC relies on some strict-dissipativity condition. We also show how this condition can be satisfied when the system is only dissipative with respect to the original objective function.**

I. Introduction

A new approach to model predictive control (MPC) where the controller directly optimizes the economic performance of the system has recently gained popularity. The main advantage of this approach, labelled 'economic MPC' (eMPC), over the conventional tracking MPC is the possible improvement in economic performance which is extracted from the transient behaviour of the process as the controller steers the system to the optimal steady state.

The use of economic performance as the controller's objective function is not entirely new [1]–[3]. However, stability proofs and performance guarantees with optimality of steady state operation have only been recently developed based on strict-dissipativity assumption [4], [5]. If the objective function is quadratic in the states and inputs, the use of economic objectives can result in indefinite costs. Analysis on the indefinite linear quadratic case has been carried out in [6] where the authors showed that any stabilizing linear quadratic regulator (LQR) can be reformulated as a positive definite LQR problem.

Unlike conventional MPC where the designer can shape the cost function to obtain a desired control performance such as good regulation against disturbances, sufficiently fast and smooth responses and control of critical signals, such flexibility is not available in eMPC since the objective function is a fixed property of the system derived mainly from the economic model of its operation which is usually to minimise economic operation cost and maximize production. This leads to economic performance optimization at the expense of control performance. Thus, a way of incorporating control performance into the objective function with minimal loss in economic performance is desired. An attempt at introducing some form of control performance tuning was introduced in [5] where a function that is positive definite around the equilibrium was added to the economic objective function. However, this implies that the objective function is no longer purely economic.

In this paper, a modified economic cost function is proposed. The approach is based on the use of a discount factor in the economic cost function to achieve strict-dissipativity and some level of control performance tuning while making a minimal modification to the cost function. In particular, we do not add a positive–definite term to it. The basic assumption made is dissipativity but not strict-dissipativity of the system with respect to the economic objective.

This paper is organized as follows: Section II contains a review of economic MPC problem formulation. The modified cost function with the proposed tuning is presented in Section III. A discussion on checking the dissipativity of the system with respect to the modified cost function and algorithms for choosing the tuning weight are presented in Section IV. An established result in the context of eMPC is that if a system is strictly dissipative with respect to the stage cost, then the closed–loop system obtained by solving the MPC problem is asymptotically stable [4], [5]. This result will be generalized for the new problem formulation in section V. Section VI contains some numerical examples while Section VII concludes the paper.

O I. Olanrewaju and J M. Maciejowski are with Department of Engineering, University of Cambridge, UK {oio24, jmm1}@cam.ac.uk

O I. Olanrewaju is sponsored by the Federal Government of Nigeria.

II. Economic MPC Problem Formulation

Consider the constrained discrete time nonlinear system

$$x_{k+1} = f(x_k, u_k) \tag{1}$$

with states $x \in \mathbb{X} \subseteq \mathbb{R}^n$, inputs $u \in \mathbb{U} \subset \mathbb{R}^m$ and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, a state transition map. The cost function is a sum of the stage costs, $l(x_k, u_k)$, and is defined as

$$J_N = \sum_{k=0}^{N-1} l(x_k, u_k) \tag{2}$$

The optimal equilibrium of the system (1) is defined as the pair (x_s, u_s) that satisfies

$$l_{(x_s,u_s)} = \min_{x,u}\{l(x,u)|x - f(x,u) = 0, (x,u) \in \mathbb{W}\} \tag{3}$$

where $\mathbb{W} \subseteq \mathbb{X}\times\mathbb{U}$ is a compact time-invariant set. (x_s, u_s) is assumed to exist and be unique throughout this paper. The stage cost $l(x_k, u_k)$ is a generic cost and not necessarily positive definite as in the case of conventional MPC (whose stage cost is positive definite by construction).

As is usually done in conventional MPC formulation, the cost function in (2) is repeatedly minimized over the horizon N in a moving horizon manner, yielding the receding horizon optimization problem

$$\begin{aligned} &\min_{\mathbf{u}} J_N(x, \mathbf{u}) \triangleq \sum_{k=0}^{N-1} l(x_k, u_k) \\ &\text{subject to} \begin{cases} x_{k+1} = f(x_k, u_k),\ \ k = 0.....N-1 \\ x_k \in \mathbb{X}, u_k \in \mathbb{U}\ ,\ \ k = 0.....N-1 \\ x_N \in \mathbb{X}_F, x_0 = x(0) \end{cases} \end{aligned} \tag{4}$$

where $x(k)$ is the measured state at time k, x_k the predicted state at time k, $\mathbb{X}_F$ is a compact terminal region containing the optimal equilibrium (3) in its interior. In this work, the origin is taken to be the optimal equilibrium.

Assuming the optimization problem (4) is feasible, it yields the optimal input sequence $\mathbf{u}^* = \{u_0^*, u_1^*, ..., u_{N-1}^*\}$. The first element of the sequence is applied to the plant yielding the control law $u(k) = u_0^* = \kappa(x(k))$ and the closed loop system

$$x(k+1) = f(x(k), \kappa(x(k))). \tag{5}$$

The standard approach to proving stability of the closed loop system (5) under conventional tracking MPC is to use the optimal cost, J_N^* as a Lyapunov function for the closed loop system [7], [8]. The basic assumptions usually made regarding the system and the objective function are:

Assumption II.1. *The system dynamics function $f(.,.)$ and the stage cost, $l(x_k, u_k)$ both satisfy $f(0,0) = 0$ and $l(0,0) = 0$.*

Assumption II.2. *The set $\mathbb{X}$ is closed and contains the origin in its interior. $\mathbb{U}$ is compact and also contains the origin in its interior. Furthermore, the set of admissible states $\mathcal{X}$ also contains the origin in its interior.*

Assumption II.3. *There exists a $\mathcal{K}_\infty$ function γ_2 such that*

$$J_N^*(x_k) \leq \gamma_2(\|x_k\|)\ \forall\ x_k \in \mathcal{X}. \tag{6}$$

A well known result in MPC literature regarding stability of the origin of the closed loop system (5) is the following:

Theorem II.1 ([8]). *Consider optimization problem (4) with the origin as a terminal constraint. Suppose Assumptions II.1–II.3 are satisfied and the stage cost is positive–definite, then the optimal cost J_N^* is a Lyapunov function for the closed loop system and the origin of the closed loop system (5) is asymptotically stable with a region of attraction $\mathcal{X}$.*

Theorem II.1 requires the stage cost $l(x_k, u_k)$ to be positive definite. However, as previously stated, the stage cost in eMPC is not necessarily positive-definite. Consequently, the stability arguments also fail. In order to prove the stability of the closed loop system, the idea of dissipativity is introduced.

Definition 1. *Following [9], the non-linear discrete-time system (1) is said to be dissipative with respect to a supply rate $\omega : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$ if there exists a storage function $\phi : \mathbb{X} \to \mathbb{R}$ such that the dissipation inequality*

$$\phi(x_{k+1}) - \phi(x_k) \leq \omega(x_k, u_k) \tag{7}$$

is satisfied for all $(x_k, u_k) \in \mathbb{W}$. Furthermore, if there exists a positive definite function $\rho : \mathbb{X} \to \mathbb{R}$ such that

$$\phi(x_{k+1}) - \phi(x_k) \leq \omega(x_k, u_k) - \rho(x_k) \tag{8}$$

then (1) is said to be strictly dissipative with respect to the supply rate ω.

The supply rate is defined as any real function that is locally absolutely summable on the bounded set $\mathbb{W}$ i.e $\sum_{k=0}^{N}|\omega(x_k, u_k)| < B$, for some $B > 0$ where $k, N \in \mathbb{Z}_+$ with $\mathbb{Z}_+$ defined as the set of all non-negative integers. For the rest of this paper, we consider

$$\omega(x_k, u_k) = l(x_k, u_k) - l(x_s, u_s).$$

III. Proposed Cost Function

In this section, we consider a modified cost function of the form:

$$J_N(x, u) = \sum_{k=0}^{N-1} (\gamma^k l(x_k, u_k)) \tag{9}$$

where γ is a positive scalar chosen such that $0 < \gamma < 1$.

Assumption III.1. *The system (1) is dissipative with respect* *to the stage cost $l(x_k, u_k)$ i.e there exists a function $\phi : \mathbb{X} \rightarrow \mathbb{R}$ such that*

$$\phi(x_{k+1}) - \phi(x_k) \leq \; l(x_k, u_k) \tag{10}$$

The 'available storage function' (ϕ_a) [9], [10] of the system with respect to the stage cost is defined as

$$\begin{aligned} \phi_a(x) = \sup_{u, x_0 = x} \quad & -\sum_{k=0}^{N} l(x_k, u_k) \\ \text{subject to} \; & \begin{cases} x_{k+1} = f(x_k, u_k) \; k = 0.....N \\ (x_k, u_k) \in \mathbb{W}, k = 0.....N \end{cases} \end{aligned} \tag{11}$$

where $-\infty < \phi_a(x) < \infty$ is equivalent to the system being dissipative [9] . Assumption III.1 guarantees $\phi_a(x)$ being finite. However, no definite statement can be made regarding the available storage function of the system with respect to the modified cost. It is possible to lose the finiteness of the available storage function when the cost function is modified (due to its indefiniteness). Hence a dissipative system can be rendered non-dissipative by our choice of γ. As such, there is the need to check whether the system is still dissipative and if yes, for what values of γ is it dissipative? Furthermore, can we make a dissipative system strictly dissipative by our choice of γ?

Remark 1. *The use of γ in the modified cost function can be given various interpretations. One of these is to view γ as a knob that helps to tune the controller. Another perspective is to treat γ as a form of discount factor which places more emphasis on the system's economic performance at the current time and less emphasis on the future performance down the prediction horizon.*

IV. Checking for Dissipativity

To ease checking of the dissipativity condition, we focus on linear quadratic cases from this section onwards. Consider the linear discrete-time system

$$x_{k+1} = Ax_k + Bu_k \tag{12}$$

with (A, B) controllable and a quadratic stage cost

$$l(x_k, u_k) = x_k^T Q x_k + u_k^T R u_k + x_k^T S u_k + u_k^T S^T x_k \tag{13}$$

with no restriction on the definiteness of the matrix $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$. Dissipativity of (12) with respect to (13) is equivalent to the existence of a storage function $\phi(x_k) = x_k^T P x_k$, $P = P^T$ such that

$$\begin{aligned} x_{k+1}^T P x_{k+1} - x_k^T P x_k \leq x_k^T Q x_k + u_k^T R u_k \\ + x_k^T S u_k + u_k^T S^T x_k \end{aligned} \tag{14}$$

(A formal proof of this equivalence when the stage cost is of the form (13) can be found in [11]). Substituting (12) for x_{k+1} in (14), the resulting expression can be written as

$$\begin{bmatrix} x_k \\ u_k \end{bmatrix}^T \begin{bmatrix} A^T P A - P - Q & A^T P B - S \\ (A^T P B - S)^T & B^T P B - R \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} \leq 0. \tag{15}$$

Thus, we can set up an LMI to find a symmetric P such that

$$\begin{bmatrix} A^T P A - P - Q & A^T P B - S \\ (A^T P B - S)^T & B^T P B - R \end{bmatrix} \leq 0. \tag{16}$$

Feasibility of the LMI (16) implies dissipativity while non-feasibility implies otherwise as feasibility of (16) is a necessary and sufficient condition for the dissipativity of a system of the form (12) with respect to a supply rate of the form (13) [11].

A. Checking for Dissipativity of the System with respect to the Modified Cost

Lemma IV.1. *Consider the linear discrete-time system (12) with the modified stage cost $\gamma^k l(x_k, u_k)$ where $l(x_k, u_k)$ is of the form (13) and $0 < \gamma < 1$. Dissipativity of (12) with respect to the modified stage cost implies the existence of a symmetric matrix P such that the LMI*

$$\begin{bmatrix} \gamma A^T P A - P - Q & \gamma A^T P B - S \\ (\gamma A^T P B - S)^T & \gamma B^T P B - R \end{bmatrix} \leq 0 \tag{17}$$

is feasible.

Proof. Dissipativity of the system (12) with respect to the modified stage cost is equivalent to the existence of a storage function $\phi(x_k) = x_k^T P x_k$ such that

$$\begin{aligned} \phi(x_{k+1}) - \phi(x_k) &\leq \gamma^k l(x_k, u_k) \\ &\leq \gamma^k (x_k^T Q x_k) + \gamma^k (u_k^T R u_k) \\ &\quad + \gamma^k (x_k^T S u_k) + \gamma^k (u_k^T S^T x_k) \\ &= \gamma^{\frac{k}{2}} x_k^T Q \gamma^{\frac{k}{2}} x_k + \gamma^{\frac{k}{2}} u_k^T R \gamma^{\frac{k}{2}} u_k \\ &\quad + \gamma^{\frac{k}{2}} x_k^T S \gamma^{\frac{k}{2}} u_k + \gamma^{\frac{k}{2}} u_k^T S^T \gamma^{\frac{k}{2}} x_k \end{aligned} \tag{18}$$

Now, consider the transformation

$$\tilde{x}_k = \gamma^{\frac{k}{2}} x_k, \tilde{u}_k = \gamma^{\frac{k}{2}} u_k$$

such that

$$\begin{aligned} \tilde{x}_{k+1} &= \gamma^{\frac{k+1}{2}} x_{k+1} \\ &= \gamma^{\frac{k+1}{2}} (Ax_k + Bu_k) \\ &= \gamma^{\frac{k+1}{2}} Ax_k + \gamma^{\frac{k+1}{2}} Bu_k \\ &= \gamma^{\frac{1}{2}} \gamma^{\frac{k}{2}} Ax_k + \gamma^{\frac{1}{2}} \gamma^{\frac{k}{2}} Bu_k \\ &= \gamma^{\frac{1}{2}} A\tilde{x}_k + \gamma^{\frac{1}{2}} B\tilde{u}_k \end{aligned} \tag{19}$$

with a storage function $\tilde{x}_k^T P \tilde{x}_k$ in the transformed state space representation (19). Using (15), dissipativity inequality (18) can be expressed in the form

$$\begin{bmatrix} \tilde{x}_k \\ \tilde{u}_k \end{bmatrix}^T \begin{bmatrix} \gamma^{\frac{1}{2}} A^T P \gamma^{\frac{1}{2}} A - P - Q & \gamma^{\frac{1}{2}} A^T P \gamma^{\frac{1}{2}} B - S \\ (\gamma^{\frac{1}{2}} A^T P \gamma^{\frac{1}{2}} B - S)^T & \gamma^{\frac{1}{2}} B^T P \gamma^{\frac{1}{2}} B - R \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ \tilde{u}_k \end{bmatrix} \leq 0 \tag{20}$$

The dissipativity (or otherwise) can then be confirmed by checking for the existence of a symmetric matrix P that guarantees feasibility of the LMI (17). □

B. Choosing γ

As earlier stated in Section III, not all values of γ can ensure dissipativity of the system with respect to the modified cost. In this section, we describe an algorithm for finding values of γ that ensure dissipativity (and strict-dissipativity) of the system with respect to the modified cost. We consider linear discrete-time systems of the form (12) with stage cost $\gamma^k l(x_k, u_k)$ where $l(x_k, u_k)$ is quadratic and of the form (13). The search will be for γ and P values that satisfy (17). Due to the product in γ and P, (17) is a Bilinear Matrix Inequality (BMI) and cannot be solved directly using linear semi-definite programming approaches. Thus, we construct a bisection algorithm to solve the problem.

Algorithm IV.2.

- *Since γ is constrained to be between 0 and 1, set the lower bound on γ as 0 and the upper bound as 1.*
- *Start a bisection to find the minimum value of γ such that there exists a symmetric P such that*
$$\begin{bmatrix} \gamma A^T P A - P - Q & \gamma A^T P B - S \\ (\gamma A^T P B - S)^T & \gamma B^T P B - R \end{bmatrix} \leq 0.$$
This is γ_m.
- *Set the lower bound on γ as γ_m. Define $M = \epsilon I_{n_x + n_u}$ for a fixed and small $\epsilon > 0$ where n_x is the number of states and n_u is the number of inputs. Start a bisection algorithm to find the minimum value of γ such that there exists a symmetric P such that*
$$\begin{bmatrix} \gamma A^T P A - P - Q & \gamma A^T P B - S \\ (\gamma A^T P B - S)^T & \gamma B^T P B - R \end{bmatrix} + M \ \leq 0.$$
This is γ_l.
- *Set the lower bound on γ as γ_l. With M as defined above, start a bisection algorithm to find the maximum value of γ such that there exists a symmetric P such that*
$$\begin{bmatrix} \gamma A^T P A - P - Q & \gamma A^T P B - S \\ (\gamma A^T P B - S)^T & \gamma B^T P B - R \end{bmatrix} + M \ \leq 0.$$
This is γ_u.

Remark 2. *The use of ϵ is to guarantee strictness of inequality (17) while searching for γ_l and γ_u and appropriate values vary depending on the system-cost-function interaction. If chosen too big, there may be no γ and P pair for which the inequality holds.*

The modified cost function as earlier defined in (9) is

$$J_N(x, u) = \textstyle\sum_{k=0}^{N-1} (\gamma^k l(x_k, u_k))$$

where $0 < \gamma < 1$. Consider the sequence

$$\begin{aligned} \{\gamma_n\} &= \gamma^k, \ \ k = 0, 1, 2, 3, ..., N-1 \\ &= 1, \gamma^1, \gamma^2, \gamma^3, ..., \gamma^{N-1} \end{aligned}$$

Define $\gamma = \gamma_u, \ \ \gamma^{m-1} = \gamma_l$ and solve to get m:

$$m = \frac{\ln \gamma_l}{\ln \gamma_u} + 1 \tag{21}$$

Thus for any value of γ that satisfies $\gamma_l \leq \gamma^k \leq \gamma_u$ where $k > 0$, we are assured of strict dissipativity of the stage cost $\gamma^k l(x_k, u_k)$ over a horizon $N \leq m$ where N is a positive integer.

Remark 3. *Having an upper bound on the horizon length may seem counter-intuitive since in conventional MPC, stability is guaranteed with longer horizons. However, when dealing with indefinite costs, the conventional MPC thinking does not always apply.*

V. Stability Analysis

We now consider the stability of the system defined by the optimization problem:

$$\min_{\mathbf{u}} J_N(x, \mathbf{u}) \triangleq \sum_{k=0}^{N-1} \gamma^k l(x_k, u_k)$$
$$\text{subject to} \begin{cases} x_{k+1} = f(x_k, u_k) \ \forall k = 0.....N-1 \\ x_k \in \mathbb{X}, u_k \in \mathbb{U} \ \forall \ k = 0.....N-1 \\ x_N = x_s, x_0 = x(0) \end{cases} \tag{22}$$

where $0 < \gamma < 1$.

Assuming feasibility of the optimization problem (22), the first element of the optimal input sequence $\mathbf{u}^* = \{u_0^*, u_1^*, ..., u_{N-1}^*\}$ is applied to the plant resulting in the feedback control law $u(k) = u_0^* = \kappa(x(k))$ and the closed loop system

$$x(k+1) = f(x(k), \kappa(x(k))). \tag{23}$$

Next, we make an assumption regarding the dissipativity of the system with respect to the modified stage cost, $\gamma^k l(x_k, u_k)$.

Assumption V.1. *Suppose that the stage cost in (22) is used and that the system is strictly dissipative with respect to that cost i.e there exists ϕ and ρ as in Definition 1 such that*

$$\phi(x_{k+1}) - \phi(x_k) \leq \gamma^k l(x_k, u_k) - \rho(x_k) \qquad \forall \ (x_k, u_k) \in \mathbb{W} \tag{24}$$

We assume that γ_l and γ_u exist as defined in Section IV-B and that the prediction horizon N is such that

$$N \leq \frac{\ln \gamma_l}{\ln \gamma_u} + 1$$

Theorem V.1. *The solution sets of the optimization problem (22) and the optimization problem defined by*

$$\min_{\boldsymbol{u}} \tilde{J}_N(x, \boldsymbol{u}) \triangleq \sum_{k=0}^{N-1} \tilde{L}(x_k, u_k)$$
$$\textit{subject to} \begin{cases} x_{k+1} = f(x_k, u_k) \ \forall k = 0.....N-1 \\ x_k \in \mathbb{X}, u_k \in \mathbb{U} \ \forall \ k = 0.....N-1 \\ x_N = x_s, x_0 = x(0) \end{cases} \tag{25}$$

are identical where

$$\tilde{L}(x_k, u_k) = \gamma^k l(x_k, u_k) + \phi(x_k) - \phi(x_{k+1}) \tag{26}$$

is defined as the 'rotated' stage cost and $\phi(.)$ is the storage function from (24) with $0 < \gamma < 1$.

Proof. The proof follows the same arguments as in [4]. From (25),

$$\begin{aligned} \tilde{J}_N(x, \mathbf{u}) &= \sum_{k=0}^{N-1} \tilde{L}(x_k, u_k) \\ &= \sum_{k=0}^{N-1} (\gamma^k l(x_k, u_k) + \phi(x_k) - \phi(x_{k+1})) \\ &= \sum_{k=0}^{N-1} (\gamma^k l(x_k, u_k)) + \phi(x_0) - \phi(x_N) \\ &= J_N(x, \mathbf{u}) + \underbrace{\phi(x_0) - \phi(x_s)}_{\text{independent of } \mathbf{u}} \end{aligned} \tag{27}$$

Since the difference between both objective functions is independent of the optimization variable ($\mathbf{u}$) and both optimization problems are subject to same constraints, it can be concluded that optimization problems (22) and (25) have equivalent solution sets. ☐

Theorem V.2. *Consider the closed loop system defined by (23). Let Assumptions II.1– II.3 and V.1 hold. Then the origin of the closed loop system is asymptotically stable.*

Proof. The proof follows similar arguments as in [5]. As a consequence of Theorem V.1, stability of the origin of closed loop system (23) can be proven by using the optimal cost $\tilde{J}_N(x, \mathbf{u}^*)$ as a candidate Lyapunov function for the closed loop system. Combining (26) with Assumption V.1, we have that

$$\tilde{L}(x_k, u_k) = \gamma^k l(x_k, u_k) + \phi(x_k) - \phi(x_{k+1}) \geq \rho(x_k) \tag{28}$$

Moreover for all $x \in \mathcal{X}$, there exists a class K function γ_1 such that $\rho(x_k) \geq \gamma_1(\|x_k\|)$. As such, we have that

$$\tilde{L}(x_k, u_k) \geq \gamma_1(\|x_k\|) \tag{29}$$

which implies that the rotated stage cost is bounded below by a $\mathcal{K}_\infty$ function with Assumption V.1 ensuring (29) holds over the horizon N. Hence, by Theorem II.1, the optimal cost $\tilde{J}_N^*$ is a Lyapunov function for the closed loop system (23) and the origin is asymptotically stable. ☐

VI. EXAMPLES

This section illustrates the results obtained using the cost function proposed in section III. The optimization problems are solved using an interior point algorithm-based solver.

A. Example 1

Consider the linear discrete-time system (12) where

$$A = \begin{bmatrix} -0.4 & 0.8 \\ -0.4 & 0.1 \end{bmatrix} \quad B = \begin{bmatrix} 0.1 \\ -1.6 \end{bmatrix}$$

and an indefinite quadratic stage cost (13) with

$$Q = \begin{bmatrix} -1 & -0.08 \\ -0.08 & 1.7 \end{bmatrix}, \ R = 0.5, \ S = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}$$

The system is only dissipative with respect to the stage cost and not strictly dissipative. Based on existing stability proofs [5], the asymptotic stability of the equilibrium cannot be guaranteed. Now consider the proposed stage cost $\gamma^k(l(x_k, u_k))$. Running Algorithm IV.2 with $\epsilon = 0.01$ gives $\gamma_m = 0.001$, $\gamma_l = 0.001$, $\gamma_u = 0.9913$ and $m \simeq 788$. This implies that strict dissipativity can be guaranteed using the modified stage cost up to a prediction horizon, $N \leq 788$ for $\gamma = 0.9913$. Hence, the origin of the closed loop system is guaranteed to be asymptotically stable.

Shown in Figure 1 are the closed loop system trajctories for different values of γ when the system is simulated from an initial condition $x_0 = [2, 1]$ over a prediction horizon $N = 40$. $\gamma = 1$ represents the nominal case i.e when no tuning is incorporated. Also shown in Table I is the average cost of each scheme computed using the actual cost over the simulation period. It shows the trade-off between γ, cost of operation and the number of time steps (T_{co}) it takes to converge to within 1% of the origin.

TABLE I
PERFORMANCE COMPARISON FOR DIFFERENT γ

ϵ	γ	$\tilde{m}$	N	T_{co}	Avg. Cost
-	1	-	40	> 50	-0.0853
0.01	0.9913	788	40	$\simeq 40$	-0.0853
0.05	0.9479	129	40	$\simeq 20$	-0.0847
0.1	0.8519	44	40	$\simeq 13$	-0.0840

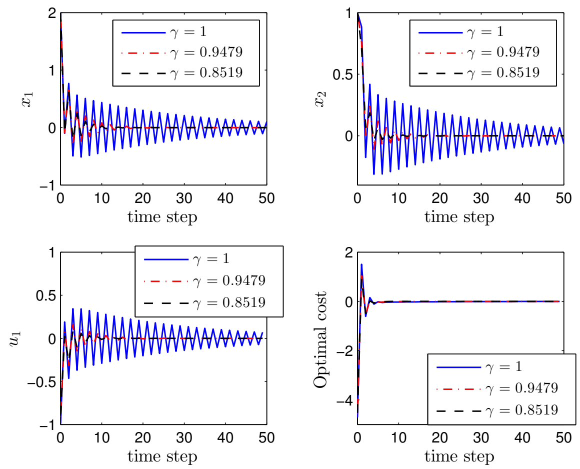

Fig. 1. Comparison of closed loop trajectories for different values of γ with $x_0 = [2, 1]$

From Figure 1 and Table I, it is seen that not only can a dissipative system be made strictly dissipative by using the proposed cost function, the rate of convergence can also be tuned with a trade-off between the rate of convergence and economic performance.

B. Example 2

In this second example, we show the effect of a wrongly chosen γ on the closed loop system. The system considered is

$$x_{k+1} = 1.6x_k - 0.6u_k \tag{30}$$

with the stage cost

$$l(x_k, u_k) = -x_k^2 + \frac{2}{3}u_k^2 + \frac{5}{3}x_k u_k \tag{31}$$

The system (30) is strictly dissipative with respect to the stage cost (31). Using the proposed stage cost with $\epsilon = 0.01$ and running Algorithm IV.2 yields $\gamma_u = 0.999$, $\gamma_l = 0.6106$, $\gamma_m = 0.6069$ and $m \simeq 493$. This shows that if γ is chosen such that $\gamma^k < 0.6069$ for some $k > 0$, the strict dissipativity property of the system may be lost.

Shown in Figure 2 are the closed loop state and input trajectories for different values of γ when initialized from $x_0 = 1$ over a prediction horizon of 10 with state and input magnitude constraints $|x_k| \leq 2$, $|u_k| \leq 2$.

As shown, as the value of γ reduces, the time of convergence to the origin increases and when $\gamma = 0.63$, the system is not asymptotically stable.

VII. Conclusion

A tuning scheme that incorporates the use of a forgetting factor was proposed in this paper in order to guarantee the strict-dissipativity condition needed to prove asymptotic stability and also incorporate some level of control performance. We also showed how the tuning weight can be chosen and its effect on the performance of the closed loop system. To ease checking of the dissipativity condition, we focused on linear quadratic cases. Checking dissipativity for nonlinear systems and non-quadratic costs is much harder and will be addressed in future work.

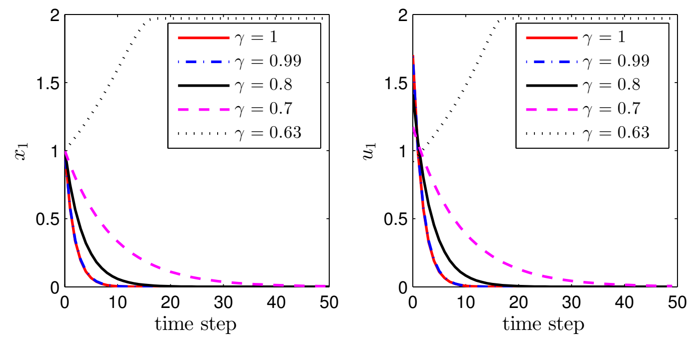

Fig. 2. Comparison of closed loop trajectories for different values of γ with $x_0 = 1$

References

[1] A. Singh, J. Forbes, P. Vermeer, and S. Woo, "Model-based real-time optimization of automotive gasoline blending operations," *Journal of Process Control*, vol. 10, no. 1, pp. 43 – 58, 2000.

[2] A. Toumi and S. Engell, "Optimization-based control of a reactive simulated moving bed process for glucose isomerization," *Chemical Engineering Science*, vol. 59, no. 18, pp. 3777 – 3792, 2004.

[3] T. Hovgaard, K. Edlund, and J. Bagterp Jorgensen, "The potential of economic MPC for power management." in *Proceedings of the 49th IEEE Conference on Decision and Control, Atlanta, USA*, Dec 2010, pp. 7533–7538.

[4] R. Amrit, J. B. Rawlings, and D. Angeli, "Economic optimization using model predictive control with a terminal cost," *Annual Reviews in Control*, vol. 35, no. 2, pp. 178 – 186, 2011.

[5] D. Angeli, R. Amrit, and J. Rawlings, "On average performance and stability of economic model predictive control," *Automatic Control, IEEE Transactions on*, vol. 57, no. 7, pp. 1615–1626, 2012.

[6] M. Zanon, S. Gros, and M. Diehl, "Indefinite linear MPC and approximated economic MPC for nonlinear systems," *Journal of Process Control*, vol. 24, no. 8, pp. 1273 – 1281, 2014.

[7] D. Mayne, J. Rawlings, C. Rao, and P. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789 – 814, 2000.

[8] J. B. Rawlings and D. Q. Mayne, *Model Predictive Control Theory and Design*. Nob Hill Publishers, 2009.

[9] J. C. Willems, "Dissipative dynamical systems," *European Journal of Control*, vol. 13, pp. 134 – 151, 2007.

[10] C. Scherer and S. Weiland, "Linear matrix inequalities in control," *Lecture Notes, Dutch Institute for Systems and Control, Delft, The Netherlands*, 2009.

[11] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*. Englewood Cliffs, New Jersey 07632: Prentice-Hall, Inc., 1984.