

Miscorrection-free Decoding of Staircase Codes

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Abstract We propose a novel decoding algorithm for staircase codes which reduces the effect of undetected component code miscorrections. The algorithm significantly improves performance, while retaining a low-complexity implementation suitable for high-speed optical transport networks.

Introduction

Hard-decision forward error correction (HD-FEC) can offer dramatically reduced complexity compared to soft-decision FEC, at the price of some performance loss. HD-FEC is used, for example, in regional/metro optical transport networks (OTNs)¹ and has also been considered for other cost-sensitive applications such as optical data center interconnects². Our focus is on staircase codes³, which provide excellent performance and have received considerable attention in the literature.

Similar to classical product codes, staircase codes are built from short component codes and decoded by iteratively applying bounded-distance decoding (BDD) to the component codes. For the purpose of this paper, BDD of a t -error-correcting component code can be seen as a black box that operates as follows. Let $\mathbf{r} = \mathbf{c} + \mathbf{e}$, where $\mathbf{c}, \mathbf{e} \in \{0, 1\}^n$ denote a component codeword and random error vector, respectively, and n is the code length. BDD yields the correct codeword \mathbf{c} if $d_{\text{H}}(\mathbf{r}, \mathbf{c}) = w_{\text{H}}(\mathbf{e}) \leq t$, where d_{H} and w_{H} denote Hamming distance and weight, respectively. On the other hand, if $w_{\text{H}}(\mathbf{e}) > t$, the decoding either fails or there exists another codeword \mathbf{c}' such that $d_{\text{H}}(\mathbf{r}, \mathbf{c}') \leq t$. In the latter case, BDD is technically successful but the decoded codeword \mathbf{c}' is not the correct one. Such *miscorrections* are highly undesirable because they introduce additional errors into the iterative decoding process and significantly degrade performance.

In this paper, we propose a novel iterative HD decoding algorithm for staircase codes which can detect and avoid most miscorrections. The algorithm provides significant post-FEC bit error rate improvements, in particular when t is small (which is typically the case in practice). As an example, for $t = 2$, the algorithm can improve performance by roughly 0.4 dB and reduce the error floor by over an order of magnitude, up to the point where the iterative decoding process is virtually miscorrection-free. Error floor improvements are particularly important for applications with stringent reliability constraints such as OTNs.

Staircase codes and iterative decoding

Let \mathcal{C} be a binary linear component code with length n and dimension k . Assuming that n is even, a staircase code with rate $R = 2k/n - 1$ based on \mathcal{C} is defined as the set of all matrix sequences $\mathbf{B}_k \in \{0, 1\}^{a \times a}$, $k = 0, 1, 2, \dots$, such that the rows in $[\mathbf{B}_{k-1}^T, \mathbf{B}_k]$ for all $k \geq 1$ form valid codewords of \mathcal{C} , where $a = n/2$ is the block size and \mathbf{B}_0 is the all-zero matrix.

We use extended primitive Bose–Chaudhuri–Hocquenghem (BCH) codes as component codes, i.e., a BCH code with an additional parity bit formed by adding (modulo 2) all $2^\nu - 1$ coded bits of the BCH code, where ν is the Galois field extension degree. The overall extended code then has length $n = 2^\nu$ and guaranteed dimension $k = 2^\nu - \nu t - 1$. The extra parity bit increases the guaranteed minimum distance to $d_{\text{min}} = 2t + 2$.

The conventional decoding procedure for staircase codes uses a sliding window comprising W received blocks $\mathbf{B}_k, \mathbf{B}_{k+1}, \dots, \mathbf{B}_{k+W-1}$. This is illustrated in Fig. 1 for $W = 5$ and $a = 6$. It is convenient to identify each component code in the window by a tuple (i, j) , where $i \in \{1, 2, \dots, W - 1\}$ indicates the position relative to the current decoding window and $j \in \{1, 2, \dots, a\}$ enumerates all codes at a particular position. As an example, the component codes (1, 3) and (4, 4) are highlighted in blue in Fig. 1. Pseudocode for the conventional decoding procedure is given in Algorithm 1 below. Essentially, all component codes are decoded ℓ times, after which the decoding window shifts to the next position. Note that after the window shifts, the same component code is identified by a different position index.

Performance analysis

Analyzing the post-FEC bit error rate of staircase codes under the conventional decoding procedure is challenging. A major simplification is obtained by assuming that no miscorrections occur in the BDD of the component codes. In this case, it is possible to rigorously characterize the asymptotic performance as $a \rightarrow \infty$ using a technique called density evolution⁴. Moreover, the error floor can be estimated by enumerating stopping sets, also known as stall patterns³. However, if miscorrections are taken into account, both the asymptotic and error floor predictions are nonrigorous and become inaccurate.

Algorithm 1: Window decoding of staircase codes

```
1  $k \leftarrow 0$ 
2 while true do
3   for  $l = 1, 2, \dots, \ell$  do
4     for  $i = W, W - 1, \dots, 1$  do
5       for  $j = 1, 2, \dots, a$  do
6         apply BDD to component code  $(i, j)$ 
7   output decision for  $\mathbf{B}_k$  and shift window
8    $k \leftarrow k + 1$ 
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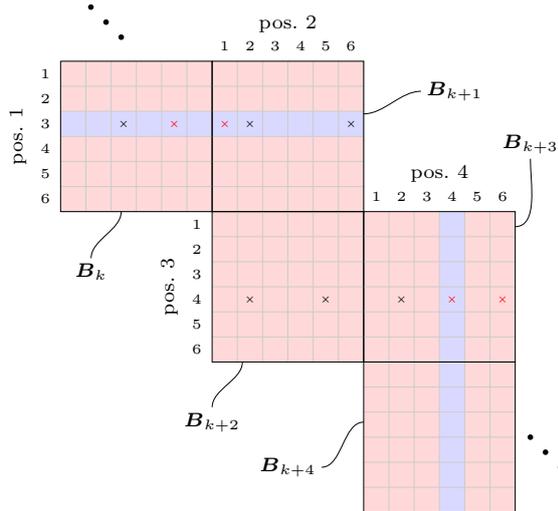


Fig. 1: Staircase decoding window of size $W = 5$

Example 1: Let $\nu = 8$ and $t = 2$, which gives a staircase code with $a = 128$ and $R = 0.867$. For window decoding parameters $W = 8$ and $\ell = 7$, the density evolution and error floor predictions are shown in Fig. 2 by the dashed lines. The analysis can be verified by performing idealized decoding, where miscorrections are prevented during BDD. The results are shown by the blue line (triangles) in Fig. 2 and accurately match the theoretical predictions. However, the actual performance with true BDD deviates from the idealized decoding, as shown by the red line (squares). \triangle

The performance degradation with respect to idealized decoding becomes less severe for larger values of t . Unfortunately, small values of t are commonly used in practice because BDD can be implemented very efficiently in this case. We note at this point that there exist several works that attempt to quantify the performance loss due to miscorrections. In terms of error floor, the work in³ introduces a heuristic parameter, whose value unfortunately has to be estimated from simulative data. In terms of asymptotic performance, the authors are aware of two works^{5,6}, both of which do not directly apply to staircase codes, but to a related code ensemble.

Proposed algorithm

The main idea in order to improve performance is to systematically exploit the fact that miscorrections lead to inconsistencies, in the sense that two component codes that protect the same bit may disagree on its value. In the following, we show how these inconsistencies can be used to (a) reliably prevent miscorrections and (b) identify miscorrected codewords in order to revert their decoding decisions.

Our algorithm relies on so-called anchor codewords, which have presumably been decoded without miscorrections. Roughly speaking, we want to make sure that bit flips do not lead to inconsistencies with anchor codewords. Consequently, decoding decisions from codewords that are in conflict with anchors are not applied. However, a small number of anchor codewords may be miscorrected and we allow for the decoding decisions

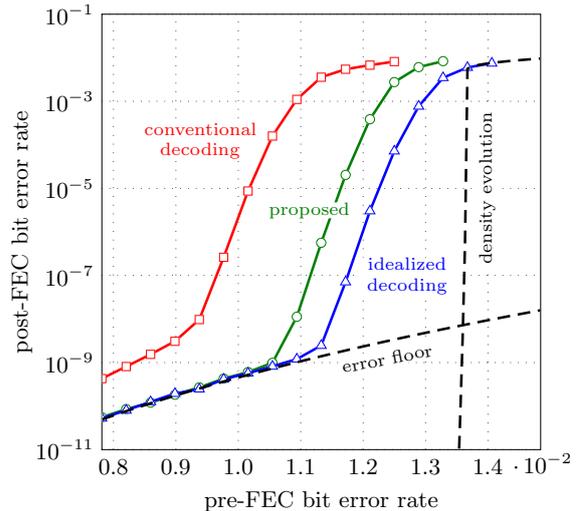


Fig. 2: Results for component codes with $n = 256$ and $t = 2$

of anchors to be reverted if too many other codewords are in conflict with a particular anchor.

In order to make this more precise, we regard the BDD of a component code (i, j) as a two-step process. In the first step, the decoding is performed and the outcome is either a set of error locations $\mathcal{E}_{i,j} \subset \{1, 2, \dots, n\}$, where $|\mathcal{E}_{i,j}| \leq t$, or a decoding failure. In the second step, error-correction is performed by flipping the bits corresponding to the error locations. Initially, we only perform the decoding step for all component codes, i.e., all component codes in the decoding window are decoded without applying any bit flips. We then iterate ℓ times over the component codes in the same fashion as in Algorithm 1, but replacing line 6 with the following four steps:

1. If no decoding failure occurred for the component codeword (i, j) , we proceed to step 2, otherwise, we skip to the next codeword.
2. For each $e \in \mathcal{E}_{i,j}$, check if e corresponds to an anchor codeword. If so, let C be the number of other conflicts that this anchor is involved in. If $C < T$, where T is a fixed threshold, the codeword (i, j) is frozen and we skip to the next codeword. Frozen codewords are always skipped (in the loop of Algorithm 1) for the rest of the decoding unless any of their bits change. If $C \geq T$, the anchor is marked for backtracking.
3. Error-correction for codeword (i, j) is applied, i.e., the bits at all error locations in $\mathcal{E}_{i,j}$ are flipped. We also apply the decoding step again for codewords that had their syndrome changed due to a bit flip. Finally, the codeword (i, j) becomes an anchor.
4. Lastly, previously applied bit flips are reversed for all anchor codewords that were marked for backtracking during step 2. These codewords are no longer anchors and all frozen codewords that were in conflict with these codewords are unfrozen.

Note that steps 3 and 4 are not reached for codeword (i, j) if the corresponding bit flips of that codeword are inconsistent with any anchor for which $C < T$ holds.

The following two examples illustrate the above steps for $t = 2$ and $T = 1$ with the help of Fig. 1.

Example 2: Assume that we are at $(i, j) = (3, 4)$, corresponding to a component code with three attached errors shown by the black crosses. The codeword is miscorrected with $\mathcal{E}_{3,4} = \{10, 12\}$ shown by the red crosses. Assuming that the codeword $(4, 4)$ is an anchor without any other conflicts, the codeword $(3, 4)$ is frozen during step 2 and no bit flips are applied. \triangle

Example 3: Let the codeword $(1, 3)$ in Fig. 1 be a miscorrected anchor without conflicts and error locations $\mathcal{E}_{1,3} = \{5, 7\}$. Assume that we are at $(i, j) = (2, 1)$. The codeword $(2, 1)$ has one attached error, thus $\mathcal{E}_{2,1} = \{3\}$. During step 2, the codeword $(2, 1)$ is frozen and we skip to codeword $(2, 2)$ with $\mathcal{E}_{2,2} = \{3, 10\}$. The bit flip at $e = 3$ is inconsistent with the anchor $(1, 3)$, but, since this anchor is already in conflict with $(2, 1)$ (and, hence, $C = T = 1$), the anchor is marked for backtracking. The bits according to $\mathcal{E}_{2,2}$ are then flipped in step 3 and the anchor $(1, 3)$ is backtracked in step 4. \triangle

The previous example shows how a miscorrected anchor is backtracked. Since we do not know if an anchor is miscorrected or not, it is also possible that we mistakenly backtrack “good” anchors. Fortunately, this is unlikely to happen for long component codes because the additional errors due to miscorrections are approximately randomly distributed within the codeword. This implies that errors of two (or more) miscorrected codewords rarely overlap.

For the algorithm to work well, a sufficiently large fraction of codewords at each position should be “good” anchors. However, when the decoding window shifts and a new block is added, no anchors exist at the last position $W - 1$. We found that it is therefore beneficial to artificially restrict the error-correcting capability of these component codes in order to avoid anchoring too many miscorrected codewords. For example, for $t = 2$, all component codes at position $W - 1$ are treated as single-error-correcting. This restriction reduces the probability of miscorrecting a component code by roughly a factor of n , which is significant for long component codes⁷. Note that due to the window decoding, we are merely gradually increasing the error-correction capability: once the decoding window shifts, the component codes shift as well and they are then decoded with their full error-correcting capability.

We remark that essentially the same algorithm can also be applied to product codes and other related code constructions, e.g., half-product or braided codes.

Decoding complexity

In terms of decoding complexity, one of the main advantages of iterative HD decoding of staircase codes compared to message-passing decoding of LDPC codes is the significantly reduced decoder data flow requirement³. While a thorough complexity analysis for the proposed algorithm is beyond the scope of this paper, we note that the algorithm can operate entirely in the syndrome domain, thereby leveraging the syndrome compression effect that is described in³. How-

ever, additional storage is needed compared to the conventional decoding to keep track of the error locations of anchor codewords (in case they are backtracked) and to store the conflicts between codewords.

Results and Discussion

We consider the same parameters as in Example 1, i.e., $\nu = 8$, $t = 2$, $W = 8$, and $\ell = 7$. The conflict threshold is set to $T = 1$ and we apply the error-correction capability restriction for component codes at position $W - 1$ as described above. Simulation results for the proposed algorithm are shown in Fig. 2 by the green line (circles). It can be seen that the performance is significantly improved compared to the conventional decoding. In particular in terms of the error floor, the performance is virtually identical to the idealized decoding where miscorrections are prevented. Overall, the improvements translate into an additional coding gain of around 0.4 dB at a post-FEC bit error rate of 10^{-9} over the conventional decoding.

Note that the staircase code parameters were chosen such that the error floor is high enough to be within the reach of software simulations. For OTNs, post-FEC bit error rates below 10^{-15} are typically required. In this case, other code parameters should be used or one may apply post-processing techniques to reduce the error floor below the application requirements⁸.

Conclusion

We have shown that the post-FEC performance of staircase codes can be significantly improved by adopting a modified iterative HD decoding algorithm that reduces the effect of miscorrections. For component codes with error-correcting capability $t = 2$, an additional coding gain of around 0.4 dB can be achieved. Moreover, the error floor can be reduced by over an order of magnitude, giving virtually miscorrection-free performance.

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