Performance Metrics for Communication Systems with Forward Error Correction

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Abstract We revisit performance metrics for optical communication systems with FEC. We illustrate the concept of universality and discuss the most widespread performance thresholds. Finally, we show by example how to include FEC into transmission experiments.

Introduction

The design of many optical communication systems requires the heavy use of transmission experiments to verify models, assumptions and complete systems. This is mostly due to the absence of a rigorous and widely accepted channel model that includes all the effects and impairments of transceivers and optical fibers. The fiber-optical communication channel is however rather static, enabling the relatively easy reproducibility of the experiments.

With the advent of coherent optical communications and in particular soft decision decoding (SDD)¹, the widely used pre-FEC bit error rate (BER) threshold was no longer an acceptable metric. This was first realized in² for early coherent system and performance metrics based on information theory were proposed. In³, this approach was extended to bit-interleaved coded modulation (BICM), the de-facto coded modulation (CM) standard for today's systems. It has now become customary to characterize a forward error correction (FEC) code by an achievable rate and output BER, where the achievable rate depends on the CM scheme used. In transmission experiments, an estimate of the achievable rate is computed and used to evaluate if the BER can be achieved or not. This assumes (often without stating so) that the utilized FEC code is universal.

In this paper, we will first introduce the concept of *universality* and then give some hints on the achievable rate to be used in various circumstances. We wrap the paper by illustrating how to include FEC into transmission experiments.

Threshold-based FEC Performance Prediction While thresholds are a perfectly fine tool for predicting the performance of some FEC schemes with hard decision decoding (HDD), the use of FEC schemes with SDD together with varying modulation formats and transmission links requires more caution. This is due to the fact that SDD is a statistical estimation process that critically relies on the knowledge of the channel model. In this scenario, the concept of FEC universality is crucial when using *thresholds*. A pair of FEC code and its decoder are said to be *universal*, if the performance of the code (in terms of post-FEC BER) does not depend on the channel (whole chain between the FEC encoder output x and the decoder log-likelihood ratio (LLR) input l, including modulation/demodulation, DSP, ADC and DAC, fiber transmission, filtering and amplification including noise), provided that the CM scheme and the achievable rate (e.g., the GMI) are fixed and identical.

Many practical low-density parity-check (LDPC) codes are conjectured to be approximately universal⁴. Polar codes are examples of nonuniversal codes that need to be redesigned for every different channel. We can define universality⁵ using Fig. 1. We assume BICM and bit-wise SDD. An FEC encoder generates a codeword of *n* bits. We transmit these bits over 2 channels with different (memoryless) channel transition probability density functions (PDFs): Channel 1 with PDF $p(y_1|x_1)$ and channel 2 with PDF $p(y_2|x_2)$. Both channels have *identical* GMI. A fraction γn of the bits is transmitted over channel 1, while the remaining bits are transmitted over channel 2. At the receiver, bit metric decoding (BMD) is carried out (Φ^{-1}) and the combined sequence is decoder. A code is universal for channels 1 and 2 if the post-FEC BER is *independent* of γ .

In⁶, the impact of strong quantization of the channel output on universality was shown for CM with nonbinary LDPC codes: quantization led to significantly different performance for constant achievable rate (MI). We illustrative the concept of non-universality of a common FEC decoder with BICM. We consider a regular QC-LDPC code of rate R = 0.8 with BICM and 5 different constellations: QPSK, 8QAM (C_3 of Fig. 3 in⁶), 16QAM, 32QAM, and 64QAM. Let *m* denote the number of bits mapped to each 2D symbol. We consider transmission over both AWGN and the Laplace

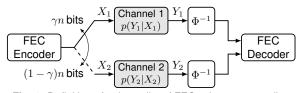


Fig. 1: Definition of universality of FEC schemes according to 5 . Channels 1 and 2 have the same GMI (achievable rate).

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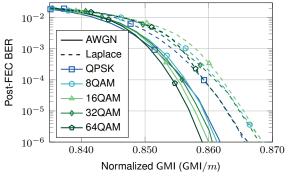


Fig. 2: Illustration of the non-universality for BICM.

channel, which adds noise (per dimension) with

$$p_{Y|X}(y|x) = \frac{1}{2b} \exp\left(-\frac{|y-x|}{b}\right), \ b = \frac{(\mathsf{E}_{\mathsf{s}}/\mathsf{N}_0)^{-\frac{1}{2}}}{2}.$$

At the receiver, we employ, after BMD, scaled min-sum decoding with 10 iterations. All setups are compared at the same normalized GMI/m. The post-FEC BER results are shown in Fig. 2. The GMI is only an approximately good threshold of the performance. If the channel law changes, the thresholding effect is compromised and a higher GMI is required. This can lead to misleading conclusions, e.g., in terms of reach prediction.

We conclude that performance predictors based on GMI (or variations thereof) should be used with caution. They can give rough first order estimates of the decoding performance, even if we introduce drastic changes into the channel (like, e.g., strong quantization, adding inline dispersion compensation, changing detection). Thresholds should only be used to quickly assess the performance and to determine the range of fine measurements. The accuracy is improved by modeling fairly close the channel of the system during measurement of the threshold. In all cases, actual decoding mimicking as closely as possible the true FEC should be used.

Performance Prediction Thresholds

In this section, we briefly discuss some of the thresholds that are commonly used today.

<u>Pre-FEC SER</u>: The pre-FEC symbol error rate (SER) metric is easy to compute directly from the experimental measurement. After symbol decision (e.g., using nearest-neighbor), the pre-FER SER is estimated as the fraction of wrongly decided modulation symbols. The Pre-FEC SER should only be used as a threshold when symbolwise HDD is used and the constellation size is matched to the symbol size of the code. In this case, the achievable rate R_{HDD-SW} given by Eq. (4) in⁷ is directly linked to the SER.

<u>Pre-FEC BER</u>: The easiest method to compute the pre-FEC BER is to start from estimated symbols and to use the inverse Φ_h^{-1} of the binary labeling function Φ to generate bit sequences, *implicitly* assuming BICM as CM technique. The pre-FEC BER is the fraction of wrongly estimated bits after generating bit sequences for both transmit and receive symbols. The pre-FEC BER should *only* be used if bit-wise HDD is utilized. The pre-FEC BER threshold can be used in some circumstances with SDD, but only if the FEC that shall be evaluated has been thoroughly simulated with a model that is sufficiently close to the experimental setup (e.g., using the same modulation format, quantization, fiber model, neighboring channel setup, etc.). In that case, the use of pre-FEC BER can be tolerated.

<u>GMI</u>: Recently, the use of generalized mutual information (GMI) has become ubiquitous. We would like to point out that the notion of GMI is a much broader concept, introduced as bound on the achievable rate for mismatched decoding⁸. In the case of BICM, the GMI equals the achievable rate. Hence, the term GMI is often used interchangeably with the achievable rate of BICM. For computing the GMI, we refer to³. The GMI should *only* be used for FEC with bit-wise SDD (i.e., for BICM) and if there is sufficient evidence that the code is approximately *universal*.

<u>NGMI</u>: Recently, the use of probabilistic amplitude shaping become popular in optical communications⁹. In this case, the GMI cannot be directly used¹⁰. A performance metric that works well in this case is the "NGMI" as defined in¹¹.

<u>MI</u>: In contrast to the GMI, the mutual information (MI) is computed on symbol-level and requires the availability of the channel PDF⁶. The MI should only be used when SDD with non-binary codes, e.g., nonbinary LDPC codes, matched to the constellation size are used as CM scheme. The MI can also be used when multilevel coding with multi-stage decoding is employed. In many cases, the MI and GMI are relatively close (square constellations with Gray coding). In these cases, the GMI threshold can be replaced by the MI without loosing much accuracy. This has advantages as the MI is often easier to compute¹².

Implementing Actual Decoding

Due to the pitfalls of thresholds, especially when unsure about universality, we suggest to include FEC into the transmission experiments. We suggested in ¹³ to reuse a database of measurements to evaluate multiple FEC schemes. We assume that the transmission experiment yields a measurement consisting of N_M aligned data points (x_{κ}, y_{κ}) that are given as the original complex transmit sequence $x_{\kappa} \in \mathbb{C}$ and the corresponding received complex sequence $y_{\kappa} \in \mathbb{C}$. Our method is based on the fact that the performance of most practical FEC codes and decoders (e.g., LDPC codes) does not depend on the codeword. Hence, we assume transmission of a codeword ctypically used, e.g., one that leads to the symbols

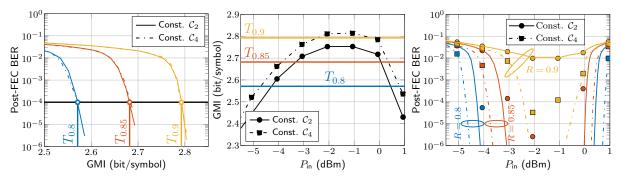


Fig. 3: AWGN simulation results of regular LDPC codes ($R \in \{0.8, 0.85, 0.9\}$), layered scaled min-sum decoding, 10 iterations (left). Estimated GMI from experiment (middle) and performance after actual decoding and interpolation (right).

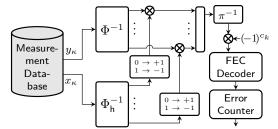


Fig. 4: Evaluating FEC performance from measurements for a BICM-based CM system.

following the distribution of probabilistic shaping.

The method is illustrated in Fig. 4 for the BICM case, also used in PAS and can be easily extended to other CM schemes. The first step of the method consists in generating an equivalent bit-stream of length mN_M corresponding to the transmit sequence from the experimental database by using the inverse bit mapping function Φ_h^{-1} . Similarly, the BMD Φ^{-1} computes a sequence of LLRs. Using both sequences, we generate a set of equivalent LLRs, corresponding to the transmission of the selected codeword c which can be fed to an FEC decoder.

Performance Example

To illustrate the proposed method, we consider a system using 8QAM together with BICM. We reuse the measurement database already used in⁶ with the two 8QAM constellations denoted C_2 and C_4 in⁶ together with their GMI-maximizing bit labeling. We use 3 regular QC-LDPC codes with rates $R \in \{0.8, 0.85, 0.9\}$ of length n = 38400 bit. Decoding takes place using I = 10 iterations with a layered scaled min-sum decoder ($\alpha = 0.75$). The AWGN performance of the codes is shown in Fig. 3 (left) as a function of the GMI. We see that the codes act approximately universal and their performance only marginally depends on the chosen constellation. For a post-FEC BER of 10^{-4} , we find equivalent achievable rate thresholds T_R .

Transmission takes place over a coherent, dual-polarization fiber-optic communication system at a symbol rate of 41.6 Gbaud over 8 round trips in a re-circulating loop (3200 km), described in⁶. The transmission test-bed consists of one laser under test and additionally 63 loading channels spaced by 50 GHz. Fig. 3 (middle) shows the estimated GMI as function of the input power P_{in} per wavelength division multiplex (WDM) channel with the thresholds T_R for $R \in \{0.8, 0.85, 0.9\}$. Whenever the estimated achievable rate lies above the threshold T_R , successful transmission is possible (i.e., post-FEC BER below 10^{-4}). For example, consider $T_{0.9}$: with constellation C_4 , we are just barely above the line for $P_{\text{in}} \in \{-2 \text{ dBm}, -1 \text{ dBm}\}$, which means that decoding is also only barely possible. Contrary, with C_2 , reliable decoding is not possible.

In Fig. 3 (right), we use the AWGN simulation to estimate the post-FEC BER of the transmission system by interpolation (solid and dashdotted lines). Additionally, we carried out actual decoding using the LDPC codes (solid markers). We can see that the estimates from interpolation match the actual decoding performance closely, especially for high BERs. However, we see another deviation at low BERs which are caused mostly by non-stationarity in the measurements.

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