Resource Efficient Isolation Mechanisms in Mixed-Criticality Scheduling

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Abstract-Mixed-criticality real-time scheduling has been developed to improve resource utilization while guaranteeing safe execution of critical applications. These studies use optimistic resource reservation for all the applications to improve utilization, but prioritize critical applications when the reservations become insufficient at runtime. Many of them however share an impractical assumption that all the critical applications will simultaneously demand additional resources. As a consequence, they under-utilize resources by penalizing all the low-criticality applications. In this paper we overcome this shortcoming using a novel mechanism that comprises a parameter to model the expected number of critical applications simultaneously demanding more resources, and an execution strategy based on the parameter to improve resource utilization. Since most mixedcriticality systems in practice are component-based, we design our mechanism such that the component boundaries provide the isolation necessary to support the execution of low-criticality applications, and at the same time protect the critical ones. We also develop schedulability tests for the proposed mechanism under both a flat as well as a hierarchical scheduling framework. Finally, through simulations, we compare the performance of the proposed approach with existing studies in terms of schedulability and the capability to support low-criticality applications.

I. INTRODUCTION

An increasing trend in embedded systems is towards open computing environments, where multiple functionalities are developed independently and integrated together on a single computing platform [1]. An important notion behind this trend is the safe isolation of separate functionalities, primarily to achieve fault containment. This raises the challenge of how to balance the conflicting requirements of isolation for safety assurance and efficient resource sharing for economical benefits. The concept of *mixed-criticality* appears to be important in meeting those two goals.

In many safety-critical systems, the correct behavior of some functionality (e.g., flight control) is more important ("critical") to the overall safety of the system than that of another (e.g., in-flight cooling). In order to certify such systems as being correct, they are conventionally assessed under certain assumptions on the worst-case run-time behavior. For example, the estimation of Worst-Case Execution Times (WCETs) of code for highly critical functionalities involves very conservative assumptions that are unlikely to occur in practice. Such assumptions make sure that the resources reserved for critical functionalities are always sufficient. Thus, the system can be designed to be fully safe from a certification perspective, but the resources are in fact severely under-utilized in practice.

In order to close such a gap in resource utilization,

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Vestal [2] proposed the mixed-criticality task model that comprises of different WCET values. These different values are determined at different levels of confidence ("criticality") based on the following principle. A reasonable low-confidence WCET estimate, even if it is based on measurements, may be sufficient for almost all possible execution scenarios in practice. In the highly unlikely event that this estimate is violated, as long as the scheduling mechanism can ensure deadline satisfaction for highly critical applications, the resulting system design may still be considered as safe.

To ensure deadline satisfaction of critical applications, mixed-criticality studies make pessimistic assumptions when a single high-criticality task executes beyond its expected (lowconfidence) WCET. They assume that the system will either immediately ignore all the low-criticality tasks [3], [4], [5], [6], [7], [8] or degrade the service offered to them [9], [10], [11], [12]. They further assume that all the high-criticality tasks in the system can thereafter request for additional resources, up to their pessimistic (high-confidence) WCET estimates. Although these strategies ensure safe execution of critical applications, they have a serious drawback as pointed out in a recent article [9]. When a high-criticality task exceeds its expected WCET, the likelihood that all the other high-criticality tasks in the system will also require more resources is very low in practice. For instance, it is unlikely that adaptive cruise control and anti-lock braking, both of which are critical, would simultaneously require additional resources because their execution time depends on different inputs. Cruise control would most likely require additional resources when the cameras and lidars provide dense data, whereas the execution of anti-lock braking mainly depends on speed of the vehicle and friction on the tyres. Therefore, to penalize all the low-criticality tasks in the event that some high-criticality tasks require additional resources seems unreasonable.

In practice, most mixed-criticality systems are componentbased wherein different vendors independently design and develop the various applications. For wide applicability, it is then natural that mixed-criticality scheduling strategies must consider the impact of WCET violations across component boundaries. To the extent possible, these strategies must limit this impact to within components, so that other components in the system can continue their execution uninterrupted. One extreme manifestation of this view is the reservation-based approach that completely isolates components but severly under-utilizes the resources. On the other hand, most of the recent mixed-criticality studies such as those mentioned above, completely ignore these component boundaries but still underutilize resources due to unrealistic assumptions.

Contributions. Addressing the two central issues described above, in this paper we propose a resource efficient mechanism to support low-criticality tasks while still ensuring isolation of high-criticality tasks. This mechanism comprises the following.

- A new parameter to model the expected number of simultaneous violation of low-confidence WCET by high-criticality tasks.
- 2) A corresponding execution strategy that maximizes low-criticality task executions as long as this number is not exceeded.
- 3) To efficiently support component-based mixedcriticality systems, we employ our mechanism at the component level. We ensure that as long as the number of low-confidence WCET violations within a component does not exceed the component's expected limit, task executions in other components, including low-criticality ones, remain unaffected.

It is worth noting that this mechanism generalizes both the reservation based approach in which high-criticality tasks are allocated resources based on their high-confidence WCETs [13], as well as the classical mixed-criticality studies that penalize all the low-criticality tasks (e.g., [8]). Considering a mixed-criticality scheduling strategy based on the Earliest Deadline First (EDF) policy (e.g., [6], [7], [8]), we also derive schedulability tests for the proposed mechanism. We derive these tests for a flat (non-hierarchical) as well as a hierarchical scheduling framework. While both these frameworks ensure isolation for high-criticality tasks as a result of employing criticality-aware scheduling, the hierarchical framework additionally supports *compositionality*, i.e., the ability of a system to derive properties (e.g., schedulability) for higher level components using derived properties of lower level components. We evaluate the performance of the proposed mechanism in terms of schedulability and the ability to support low-criticality executions through extensive simulations. These results show that our proposed mechanism outperforms all the other existing studies in terms of this dual objective.

Related Work. Since Vestal's seminal work in 2007 [2], a growing number of studies have been introduced for mixedcriticality real-time scheduling, e.g., [3], [4], [5], [6], [7], [8], sharing the pessimistic strategy that all the low-criticality tasks will be immediately dropped upon WCET violation of a single high-criticality task. Some recent studies have presented solutions to improve support for low-criticality executions [9], [10], [11], [14], [12], [15]. The elastic mixed-criticality model allows for a flexible release pattern of low-criticality tasks depending on the runtime resource consumption of highcriticality tasks, essentially treating the low-criticality workload as background [11], [9], [10]. This was improved by the service adaptation strategy that decreased the dispatch frequency of low-criticality tasks only when a high-criticality task violated its low-confidence WCET. All the above studies however, share the unrealistic assumption that once a single high-criticality task violates its low-confidence WCET, all the other high-criticality tasks in the system will also exhibit similar behavior. The interference constraint graph strategy partially relaxes this assumption, at least in terms of its online strategy for penalizing low-criticality tasks [14]. The constraint graph is used to specify execution dependencies between highand low-criticality tasks, and a response-time based approach was presented to determine graph constraints that improve low-criticality executions at runtime. However, it still uses high-confidence WCET estimates for all the high-criticality tasks when determining schedulability (test based on [2]), which again leads to the same unrealistic assumption and therefore results in resource under-utilization. Further, none of the above studies consider the impact of WCET violations in the context of component-based systems. A couple of recent studies proposed techniques to support hierarchical scheduling for component-based mixed-criticality systems [16], [13]. These studies focused on implementation issues however, and therefore did not consider the problems discussed above.

II. SYSTEM MODEL

A. Task and Component

In this paper we consider constrained deadline mixed criticality sporadic tasks (or *tasks* for short). Such a task can be specified as $\tau_i = (T_i, L_i, C_i, D_i)$, where T_i denotes the minimum separation between job releases, L_i denotes the criticality level, C_i is a list of WCET values, and D_i ($\leq T_i$) denotes the relative deadline. We assume that tasks have only two criticality levels, LC denoting low-criticality and HC denoting high-criticality. Hence $L_i \in \{LC, HC\}$ and $C_i = \{C_i^L, C_i^H\}$, where C_i^L denotes LC WCET and C_i^H denotes HC WCET. If $L_i = HC$, then τ_i is called a HC task, otherwise τ_i is called a LC task. We also assume that $C_i^L < C_i^H$ for all the HC tasks, and $C_i^L = C_i^H$ for all the LC tasks. Jobs of τ_i are released with a minimum separation of T_i time units, and each job can execute for no more than C_i^H time units (C_i^L in the case of LC task) within D_i time units from its release. Let $\mathcal{T} = \{\tau_1, \ldots, \tau_n\}$ denote a set of such mixed-criticality tasks that are scheduled on a single-core processor.

We assume that the tasks are partitioned into *components*, where each component $\mathbb{C} = (\mathcal{W}, TL)$ comprises the following.

- A *real time workload* W denoting a subset of tasks from T, and
- A HC Tolerance Limit TL ∈ N denoting the maximum HC workload isolation limit of the component. As long as no more than TL_i tasks in the component simultaneously exhibit HC behavior (execution requirement is more than LC WCET), we must ensure that all the job deadlines in the other components, including those of LC jobs, are met. More details about this parameter are presented later in this section.

Partitioning the task set into components is mainly driven by practical considerations as mentioned in the introduction. Since these components are developed independently, it is desirable to limit the impact of WCET violations to within components as much as possible, while still efficiently utilizing the resources. The HC tolerance limit TL precisely does that in our model. It could be set based on component properties if information about the runtime behavior of HC jobs is available, e.g., probability of execution requirement exceeding LC WCET. It can also be determined such that the limit is maximized so as to support more LC job executions, while still maintaining system schedulability. $\mathbb{C} = (\mathcal{W}, TL)$ is called a LC component if every task in its workload is a LC task, and for such components we assume that TL = 0. Otherwise, \mathbb{C} is called a HC component.

B. Task and Component Execution Model

The execution semantics of a mixed-criticality task has been presented previously [3], and we summarize it as follows. A task τ_i is said to be in low-criticality mode (or *LC mode* for short) as long as no job of the task has executed beyond its *LC* WCET C_i^L . If τ_i is a *LC* task, then this is the only available criticality mode. Whereas if τ_i is a *HC* task, then it switches to high-criticality mode (or *HC mode* for short) at the time instant when some job of the task requests to execute for more than its *LC* WCET. In *HC* mode, jobs of τ_i can request to execute for no more than C_i^H time units.

We now define the execution semantics of a component $\mathbb{C} = (\mathcal{W}, TL)$. \mathbb{C} has two execution modes, an *internal mode* that concerns the behavior of tasks in \mathbb{C} , and an *external mode* that concerns the behavior of tasks in the other components. We first describe these two modes, and then discuss their implications.

Internal Mode. Component \mathbb{C} experiences *Internal Mode Switch* (or IMS for short) at the earliest time instant when any *HC* task in \mathbb{C} switches to *HC* mode. The component switches its internal mode from *LC* to *HC* at this time instant. Prior to this mode switch, all the task deadlines are required to be met. After this switch however, all the *LC* tasks in \mathbb{C} can be dropped, and only the *HC* task deadlines are required to be met. There is no impact of this mode switch on the other components in the system.

External Mode. Component \mathbb{C} experiences *External Mode Switch* (or EMS for short) at the earliest time instant when the $(TL + 1)^{st}$ HC task in \mathbb{C} switches to HC mode. The component switches its external mode from LC to HC at this time instant. Prior to this mode switch, at most TL tasks in \mathbb{C} were executing in HC mode. After this switch however, all the HC tasks in \mathbb{C} may execute in HC mode. Further, all the LC tasks in the system, including the LC tasks in the other components, are no longer required to meet deadlines. Component \mathbb{C} 's internal as well as external modes could switch back to LC mode when there are no pending jobs in the system at some time instant.

Note that the intra- and inter-component execution requirements based on their internal and external modes respectively, are consistent with the mixed-criticality requirements in the existing literature (e.g, [6]). If \mathbb{C} is a LC component, then its internal and external modes are identical and equal to LC. On the other hand if \mathbb{C} is a HC component, then these modes, together with the HC tolerance limit TL, are key mechanisms for supporting LC job executions. If TL > 0, it is possible for IMS and EMS to occur at different time instants (asynchronously). Then, during the interval when component \mathbb{C} 's internal mode is HC while its external mode is LC, LCtasks in the other components are isolated from the internal mode switch of \mathbb{C} . That is, these LC tasks can continue their execution even though some HC tasks in \mathbb{C} are already executing in HC mode. The proposed model and execution strategy generalizes both the worst-case reservation based approach in which HCtasks are allocated resources based on their HC WCETs [13], as well as the classical mixed-criticality studies that drop all the low-criticality tasks upon WCET violation by a single HC task (e.g., [8]). The former can be modeled by setting TL = |H|, where |H| denotes the total number of HC tasks in the component, while the latter can be modeled by setting TL = 0.

Scheduling Strategy. In this paper we focus on the Earliest Deadline First (EDF) strategy, and assume that LC tasks are dropped (not considered for scheduling) once it becomes known that their deadlines are not required to be met. We have chosen this scheduling strategy because it has been successfully employed in the past for mixed criticality systems (e.g., [6], [7], [8]). To accommodate the sudden increase in demand when tasks start executing in HC mode, these existing studies artificially tighten the deadlines of HC tasks when they are executing in LC mode. This ensures that when a task switches to HC mode, it has some amount of time left until its real deadline to execute any additional demand. In this paper we assume that such deadline tightening strategies are employed.

For a task $\tau_i = (T_i, L_i, C_i, D_i)$, we let D_i^L denote the artificially *tightened deadline* in LC mode of execution. By definition, $D_i^L \leq D_i$ for all tasks, and $D_i^L = D_i$ if $L_i = LC$ because no tightening is required for such LC tasks. While a HC task $\tau_i = (T_i, HC, C_i, D_i)$ is executing in LC mode, τ_i must receive at least C_i^L processor units before its tightened deadline D_i^L . When the task τ_i switches to HC mode, it must receive at least C_i^H processor units before the actual deadline D_i . Note that a HC task in component \mathbb{C} that executes in LC mode after IMS of \mathbb{C} will continue to be scheduled using its tightened deadline D_i^L , unless it switches to HC mode. After EMS of \mathbb{C} however, all the HC tasks are assumed to switch to HC mode and will be scheduled using their actual deadlines.

We consider two different scheduling frameworks in this paper; a flat (non-hierarchical) framework in which all the tasks in all the components are collectively scheduled by a single scheduler, and a hierarchical framework in which the tasks in components are scheduled by intra-component schedulers and the components themselves are scheduled by a inter-component scheduler. The flat framework is relevant in applications that do not use hierarchical scheduling (e.g., Deos Real-Time Operating System for avionics [17]), whereas the hierarchical framework is relevant in applications that require compositionality (e.g., ARINC653 in avionics [18]). Note that a criticality-aware flat scheduler also ensures isolation for high-criticality tasks, and hence from that perspective provides similar functionality as a hierarchical scheduler. In Section III we present the schedulability test under a flat scheduling framework, and in Section IV we present the schedulability test under a hierarchical scheduling framework. Finally, the capability of the proposed mechanism and the corresponding schedulability tests to support LC job executions are evaluated through extensive simulations in Section V.

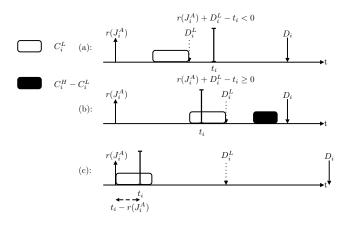


Fig. 1. Execution pattern for J_i^A that generates maximal demand

III. SCHEDULABILITY TEST FOR FLAT SCHEDULING FRAMEWORK

Demand bound function (dbf), which gives an upper bound on the maximum possible execution demand of tasks in given time interval length, was first proposed to characterize the maximal demand of workloads comprising non-mixedcriticality tasks [19]. Since then dbf has been extended to mixed-criticality tasks as well [7], [8].

In this section, for the task and component model presented earlier, we propose a dbf-based schedulability test under an EDF-based flat scheduling framework. In Section III-A we present the functions to calculate the demand of two special jobs of a task, and in Section III-B we use this to compute the dbf of a task (this dbf has already been developed in [8]). In Section III-C, we present the dbf of a component, and finally in Section III-D we present the dbf-based schedulability test.

Let t denote the time interval length and without loss of generality we assume the time interval is [0, t). Let $t_E (\leq t)$ denote the time instant for External Mode Switch or EMS of \mathbb{C} , and $t_I (\leq t_E)$ denote the time instant for Internal Mode Switch or IMS of \mathbb{C} . If \mathbb{C} is a *LC* component, then it has no IMS or EMS, and tasks within it will be dropped after the earliest EMS of any component in the system. For a HC task τ_i in the workload of \mathbb{C} , let t_i denote the time instant when it switches to HC mode. By definition $t_I \leq t_i \leq t_E$. For a LC task τ_i in the workload of \mathbb{C} , let t_i denote the time instant when it is dropped. Note that t_i in the LC case is either equal to t_I or the earliest EMS of any HC component, whichever is earlier. We use J_i to denote any job of τ_i , and $r(J_i)$ to denote its release time.

A. Demand of two special jobs

We now introduce how to compute the demand of the first special job which is the last one released by HC task τ_i before it switches to HC mode at t_i . As shown in Figure 1, this is a job such that $r(J_i) \leq t_i$ and $r(J_i) + T_i > t_i$, and we denote such a job as J_i^A . The following lemma bounds the demand of J_i^A when its deadline is greater than t.

Lemma 1: If $r(J_i^A) + D_i^L > t$, then J_i^A will generate zero demand during [0, t). Further, if $r(J_i^A) + D_i > t$, then J_i^A will not generate any demand after t_i .

Proof: Since $r(J_i^A) + D_i^L > t \Rightarrow r(J_i^A) + D_i > t$ $(D_i \ge t)$ D_i^L), J_i^A does not generate any demand in the interval of interest. On the other hand, if $r(J_i^A) + D_i > t$ and $r(J_i^A) + D_i > t$ $D_i^L \leq t$, then even if J_i^A does not finish before t_i , it does not generate any demand in the interval $[t_i, t)$ because after t_i its deadline is outside the interval of interest.

If J_i^A satisfies the condition $r(J_i^A) + D_i^L < t_i$ (Figure 1(a)), then J_i^A must finish by t_i , and hence it can generate a demand of up to C_i^L during [0, t). However if $r(J_i^A) + D_i^L \ge t_i$ (Figure 1(b)), then J_i^A will generate maximal demand during [0,t) if it executes as late as possible. In this case it can generate a demand of up to C_i^H . One special case is when $t_i \leq r(J_i^A) + D_i^L \leq t$ and $r(J_i^A) + D_i > t$ (Figure 1(c)). In this case, J_i^A will not generate any demand after t_i according to Lemma 1. Thus, the demand of job J_i^A for the interval [0,t)can be bounded as follows.

$$dbf(J_{i}^{A}, t, t_{i}) = \begin{cases} C_{i}^{L}, & r(J_{i}^{A}) + D_{i}^{L} < t_{i} \\ C_{i}^{H}, & r(J_{i}^{A}) + D_{i}^{L} \ge t_{i} \\ & \text{and } r(J_{i}^{A}) + D_{i} \le t \\ & \text{and } r(J_{i}^{A}) + D_{i} \le t \\ & \text{and } (J_{i}^{A}) + D_{i} > t \\ 0, & r(J_{i}^{A}) + D_{i}^{L} > t \end{cases}$$

$$(1)$$

Another special job is the last job released by a LC task τ_i before it is dropped at t_i , and we denote such a job as J_i^B . The release time of J_i^B satisfies the conditions $r(J_i^B) \leq t_i$ and $r(J_i^B) + T_i > t_i$.

If $r(J_i^B) + D_i^L > t$ $(D_i^L = D_i)$, J_i^B will generate zero demand during [0, t) because its deadline is outside the interval. Otherwise, it may generate some demand in the interval $[0, t_i)$, because it will be dropped after t_i . In order to maximize the demand of J_i^B in this interval, we assume that J_i^B will execute continuously from $r(J_i^B)$. Thus, the demand of job J_i^B for the interval [0, t) can be bounded as follows.

$$dbf(J_i^B, t, t_i) = \begin{cases} \min\left\{t_i - r(J_i^B), C_i^L\right\}, & r(J_i^B) + D_i^L \le t\\ 0, & \text{otherwise} \end{cases}$$
(2)

B. Dbf of task τ_i

In this section we derive the dbf of a task au_i using Equations 1 and 2 presented above. Let $dbf(\tau_i, t, t_i)$ denote the dbf of task τ_i for a given time interval length t and instant t_i . We present dbf (τ_i, t, t_i) using four sub-cases dbf $(\tau_i, t, t_i)_{[x]}$, where $x \in \{a, b, c, d\}$, defined as follows.

- a:
- b:
- $L_i = LC,$ $L_i = HC \text{ and } t t_i < D_i D_i^L,$ $L_i = HC \text{ and } t t_i \ge D_i, \text{ and } t t_i \ge D_i,$ c:

d:
$$L_i = HC$$
 and $D_i - D_i^L \leq t - t_i < D_i$.

If τ_i satisfies condition a, then it is a LC task. The total demand that τ_i can generate during [0, t) is then the sum of demand of jobs released before $r(J_i^B)$ and the demand of J_i^B itself. τ_i generates maximal demand during [0,t) if the release time of the first job is equal to zero, and all successive jobs are released as soon as possible with period T_i . Therefore $dbf(\tau_i, t, t_i)_{[a]}$ is given as follows.

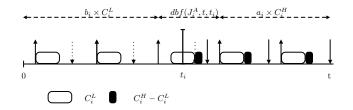


Fig. 2. Execution pattern for condition c

$$dbf(\tau_i, t, t_i)_{[a]} = \left\lfloor \frac{t_i}{T_i} \right\rfloor C_i^L + dbf(J_i^B, t, t_i)$$
(3)

If τ_i satisfies condition b, c or d, then τ_i is a HC task. Therefore, the total demand it generates is the sum of demand of all the jobs released before t. Among these jobs, the ones released before $r(J_i^A)$ will generate a demand of C_i^L , and the ones released after $r(J_i^A) + T_i$ will generate a demand of C_i^H . The demand of job J_i^A itself is given in Equation 1. In the following lemmas we derive the dbf of τ_i for the three conditions.

Lemma 2: If τ_i satisfies condition b $(t - t_i < D_i - D_i^L)$, no job of τ_i can execute for C_i^H time units. Therefore τ_i can generate maximal demand during [0, t) if the first job of τ_i is released at time instant 0 and all the successive jobs are released as soon as possible.

Proof: We prove this lemma by contradiction. Suppose there exists a job J_i of τ_i that can generate a demand of C_i^H time units in the interval [0, t). Then it must be true that $r(J_i) +$ $D_i \leq t$ and $r(J_i) + D_i^L \geq t_i \Rightarrow t - t_i \geq D_i - D_i^L$, because τ_i is a *HC* task that switched to *HC* mode at t_i . This contradicts our assumption that $t - t_i < D_i - D_i^L$. Thus no job of τ_i that satisfies condition b can generate a demand of C_i^H time units in the interval [0, t). Therefore τ_i essentially behaves like a *LC* task, and this proves the lemma.

Thus $dbf(\tau_i, t, t_i)_{[b]}$ is given as follows.

$$dbf(\tau_i, t, t_i)_{[b]} = \left\lfloor \frac{t_i}{T_i} \right\rfloor C_i^L + dbf(J_i^A, t, t_i)$$
(4)

Lemma 3: If τ_i satisfies condition c $(t - t_i \ge D_i)$, it generates maximal demand during [0,t) if the first job of τ_i is released at $t - D_i - \lfloor (t - D_i)/T_i \rfloor \times T_i$, and all the successive jobs are released as soon as possible (scenario shown in Figure 2).

Proof: If $t - t_i \ge D_i$ and the first job of τ_i is released at $t - D_i - \lfloor (t - D_i)/T_i \rfloor \times T_i$, then the last job released before t will have its deadline at t. In this case, t_i happens before the release time of this last job. Therefore the last job can generate a demand of C_i^H in the interval. Additionally, the number of jobs with deadline before t as well as the number of jobs that can generate C_i^H demand during [0, t) are maximized with this pattern. This proves the lemma.

Intuitively speaking, the demand is maximized when the deadline of a job of τ_i coincides with t, because it maximizes the possible executions for τ_i in HC mode. Thus, $dbf(\tau_i, t, t_i)_{[c]}$ is given as follows.

$$dbf(\tau_i, t, t_i)_{[c]} = b_i C_i^L + dbf(J_i^A, t, t_i) + a_i C_i^H , \text{ where}$$

$$b_i = \left\lfloor \frac{t_i - (t - D_i - \lfloor (t - D_i)/T_i \rfloor \times T_i)}{T_i} \right\rfloor, \text{ and}$$

$$a_i = \left\lfloor \frac{t - D_i}{T_i} \right\rfloor - b_i.$$
(5)

If τ_i satisfies condition d, it does not have a single execution pattern that maximizes its demand as stated in the following lemma.

Lemma 4: If τ_i satisfies condition d $(D_i - D_i^L \le t - t_i < D_i)$, it generates maximal demand if its first job is either released at 0 (condition b) or at $t - D_i - \lfloor (t - D_i)/T_i \rfloor \times T_i$ (condition c).

Proof: Since $D_i - D_i^L \leq t - t_i < D_i$, τ_i can have at most one job that can generate a demand of C_i^H in the interval [0, t). If the first job of τ_i is released at $t - D_i - \lfloor (t - D_i)/T_i \rfloor T_i$ and all the successive jobs are released as soon as possible (release pattern of condition c), then the last job is a special job J_i^A and is the only job generating C_i^H demand. The only way to further increase the demand of τ_i is to add a new job in the interval by shifting the pattern left to the point when the first job is released at time instant 0.

Thus, $dbf(\tau_i, t, t_i)_{[d]}$ is given as follows.

$$dbf(\tau_i, t, t_i)_{[d]} = \max \left\{ dbf(\tau_i, t, t_i)_{[b]}, dbf(\tau_i, t, t_i)_{[c]} \right\}$$
(6)

C. Dbf of component \mathbb{C}

In this section we present the dbf of a component $\mathbb{C} = \{\mathcal{W}, TL\}$. Let dbf $(\mathbb{C}, t, t_E, t_I)$ denote the dbf of component \mathbb{C} for a given time interval length t, with mode-switch instants t_I (IMS) and t_E (EMS).

We first present dbf for the case when TL = 0 and then for the case when TL > 0. Note that among all the HCtasks in \mathbb{C} , at most TL of them can switch to HC mode in the interval $[t_I, t_E)$, while all the remaining HC tasks are assumed to switch to HC mode at t_E .

If TL = 0, then this means t_E (EMS) is equal to t_I (IMS), because \mathbb{C} 's internal and external modes will switch at the same time. Thus, each HC task τ_i in \mathbb{C} will switch to HCmode at $t_i = t_I = t_E$, and hence $dbf(\mathbb{C}, t, t_E = t_I, t_I)$ is given as follows.

$$dbf(\mathbb{C}, t, t_E = t_I, t_I) = \sum_{\tau_i \in \mathbb{C}} dbf(\tau_i, t, t_I) \quad (TL = 0) \quad (7)$$

If TL > 0, then at most $TL \ HC$ tasks can switch to HCmode before t_E . To compute the dbf of \mathbb{C} , we then need to determine which HC tasks should switch to HC mode before t_E so as to maximize the total demand. The following lemma asserts that for any HC task, its demand is maximized when it switches to HC mode either at t_I or t_E .

Lemma 5: If a HC task τ_i switches to HC mode at some time $t_i \in [t_I, t_E]$, then dbf (τ_i, t, t_i) is maximized when t_i is either equal to t_E or t_I .

Proof: Suppose τ_i satisfies condition b when $t_i = t_E$, i.e., $t - t_E < D_i - D_i^L$. Then as t_i decreases, τ_i could eventually satisfy condition d, i.e., $D_i - D_i^L \le t - t_i < D_i$, and finally condition c, i.e., $t - t_i \ge D_i$. Without loss of generality, assume that τ_i satisfies condition b for $t_i \in (t_b, t_E]$, condition d for $t_i \in (t_d, t_b]$, and condition c for $t_i \in [t_I, t_d]$, where $t_I \le t_d \le t_b \le t_E$.

Case 1 $(t_i \in [t_I, t_d])$: In this case, dbf $(\tau_i, t, t_i)_{[c]}$ (see Equation 5) is maximized if $t_i = t_I$. This is because as t_i decreases from t_d to t_I , the number of jobs generating C_i^H demand will remain the same or increase, while the total number of jobs that generate demand for this time interval remains unchanged. **Case 2** $(t_i \in (t_b, t_E])$: In this case, dbf $(\tau_i, t, t_i)_{[b]} = \left\lfloor \frac{t_i}{T_i} \right\rfloor C_i^L + dbf(J_i^A, t, t_i)$. Then as t_i increases from t_b to t_E , dbf (J_i^A, t, t_i) and $\left\lfloor \frac{t_i}{T_i} \right\rfloor \times C_i^L$ will stay the same or increase. Thus dbf $(\tau_i, t, t_i)_{[b]}$ is maximized when $t_i = t_E$. **Case 3** $(t_i \in (t_d, t_b])$: From Lemma 4 we know that dbf $(\tau_i, t, t_i)_{[b]} | t_i \in (t_d, t_b]$ is maximized if $t_i = t_b$, dbf $(\tau_i, t, t_i)_{[c]} | t_i \in (t_d, t_b]$ stays the same. Since dbf $(\tau_i, t, t_i)_{[c]} | t_i \in (t_d, t_b]$ and dbf $(\tau_i, t, t_b)_{[c]} \leq dbf(\tau_i, t, t_i)_{[c]}$, combining the above three cases, we conclude that dbf (τ_i, t, t_i) is maximized when $t_i = t_I$ or $t_i = t_E$.

Let $\Delta_i = \max\{0, \operatorname{dbf}(\tau_i, t, t_I) - \operatorname{dbf}(\tau_i, t, t_E)\}$. From Lemma 5 we know that task τ_i generates maximum demand when $t_i = t_E$ or $t_i = t_I$. Therefore Δ_i denotes the maximum possible increase in the demand of τ_i (if it increases) for a time interval length t when τ_i is chosen as one of the TLtasks to switch to HC mode before t_E . Once we compute Δ_i for all the HC tasks in component \mathbb{C} , we sort these values in descending order and select the first TL elements. Let the corresponding set of TL HC tasks be denoted by \mathcal{G} . The total maximum demand of all the tasks in \mathbb{C} is then given by the following equation.

$$dbf(\mathbb{C}, t, t_E, t_I) = \sum_{L_i = HC} dbf(\tau_i, t, t_E) + \sum_{\tau_i \in \mathcal{G}} \Delta_i + \sum_{L_i = LC} dbf(\tau_i, t, t_I)$$
(8)

A tighter bound for the dbf of component \mathbb{C} can be obtained using an optimization presented in Section A of the Appendix.

D. Schedulability Test and Tolerance Limit

In this section we derive the schedulability test for a mixed-criticality system comprising multiple components and scheduled under a flat scheduling framework. Consider a system with p HC components $\mathbb{C}_1, \mathbb{C}_2, \ldots, \mathbb{C}_p$ and q LC components $\mathbb{C}_{p+1}, \mathbb{C}_{p+2}, \ldots, \mathbb{C}_{p+q}$. Each HC component \mathbb{C}_i can independently switch its internal mode to HC at t_{Ii} . Once the first HC component switches its external mode to HC at t_E , all the LC tasks in the system are immediately dropped. We assume that all the HC tasks in the system can thereafter execute in HC mode.

Suppose there is a first deadline miss in the system at some time instant t. Then, the total maximum demand generated by the system in [0, t) must be greater than t. This assertion immediately leads to the following theorem that presents the schedulability test.

Theorem 1: A mixed-criticality system comprising p HC components and q LC components is schedulable under a flat scheduling framework if, $\forall t: 0 \le t \le t_{MAX}, \forall t_E: 0 \le t_E \le t, \forall t_{Ii}: 0 \le t_{Ii} \le t_E$,

$$\sum_{i=1}^{i \le p+q} \operatorname{dbf}(\mathbb{C}_i, t, t_E, t_{Ii}) \le t,$$
(9)

where t_{MAX} is a pseudo-polynomial in the size of the input, and is defined in Section B of the Appendix.

The complexity of the schedulability test in Theorem 1 is exponential in the number of HC components, because we need to consider a separate internal mode switch instant for each component. In practice however, we expect the number of HC components scheduled on a single processor to be relatively small, and then the complexity of the proposed test is pseudo-polynomial in the size of the input. Besides, if there is freedom to select the allocation of system tasks to components, then it is feasible to create a component structure comprising only two components, while still fully supporting LC task executions. All the HC tasks in the system are allocated to a single HC component $\mathbb{C}_H = \{\mathcal{W}_H, TL_H\},\$ and each LC task can be either allocated to \mathbb{C}_H or to a LC component $\mathbb{C}_L = \{\mathcal{W}_L, TL_L = 0\}$. This two-component system is sufficient to consider all the possible design choices for isolating HC and LC task executions. This can be done by considering different values for the tolerance limit TL_H , and by considering different allocations of LC tasks to the two components. We can choose the maximum possible value for these tolerance limit as long as the resulting system is still schedulable. Higher tolerance limit indicates support for more LC task executions, and thus better resource utilization. In Section V, we show through simulations that our mechanism outperforms existing studies even with this two-component structure. However, if the allocation of tasks to components is fixed and the number of HC components is not small, then the hierarchical scheduling framework presented in the following section can be used to reduce the complexity of the test.

IV. SCHEDULABILITY TEST FOR HIERARCHICAL SCHEDULING FRAMEWORK

Hierarchical scheduling has emerged as an effective mechanism to support temporal partitioning between applications, serving as a common scheduling paradigm in many mixedcriticality systems in practice [18]. It is preferred in practice because it supports *compositionality* so that higher-level properties can be derived from verified component-level properties. Therefore, to increase the practical relevance of the proposed mechanism, we develop a schedulability test under a hierarchical scheduling framework in this section.

A. Execution Strategy under Hierarchical Scheduling

For hierarchical systems, each component will have an additional parameter S denoting its local scheduler. We specify such a component as $\mathbb{C} = (\mathcal{W}, TL, S)$. The component workload \mathcal{W} is comprised of regular mixed-criticality tasks as well as interface tasks representing the child components. The tasks in the workload \mathcal{W} are scheduled by the local scheduler S, independently of all the other components in the system.

Component interfaces have been widely used in traditional hierarchical systems to abstractly represent the resource demand and supply of components (see for example [20]). From the component's perspective, its interface represents the resource demand of its workload. While from the perspective of its parent component or system, the interface represents the resource supply that the parent guarantees. These interfaces of components are essential for satisfying the property of compositionality.

Resource models such as periodic have been previously defined as interfaces for components in traditional hierarchical systems [20]. Analogously, we now present the mixed-criticality periodic resource (MCPR) model for mixedcriticality components. Since we focus on systems with two criticality levels, we assume that the MCPR model can have at most two criticality levels.

Definition 1: A Mixed-Criticality Periodic Resource (MCPR) is defined as $\mathbb{I} = (T, L, C)$, where T denotes the period, $L \in \{LC, HC\}$ denotes the criticality level, and $C = \{C^L, C^H\}$ is a list of resource capacities. C^L denotes LC resource capacity and C^H denotes HC resource capacity.

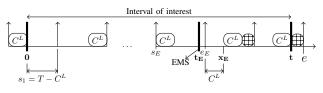
A component \mathbb{C} can be abstracted as an MCPR interface $\mathbb{I} = (T, L, C)$, and the corresponding task (T, L, C, T) (denoted as *interface task*) represents \mathbb{C} in the workload of its parent component. We assume that period T of this interface is already specified by the system designer as in the standard literature on hierarchical scheduling (e.g., see [20]). For instance, this period could be determined based on either component-level requirements or considerations for overheads such as context-switches. The criticality level L is directly determined by the criticality level of the component it is representing. If \mathbb{C} is a LC component, then L = LC, otherwise L = HC.

Mode of the interface. The semantics of interface \mathbb{I} (and the corresponding interface task) depend on its *criticality mode* at run time, which in turn depends on the criticality mode of component \mathbb{C} . In fact, we assume that the criticality mode of \mathbb{I} is identical to the external mode of \mathbb{C} . When \mathbb{C} experiences EMS, the mode of the interface and interface task switches from LC to HC. While the interface is in LC mode, it is guaranteed to request no more than C^L time units of resource periodically every T time units from the parent component. But when it switches to HC mode, it can thereafter request up to C^H time units of resource periodically.

B. MCPR Supply Bound Function

The supply bound function (sbf) of a resource model characterizes the minimum resource supply guaranteed by the model to the underlying component. In this section, we derive the sbf for a MCPR interface $\mathbb{I} = (T, L, C)$ of a component $\mathbb{C} = (W, TL, S)$. We let $sbf_{\mathbb{I}}(t_E, t)$ denote the sbf for a time interval of length t, where $t_E(\leq t)$ denotes the time instant for EMS of component \mathbb{C} . As the resource is supplied periodically, component \mathbb{C} is guaranteed to receive either C^L or C^H units of resource every T time units in LC or HC mode, respectively. We use the following additional notations in this section.

• s_1 denotes the start time of the first interface period within time interval [0, t).



Executions beyond C^L and up to C^H

Fig. 3. MCPR worst-case resource supply pattern A

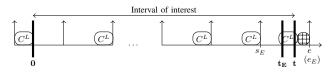


Fig. 4. Boundary case for MCPR worst-case resource supply pattern A

- *n* denotes the number of interface periods within interval [0, t).
- n_E denotes the number of interface periods within interval $[0, t_E)$.
- s_E denotes the start of a interface period that experiences EMS (t_E) , i.e., $s_E \leq t_E < e_E$, where $e_E = s_E + T$.
- e denotes the start of interface period after t, i.e., $e = s_1 + n \times T + T$.
- For simplicity of presentation, we also use the shortcut notation $[x]_0 = \max\{0, x\}$.

When $t_E = t$, there is no external mode switch for component \mathbb{C} in the interval of interest, and the component and interface are only executing in LC mode. Therefore, $\mathrm{sbf}_{\mathbb{I}}(t_E, t)$ in this case is identical to the sbf defined for periodic resource models with \mathbb{I} supplying C^L units of resource periodically [20]. Thus, in this case, minimal resource is supplied when $s_1 = T - C^L$ and $n = \left[\left\lfloor \frac{t - (T - C^L)}{T} \right\rfloor \right]_0$. We record this sbf in the following equation.

$$\operatorname{sbf}_{\mathbb{I}}(t_E, t) = n \times C^L + \left[t - 2(T - C^L) - n \times T\right]_0 \quad \text{If } t_E = t \tag{10}$$

For the case when $t_E < t$, there are two possible resource supply patterns, denoted A and B, that can lead to the minimum resource supply. We now present these two patterns and the corresponding sbf equations, $sbf_{\mathbb{I}}(t_E, t)_{[A]}$ and $sbf_{\mathbb{I}}(t_E, t)_{[B]}$.

Pattern A: $s_1 = T - C^L$. The scenario of pattern A is shown in Figure 3, where $n_E = \left[\left\lfloor \frac{t_E - (T - C^L)}{T} \right\rfloor \right]_0$ and $n = \left[\left\lfloor \frac{t - (T - C^L)}{T} \right\rfloor \right]_0$. In the first period, C^L units of resource are supplied as early as possible and hence during $[0, 2 \times (T - C^L)]$, no resource is supplied. In the following periods until time instant $s_E(=n_E \times T + T - C^L)$, C^L units are supplied as late as possible. In the period $[s_E, e_E]$, the amount of supply depends on the distance of t_E from s_E . If $t_E - s_E < C^L$, then the resource supply in this period cannot be exhausted when component \mathbb{C} has EMS at t_E . Therefore interface I will provide C^H units of resource in this period, because it can signal its mode switch to the parent component. On the other hand, if $t_E - s_E \ge C^L$ as in the example figure, then the resource supply in $[s_E, e_E]$ can be exhausted before component \mathbb{C} experiences EMS, and hence the interface may only provide C^L units in this period. After time instant e_E , the interface is guaranteed to provide C^H units of resource in every period. An important boundary case to consider is when $e = n \times T + 2T - C^L = e_E$ and $t_E - s_E \ge C^L$. That is, when t_E and t are in the same period and the interface can exhaust its resource supply before EMS of component \mathbb{C} (scenario shown in Figure 4). In this case, the minimum supply in this period can happen when it is provided as late as possible (for instance when $e - t > C^H - C^L$). We record the sbf corresponding to the pattern of Figures 3 and 4 below.

$$\begin{split} \text{sbf}_{\mathbb{I}}(t_{E},t)_{[A]} &= \\ \begin{cases} n_{E} \times C^{L} + (n - n_{E}) \times C^{H} \\ + \left[t - (2T - C^{L} - C^{H}) - n \times T\right]_{0} & t_{E} - s_{E} < C^{L} \\ (n_{E} + 1) \times C^{L} + (n - n_{E} - 1) \times C^{H} \\ + \left[t - (2T - C^{L} - C^{H}) - n \times T\right]_{0} & e \neq e_{E} \wedge \\ & t_{E} - s_{E} \ge C^{L} \\ n_{E} \times C^{L} \\ + \min\left\{C^{L}, \left[t - (2T - C^{L} - C^{H}) - n \times T\right]_{0}\right\} & e = e_{E} \wedge \\ & t_{E} - s_{E} > C^{L} \end{split}$$

Pattern B: $s_1 = T - C^L - (x_E - t_E)$, where $x_E = \left\lfloor \frac{t_E}{T} \right\rfloor \times T$. Scenario of pattern B is shown in Figure 5, which is obtained by shifting pattern A in Figure 3 by $x_E - t_E$. In this case,

$$n_E = \left[\left\lfloor \frac{t_E - s_1}{T} \right\rfloor \right]_0, n = \left[\left\lfloor \frac{t - s_1}{T} \right\rfloor \right]_0, e_E = t_E - C^L + T$$

and $e = n \times T + T + s_1$.

The sbf corresponding to this shifted supply pattern is given below. It is similar to the previous case, except that the interface period containing t_E is now guaranteed to supply no more than C^L time units.

$$sbf_{\mathbb{I}}(t_{E},t)_{[B]} = \begin{cases} (n_{E}+1) \times C^{L} + (n-n_{E}-1) \times C^{H} \\ + [t-s_{1}-(T-C^{H})-n \times T]_{0} & e \neq e_{E} \end{cases}$$

$$n_{E} \times C^{L} \\ + \min \left\{ C^{L}, \left[t-s_{1}-(T-C^{H})-n \times T \right]_{0} \right\} \quad e = e_{E} \end{cases}$$
(12)

The following lemma proves that it is sufficient to consider the above two supply patterns for determining the sbf.

Lemma 6: When $t_E < t$, pattern A or B are the only two possible supply patterns that can result in the minimal resource supply from interface \mathbb{I} .

Proof: Suppose there exists a $s_1 \in [0, T)$ such that $s_1 \neq T - C^L$ (pattern A) and $s_1 \neq T - C^L - (x_E - t_E)$ (pattern B), but s_1 leads to the minimal supply pattern for time interval length t. **Case 1** ($s_1 = T - C^L + \epsilon | 0 < \epsilon \leq C^L$): In this case, it is easy to see that the supply will be greater than or equal to $sb_{I}(t_E, t)_{[A]}$, because the supply for the first interface period will increase by ϵ and the supply for the last interface period will decrease by at most ϵ . **Case 2** ($s_1 = T - C^L - (x_E - t_E) + \epsilon | 0 < \epsilon < (x_E - t_E)$): In this case, the supply for the interface period containing t_E will stay the same or

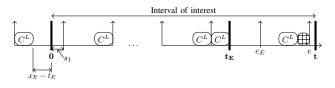


Fig. 5. MCPR worst-case resource supply pattern B

increase by $C^H - C^L$ while the supply for the last interface period may decrease by at most ϵ compared with the case when $s_1 = T - C^L - (x_E - t_E)$. Therefore this supply is also minimized when $\epsilon \to x_E - t_E$ or $\epsilon \to 0$. **Case 3** $(s_1 = T - C^L - (x_E - t_E) - \epsilon | 0 < \epsilon \le T - C^L - (x_E - t_E))$: In this case, the supply for the interface period containing t_E will stay the same, while the supply for the last interface period may stay the same or increase compared with the case when $s_1 = T - C^L - (x_E - t_E)$. Therefore in this case as well, the supply is minimized when $\epsilon \to 0$. Combining the above cases, we can conclude that the supply is minimized with either pattern A or pattern B.

Thus, a safe lower bound for $\text{sbf}_{\mathbb{I}}$ for the case when $t_E < t$ can be stated as follows.

$$\operatorname{sbf}_{\mathbb{I}}(t_E, t) = \min\left\{\operatorname{sbf}_{\mathbb{I}}(t_E, t)_{[A]}, \operatorname{sbf}_{\mathbb{I}}(t_E, t)_{[B]}\right\}$$
(13)

C. Interface Generation

(11)

In this section we use the sbf, together with the dbf of component \mathbb{C} , to generate interface \mathbb{I} . For component \mathbb{C} to be schedulable using interface \mathbb{I} , it is sufficient to ensure that $dbf(\mathbb{C}, t, t_E, t_I) \leq sbf_{\mathbb{I}}(t_E, t)$ for various time interval lengths. Below we first present the schedulability test for the case when component \mathbb{C} does not experience EMS. That is, the interface only executes in LC mode supplying C^L resource capacity periodically.

Theorem 2: A mixed-criticality component \mathbb{C} is schedulable in LC mode with $\text{sbf}_{\mathbb{I}}(t_E = t, t)$ if, $\forall t: 0 \le t \le t_{MAX}, \forall t_I: 0 \le t_I \le t$,

$$dbf(\mathbb{C}, t, t_E, t_I) \le sbf_{\mathbb{I}}(t_E = t, t)$$
 If $t_E = t$ (14)

where t_{MAX} is a pseudo-polynomial in the size of the input that can be derived using similar techniques in Section B in the appendix, and $\text{sbf}_{\mathbb{I}}(t_E, t)$ is given by Equation (10) in Section IV-B.

For a given t and t_I , dbf(\mathbb{C}, t, t_E, t_I) can be computed using techniques described in Section III-C. The only unknown quantity in Equation (14) is the *LC* resource capacity C^L . This capacity can then be computed exactly using existing techniques [21].

To compute the HC resource capacity C^H , we need to consider the schedulability test when component \mathbb{C} experiences EMS at some time instant $t_E(< t)$. The following theorem presents this test.

Theorem 3: A mixed-criticality component \mathbb{C} is schedulable in HC mode with $sbf_{\mathbb{I}}(t_E, t)$ if $\forall t : 0 \le t \le t_{MAX}, \forall t_E : 0 \le t_E \le t, \forall t_I : 0 \le t_I \le t_E$,

$$dbf(\mathbb{C}, t, t_E, t_I) \le sbf_{\mathbb{I}}(t_E, t)$$
(15)

where $sbf_{\mathbb{I}}(t_E, t)$ is given by Equation (13) in Section IV-B.

The only unknown quantity in Equation (15) is the HC resource capacity C^H , assuming we have already computed C^L using Theorem 2. C^H can then be computed similar to C^L using existing techniques [21].

V. EVALUATION

In this section we evaluate the performance of the proposed mechanism in terms of offline schedulability as well as its ability to support LC task executions online. Tasksets are generated using the following settings, where each parameter is randomly drawn from the given range using an uniform distribution.

- $u_i^L = C_i^L/T_i$ is in the range [0.02, 0.1]. C_i^H/C_i^L is in the range [2, 3]. T_i is in the range [10, 150]. •
- •
- •
- $D_i = T_i$ as service adaption strategy, one of the mechanisms being compared, can only support implicit deadline tasks.
- Task τ_i is deemed to be a HC task with probability 0.5.
- For a HC task τ_i , D_i^L is determined by the deadline tuning algorithm in [8].
- For the proposed mechanism, we assume that all the |H| HC tasks in the generated taskset are allocated to a *HC* component $\mathbb{C}_H = \{\mathcal{W}_H, TL_H\}$, and all the LC tasks are allocated to a LC component \mathbb{C}_L .

We have chosen relatively small values for u_i^L and C_i^H/C_i^L so that sufficient number of HC tasks are generated. This enables us to evaluate the online performance of various approaches when different number of HC tasks synchronously switch to HC mode. The generated taskset is evaluated for offline schedulability as well as online performance in terms of support for LC execution under four different mechanisms. These include the mechanism presented in this paper ("Proposed Mechanism"), service adaptation strategy [12] ("Service Adaptation"), Interference Constraint Graph [14] ("ICG"), and the classical mixed-criticality studies in which all the LC jobs are dropped at the moment any HC job switches to HCmode [8] ("Classical Model"). Note that the classical model can be obtained by setting $TL_H = 0$ in our mechanism. In Section V-A we present our results for offline performance based on schedulability tests, and in Section V-B we compare their online performance through simulations.

A. Offline Schedulability

In order to generate feasible tasksets, we consider different bounds for the term $\max\{U_L^L + U_H^L, U_H^H\}$, where $U_L^L = \sum_{\substack{L_i = LC \\ L_i = HC}} C_i^L/T_i$, $U_H^L = \sum_{\substack{L_i = HC \\ L_i = HC}} C_i^L/T_i$ and $U_H^H = \sum_{\substack{L_i = HC \\ L_i = HC}} C_i^L/T_i$. For each bound value, we generate 1000

tasksets based on the procedure described above, and evaluate their off-line schedulability. For the elastic model [11] in which the LC task periods are extended, any generated taskset with U_H^H is always schedulable, because in the worst-case all the $L\overline{C}$ task periods can be extended to infinity. The schedulability test for the service adaption strategy [6] is a utilization based test. ICG uses the well known Audsley's algorithm to assign task priorities, and its schedulablity is maximized when the interference graph is fully connected, i.e., each HC task has an execution dependency with every LC task in the system. For our mechanism, if a hierarchical scheduling framework is considered, then we assume that the MCPR interface period Tfor both \mathbb{C}_H and \mathbb{C}_L is equal to 5 time units. This is reasonable because the smallest task period in any taskset is 10 time units.

Figures 6 and 7 show the schedulability performance for the tasksets under various mechanisms. In Figures 6 we present results for our mechanism under a flat scheduling framework, and in In Figures 6 we present results for our mechanism under a hierarchical scheduling framework. In these figures, the x-axis denotes the bound value for $\max\{U_L^L + U_H^L, U_H^H\}$, and the y-axis denotes schedulability ratio, i.e., percentage of tasksets deemed schedulable by the different mechanisms. For our mechanism, we generate the schedulability results for various values of the tolerance limit: $TL_H = 0, |0.2|H||, |0.4|H||, |0.6|H||, |0.8|H||$ and |H|.

As shown in Figure 6, the schedulability performance of our mechanism clearly depends on the tolerance limit; a higher limit generally implies lower schedulability, because it uses additional resources to support LC executions. For values of TL_H up to |0.4|H||, our mechanism outperforms both service adaptation and ICG on an average. Similar trends can also be observed for our mechanism under a hierarchical framework, except that the schedulability drops more rapidly due to the overhead of hierarchical scheduling. The classical model is represented by the curve with $TL_H = 0$ and it has the highest schedulability, but offers no support for LC executions when HC tasks switch to HC mode. Thus we can conclude that as long as no more than |0.4|H|| of the HC tasks execute in HC mode at each time instant, our mechanism offers the best performance in terms of offline schedulability as well as online support for LC executions.

B. Online Support for LC Executions

In this section, we compare the performance of our mechanism in terms of its ability to support LC executions with the other mechanisms described above. We use the following quantitative parameter to measure this online LC performance.

Definition 2 (Percentage of Finished LC Jobs (PFJ)): Let MAX_t denote the maximum possible number of LCjobs that a taskset \mathcal{T} can generate in the time interval [0, t). By definition, $MAX_t = \sum_{L_i = LC} \lceil t/T_i \rceil$. Let FIN_t denote the number of LC jobs that successfully finish by their deadlines in the time interval [0, t) using some mechanism. Then, PFJis equal to FIN_t/MAX_t .

Tasksets are generated using the procedure described earlier, and the various mechanisms are simulated to measure their online performance. The following additional settings and restrictions are used for this purpose.

- $$\label{eq:max} \begin{split} \max\{U_L^L+U_H^L,U_H^H\} &= 0.8, 0.85 \text{ and } 0.9.\\ \text{Tolerance limit } TL_H \text{ is chosen to be the largest value} \end{split}$$
 that still guarantees schedulability of our mechanism under a flat scheduling framework.
- Tasksets are simulated for t = 10,000 time units.

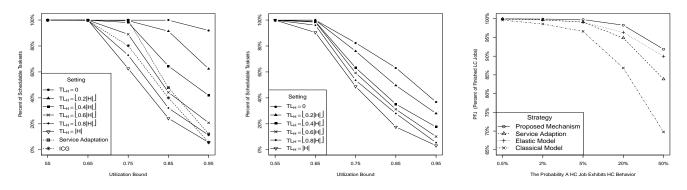


Fig. 6. Schedulability under a Flat Scheduling Fig. 7. Schedulability under a Hierarchical Fig. 8. $\max\{U_L^L + U_H^L, U_H^H\} = 0.8$ Framework Scheduling Framework

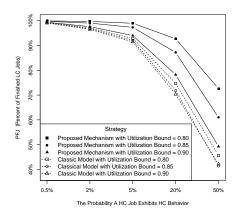


Fig. 9. $\max\{U_L^L + U_H^L, U_H^H\} = 0.8, 0.85 \text{ and } 0.9$

- Each *HC* job independently switches to *HC* mode, i.e., executes for more than *LC* WCET, with a probability of 0.005, 0.02, 0.05, 0.2 or 0.5.
- All the mechanisms will transition back to *LC* mode of execution when there are no pending jobs.

We have chosen a relatively high value for $\max\{U_L^L +$ U_{H}^{L}, U_{H}^{H} , because at smaller values there is sufficient spare capacity so that all the mechanisms are easily able to support LC executions. Simulation results are shown in Figures 8 and 9. The x-axis denotes the probability that a HC job independently switches to HC mode, and the y-axis denotes PFJ for each mechanism. Each point in these figures is generated by taking an average value of PFJ over 1000 tasksets. In Figure 8, we consider only those tasksets that are deemed to be offline schedulable by all the presented mechanisms. As shown in the figure, our mechanism consistently outperforms all the other mechanisms for different values of mode switch probability, and the performance gap improves with increasing probability values. One should note that the results in Figure 8 may not be truly representative of the performance of our mechanism in terms of its ability to support LC jobs, and this can be explained as follows. To compare our mechanism's ability to support LC executions with the other mechanisms, we have to simulate using tasksets that are schedulable by all these mechanisms. In particular, it does not include many

tasksets that are schedulable under our mechanism, but not under one of the other mechanisms. From our observation, in the tasksets that are schedulable by all these mechanisms, the average percentage of HC tasks is much higher than that of LC tasks. Hence to show the ability of our mechanism to support LC executions in a more objective way, we compare the proposed mechanism alone with the classical model, with utilization bound $\max\{U_{L}^{L} + U_{H}^{L}, U_{H}^{H}\} = 0.8, 0.85 \text{ and } 0.9$ as shown in Figure 9. In this case, any taskset schedulable by the classical model can be used in the simulation. It can be seen that the performance of both our mechanism and the classical model drops when compared with the results in Figure 8. However, it can also been seen that, our mechanism still dominates the classical model and the corresponding performance gap does not decrease compared with the gap in Figure 8.

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VI. CONCLUSIONS

In this paper we proposed a novel mechanism to improve the service levels of low-criticality tasks by allowing them to execute even when some high-criticality tasks have exceeded their estimated WCETs. We developed schedulability tests for our mechanism under the mixed-criticality EDF scheduling strategy, considering both a flat as well as an hierarchical scheduling framework. We also evaluated the performance of our mechanism in terms of offline schedulability and online support for low-criticality executions. Simulation results clearly show that the proposed mechanism outperforms all the existing approaches.

In the evaluation section we only consider the performance of our mechanism when all the high-criticality tasks are in one component and all the low-criticality tasks are in another component. In fact, its performance can be further improved if we also consider scenarios in which the low-criticality tasks are allocated to the same component as the high-criticality ones, especially in terms of offline schedulability. In our future work we will consider this problem of optimally allocating the lowcriticality tasks so as to maximize offline schedulability as well as online performance.

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Appendix

A. Dbf Optimization

When component \mathbb{C} experiences EMS, i.e., the case when $t_E < t$, it is pessimistic to simply add up the demand of

all the tasks. Here we introduce an optimization that can be applied in the schedulability test to reduce this pessimism. We split $dbf(\tau_i, t, t_i)$ into two elements, $DL(\tau_i, t, t_i)$ denoting the demand for the interval $[0, t_E)$, and $DH(\tau_i, t, t_i)$ denoting the demand for the interval $[t_E, t)$.

$$dbf(\tau_i, t, t_i) = \mathsf{DL}(\tau_i, t, t_i) + \mathsf{DH}(\tau_i, t, t_i)$$
(16)

Below we present a key observation that provides some insight into this split. Since the first deadline miss is assumed to happen at time instant t in our schedulability test, the demand before $t_E| < t$ cannot exceed t_E . Otherwise, the first deadline miss would happen at or before t_E . Thus the total demand during $[0, t_E)$ can be bounded by t_E , and as a consequence dbf(\mathbb{C}, t, t_E, t_I) can be more tightly bounded as follows.

$$dbf(\mathbb{C}, t, t_E, t_I) = DL + DH + \sum_{\Delta_i \in \mathcal{G}} \Delta_i, \text{ where}$$

$$DH = \sum_{L_i = LC} DH(\tau_i, t, t_I) + \sum_{L_i = HC} DH(\tau_i, t, t_E), \text{ and}$$

$$DL = \min\left\{t_E, \sum_{L_i = LC} DL(\tau_i, t, t_I) + \sum_{L_i = HC} DL(\tau_i, t, t_E)\right\}$$
(17)

In Equation 17, we use DL to bound the total demand of \mathbb{C} for the interval $[0, t_E)$, and DH to bound the total demand for the interval $[t_E, t)$. In order to maximize the total demand, we must then split the demand between DL and DH such that DH is maximized (or equivalently DL is minimized). This is because the total demand for the interval $[0, t_E)$ is bounded by t_E .

In Section III-B we already present $dbf(\tau_i, t, t_i)$ when task τ_i satisfies condition a, b, c or d. Here we present $DL(\tau_i, t, t_i)$ and $DH(\tau_i, t, t_i)$ for these cases, such that $DH(\tau_i, t, t_i)$ is maximized. If τ_i is a LC task, then it cannot execute after t_E (dropped at $t_i = t_I \leq t_E$). Hence for **condition a**,

$$DL(\tau_i, t, t_I)_{[a]} = dbf(\tau_i, t, t_I)_{[a]}$$

$$DH(\tau_i, t, t_I)_{[a]} = 0$$
(18)

Consider the case when τ_i satisfies **condition b**, i.e., $L_i = HC$ and $t - t_i < D_i - D_i^L$. Here as well τ_i cannot execute after t_E as given in Lemma 2. Hence,

$$DL(\tau_i, t, t_E)_{[b]} = dbf(\tau_i, t, t_E)_{[b]}$$

$$DH(\tau_i, t, t_E)_{[b]} = 0$$
(19)

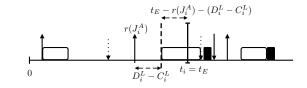


Fig. 10. $DL(\tau_i, t, t_E)_{[c]}$ and $DH(\tau_i, t, t_E)_{[c]}$

Consider the case when τ_i satisfies **condition c**, i.e., $L_i = HC$ and $t - t_i \ge D_i$. In this case $t_i(=t_E)$ occurs after the release of special job J_i^A and this scenario is shown in

Figure 10. To minimize the demand of J_i^A before t_E , we assume that it executes as late as possible. Thus, J_i^A 's demand before t_E can be bounded by $t_E - r(J_i^A) - (D_i^L - C_i^L)$, and we have,

$$DL(\tau_{i}, t, t_{E})_{[c]} = \min \left\{ \left[t_{E} - r(J_{i}^{A}) - (D_{i}^{L} - C_{i}^{L}) \right]_{0}, C_{i}^{L} \right\} \\ + b_{i} \times C_{i}^{L}, \\ DH(\tau_{i}, t, t_{E})_{[c]} = -\min \left\{ \left[t_{E} - r(J_{i}^{A}) - (D_{i}^{L} - C_{i}^{L}) \right]_{0}, C_{i}^{L} \right\} \\ + dbf(J_{i}^{A}, t, t_{E}) + a_{i} \times C_{i}^{H}, \text{ where} \\ b_{i} = \lfloor (t_{E} - (t - D_{i} - \lfloor (t - D_{i})/T_{i} \rfloor \times T_{i}))/T_{i} \rfloor, \\ a_{i} = \lfloor (t - D_{i})/T_{i} \rfloor - b_{i}, \text{ and} \\ r(J_{i}^{A}) = t - D_{i} - \lfloor (t - D_{i})/T_{i} \rfloor \times T_{i} + b_{i} \times T_{i}. \end{cases}$$
(20)

Finally, consider the case when τ_i satisfies **condition d**, i.e., $L_i = HC$ and $D_i - D_i^L \leq t - t_i < D_i$. In this case as well $DH(\tau_i, t, t_i = t_E)_{[d]}$ is maximized if the first job is released at $t - D_i - \lfloor (t - D_i)/T_i \rfloor \times T_i$ (pattern of condition c), and therefore we have,

$$DH(\tau_i, t, t_E)_{[d]} = DH(\tau_i, t, t_E)_{[c]}$$

$$DL(\tau_i, t, t_E)_{[d]} = dbf(\tau_i, t, t_E)_{[d]} - DH(\tau_i, t, t_E)_{[d]}$$
(21)

B. Upper bound for t_{MAX}

Consider a mixed-criticality system with p HCcomponents $\mathbb{C}_1, \mathbb{C}_2, \dots, \mathbb{C}_p$ and q LC components $\mathbb{C}_{p+1}, \mathbb{C}_{p+2}, \dots, \mathbb{C}_{p+q}$. Let $U_L^L(j) = \sum_{\substack{I = LC \\ Li = LC}}^{\tau_i \in \mathbb{C}_j} C_i^L/T_i$, $U_H^L(j) = \sum_{\substack{Li = HC \\ Li = HC}}^{\tau_i \in \mathbb{C}_j} C_i^L/T_i$ and $U_H^H(j) = \sum_{\substack{Li = HC \\ Li = HC}}^{\tau_i \in \mathbb{C}_j} C_i^H/T_i$.

Case 1: If component \mathbb{C}_j experience IMS at t_{Ij} , then the demand of a LC task τ_i in the time interval [0, t) is upper bounded by $(t_{Ij}/T_i + 1) \times C_i^L$, because τ_i will be dropped after t_{Ij} .

A *HC* task τ_i in \mathbb{C}_j switches to *HC* mode at some time instant $t_i \in [t_{Ij}, t_E]$. The demand of τ_i before job J_i^A is bounded by $t_i/T_i \times C_i^L$, the demand of job J_i^A is bounded by C_i^H , and the demand after t_i is bounded by $(t - t_i - D_i + T_i)/T_i \times C_i^H$. Thus the total demand of τ_i in the time interval [0, t) is bounded by

$$\frac{t_i}{T_i} \times C_i^L + C_i^H + \frac{t - t_i - D_i + T_i}{T_i} \times C_i^H$$
(22)

Since $C_i^H > C_i^L$ and $t_i \in [t_{Ij}, t_E]$, the value of Expression (22) is maximized when $t_i = t_{Ij}$. Therefore the total demand of \mathbb{C}_j is bounded by

$$\sum_{L_{i}=HC}^{\tau_{i}\in\mathbb{C}_{j}} \left(t_{Ij} \times C_{i}^{L} + C_{i}^{H}(t - t_{Ij} - D_{i} + 2T_{i}) \right) / T_{i} + \sum_{L_{i}=LC}^{\tau_{i}\in\mathbb{C}_{j}} \left(t_{Ij} / T_{i} + 1 \right) \times C_{i}^{L} \\ \leq U_{H}^{H}(j) \times t + \max_{\tau_{i}\in\mathbb{C}_{j}} \{ 2T_{i} - D_{i} \} \times U_{H}^{H}(j) + \sum_{L_{i}=LC}^{\tau_{i}\in\mathbb{C}_{j}} C_{i}^{L} \\ + \left(U_{L}^{L}(j) + U_{H}^{L}(j) - U_{H}^{H}(j) \right) \times t_{Ij}$$

Case 2: Suppose component \mathbb{C}_j does not experience IMS, i.e., all the LC tasks within \mathbb{C}_j are dropped after t_E , and all the HC tasks switch to HC mode at t_E . In this case, the demand of a LC task τ_i in the time interval [0,t) is upper bounded by $(t_E/T_i+1) \times C_i^L$, and the demand of a HC task τ_i in the time interval [0,t) is upper bounded by $\frac{t_E}{T_i} \times C_i^L + C_i^H + \frac{t-t_E-D_i+T_i}{T_i} \times C_i^H$. Therefore the total demand of \mathbb{C}_j is bounded by

$$\sum_{Li=HC}^{\tau_i \in \mathbb{C}_j} \left(t_E \times C_i^L + C_i^H \times (t - t_E - D_i + 2T_i) \right) / T_i$$

+
$$\sum_{L_i=LC}^{\tau_i \in \mathbb{C}_j} \left(t_E / T_i + 1 \right) \times C_i^L$$

$$\leq U_H^H(j) \times t + \max_{\tau_i \in \mathbb{C}_j} \{ 2T_i - D_i \} \times U_H^H(j) + \sum_{L_i=LC}^{\tau_i \in \mathbb{C}_j} C_i^L$$

+
$$\left(U_L^L(j) + U_H^L(j) - U_H^H(j) \right) \times t_E$$

Let A denote the set of components \mathbb{C}_j with $U_L^L(j) + U_H^L(j) - U_H^H(j) < 0$, and B denote the remaining set of components. Then if $\mathbb{C}_j \in A$, its demand bound given above is maximized when $t_{Ij} = 0$ or $t_E = 0$. On the other hand, if $\mathbb{C}_j \in B$, its demand bound is maximized when $t_{Ij} = t$ or $t_E = t$. Thus, an upper bound on the total demand of \mathbb{C}_j is equal to

$$\begin{cases}
U_{H}^{H}(j) \times t + \max_{\tau_{i} \in \mathbb{C}_{j}} \{2T_{i} - D_{i}\} \times U_{H}^{H}(j) \\
+ \sum_{L_{i} = LC}^{\tau_{i} \in \mathbb{C}_{j}} C_{i}^{L} & \text{if } \mathbb{C}_{j} \in A \\
\max_{\tau_{i} \in \mathbb{C}_{j}} \{2T_{i} - D_{i}\} \times U_{H}^{H}(j) + \sum_{L_{i} = LC}^{\tau_{i} \in \mathbb{C}_{j}} C_{i}^{L} \\
+ (U_{L}^{L}(j) + U_{H}^{L}(j)) \times t & \text{if } \mathbb{C}_{j} \in B
\end{cases}$$
(23)

Suppose $\sum_{j=1}^{j \le p+q} dbf(\mathbb{C}_j, t, t_E, t_{Ij}) > t$ for some t. Then it must be the case that

$$\begin{split} &\sum_{j=1}^{j \leq p+q} \left(\max_{\tau_i \in \mathbb{C}_j} \{ 2T_i - D_i \} \times U_H^H(j) + \sum_{L_i = LC}^{\tau_i \in \mathbb{C}_j} C_i^L \right) \\ &> t \left(1 - \sum_{\mathbb{C}_j \in A} U_H^H(j) - \sum_{\mathbb{C}_j \in B} (U_L^L(j) + U_H^L(j)) \right) \\ &\Rightarrow t < \frac{\sum_{j=1}^{j \leq p+q} \left(\max_{\tau_i \in \mathbb{C}_j} \{ 2T_i - D_i \} \times U_H^H(j) + \sum_{L_i = LC}^{\tau_i \in \mathbb{C}_j} C_i^L \right)}{1 - \sum_{\mathbb{C}_j \in A} U_H^H(j) - \sum_{\mathbb{C}_j \in B} (U_L^L(j) + U_H^L(j))} \end{split}$$

Thus we can conclude that the upper bound of t, i.e., t_{MAX} , is given as

$$\frac{\sum_{j=1}^{j \le p+q} \left(\max_{\tau_i \in \mathbb{C}_j} \left\{ 2T_i - D_i \right\} \times U_H^H(j) + \sum_{L_i = LC}^{\tau_i \in \mathbb{C}_j} C_i^L \right)}{1 - \sum_{\mathbb{C}_j \in A} U_H^H(j) - \sum_{\mathbb{C}_j \in B} (U_L^L(j) + U_H^L(j))}$$