Pseudo-Exhaustive Word-Oriented DRAM Testing

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This work was partially supported by the NSF, grant MIP9208487, and the NATO, grant 910411.

Abstract

This paper presents a new methodology for RAM testing based on the $PS(n,k)$ q-ary fault model ($q = 2^w$) which includes most classical fault models for SRAMs and DRAMs. According to this fault model, the contents of any w-bit memory word of a memory with n words, or ability to change this contents, is influenced by the contents of any other $k-1$ words of the memory. The proposed methodology uses a pseudo-exhaustive technique based on Reed-Solomon *codes, which can be efficiently applied to a word-oriented RAMs, assuming small values of k. The methodology ensures the detection of any number of disjoint (not linked) k-coupling faults, whereby the involved k words may be located anywhere in the memory; i.e., no assumptions have to be made on the physical topology of the cells in the memory cell array. Because of the systematic structure of the proposed tests, they are well suited for BIST implementations.*

Key words: *Memory testing, pattern sensitive faults, pseudo-exhaustive memory testing, random access memory.*

1 Introduction

The increasing densities in memory technology has resulted in a dramatically increasing test cost [1,2] caused by the increased number of cells to be tested, as well as the more complex fault models. The latter applies especially to DRAMs, where in addition to the traditional faults for SRAM chips [3,4], neighborhood pattern sensitive faults 'NPSFs' [5-8] have to be considered.

The well-known tests for NPSFs usually require that the physical topology of the cells in the memory cell array is known, while they assume that the memory words usually consist of a single bit. In addition, tests for NPSFs do not detect many of the classical faults which also apply to SRAMs [3]; e.g., address decoder faults 'AFs', data retention faults 'DRFs' [9], stuck-open faults 'SOFs' [9], and coupling faults. Pseudo-random memory tests [10,11] do not require knowledge of the physical topology of the memory cell array and can be applied to memories with w-bit words ($w \geq 2$); however, they have the disadvantage that their fault coverage is probabilistic. Tests for k-coupling faults (for $k = 4$ and 5) have been proposed in [16,17,18]; however, those tests are restricted to memories with 1-bit words and are based on combinational, rather than analytical, techniques. The capability of a test to cope with memories with w-bit words ($w \geq 2$) is of increasing importance; whereas early memory chips have a $n * 1$ (where n is the number of words) organization; currently, many chips have a $n * 4$ organization while $n * 8$ chips are expected to reach high volume production soon [13].

This paper proposes a new fault model which has the following properties:

1. It is modular in terms of k, the number of words involved in the fault.
2. Words are w-bits ($w \geq 1$) wide.
3. No assumptions have to be made on the physical location of the k words.
4. It includes many of the traditional SRAM and DRAM faults.

The organization of this paper is as follows: Section 2 introduces the fault model, Section 3 describes the test approach which is based on pseudo-exhaustive testing, Section 4 gives the mathematical background for the proposed tests, Section 5 describes the pseudo-exhaustive tests, and Section 6 concludes this paper.

2 Fault model

This section describes the new fault model for pseudo-exhaustive testing of DRAMs. First, the fault models used for testing SRAMs, together with an explanation concerning their applicability to DRAMs, will be presented. Next, the classical DRAM fault models are presented. And last, the new $PS(n, k)$ q-ary fault model will be introduced; it will be shown which of the classical SRAM and DRAM fault models it covers; for those faults, considered important for DRAMs, which are not covered by the new fault model a separate set of tests will be proposed.

2.1 Classical SRAM fault models

The classical SRAM faults which have been found to be important [4,9] are listed below; a motivation is given when they do not appy to DRAMs.

- Stuck-at fault 'SAF'.
- Stuck-open fault 'SOF' [9]
  SRAMs need special test provisions to cope with SOFs when the sense amplifiers are not transparent to SOFs. In case of DRAMs this problem does not occur because the differential sense amplifier has only one input from the cell being read such that SOFs behave as SAFs.
- Transition faults 'TFs'
  These faults cannot occur in the memory cell array of the DRAM because the cells are not implemented as bistable elements.
- Coupling faults 'CFs'
  The CFs of interest are the idempotent CF 'CFid" and the state CF 'CFst' [9].
- Data retention faults 'DRFs' [9]
  The SRAM type of DRFs cannot occur in DRAMs because of the absense of pull-up devices. However, leakage currents may cause loss of information. A refresh test, using a checkerboard pattern, has to be used for this [3].
- Address decoder faults 'AFs'.

Considering the above, the SRAM faults which also apply to DRAMs are the SAFs, the CFs and the AFs.

2.2 Classical DRAM faults

Pattern sensitive faults 'PSFs' [5-8,3] are considered typical for DRAMs. They involve a group of k cells whereby $k-1$ cells influence a given target cell, called the *base cell*. In order to keep the test time within acceptable limits for larger chips, the assumption is made that the $k-1$ cells, which influence the base cell, physically surround the base cell; this simplifies the PSF model to a *neighborhood* PSF 'NPSF' model; the $k-1$ cells influencing the base cell are called the *deleted neighborhood cells*. This is a realistic simplification because of the underlying assumption that PSFs are caused by leakage currents which can only occur between cells in a physical neighborhood. The disadvantage of the NPSF model is that the physical topology of the cells in the memory cell array has to be known; this is not always so: the use of spare rows and columns already violates this, even for tests performed by the manufacturer; the user usually does not have access to the physical topology which, in addition, may differ between functionally equivalent parts of different manufacturers.

The classical NPSFs usually considered are [3]:

- Active NPSF 'ANPSF' [8]
  The base cell changes its contents due to a change in the $k-1$ deleted neighborhood patterns (i.e. the value of the $k-1$ cells).
- Passive NPSF 'PNPSF' [12]
  The content of the base cell cannot be changed due to a certain deleted neighborhood pattern.
- Static NPSF 'SNPSF' [8]
  The base cell is forced to a certain state due to a certain deleted neighborhood pattern.

2.3 The $PS(n, k)$ q-ary fault model

Given a memory with n words consisting of w-bits per word, whereby q is defined as $q = 2^w$. Then the following fault definitions can be given.

1. *Stuck-at q-ary faults 'SAFq'*
   A permanent stuck-at q-ary fault reduces the number of faulty memory word states. A faulty word i of the memory may contain only one q-ary digit, or a subset S of all possible q-ary digits $0, 1, 2, \ldots, q-1$. This fault model covers the classical SAFs.
2. *Transition q-ary faults 'TFq'*
   A memory word i in the state $W_i(t)$ fails to undergo a $W_i(t)$ to $W_i(t+1)$ transition while $W_i(t) \neq W_i(t+1)$ ($W_i(t)$ and $W_i(t+1) \in \{0, 1, 2, \ldots, q-1\}$) and $W_i(t+1)$ is to be written in the i-th memory word; however, both states are possible for the i-th memory word, for instance at power-on time. This fault model covers the classical TFs.
3. *Coupling q-ary faults 'CFq'*
   A coupling q-ary fault is present from a memory word i to a word j if, when the words contain a particular pair of q-ary values $W_i(t)$ and $W_j(t)$, and $W_i(t+1)$ is written into word i, then word j, as well as word i, change state. This fault model covers the classical CFids and CFsts.
4. *Pattern sensitive q-ary faults 'PSFq'*
   The base word changes its contents, or cannot be changed, due to a pattern, or a change, in the $k-1$ other words. This definition covers the classical NPSFs of Section 2.2.

The above 4 fault models are covered by the $PS(n,k)$ q-ary fault model, which has the following properties:

1. k w-bit words, whereby each word can be in q ($q = 2^w$) states, are involved in the fault model.

2. the base word will take on all 2^w states and each cell in the base word will make an up and a down transition for each of 2^{w-1} states of the $w-1$ other cells in the word.

3. each of the $k-1$ non-base words will take on all 2^w states for each state or transition of the base word; and for any one of the $2^{(k-1)w}$ internal states of the $k-1$ non-base cells, all $2w$ transitions in the base cell may occur.

The above fault model will detect the 4 q-ary faults:

1. $SAFq$ and TFq faults will be detected because of property 1, for $k \geq 1$.
2. CFq faults will be detected because of property 1 and 2, for $k \geq 2$.
3. $PSFq$ faults will be detected because of properties 1 through 3, and $k = k$.

3 Pseudo-exhaustive memory testing

Pseudo-exhaustive testing [14] of combinational devices has several attractive features. In addition to the fact that test patterns can be generated quite easily, the process and its fault coverage are basically dependent neither on the fault model assumed nor on its specific circuit under test.

Let us give some basic definitions of pseudo-exhaustive memory testing.

Definition 3.1 A background for a $(w \times n)$ memory (w-bits per word, n words) is a vector $B = (B^{(0)}, B^{(1)}, \ldots, B^{(n-1)})$, where $B^{(j)} \in GF(2^w)$, $j \in \{0,1,2,\ldots,n-1\}$ and $GF(2^w)$ is the field of w-dimensional binary vectors.□

Definition 3.2 A set of k-pseudo-exhaustive backgrounds is a matrix $B(n,k,w)$, where rows are backgrounds $B_i = (B_i^{(0)}, B_i^{(1)}, \ldots, B_i^{(n-1)})$, where $B_i^{(j)} \in GF(2^w)$, $i = 0,1,\ldots,T_k-1$, and $j = 0,1,\ldots,n-1$, such that in the matrix $B(n,k,w)$ all q^k k-digit q-ary ($q = 2^w$) vectors $(y_0, y_1, \ldots, y_{k-1})$ (where $y_l \in GF(2^w)$, and $l = 0,1,\ldots,k-1$) appear at least once in any k columns.□

By the definition of k-pseudo-exhaustive backgrounds $B(n,k,w)$ we have the lower bound on the number $T_k = T_k(n)$ of backgrounds $T_k(n) \geq q^k = 2^{wk}$.

Techniques for the construction of k-pseudo-exhaustive data backgrounds $B(n,k,w)$ and estimations on minimal numbers of pseudo-exhaustive patterns can be found for the binary case $(w=1)$ in [14]. Techniques for the construction of k-pseudo-exhaustive data backgrounds $B(n,k,w)$ and estimations on their minimal sizes for the q-ary case $(w>1)$ are not known. We will present in this paper optimal solutions, satisfying to the lower bound, of this problem for small k.

As a systematic approach for generating k-pseudo-exhaustive data backgrounds we propose to use *Reed-Solomon* 'RS' codes over $GF(2^w)$ [15].

The extended $(q+1, q+1-k, k+1)$ RS code over $GF(2^w)$ is defined by the check matrix [15]:

$$H = \begin{vmatrix} 1 & 0 & 1 & 1 & 1 & \ldots & 1 \\ 0 & 0 & 1 & \alpha & \alpha^2 & \ldots & \alpha^{q-2} \\ 0 & 0 & 1 & \alpha^2 & \alpha^4 & \ldots & \alpha^{2(q-2)} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 1 & 1 & \alpha^{k-1} & \alpha^{2(k-1)} & \ldots & \alpha^{(k-1)(q-2)} \end{vmatrix} \tag{1}$$

where α is primitive in $GF(2^w)$ ($\alpha^i \neq \alpha^j$ for $i \neq j \in \{0,1,\ldots,q-2\}$). Since any k columns of H are linearly independent over $GF(q)$, the linear span of rows of H will be an optimal k-pseudo-exhaustive background $B(2^w+1,k,w)$ with $T_k = q^k = 2^{wk}$.

Example 3.1 Let $q = 2^w = 4$ and $GF(2^2) = \{0,1,\alpha,\alpha^2\}$, where α is a root of polynomial $\varphi(x) = x^2+x+1$ $(\alpha^3 = 1)$, then the operations of addition and multiplication in the field $GF(2^2)$ are described by the following tables for which $0 = 00, 1 = 10, \alpha = 01, \alpha^2 = 11, \alpha^3 = 1 = 10, \alpha^4 = \alpha = 01$.

Addition (+)				
$+$	0	1	α	α^2
0	0	1	α	α^2
1	1	0	α^2	α
α	α	α^2	0	1
α^2	α^2	α	1	0

Multiplication ($\times$)				
$\times$	0	1	α	α^2
0	0	0	0	0
1	0	1	α	α^2
α	0	α	α^2	1
α^2	0	α^2	1	α

For the construction of the optimal 2-pseudo-exhaustive backgrounds over $GF(2^2)$ we use the check matrix H. Then, any background $B = (B^{(0)}, B^{(1)}, B^{(2)}, B^{(3)}, B^{(4)})$ can be generated as

$$(v_0, v_1) \cdot \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & \alpha & \alpha^2 \end{vmatrix} = (v_0, v_1, v_0+v_1, v_0+\alpha v_1, v_0+\alpha^2 v_1), \tag{2}$$

where $v_0, v_1 \in GF(2^2)$.

For example, $(\alpha, \alpha^2) \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & \alpha & \alpha^2 \end{vmatrix} = (\alpha, \alpha^2, \alpha+\alpha^2, \alpha+\alpha^3, \alpha+\alpha^4) = (\alpha, \alpha^2, 1, \alpha^2, 0)$ or in the binary notation:

Table 1: 2-Pseudo-exhaustive backgrounds $B(5,2,2)$

i	β^{i-1}	$v_0(i)$	$v_1(i)$	$B_i^{(0)}$	$B_i^{(1)}$	$B_i^{(2)}$	$B_i^{(3)}$	$B_i^{(4)}$
$-$	$-$	0	0	0	0	0	0	0
1	β^0	1	0	1	0	1	1	1
2	β^1	0	1	0	1	1	α	α^2
3	β^2	α	1	α	1	α^2	0	1
4	β^3	α	α^2	α	α^2	1	α^2	0
5	β^4	1	1	1	1	0	α^2	α
6	β^5	α	0	α	0	α	α	α
7	β^6	0	α	0	α	α	α^2	1
8	β^7	α^2	α	α^2	α	1	0	α
9	β^8	α^2	1	α^2	1	α	1	0
10	β^9	α	α	α	α	0	1	α^2
11	β^{10}	α^2	0	α^2	0	α^2	α^2	α^2
12	β^{11}	0	α^2	0	α^2	α^2	1	α
13	β^{12}	1	α^2	1	α^2	α	0	α^2
14	β^{13}	1	α	1	α	α^2	α	0
15	β^{14}	α^2	α^2	α^2	α^2	0	α	1
16	β^1	1	0	1	0	1	1	1

$$(01,11)\begin{vmatrix} 10 & 00 & 10 & 10 & 10 \\ 00 & 10 & 10 & 01 & 11 \end{vmatrix} = (01,11,10,11,00) \tag{3}$$

As a result of multiplication of all vectors $V = (v_0,\ v_1)$ by H we have 2-pseudo-exhaustive data backgrounds $B(5,2,2)$ (see Table 1).

As we can see from Table 1 for any $k = 2\ \ q = 2^2$-ary words we have all $q^2 = (2^2)^2 = 16$ combinations of data in these words.□

In the following sections we will describe test procedures based on k-pseudo-exhaustive data backgrounds $B_0, B_1, \ldots, B_{q^k-1}$, combined with the standard $MATS+$ test (to cover AFs) [4] for $k = 1, 2$ and 3.

4 Mathematical background

The following theorem can be used for construction of k-pseudo-exhaustive backgrounds for any k and $n \leq q-1$.

Theorem 4.1 Let $q = 2^w$, α is primitive in $GF(q)$ ($\alpha^l \neq \alpha^j; l \neq j; l,j = 0,1,2,\ldots,q-2$), β is primitive in $GF(q^k)$ ($\beta^l \neq \beta^j; l \neq j; l,j = 0,1,2,\ldots,q^k-2$), and

$$H = \begin{vmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & \alpha & \alpha^2 & \ldots & \alpha^{q-2} \\ 1 & \alpha^2 & \alpha^4 & \ldots & \alpha^{2(q-2)} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \alpha^{k-1} & \alpha^{2(k-1)} & \ldots & \alpha^{(k-1)(q-2)} \end{vmatrix} \tag{4}$$

Given that

$$\beta^{i-1} = (\alpha^{i_0}, \alpha^{i_1}, \ldots, \alpha^{i_{k-1}}) \in GF(q^k), \tag{5}$$

and $B_0 = (0,0,\ldots,0)$, $B_i = (B_i^{(0)}, B_i^{(1)}, \ldots, B_i^{(q-1)}) = (\alpha^{i_0}, \alpha^{i_1}, \ldots, \alpha^{i_{k-1}})H$ ($B_i^{(j)} \in GF(q), i = 1,2,\ldots,q^k$); then

1. For any $j_0 \leq j_1 \leq \ldots \leq j_{k-1}$ and any $A_{j_0}, A_{j_1}, \ldots, A_{j_{k-1}} \in GF(q)$ there exists $i \in \{0,1,\ldots,q^k-1\}$ such that

$$B_i^{(j_0)} = A_{j_0},\ B_i^{(j_1)} = A_{j_1}, \ldots,\ B_i^{(j_{k-1})} = A_{j_{k-1}}.$$

2. For any $s \in \{0,1,\ldots,q-2\}$, $j_0 \leq j_1 \leq \ldots \leq j_{k-3}$ ($s \notin \{j_0, j_1, \ldots, j_{k-3}\}$) and any $A_{j_0}, A_{j_1}, \ldots A_{j_{k-3}}, A_s, A_s^{'} \in GF(q)$, except $A_{j_0} = A_{j_1} = \ldots = A_{j_{k-3}} = A_s = A_s^{'} = 0$, there exists $i \in \{1,2,\ldots,q^k\}$ such that

$$\begin{aligned} B_i^{(j_0)} &= A_{j_0},\ B_i^{(j_1)} = A_{j_1}, \ldots, B_i^{(j_{k-3})} \\ &= A_{j_{k-3}},\ B_i^{(s)} = A_s \ and\ B_{i+1}^{(j_s)} = A_s^{'}. \end{aligned} \tag{6}$$

□

Remark 4.1 Theorem 4.1 is valid for more general case when for any subset J of $\{j_0, j_1, \ldots, j_{k-3}\}$ in (6) $B_i^{(j)}$ is replaced by $B_{i+1}^{(j)}$, $j \in J$.□

Remark 4.2 Theorem 4.1 and Remark 4.1 are valid for $k = 2$ and $n = q+1$ when we use the check matrix H, below, of the $[q+1, q+1-k, k]$ MDS code [15] instead of H defined by(4).□

$$H = \begin{vmatrix} 1 & 0 & 1 & 1 & 1 & \ldots & 1 \\ 0 & 0 & 1 & \alpha & \alpha^2 & \ldots & \alpha^{q-2} \\ 0 & 0 & 1 & \alpha^2 & \alpha^4 & \ldots & \alpha^{2(q-2)} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 1 & 1 & \alpha^{k-1} & \alpha^{2(k-1)} & \ldots & \alpha^{(k-1)(q-2)} \end{vmatrix} \tag{7}$$

Remark 4.3 Theorem 4.1 is valid for any k and $n = q$ when check matrix H, below, represents the $[q, q-k, k]$ MDS code.□

$$H = \begin{vmatrix} 1 & 1 & 1 & 1 & \ldots & 1 \\ 0 & 1 & \alpha & \alpha^2 & \ldots & \alpha^{q-2} \\ 0 & 1 & \alpha^2 & \alpha^4 & \ldots & \alpha^{2(q-2)} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 1 & \alpha^{k-1} & \alpha^{2(k-1)} & \ldots & \alpha^{(k-1)(q-2)} \end{vmatrix} \tag{8}$$

By the Theorem 4.1 and Remarks 4.1, 4.2 and 4.3 k-pseudo-exhaustive backgrounds, defined by (4), (5), combined with $MATS+$ procedure generate optimal tests with $q^k = 2^{wk}$ backgrounds and with complexity $2^{wk+1}n$ detecting static $SPS(n,k)$ faults and dynamic $DPS(n,k-1)$ faults for any $k > 2$ for $n < 2^w$; for $k = 2$ and $n = 2^w + 1$; and detecting $SPS(n,k)$ for any k and $n = 2^w$. In the next sections we will expand these procedures for the cases $n > 2^w$ and $k = 1, 2, 3$.

5 Pseudo-exhaustive memory tests

5.1 k-Pseudo-exhaustive backgrounds

For the case $k = 1$ the procedure for generation of 1-pseudo-exhaustive backgrounds consists of multiplication in $GF(2^w)$ of all q-ary vectors $V = (v_0)$, $v_0 \in \{0, 1, \alpha, \alpha^2, \ldots, \alpha^{q-2}\}$ by the first row of the *RS* check matrix (1). The row dimension is determined by the memory size n. As a result we will have the $B(n,1,w)$ optimal 1-pseudo-exhaustive backgrounds with $T_1(n) = q$ for any n.

For example, for a 2-bit wide memory with 6 cells ($w = 2$, $q = 4$, $n = 6$) we have the following backgrounds $B(6,1,2)$:

B_0	B_1	B_2	B_3	B_4	B_5
0	0	0	0	0	0
1	1	1	1	1	1
α	α	α	α	α	α
α^2	α^2	α^2	α^2	α^2	α^2

=

B_0	B_1	B_2	B_3	B_4	B_5
00	00	00	00	00	00
10	10	10	10	10	10
01	01	01	01	01	01
11	11	11	11	11	11

For the complexity of the test procedure based on $B(n,1,w)$ and MATS+ we have $L[MATS+, B(n,1,w)] = 2^{w+1}n$.

More complex is a procedure of the background generation for $k = 2$. Let $\varphi(x) = x^2 + c_1x + c_0$ ($c_0, c_1 \in GF(2^w)$) be a primitive polynomial of degree 2 over $GF(2^w)$ and β is a root of $\varphi(x)$ ($\varphi(\beta) = 0$). Then [15], there exists a one-to-one mapping $i \leftrightarrow (v_0(i), v_1(i))$, where $v_0(i), v_1(i) \in \{1, \alpha, \alpha^2, \ldots, \alpha^{q-2}\}$ $(v_0(i), v_1(i)) \neq (0,0)$; $i \in \{1, 2, \ldots, q^2\}$; and $q = 2^w$, such that

$$v_0(i) + v_1(i)\beta = \beta^{i-1}, \tag{9}$$

where $\beta^{q^2-1} = \beta^0 = 1$. This mapping for $w = 2$ and $\varphi(x) = x^2 + x + \alpha$ is given in Table 1.

According to the procedure for $n = q+1$ described by the Remark 4.2 for generation of optimal 2-pseudo-exhaustive backgrounds we have $B(q+1,2,w)$, where $T_2(q) = q^2+1$; ($B_i^{(j)} \in GF(2^w), q = 2^w$), $B_0^{(j)} = 0, (j = 0, 1, \ldots, q)$,

$$B_i^{(j)} = B(q+1,2,w) = |v_0(i), v_1(i)| \cdot \left| \begin{array}{cccccccc} 1 & 0 & 1 & 1 & 1 & \ldots & 1 & 1 \\ 0 & 1 & 1 & \alpha & \alpha^2 & \ldots & \alpha^{q-4} & \alpha^{q-2} \end{array} \right| \tag{10}$$

Thus, $B_0 = (0,0,\ldots,0)$ $B_i^{(0)} = v_0(i)$, $B_i^{(1)} = v_1(i)$, $B_i^{(j)} = v_0(i) + v_1(i)\alpha^{j-2}$, $(i = 1, 2, \ldots, q^2; j = 2, 3, \ldots, q)$. For $w = 2$, $q + 1 = 2^2 + 1 = 5$ 2-pseudo-exhaustive backgrounds $B(5,2,2)$ are shown in Table 1.

Any set $B(q+1,2,w)$ of 2-pseudo-exhaustive backgrounds consists of the $T_2(q+1) = 2^{2w} + 1$ backgrounds.

For the complexity of the test procedure based on 2-pseudo-exhaustive backgrounds $B(n,2,w)$ and MATS+ we have $L(MATS+B(n,2,3)) = 2(q^2+1)n = 2^{2w+1}n+2n$.

For any k the procedure for generating k-pseudo-exhaustive backgrounds will be described the following way. Let $\varphi(x)$ be a primitive polynomial of degree k over $GF(2^w)$ and β is a root of $\varphi(x)$ ($\varphi(\beta) = 0$). Then [15], there exists an one-to-one mapping $i \leftrightarrow (v_0(i), v_1(i), \ldots, v_{k-1}(i))$, where $v_0(i), v_1(i), \ldots, v_{k-1}(i) \in \{1, \alpha, \alpha^2, \ldots, \alpha^{q-2}\}$ $(v_0(i), v_1(i), \ldots, v_{k-1}(i)) \neq (0,0,\ldots,0)$; $i \in \{1, 2, \ldots, q^k\}$; and $q = 2^w$, such that

$$v_0(i) + v_1(i)\beta + \ldots + v_{k-1}(i)\beta^{k-1} = \beta^{i-1} \tag{11}$$

where $\beta^{q^2-1} = \beta^0 = 1$.

According to the procedure for $n = q - 1$ described by the Theorem 4.1 for the generation of optimal k-pseudo-exhaustive backgrounds we have $B(q-1,k,w)$, where $T_k(q) = q^k+1$; ($B_i^{(j)} \in GF(2^w), q = 2^w$), $B_0^{(j)} = 0, (j = 0, 1, \ldots, q)$,

$$B_i^{(j)} = |v_0(i), v_1(i), \ldots, v_{k-1}(i)| \cdot \left| \begin{array}{cccccc} 1 & 1 & 1 & \ldots & 1 & 1 \\ 1 & \alpha & \alpha^2 & \ldots & \alpha^{q-4} & \alpha^{q-2} \\ 1 & \alpha^2 & \alpha^4 & \ldots & \alpha^{2(q-4)} & \alpha^{2(q-2)} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \alpha^{k-1} & \alpha^{2(k-1)} & \ldots & \alpha^{(k-1)(q-4)} & \alpha^{(k-1)(q-2)} \end{array} \right| \tag{12}$$

Any set $B(q-1,k,w)$ of k-pseudo-exhaustive backgrounds consists of the $T_k(q-1) = 2^{kw} + 1$ backgrounds.

5.2 Restricted pseudo-exhaustive tests $RPST_{k,k-1}$

Generalization of the tests for detection of crosstalks between three or more words will require high complexity and considerable overheads for BIST implementations. In view of this we describe in this section a class of restricted (local) pseudo-exhaustive tests $RPXT_{k,k-1}, (k = 2, 3, 4, ...)$ for word-oriented memories detecting static $SPS(q-1, k)$ and dynamic $DPS(q-1, k-1)$ faults due to crosstalks between k or $k-1$ words within any block of $q-1$ neighbouring words.

To construct these tests we use k-pseudo-exhaustive backgrounds $B(q-1, k, w)$ described in Theorem 4.1. In this case $B(q-1, k, w)$ is $(q-1, k, q-k)$ q-ary RS code and $|B(q-1, k, w)| = q^k + 1$. At the first step of $RPXT_{k,k-1}$ we run pseudo-exhaustive tests $PXT_{k,k-1}$ based on $B(q-1, k, w)$ and $MATS+$ for words with addresses $0, 1, \ldots, q-2$. At the second step we repeat the same procedure for the block consisting of words with addresses $2^{w-1}, 2^{w-1}+1, \ldots, 2^{w-1}+2^w-2$. At the third step we repeat the procedure for words with addresses $2^w, 2^w+1, \ldots, 2^w+2^w-2$, etc. This approach is illustrated in Fig. 1.

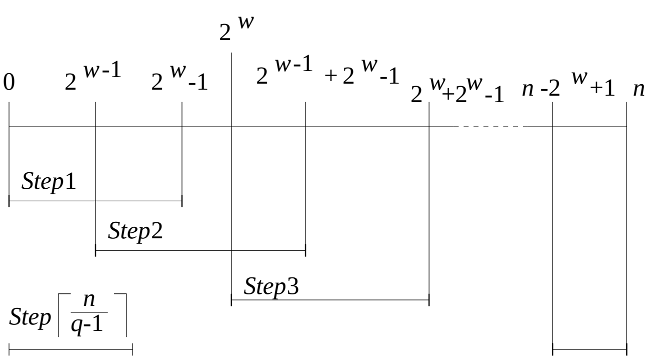

Figure 1: Test Organization for $RPXT_{k,k-1}$

Test $RPXT_{2,1}$ for $w = 2$ $(q = 4)$, $n = 5$ consisting of two steps is represented by Table 2 $(a_0, a_1, a_2, a_3, a_4,)$ is an initial state of the RAM; first block consists of words W_0, W_1, W_2 and second block consists of W_2, W_3, W_4).

We have for complexity $L(RPXT_{k,k-1})$ of these tests

$$L(RPXT_{k,k-1}) = 2(q-1)(q^k)\lceil\frac{n}{q-1}\rceil + 2n \approx 2^{wk+1}n + 2n. \tag{13}$$

Test complexities (in $sec.$) of $RPXT_{k,k-1}$ tests for different k and $w = 4$ are presented in Table 3 (assuming a cycle time of 50 ns). For example, for a 4-bit memory with $N = nw = 2^{16}$ bits detection of Static $SPS(n, 4)$ faults and Dynamic $DPS(n, 3)$ faults by $RPXT_{4,3}$ requires $107.37 sec$.

To summarise this section we note that as it follows from Table 3 tests $RPXT_{2,1}$, $RPXT_{3,2}$ and $RPXT_{4,3}$ may be efficient for 4-bit memories $(w = 4)$.

Table 2: $RPXT_{2,1}$ test for $n = 5$, $w = 2$ based on 2-pseudo-exhaustive backgrounds $B_i^{(2)}, B_i^{(3)}$ and $B_i^{(4)}$, combined with $MATS+$

t	$r(W_j), w(W_j)$	W_0	W_1	W_2	W_3	W_4	B_i
0		a_0	a_1	a_2	a_3	a_4	
1	$w(W_0)$	$\underline{0}$	a_0	a_1	a_2	a_3	
2	$w(W_1)$	0	$\underline{0}$	a_1	a_2	a_3	
3	$w(W_2)$	0	0	$\underline{0}$	a_2	a_3	
4	$w(W_3)$	0	0	0	$\underline{0}$	a_3	
5	$w(W_4)$	0	0	0	0	$\underline{0}$	B_0
6	$r(W_2), w(W_2)$	0	0	$\underline{1}$	0	0	
7	$r(W_1), w(W_1)$	0	$\underline{1}$	1	0	0	
8	$r(W_0), w(W_0)$	$\underline{1}$	1	1	0	0	B_1
9	$r(W_0), w(W_0)$	$\underline{1}$	1	1	0	0	
10	$r(W_1), w(W_1)$	1	$\underline{\alpha}$	1	0	0	
11	$r(W_2), w(W_2)$	1	α	$\underline{\alpha^2}$	0	0	B_2
...	...	...	...	...	...	...	...
48	$r(W_2), w(W_2)$	α^2	α	$\underline{1}$	0	0	
49	$r(W_1), w(W_1)$	α^2	$\underline{\alpha}$	1	0	0	
50	$r(W_0), w(W_0)$	$\underline{0}$	α	1	0	0	B_{15}
51	$r(W_0), w(W_0)$	$\underline{1}$	α	1	0	0	
52	$r(W_1), w(W_1)$	1	$\underline{1}$	1	0	0	
53	$r(W_2), w(W_2)$	1	1	$\underline{1}$	0	0	B_1
54	$r(W_4), w(W_4)$	1	1	1	0	$\underline{1}$	
87	$r(W_3), w(W_3)$	1	1	1	$\underline{1}$	1	
88	$r(W_2), w(W_2)$	1	1	$\underline{1}$	1	1	
89	$r(W_1)$	1	$\underline{1}$	1	1	1	
90	$r(W_0)$	$\underline{1}$	1	1	1	1	B_1
90	$r(W_2), w(W_2)$	1	1	$\underline{1}$	1	1	
91	$r(W_3), w(W_3)$	1	1	1	$\underline{\alpha}$	1	
92	$r(W_4), w(W_4)$	1	1	1	α	$\underline{\alpha^2}$	B_2
...	...	...	...	...	...	...	...
132	$r(W_4), w(W_4)$	1	1	0	α	$\underline{1}$	
133	$r(W_3), w(W_3)$	1	1	0	$\underline{1}$	1	
134	$r(W_2), w(W_2)$	1	1	$\underline{1}$	1	1	B_1
135	$r(W_2)$	1	1	$\underline{1}$	1	1	
136	$r(W_3)$	1	1	1	$\underline{1}$	1	
137	$r(W_4)$	1	1	1	1	$\underline{1}$	B_1

Table 3: Time complexities (in seconds) for $RPXT_{k,k-1}$ tests for $(w = 4)$, $k = 2, 3, 4, 5$ and different $N = wn$

N	2^8	2^{12}	2^{14}	2^{16}	2^{20}
$k = 2$	0.00	0.02	0.10	0.41	6.71
$k = 3$	0.02	0.41	1.67	6.71	107.37
$k = 4$	0.41	6.71	26.84	107.37	1717.98
$k = 5$	6.71	107.37	429.49	1717.98	

6 Conclusions

In this paper we have presented a unified approach for testing of word-oriented memories based on the single $PS(n,k)$ fault model which covers SAFs, TFs, CFids, CFins, APSFs, PPSFs and SPSFs. A systematic approach for generating data backgrounds $B(n,k,w)$ has been proposed, based on $Reed-Solomon$ codes over $GF(2^w)$, where w is the number bits per word. Combining k-pseudo-exhaustive backgrounds $B(n,k,w)$ with the $MATS+$ test algorithm we presented a range of optimal pseudo-exhaustive tests.

For the case when faults are restricted to a neighbourhood consisting of at most $2^{w-1}-1$ words we propose the test $RPXT_{k,k-1}$. Test $RPXT_{k,k-1}$, with complexity $2^{kw+1}n+2n$, verifies for any k words all 2^{kw} states of the words and all 2^{2w} transitions within one word for any fixed state of any other $k-2$ words for the memory-under-test block with the size $\lceil (q-1)/2 \rceil - 1$.

The deterministic 100% fault coverage, also for the complex PSFs involving a large number of words, causes it to be preferred above pseudo-random tests in many applications, while due to its systematic nature it renders itself well for BIST applications.

Acknowledgment

The authors wish thank to Prof. Lev B. Levitin of the Boston University, Boston, for his valuable comments and suggestions.

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