Applications of Complex Network Theory on Power Grids

Muhammad Jawad and Bei Gou
Department of Electrical and Computer Engineering,
North Dakota State University
Fargo, ND 58108

Abstract— Complex network statistical analysis tools are helpful to understand silent features of complex systems. One of the most important system growing intentions in recent days is power grid. Complex system like power system with specific topology can undergo a local or global cascade failure. Prediction and prevention of these cascade failures or blackouts is obligatory. In this paper some of the most important properties of power grids infrastructure are investigated using Complex Network Theory (CNT) techniques and methodologies. Discussions are conducted on different measures of network structures of IEEE 118 and Polish 2383 bus systems. These results and structure of power network has important implication on reliability and security.

NOMENCLATURE

A_{ij} Adjacency Matrix.
x_i' Eigenvector Centrality Measure
d_{ij} Geodesic distance b/w nodes 'i' and 'j'
C_i' Closeness Centrality
B_i Betweenness Centrality
p_{st}^{i} Geodesic path b/w nodes 'i' and 'j'
C_i Clustering Coeeficient
C_{WS} Clustering Coefficient for whole system
P_k Degree (k) Distribution

I. INTRODUCTION

Complex Network Theory (CNT) is an emerging field of research and it is being utilized in different fields like social sciences, inter-banking systems, computer networks, transportation and logistics systems and never the less electrical power systems. Recent advancements in network theory focus on the relationship between topological structural of networks and vulnerability of networks in different types of failures. Many classifications of network structure and behavior had been discussed in the field of statistical mechanics and complex network [1, 2], but the two mostly used stochastic models are developed by Erd˝os and Rényi [3, 4].

This study focuses on electrical transmission and distribution systems due to its large-scale infrastructure and decentralized mean growth. Power network efficiency and reliability is a major question in recent years especially after 2003 blackout in eastern coast of USA. Technical study of power transmission system mainly involves three scientific fields including Physics (Electro-mechanics), electrical engineering (single and 3-phase circuit analysis, control systems) and mathematics (differential equations and linear algebra), but 'local' and 'global' properties of network are important to study.

Wide-area cascading failures (Blackouts) in electrical power networks occur frequently than one would expect. These cascade failures are random and independent. In United States frequent occurrence of blackouts has not decreased since the creation of NERC in 1965 [5] despite of many technological advancements. A number of modeling techniques has been developed so far for the prediction and prevention of blackouts. One group of researchers studied blackouts from power systems point of view and focus only on characteristics of power systems [6, 7], another group analyzed the electric supply and demand influence on blackouts [8, 9], the research group of Dr. Ian Dobson from university of Wisconsin finds the feature of self-organized criticality (SOC) in their series of papers [10, 11, 12, 13, and 14]. Studies of power transmission in the light of complex network theory are either from the perspective of power systems or complex network theory. Half of studies focus only on node degree distribution and betweenness distribution which is no doubt a key parameters in establishing a theoretical probability model [15, 16, and 17], other half develop their models by take into account the small-world network behavior of power grids by using the properties of centrality, and path length [18, 19, 20] but there is no compact study so far that keep the view of all these parameters in their models.

The goal of the study is to apply complex network statistical analysis tools to understand features of power grid networks. We use IEEE 118-bus and Polish 2383-bus network to illustrate these properties. The paper is organized into four sections, section I describes the indexes of network theory which are used in this study. Section II contains local and global characteristics of power grid networks. Section III provides results and analysis of the study and section IV offers the conclusions and future work.

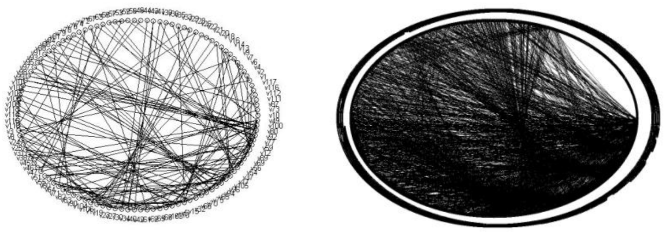

Fig.1. A circular node-edge representation of the IEEE 118-bus (left) and IEEE 2383-bus (right) test systems

II. MEASURES AND METRICS

If the structure of any network is known then a variety of useful quantities that capture particular features of network topology can be calculated. In this section some of the basic measures of complex network theory (CNT) that reveals intriguing features and patterns of power transmission networks are described.

A. Power Grid Graph

A graph G of network topology is a pair of sets G (N, E) where N is a set of Nodes (buses) and E is the set of edges (transmission lines). An edge '$e_{i,j}$' is the representation that there is an edge b/w nodes' n_i' and' n_j' as shown in figure 1.

B. Adjacency Matrix

Mathematical representation of network has a number of ways but the simplest is Adjacency Matrix. It can be represented in mathematical form for the undirected case as:

$$A_{ij} = \begin{cases} 1; \text{if there is an edge b/w vertices i \& j,} \\ 0; \text{otherwise} \end{cases}$$

The Adjacency Matrix of a directed graph has matrix elements.

$$A_{ij} = \begin{cases} 1; \text{if there is an edge from j to i,} \\ 0; \text{otherwise} \end{cases}$$

C. Centrality

Centrality of a network addresses the question, "Which are the most important or central vertices in a Network". There are a number of ways to calculate the centrality of a network; some of the important measuring techniques are as follows:

i. Degree Centrality

The simplest centrality measure of network is degree centrality which explains how much number of nodes is connected to any specific node (n_i) in the network.

ii. Eigenvector centrality

An extension of degree centrality is eigenvector centrality. Let the centrality of node 'i' is 'x_i'. Start algorithm by setting 'x_i=1', now use this assumption to measure better centrality measure 'x_i'', which is the sum of centralities of i's neighbors.

$$x'_i = \sum_j A_{ij} x_j$$

iii. Closeness Centrality

Another different way to calculate centrality is closeness centrality, which measures mean distance from a node to other nodes. Suppose 'd_{ij}' is the length of a geodesic path from 'i' to 'j', meaning the number of edges along the path. Then mean geodesic distance from 'i' to 'j' averaged over all nodes j in the network is,

$$C'_i = \frac{1}{n-1} \sum_{j(=i)} \frac{1}{d_{ij}}$$

iv. Betweenness Centrality

Betweenness centrality measures the extent to which a node lies on path between other nodes. Mathematically, let p_{st}^i be '1' if 'i' lies on the geodesic path from s to t and 0 if there is no path. Then betweenness centrality x_i is calculated as

$$\mathrm{B_i} = \sum_{st} p_{st}^i$$

D. Clustering Coefficient

The clustering coefficient measures the average probability that two neighbors of a node are themselves neighbors. In effect it measures the density of triangles in the networks and it is of interest because in many cases it is found to have values sharply different from what one could expect on the basis of chance. Clustering coefficient for a single node 'i' is defined as:

$$C_i = \frac{(\textit{number} \text{ of pairs of neighbors of i that are connected})}{\textit{number of} \text{ pairs of neighbors of i}}$$

Some researchers calculate clustering coefficient for an entire network as the mean of the local clustering coefficients for each vertex.

$$C_{WS} = \frac{1}{n} \sum_{i=1}^{n} C_i$$

E. Degree Distribution

Degree distribution is the most known fundamental property of a network. It calculates the frequency distribution of a node degree as:

$$P_k = \frac{\textit{N}\text{umber of nodes having degree 'k'}}{\textit{Total} \text{ number of nodes}}$$

For power grids, the shape of this silent characteristics is either exponential or a power-law. In exponential node degree (k) distribution, nodes having relatively high node degree decay faster in probability.

$$P_k = ae^{bk}$$

Where 'a' and 'b' are parameters of the considered network.

Power-law distribution has slow decay with high probability of having higher node degree. It is express as

$$P_\text{k} = Ck^{-\alpha}$$

where 'C' and 'α' are parameters of the considered network.

III. EXPERIMENT AND RESULTS

The topological representation of IEEE 118-bus and Polish 2383-bus systems can be observed visually from circular representation in Figure 1. Study of both systems is conducted as undirected graphs. For simplicity impendence of each transmission line exist b/w any two nodes is considered as '1Ω'. All Centrality measures of every node in IEEE 118-bus and Polish 2383-bus systems can be observed from Figure 2 and Figure 3 respectively; large spikes in the Figures are the identification of most critical (central) nodes in the networks under different centrality measures. It can be concluded that power flow on transmission lines associated to these nodes should be observed carefully to avoid any major cascade failure (blackout).

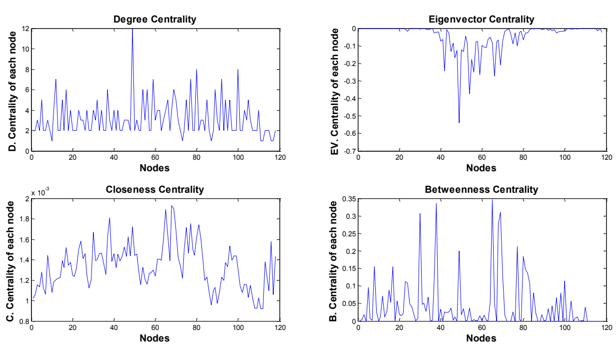

Fig.2. Centrality measures of IEEE 118-bus system.

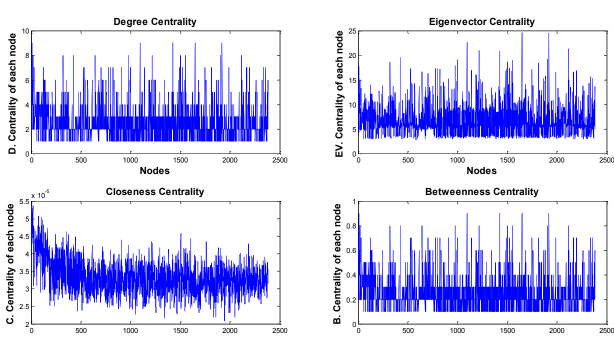

Fig.3. Centrality measures of Polish 2383-bus system

Table 1 describes some significant 'local' and 'global' properties of the networks together with complexity of networks (like density of nodes and edges), degree distribution type. A significant conclusion is obtained from the results that both networks are sparse and geodesic path lengths between random pair of nodes are longer than suggested by Watts-Strogatz in their small-world structure where less than seven even for large networks but path lengths are greater than seven in power systems.

TABLE 1
Comparison of Complex Network Statistical Analysis of IEEE 118-bus and Polish 2383-bus

NO	Network Properties	Network Analysis and comparison	
		IEEE 118-bus	IEEE 2383-bus
1	Number of Nodes (buses)	118	2383
2	Number of edges (transmission lines)	372	5792
3	Diameter (longest shortest path b/w two nodes)	14	16
4	Average Node Degree	3.1525	2.05
5	Average Clustering Coefficient	0.1651	0.0077
6	Cumulative Degree Distribution Type	Exponential partially follow power-law	Exponential partially follow power-law
7	Fitted Distribution	$y(x) \sim 2.9x^{0.02192}$	$y(x) \sim 4.665*x^{-0.09665}$

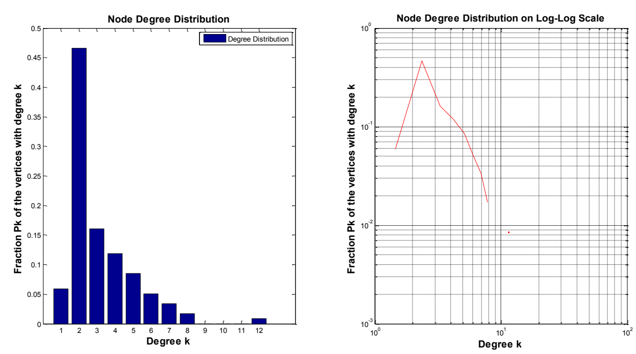

Fig.4. Probability distribution function (Histogram) for the node degree in IEEE 118-bus (left) and cumulative PDF of node degree on log-log scale (right).

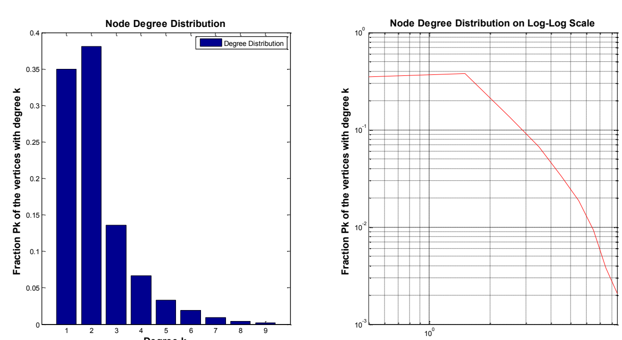

Fig.5. Probability distribution function (Histogram) for the node degree in Polish 2383-bus (left) and cumulative PDF of node degree on log-log scale (right).

Graphs of power networks considered for this study are large but finite in terms of order and size, thus only provide limited and finite probability statistics. Degree distribution provides small information about static condition of network. These graphs do not reveal any highly connected Hubs at first glance that could consider them as a scale-free networks like World Wide Web and airline network. Degree distribution of these two networks does not fit well with a power-law distribution as shown in Figure 4 and Figure 5. When observed on log scale then they exhibit some properties of power-law in large-k region which is precisely the region in which the power law is normally followed most closely. Previous study [10, 11, 12, 13, and 14] explains a reason of cascade failure blackouts in power systems and found a power law b/w PDFs of blackouts and number of customers' unserved (loss of load). Power law profoundly increases the risk of large blackouts and also show evidence of critical loading where probability of cascading failure rapidly increases. In this study a power-law is found up-to some extend b/w degree distribution as an evidence of critical node degree and overall blackout risks due to loss in structure of the system.

IV. CONCLUSION AND DISCUSSION

Our analysis shows that network analysis tools applied to power networks can display some general features of power systems, but it is still in lock of metrics to quantitate the unique characteristics of power systems, for example, the energy balance at each node, the impact of reactive power on blackouts, etc. Complex network theory (CNT) also deals with the design aspect of power systems. In the light of CNT power grid topologies can be improved to avoid any cascade failure.

Network Analysis clearly states link between network structure and types of failures to which these networks are vulnerable. Scale-free networks are more robust to failures at their highly connected Nodes. These made them highly vulnerable to premeditated attacks on these hubs. Analysis exhibits that power networks illustrate some semblance to scale-free network when observed more carefully on large scale.

Complex network theory is a useful tool in power system domain, especially in creating models that consider physical parameters of network data and also the information regarding electrical parameters. Our next step is to develop a comprehensive model by keeping in view the valuable information of network topological characteristics and also the electrical engineering perspective of power grid like Kirchhoff's Laws and demand and supply relationships.

V. REFERENCES

[1] Newman, M. E. J., "The Structure and Function of Complex Networks," *SIAM Review* 45:2, pp. 167– 256, 2003.

[2] Albert, R. and A. Barabasi, "Statistical Mechanics of Complex Networks," *Reviews of Modern Physics* 74, pp. 47 – 97, 2002.

[3] P. Erd˝os and A. R´enyi,"On random graphs. I," Publ. Math. Debrecen, vol.6, pp. 290–297,1959.

[4] P. Erd˝os and A. R´enyi, "On the evolution of random graphs," in Publication of the mathematical institute of the Hungarian academy of Sciences, 1960, pp. 17–61.

[5] Hines, P., J. Apt, H. Liao, and S. Talukdar, "The Frequency of Large Blackouts in the United States Electrical Transmission System: An Empirical Study," presented at the 2nd Carnegie Mellon Conference on Electric Power, January 2006.

[6] M. L. Sachtjen, B.A. Carreras, V.E. Lynch,"Disturbances in a Power Transmission system," phys. Rev. E, vol.61. No. 5, Part A, pp. 4877-4882, May 2000.

[7] Fang Yang, A. P. Sakis Meliopoulos, George J. Cokkinides, and Q. Binh Dam, "Effects of Protection System Hidden Failures on Bulk Power System Reliability," IEEE 38th North American Power Symposium, pp. 517-523, 2006

[8] S. H. Li, M. Ding, M. Wang, and X. Q. Wang, "Search power system chained failure mode considering uncertainty of element fault and performance of protective relaying", power system technology, Beijing, vol. 28, pp. 27-31, July 2004.

[9] J. Yi and X. X. Zhou, "Cascading failure model of power grids considering frequency response characteristics and hidden failures", Automation of electrical power system, Nanjing, vol. 30, pp. 1-6, July 2005.

[10] I. Dobson, B. A. Carreras, V. Lynch, and D. E. Newman, "An initial model for complex dynamics in electric power system blackouts," 34th Hawaii International Conference on System Sciences, Maui, Hawaii, January 2001.

[11] B. A. Carreras, D. E. Newman, I. Dobson, and A. B. Poole, "Modeling Blackout dynamics in power transmission networks with simple structure," 34th Hawaii International Conference on System Sciences, Maui, Hawaii, January 2001.

[12] B. A. Carreras, V. Lynch, I. Dobson, and D. E. Newman, CHAOS, **12**, 985 (2002).

[13] B. A. Carreras, V. Lynch, I. Dobson, and D. E. Newman, "Blackout mitigation Assessment in power transmission systems," 36th Hawaii International Conference on System Sciences, Hawaii, January 2003.

[14] B. A. Carreras, D. E. Newman, I. Dobson, and A. B. Poole, "Evidence for self-organized Criticality in electric power system blackouts," IEEE Transactions on Circuits and Systems, vol. 51, no. 9, September 2004.

[15] R. Albert, I. Albert, and G. Nakarado, "Structural vulnerability of the north american power grid," Physical Review E, vol. 69, 2004.

[16] P. Crucitti, V. Latora, and M. Marchiori, "A topological analysis of the italian electric power grid," Physica A: Statistical Mechanics and its Applications, vol. 338, no. 1-2, pp. 92 – 97, 2004, proceedings of the conference A Nonlinear World: the Real World, 2nd International Conference on Frontier Science.

[17] D. P. Chassin and C. Posse, "Evaluating North American electric grid reliability using the Barab´asi Albert network model," Physica A Statistical Mechanics and its Applications, vol. 355, pp. 667–677, Sep. 2005.

[18] G. A. Pagani and M. Aiello, "Towards decentralization: A topological investigation of the medium and low voltage grids," Smart Grid, IEEE Transactions on, vol. 2, no. 3, pp. 538 –547, sept. 2011.

[19] S. Mei, X. Zhang, and M. Cao, Power Grid Complexity. Springer, 2011.

[20] Hines, Paul, Blumsack, Seth.,"A centrality Measure for Electrical Networks", IEEE 41st Hawaii International conference on system sciences, 2008.