Disturbance Observer based Repetitive Controller for Time-Delay Systems

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Abstract

This paper presents a discrete control design for time-delay systems subjected to the periodical command signal or exogenous disturbances. Unlike other dead-time compensators (DTC), we take profit of the system components to construct an internal model. In addition, a novel disturbance observer is developed to compensate the effect of disturbances, and thus to achieve tracking and disturbance rejection simultaneously. The possible fractional delay from discretization is also handled by using a fractional delay filter. The stability conditions and robustness analysis under model uncertainties are provided. Two numerical examples including a supply chain management (SCM) is provided to illustrate the feasibility of the results.

1. Introduction

Time-delay as well as external disturbances is often encountered in control systems such as teleoperator, process control, networked control and among others [1]. The presence of time-delay, as known, may render the control design much more difficult. Although many significant results have been reported on the stability analysis and stabilization control for time-delay systems [2], the tracking control design is still general and more challenging, which has drawn considerable attentions until now.

The well-known Smith predictor [3-5] was proposed to deal with the effect of time-delay. The key merit is to deprive the time-delay from the denominator of the closed-loop character equation. Many extensions and modifications of Smith predictor (or dead-time compensators (DTC) [6-11]) were also developed to enhance the robustness and to improve the performance. For the time-delay system with external disturbances, the idea of disturbance observer is introduced in [8-9], thus the disturbance attenuation can be improved. Tsang and Rad et al.[10-11] established a class of time-delay controllers for first order plus dead-time (FOPDT) and second order plus dead-time (SOPDT) plants. Here, most of control structures in literature for time-delay systems are especially suitable for tracking constant references and for attenuating constant disturbances [8].

In control practice, the periodical references and disturbances to be tracked or rejected are often encountered such as motor control [13], compact disk [14], and power system [15-16]. The internal model principle (IMP) [12] and its corresponding repetitive control concept [13-19] have been considered as a powerful method for periodical tracking and disturbance rejection. According to IMP, an internal model (or generator) of the signal to be tracked or rejected must be inside the control loop, in the controller, or in the plant itself. Compared with DTC, the time-delay in repetitive control is, in fact, considered as a useful memory term to construct an internal model for periodic signals. Recently, the delayed feedback (named delayed resonator) was also introduced in vibration suppression [20-21]. However, it should be pointed out that, all aforementioned delay- based feedback control schemes [13-21], in general, are only valid for systems without inherent delay, and little progress has been made towards their application to systems with input-delay. In addition, in most of repetitive controllers [13-19], the repetitive control loop is injected into system with Plug-in form, which may result in a complex control design.

For time-delay systems, Watanabe [23] proposed the modified repetitive controllers for time-delay systems, in which the time-delay can be handled in the closed-loop. Taking profit of the time-delay, Alvarez et al. [24] developed a repetitive controller for tubular heat exchangers with delayed resonance dynamics. The latest work of Tan et al. [25-26] extended the principle of repetitive control to the system with delayed control input, in which the system intrinsic delay is utilized advantageously for iterative learning. Relay feedback experiment and nonlinear least squares algorithm are used to obtain the control parameters.

In this paper, we develop a novel discrete control scheme using repetitive control principle for the system with time-delay (dead-time) in its control input, which is also subjected to the periodical reference and exogenous disturbances. The system including the intrinsic delay is

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combined with a model-based compensator to construct an internal model for signals with fixed periods. A novel disturbance observer is then introduced inside the internal model, such that the effect of disturbances can be compensated successfully. Consequently, our method can achieve tracking and disturbance rejection simultaneously. Different from other repetitive controllers with Plug-in structure, the proposed internal model is designed corresponding to a simple stabilizing control, thus provide a simpler control structure. The possible fractional delay that comes from the discretization is also considered by using the optimized fractional delay filter approximation, and a recipe of the sampling interval selection is provided. In addition, the paper states sufficient conditions for the closed-loop stability via small gain theorem, and the robustness stability condition is also obtained to analyze the modeling uncertainties. Numerical examples with frequency and time response are included to illustrate the feasibility of our methods.

The paper is organized as follows, Section 2 provides the problem statement and repetitive control basis; Section 3 introduces the construction of internal model and the control structure. The fractional delay is also discussed. Section 4 describes a disturbance observer and Section 5 proposes the robustness analysis; Simulation and conclusions are included in Section 6 and Section 7.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider a class of linear time-delay systems as

$$\begin{aligned} Y(s) &= \frac{(b_0 s^m + b_1 s^{m-1} + \cdots + b_m)e^{-\tau s}}{s^n + a_1 s^{n-1} + \cdots + a_n}[U(s) + D_1(s)] + D_2(s) \\ &= G(s)e^{-\tau s}[U(s) + D_1(s)] + D_2(s) \end{aligned} \tag{1}$$

where $Y(s)$ and $U(s)$ are the input and output of the system; $a_i, b_i \in \square$ are the plant parameters with $m \le n$, and τ is the intrinsic time-delay. $D_1(s)$ and $D_2(s)$ are external disturbances injected into the system.

In order to design a digital controller, the plant (1) can be discretized with a zero-order hold (ZOH) at its input. If the sampling time T_s is chosen as $\tau = M \cdot T_s$ with $M \in \square$, the following discrete-time plant is obtained

$$\begin{aligned} Y(z) &= \frac{(d_1 z^{n-1} + d_2 z^{n-2} + \cdots + d_n)z^{-M}}{z^n + c_1 z^{n-1} + \cdots + c_n}[U(z) + D_1(z)] + D_2(z) \\ &= G(z)z^{-M}[U(z) + D_1(z)] + D_2(z) \end{aligned} \tag{2}$$

where $c_i, d_i \in \square$ are the plant parameters. It is noted that most continuous-time plants can be converted to the corresponding discrete-time plant with relative degree one using a zero-order hold at their inputs.

The objective is to design a controller, such that the output $Y(z)$ tracks a periodical reference signal $R(z)$, while rejecting external periodical disturbances $D_i(z)$ in both control and output, which are all with the fixed period $N \in \square$.

Remark 1. Many physical processes can be modeled as time- delay system (1), such as recycle reactors, cold rolling mills [1-2]. The stability analysis and stabilization control for such systems has bee well-studied. However, due to the presence of time delay, a unity feedback control structure with the traditional controllers is, in general, difficult to achieve a good tracking performance when there are exogenous disturbances.

2.2. Repetitive control basis

A periodic signal $r(t)$ (with period T_p) can be developed in Fourier series as

$$r(t) = \sum_{n=-\infty}^{\infty} a_n e^{j\frac{2n\pi t}{T_p}} \tag{3}$$

where $a_n \in \square$ are the Fourier series coefficients. To track or reject such signals, according to Internal Model Principle [12], the following generator (internal model) should be included in the control loop

$$R(s) = \frac{T_p e^{-T_p s/2}}{1 - e^{-T_p s}} \tag{4}$$

Since $T_p e^{-T_p s/2}$ is a delay term with a gain T_p, it is enough to include $e^{-T_p s}/(1 - e^{-T_p s})$ inside the closed-loop [13]. For digital implementation[1], the time-delay $e^{-T_p s}$ can be replaced by a discrete delay as z^{-N} with $N = T_p / T_s \in \square$. Consequently, the following internal model [17-18] is employed

$$R(z) = \frac{z^{-N}}{1 - z^{-N}} \tag{5}$$

which can be implemented by a positive feedback loop with z^{-N} in the forward path.

In order to improve the robustness or repetitive control, a low pass filter $Q(z)$ is usually introduced in the internal model [19]. Hence, the internal model (5) with a low pass filter $Q(z)$ can be shown in Fig. 1.

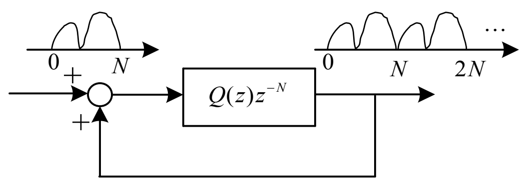

Figure1. Internal model with filter (Q(z)=1).

From a frequency point of view, the internal model (5) (shown in Fig.1 with $Q(z) = 1$) introduces infinity gains at frequencies $\omega = 2\ell\pi / T_p$ rad/sec for $\forall \ell \in \square$. This property assures the zero-error tracking at these frequencies as long as the complete closed-loop system is stable.

[1] The possible fractional delay from discretization and the design of sampling interval will be discussed in Section 3.2.

3. Repetitive control for time-delay system

3.1. Construction of internal model

As seen in Fig.1, the proposed internal model is composed of a time-delay z^{-N} and a linear system $Q(z)$ (low-pass filter), which has the similar formulation as system (2). Inspired by this fact, by introducing an appropriate compensator $H(z)$ and an additional delay z^{-O}, the system (2) can be utilized to construct an internal model for signals with the period $\ell \cdot N, \ell \in \mathbb{N}$ for $N = T_p / T_s$. The proposed internal model is depicted in Fig.2.

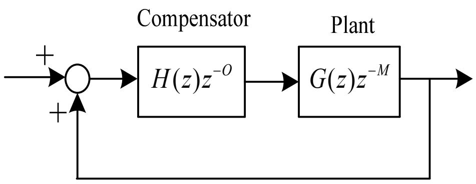

Figure 2. Constructed internal model.

In order to obtain the desired frequency response, according to IMP, the concrete time delay inside the internal model must be tuned to the desired value, i.e., $\ell \cdot N$. Therefore, as shown in Fig.2, an additional delay z^{-O} is introduced in the proposed internal model, which can be specified as $O = \ell \cdot N - M$, where ℓ is the smallest integral to obtain $O \geq 0$.

In addition, a compensator $H(z)$ is used to shape the frequency response. These elements are connected in series with the original plant, and then provide an internal model as

$$G_I(z) = \frac{H(z)G(z)z^{-\ell \cdot N}}{1 - H(z)G(z)z^{-\ell \cdot N}} \tag{6}$$

Note that if the compensator $H(z)$ is designed to fulfill $H(z)G(z) = 1$, the transfer function (6) corresponds to the ideal internal model (5) for $\ell \cdot N$-periodic signal. In practice, the compensator $H(z)$ can be appropriately designed, such that $H(z)G(z)$ plays the same role as $Q(z)$ to obtain the desired low-pass property.

Remark 2. If the intrinsic delay fulfills $M \leq N$, we can choose $\ell = 1$ and $O = N - M$, then the total delay in the internal model is N. This corresponds to the case that the signal to be tracked has a period greater than the plant intrinsic delay. On the contrary, if $M > N$ where the signal period is smaller than the plant delay, the total delay is $\ell \cdot N$ with $\ell > 1$. It should be pointed out that the additional delay O will introduce some increments of control time response. However, the final tracking can be achieved since an internal model is included.

3.2. Fractional delay approximation

It is stated in Section 2.1 that the sampling interval T_s should be chosen to fulfill $\tau = M \cdot T_s$. On the other hand, the period of exogenous signals (reference trajectory, external disturbances) should be multiple of sampling points, i.e., $T_p = \ell \cdot N \cdot T_s$. In some cases, these may not be simultaneously satisfied for $\ell, N, M \in \mathbb{N}$. Without loss of generality, the following steps are proposed to to overcome this possible problem:

i) Select a sampling time fulfilling $\tau = M \cdot T_s$ for $M \in \mathbb{N}$.

ii) Denote $n = T_p / T_s$ as the number of samples in each period.

a) If n is an integer, e.g., $n \in \mathbb{N}$, then the total delay $\ell \cdot N$ can be selected as $\ell \cdot N = n$.

b) If n is not an integer, we can rewrite $n = T_p / T_s$ as an integer part and a fractional part

$$n = \left\lfloor \frac{T_p}{T_s} \right\rfloor + \alpha = \ell \cdot N + \alpha \tag{7}$$

where the rounded integer $\ell \cdot N = \left\lfloor \frac{T_p}{T_s} \right\rfloor$ is used, and $\alpha \in (0,1)$ is a fractional delay.

To improve the performance, the remainder fractional delay α of (7) should be handled. One of the promising approaches to deal with fractional delays is the fractional delay filter approximation, which is a technique for bandlimited interpolation method in digital processing theory [27-28]. Here, this technique is introduced

$$z^{-\alpha} \approx F(z,\alpha) = \sum_{i=0}^{m} h(i) z^{-i} \tag{8}$$

where $F(z,\alpha)$ is a m order FIR filter used to approximate the ideal response of fractional delays. The coefficients $h(i)$ can be determined using an optimization procedure. By doing so, the complete internal model can be provided as Fig.3.

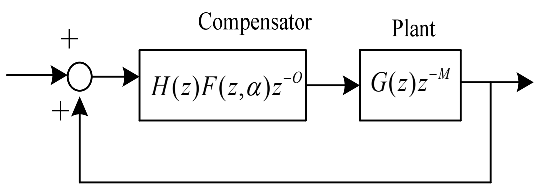

Figure 3. Internal model with fractional delay.

Although different algorithms can be used to determine $h(i)$, the most common ones are Lagrange interpolator method and the least square FIR optimized approximation method [28]. The advantage of Lagrange Interpolator method lies in easy explicit formulas for the coefficients. The least square FIR method has, in principle, the smallest least square error. As an example, Fig.4 shows the approximation result of $z^{-2/3}$ using 4th order FIR filters. As it can be seen, a fairly good approximation is provided in the low frequency range, in which the control system usually works.

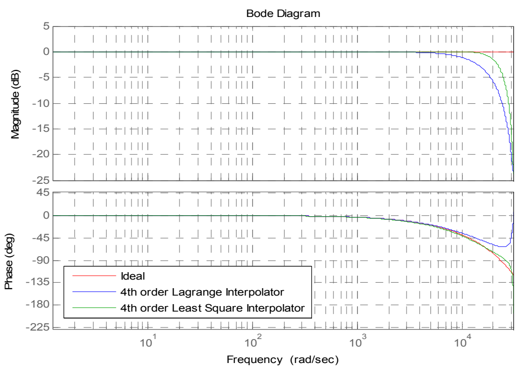

Figure 4. Fractional delay approximation.

3.3. Complete control scheme

It is noted that the internal model (6) is not stable (marginally stable) and thus not directly suitable as a complete closed-loop system. Then a stabilizing controller $K(z)$ is introduced to include the proposed internal model into a closed-loop while guaranteeing the system stability. The concrete scheme is presented in Fig. 5.

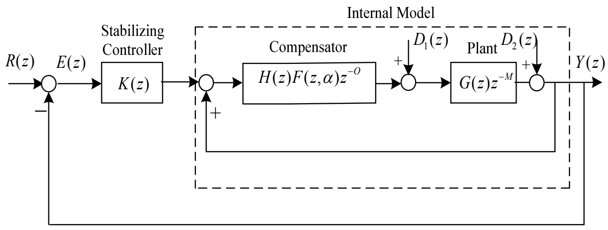

Figure 5. Complete control structure.

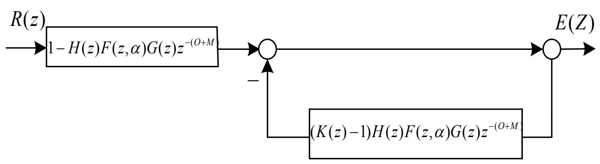

Figure 6. Alternative diagram of control.

We have the following Theorem:

Theorem 1. For the time-delay system (2) with the repetitive control scheme in Fig.5, the system is stable if

i) $H(z), F(z,\alpha)$ and $G(z)$ are stable systems fulfilling $\| H(z), F(z,\alpha)G(z) \|_\infty < 1$;

ii) $K(z)$ is a stable system with $\| K(z)-1 \|_\infty < 1$.

Proof. The transfer function from the reference $R(z)$ to the error $E(z)$ in Fig. 5 can be given as

$$\frac{E(z)}{R(z)} = \frac{1-H(z)F(z,\alpha)G(z)z^{-(O+M)}}{1+(K(z)-1)H(z)F(z,\alpha)G(z)z^{-(O+M)}} \tag{9}$$

This transfer function can be decomposed in the series connection of two systems [13, 16], which can be depicted in Fig.6. The first part $1-H(z)F(z,\alpha)G(z)z^{-(O+M)}$ is stable if $\| H(z), F(z,\alpha)G(z) \|_\infty < 1$.

Furthermore, the second term is a feedback closed-loop with the term $(K(z)-1)H(z)F(z,\alpha)G(z)z^{-(O+M)}$ in the feedback channel. By means of the *Small Gain Theorem* [29], a sufficient stability condition of this feedback loop can be interpreted as

$$\| (K(z)-1)H(z)F(z,\alpha)G(z) \|_\infty \leq \| K(z)-1 \|_\infty \| H(z)F(z,\alpha)G(z) \|_\infty < 1$$

which will be fulfilled with $\| H(z), F(z,\alpha)G(z) \|_\infty < 1$ and $\| K(z)-1 \|_\infty < 1$. This concludes the proof. □

Remark 3. Compared to traditional repetitive control schemes, in which the internal model is involved into the system as a Plug-in form and then introduced in the control side [13-19], a novel internal model taking profit of the plant time-delay and placing in the plant inside is constructed. By introducing an additional delay z^{-O}, the system inherent delay z^{-M} can be taken as a useful element in the proposed internal model. This is clearly different from previous dead-time comparators (DTC) [3-9], which consider the delay as a malicious phenomenon in the closed-loop system.

Remark 4. As shown in Fig.5, two feedback loops are employed in our scheme by introducing a compensator in the inner loop and a stabilizing controller in the outer loop, which is different from typical control using delayed feedback [20-21]. In fact, the compensator $H(z)$ is used to obtain the desired low-pass filter and to shape the frequency response, and the stabilizing control $K(z)$ is dedicated to provide the stability.

The remainder of this section is to design the compensator $H(z)$ and the control $K(z)$, which fulfill stability conditions of Theorem 1 and to provide the ideal low-pass property. For minimum-phase plants, a constructive way is given as

i) Select $H(z) = Q(z)G^{-1}(z)$ where $Q(z)$ is a low pass filter satisfying $\| Q(z) \|_\infty < 1$;

ii) Select $K(z) = k_r \in (0,2)$ as a proportional control.

Since FIR is also utilized to approximate the fractional delay $F(z,\alpha)$, with the above selection, the conditions of Theorem 1 can be satisfied, and the open-loop transfer function can be given as

$$G_o(z) = \frac{k_r Q(z)F(z,\alpha)z^{-(O+M)}}{1-Q(z)F(z,\alpha)z^{-(O+M)}} \tag{10}$$

From (10), if $\| Q(z) \| = 1$ and $\| F(z,\alpha) \| = 1$, infinite open-loop gains can be achieved at certain frequencies $\omega = 2\ell\pi / T_p$ rad/s to preserve the tracking control. However, the introduction of $F(z,\alpha)$ and $Q(z)$, as known, may reduce the open-loop gain and slightly modify the phase at these specified frequencies. To minimize the frequency shift, a null phase FIR filter is utilized as $Q(z)$ as well as a FIR filter $F(z,\alpha)$ in this paper. The value of k_r can be chosen as a trade-off between robustness and time response.

Remark 5. It should be note that, although the employed FIR filters in $F(z,\alpha)$, $Q(z)$ may be noncausal, the proposed control scheme is still causal owing to the filters are connected with an additional delay z^{-O} in series.

Remark 6. The Lagrange interpolator-based fractional delay approximation $F(z,\alpha)$ may modify the internal model frequency response, especially in the high frequency range as shown in Fig.4. To further improve the tracking performance, the more complex least square interpolator method proposed in [28] can be used, which combines the design of the fractional delay filter and the low-pass filter $Q(z)$ together.

Remark 7. The aforementioned design of the compensator $H(z)$ is only valid for minimum-phase plants. For the case of nonminimum phase plants, other techniques such as phase cancellation [8] can be employed.

4. Disturbance observer design

The proposed control scheme in Section 3, as shown in Fig.5, can achieve the tracking performance. However, if there are external periodical disturbances $D_1(z)$ and $D_2(z)$, the disturbance rejection can not be guaranteed. The key reason is that these exogenous disturbances are involved into the proposed internal model.

To overcome this problem, a novel disturbance observer is developed inside the internal model, such that the tracking and disturbance rejection can be preserved simultaneously. The modified repetitive control scheme with a disturbance observer is depicted in Fig.7, where $G_m(z)z^{-M_m}$ is the nominal model of the system $G(z)z^{-M}$.

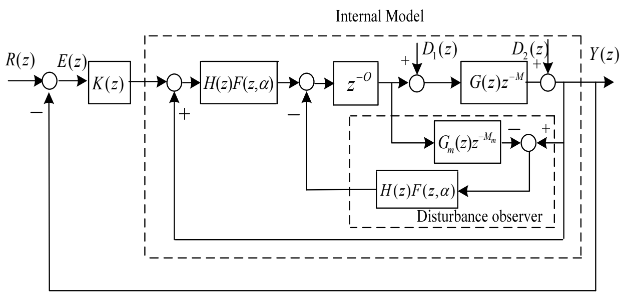

Figure 7. Control with disturbance observer.

The disturbance observer is comprised of the nominal system model $G_m(z)z^{-M_m}$ and the compensator $H(z)F(z,\alpha)$. Consequently, the error transfer functions of the input and disturbances in Fig.7 can be given as

$$\frac{E(z)}{R(z)}=\frac{1-H(z)F(z,\alpha)G_m(z)z^{-(O+M_m)}}{1+(K(z)G(z)z^{-(O+M)}-G_m(z)z^{-(O+M_m)})H(z)F(z,\alpha)} \tag{11}$$

$$\frac{E(z)}{D_1(z)}=\frac{-(1-H(z)F(z,\alpha)G_m(z)z^{-(O+M_m)})}{1+(K(z)G(z)z^{-(O+M)}-G_m(z)z^{-(O+M_m)})H(z)F(z,\alpha)} \tag{12}$$

$$\frac{E(z)}{D_2(z)}=\frac{-(1-H(z)F(z,\alpha)G_m(z)z^{-(O+M_m)})G(z)z^{-M}}{1+(K(z)G(z)z^{-(O+M)}-G_m(z)z^{-(O+M_m)})H(z)F(z,\alpha)} \tag{13}$$

From (11)-(13), if the plant model is known precisely, i.e., $G_m(z)=G(z)$ and $M_m=M$, the closed-loop characteristic equation is preserved as (9), thus the introduction of the disturbance observer does not modify the system stability. In addition, by choosing $H(z)=Q(z)G_m^{-1}(z)$, $K(z)=k_r\in(0,2)$ with $Q(z)$ a null phase FIR filter, high gains are still introduced at the specified frequencies $\omega=2\ell\pi/T_p$ rad/s. Consequently, the tracking and disturbance rejection of signals with period $\ell\cdot N,\ell\in\mathbb{N}$ can be achieved simultaneously. We have:

Theorem 2. For the disturbance observer-based control scheme in Fig.7 with $G_m(z)=G(z)$ and $M_m=M$, select $H(z)=Q(z)G_m^{-1}(z)$ and $K(z)=k_r\in[0,2]$ with $Q(z)$, a null phase FIR filter and $F(z,\alpha)$ the introduced fractional delay FIR filter, then

i) The closed-loop system is stable;

ii) The tracking and disturbance attenuation can be guaranteed simultaneously.

Proof. The proof is similar to Theorem 1 and follows the above analysis, thus is omitted here.

Remark 8. Compared with the control scheme proposed in Section 3, $1-H(z)F(z,\alpha)G_m(z)z^{-(O+M_m)}$ is involved in the numerator of the transfer functions (12)-(13). Consequently, by choosing $H(z)=Q(z)G_m^{-1}(z)$ with $Q(z)$ a null phase FIR filter and $F(z,\alpha)$ a fractional delay FIR filter, the effect of disturbances can be compensated inside the internal model and thus the disturbance rejection can be preserved.

5. Robustness analysis

The system model $G_m(z)z^{-M_m}$ is employed in both compensator $H(z)$ and disturbance observer in Fig.7. However, in practical implementation, there may be modeling errors, i.e., $G_m(z)z^{-M_m}\neq G(z)z^{-M}$. The robustness stability condition is then provided.

We define the plant as $P(z)=G(z)z^{-M}$ and its nominal model as $P_m(z)=G_m(z)z^{-M_m}$, such that $P(z)$ belongs to a family of models as $P(z)=P_m(z)(1+\delta P(z))$, where $\delta P(z)$ is the multiplicative uncertainties [8]. Denote $\overline{\delta P}(w)$ as the upper bounds of the multiplicative uncertainty such that $|\delta P(jw)|\le\overline{\delta P}(w),\forall w\in[0,\pi/T_s]$.

From (11)-(13), the characteristic equation for the closed-loop system in Fig.7 with uncertainties is

$$1+(K(z)G(z)z^{-(O+M)}-G_m(z)z^{-(O+M_m)})H(z)F(z,\alpha)=0 \quad (14)$$

Considering $P(z)=P_m(z)(1+\delta P(z))$, Eq. (14) can be rewriten as

$$\begin{aligned}1+(K(z)-1)H(z)F(z,\alpha)P_m(z)z^{-O}&\\ +K(z)H(z)F(z,\alpha)P_m(z)\delta P(z)z^{-O}&=0\end{aligned} \quad (15)$$

As shown in [8], the system is stable, if and only if the point (-1, 0) is not surrounded by Nyquist curve of (15). Hence, with $H(z)=Q(z)G_m^{-1}(z)$ and $K(z)=k_r$, the system is robust stable for $\forall w\in[0,\pi/T_s]$ if

$$\begin{aligned}\overline{\delta P}(w)<dP(jw)&=\frac{|1+(K(e^{jw})-1)H(e^{jw})F(e^{jw},\alpha)P_m(e^{jw})e^{-Ojw}|}{|K(e^{jw})H(e^{jw})F(e^{jw},\alpha)P_m(e^{jw})e^{-Ojw}|}\\ &=\frac{|1+(K(e^{jw})-1)Q(e^{jw})F(e^{jw},\alpha)e^{-(O+M_m)jw}|}{|k_rQ(e^{jw})F(e^{jw},\alpha)|}\end{aligned} \quad (16)$$

where $dP(jw)$ is the robustness stability index.

According to (16), the robustness of the proposed repetitive control depends on the low-pass filter $Q(z)$ and the controller $K(z)$. From (10) and (16), a larger control gain k_r will reduce the robustness bounds of the system while providing a better tracking response. Thus it should be tuned as a tradeoff between the robustness and control performance.

6. Simulation study

Two examples are included in this section to illustrate the validity and effectiveness of the proposed results.

Example 1. Consider the system with stable zeros as

$$G(s)e^{-\tau s}=\frac{2s+1}{s^2+4s+4}e^{-0.25s} \quad (17)$$

The reference trajectory is set as

$$r(t)=\sin(2\pi t/T_p)+2\sin(4\pi t/T_p)+0.2\sin(6\pi t/T_p) \quad (18)$$

with the period $T_p=3.02/3$ s. According to the recipe given in Section 3, the sampling interval is chosen as $T_s=\frac{\tau}{M}=\frac{0.25}{25}=0.01$ s with $M=25$. Consequently, the system can be presented in the discrete form as

$$G(z)z^{-M}=\frac{0.01965z-0.01956}{z^2-1.96z+0.9608}z^{-25} \quad (19)$$

In this case, since $n=T_p/T_s=302/3$, we can choose $\ell\cdot N=100$ with $\ell=1, N=100$ and the additional delay as $z^{-O}=z^{-75}$ (i.e., $O=\ell\cdot N-M=75$). Therefore, a fractional delay $z^{-\alpha}=z^{-2/3}$ should be approximated using the FIR filter introduced in Section 3.2. The 4th order Least Square Interpolator filter [28] is employed as

$$F(z,\alpha)=0.0292z^2-0.1518z+0.4681+0.6839z^{-1}-0.0294z^{-2} \quad (20)$$

The null-phase FIR filter is $Q(z)=0.2z+0.6+0.2z^{-1}$.

Consequently, the compensator $H(z)$ can be given as

$$\begin{aligned}H(z)F(z,\alpha)&=Q(z)G^{-1}(z)F(z,\alpha)\\ &=\frac{(z^2-1.96z+0.9608)(0.2z^2+0.6z+0.2)}{(0.01965z-0.01956)z}F(z,\alpha)\end{aligned} \quad (21)$$

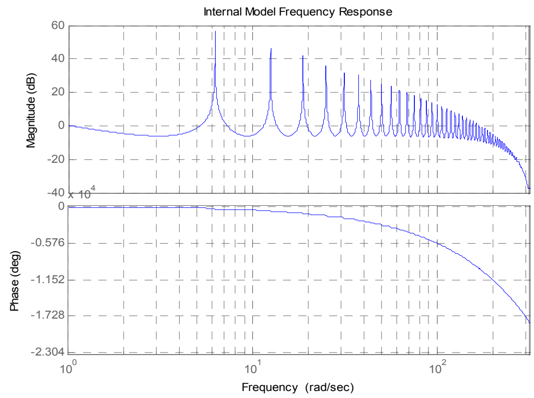

Figure 8. Frequency response of internal model.

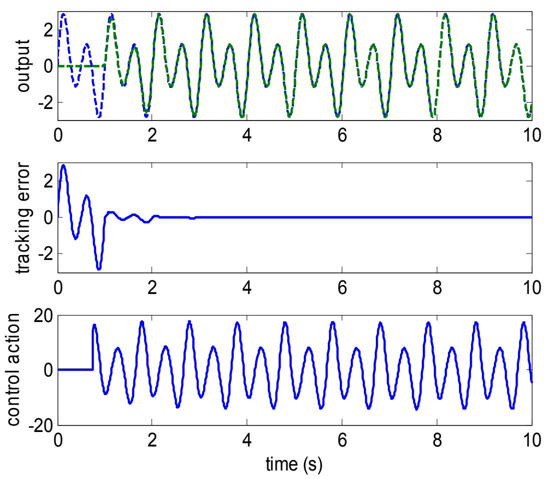

Figure 9. Control performance with $k_r=0.9$.

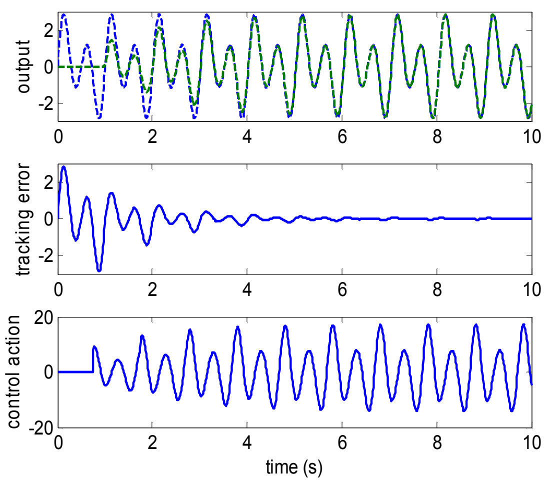

Figure 10. Control performance with $k_r=0.5$.

Although the compensator (21) is noncausal, it can be implemented combining with the additional delay

$z^{-O} = z^{-75}$. By doing so, the frequency response of the proposed internal model can be observed in Fig.8. It is shown that the internal model introduces high-gains at the specific frequencies at $\omega = 2\ell\pi / T_p$ rad/sec for $\forall \ell \in$ □ , which in turn guarantee the tracking performance.

The repetitive control proposed in Section 3 without disturbance observer is utilized and Fig. 9-Fig. 10 depict the closed-loop time response. As shown, the system output converges to the reference signal without steady-state error for different values of k_r. A large control gain, e.g., $k_r = 0.9$ can improve the tracking capability in the time response, while a lower $k_r = 0.5$ giving a more sluggish transient.

Example 2. Efficient supply chain management (SCM) has become imperative for many enterprises. Recently, some automatic control-based approaches have been proposed to deal with the supply chain management subjected to a significant time-delay [8]. Here, a simple model of a manufacturing supply chain is used

$$y(k) = cy(k-1) + du(k-M-1) - q(k) \quad (22)$$

where y is the stock level, u is the factory starts, q is the exogenous disturbance signal, respectively. d is the factory production and M is the time-delay between the control action (factory starts) and the controlled variable (the stock level). This system is rewritten as

$$\frac{Y(z)}{U(z)} = \frac{dz^{-M}}{z-c} + D_2(z) \quad (23)$$

In this example, the disturbance signal is select as ones in [8], i.e., $D_2(k) = \mu \sin(2\pi / 120)$ with the amplitude μ and the period $T_p = 120$, to simulate the effect of periodic changes caused by seasonality. Other system parameters are specified as $c = 0.85, d = 1$, $M = 50$. In this case, we can set the additional delay as $O = T_p - M = 70$, and the null-phase FIR filter as $Q(z) = 0.2z + 0.6 + 0.2z^{-1}$. Consequently, the compensator can be given as

$$H(z) = Q(z)G^{-1}(z) = (z-0.85)(0.2z + 0.6 + 0.2z^{-1}) \quad (24)$$

The compensator (24) can be implemented combining with the delay $z^{-O} = z^{-70}$. The proportional stabilizing control is set as $k_r = 0.9$ for the disturbance observer-based control proposed in Section 4.

To verify the effectiveness of the disturbance rejection, the command signal is first set as $r(k) = 0$ and the exogenous disturbance as $D_2(k) = 5\sin(2\pi k / 120)$. The simulation result is depicted in Fig. 11, which shows the closed-loop time response, the disturbance and the control signal. As can be seen, the tracking error converges to zero after a transient with short duration, which means the effect of the exogenous disturbance $D_2(k)$ on the inventory is properly rejected by the control system.

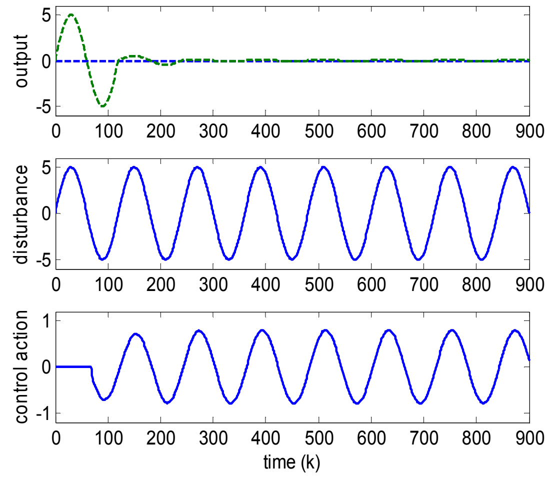

Figure 11. Disturbance rejection performance.

In addition, the demand signal $r(k) = 5\sin(2\pi k / 60)$ and the exogenous disturbance $D_2(k) = \sin(2\pi k / 120)$ are utilized simultaneously. In this case, the period of reference signal $r(k)$ is $T_p = 60$. To fulfill the condition of the internal model, the delay $O = \ell T_p - M = 70$ is still introduced with $\ell = 2$ for $r(k)$ and $\ell = 1$ for $D_2(k)$. With the same parameters, Fig. 12 provides the profile of system performance subjected to the command signal and disturbance. It is also shown that a satisfactory tracking and disturbance rejection performance can be achieved.

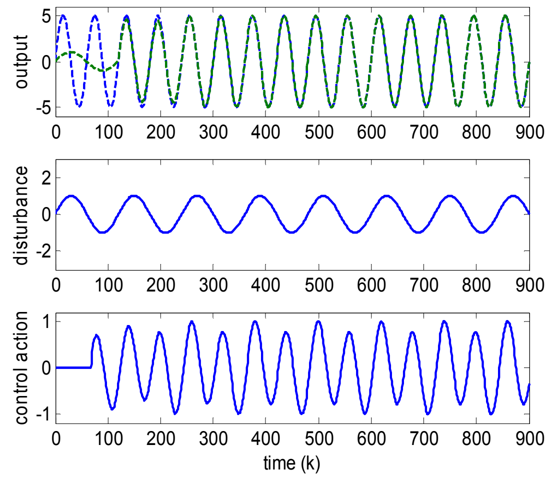

Figure 12. Tracking and disturbance rejection.

7. Conclusion

This paper presents a novel digital control for time-delay systems subjected to the periodical reference signal and external disturbances. The system intrinsic delay combing with an additional time-delay is utilized advantageously to construct an internal model. A model-based compensator with a low-pass filter is developed to shape the system response and a simple stabilizing control is utilized to close the control loop. We also develop a novel disturbance observer to deal with disturbances inside the internal model, such that the tracking and disturbance rejection can be achieved simultaneously. The selection of the sampling interval and the fractional delay approximation are all introduced. Sufficient stability conditions as well as robustness analysis are provided. Simulation results verify the effectiveness of the proposed methods.

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