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# Group Decision Support by Interval AHP With Uncertainty-based Hierarchical Clustering

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***Abstract*—The group decision in this study follows the structure of AHP, where two kinds of criteria-comparisons and alternatives-comparisons are given by a member. Reflecting the difference of the comparisons, two models to aggregate individual decisions into a group decision are proposed and compared. The grouping process is achieved by hierarchical clustering, in which an individual is merged into the nearest cluster one by one. The similarity of individuals is measured by the uncertainty of the group decision, since it tends to be uncertain in case of different thinking individuals. The uncertainty is quantified by Interval AHP, which uses interval weights to reflect the uncertainty of the decision problem. Then, based on the increase of uncertainty by each step, the sub-groups are noticed. In order for individuals to recognize their standpoints and reconsider their judgments if necessary, the group decision in progress is open to them.**

## I. Introduction

The desirable group decision is that all group members agree without hesitations. The whole group decision in this study is reached by grouping a pair of individuals and/or sub-groups until all individuals become a group. The grouping process is achieved by hierarchical clustering [1], in which individuals are merged into clusters one by one in a bottom up scheme. In the nearest cluster merging principle, inter-cluster similarity is measured by such formulations as nearest neighbor (single-linkage), furthest neighbor (complete-linkage) and so on. The sequence of being groups shows the divisions of a group.

Caring for the majority and/or centrality of the group, individuals often change their judgments in group discussions [2]. On one hand, the change may deprive the crucial information which prevent group decision from being misled. On the other hand, the change helps individuals to satisfy the group decision gradually without giving up, since they understand how their decisions are reflected in the group one. From this viewpoint, the difference of an individual decision from a sub-group decision should be open to him/her. Since a decision maker is not fully confident in all comparisons he/she gives, there is often room for revising them. Checking and changing in progress release a decision maker from being discouraged by facing the difference of the whole group decision from his/hers at the end. This concept follows Delphi method which is a well-known technique to stimulate communication for a consensus and one of its essences is to encourage to revise individual answers reflecting others replies [3].

The sequence of reaching a whole group decision is discussed referring to the structure of AHP (Analytic hierarchy Process). AHP is an approach to multi-criteria decision making problems and the problem is decomposed into hierarchy by criteria and alternatives [4]. A decision maker needs to give two kinds of comparisons among alternatives and criteria, which are at the same hierarchy, and the decisions are obtained as scores of alternatives and importance of criteria, respectively. In order to characterize the level of group decision making, clustering is used in conjunction with the conventional (crisp) AHP [5]. Although the technique is useful for gathering experts' knowledge, it is not possible for individuals to recognize their standpoints in the group decision. This paper proposes a new approach of gathering individuals based on the uncertainty of the group decision and it clarifies their standpoints and the divisions of a group. In AHP, there are two kinds of decisions of criteria and alternatives so that two models to aggregate individual decisions are proposed. By both models, the pair of individuals whose group decision is less uncertain than the other pairs' becomes a group primarily. Based on the increase of uncertainty by each step, a whole group might be divided into several sub-groups. The divisions might depend on the models for criteria and alternatives. The group decision at each step is open to all individuals in order for individuals to recognize their standpoints and sometimes to change their opinions.

## II. Interval Analytic Hierarchy Process

The problem in AHP is decomposed into hierarchy by criteria and alternatives as in Fig. 1 [4]. The decision maker compares items at the same hierarchy in Fig.1, i.e., alternatives at the bottom and the criteria at the middle, so that there are two kinds of comparisons. All pairs of criteria and alternatives under each criterion are compared at the middle and bottom hierarchies, respectively. In comparing criteria, a decision maker considers how much more important the criterion in evaluation is than the other. In comparing alternatives, he/she considers how much better the alternative under the criterion is over the other. The criteria- and alternative-comparisons are

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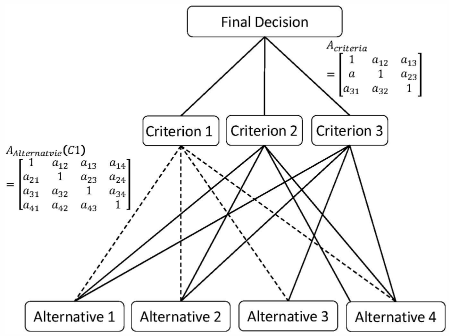


Fig. 1. Structure of AHP

denoted as the following pairwise comparison matrix.

$$A = \begin{bmatrix} 1 & \cdots & a_{1n} \\ \vdots & a_{ij} & \vdots \\ a_{n1} & \cdots & 1 \end{bmatrix}, \tag{1}$$

where $a_{ij}$ shows the importance ratio of alternative/criterion $i$ comparing to alternative/criterion $j$.

The comparison matrix satisfies the following relations so that the number of given comparisons is $n(n-1)/2$ in case of $n$ alternatives/criteria;

$$\begin{array}{ll} a_{ii} = 1 & \text{identical,} \\ a_{ij} = 1/a_{ji} & \text{reciprocal.} \end{array} \tag{2}$$

When the comparison matrix is consistent, the following transitivity relations are satisfied;

$$a_{ij} = a_{ik}a_{kj} \ \forall (i,j,k). \tag{3}$$

Though, since a decision maker gives comparisons one by one intuitively, the relative relations of all comparisons are not always consistent, i.e., (3) is not satisfied. In order to reflect such inconsistency among given comparisons into the obtained weights, Interval AHP model has been proposed [6], [7]. In Interval AHP, the weights are assumed as interval $W_i = [\underline{w}_i, \overline{w}_i]$ extended from crisp $w_i$ in the conventional AHP. The procedures of obtaining the importance of criteria and the scores of alternatives under each criterion from the corresponding comparison matrices are technically the same. The problem to obtain interval weights $W_i$ from comparisons $A$ is formulated as follows.

$$\begin{array}{rl} I = & \min \ \sum_i (\overline{w}_i - \underline{w}_i) \\ \text{s.t.} & \sum_{i \neq j} \overline{w}_i + \underline{w}_j \geq 1 \ \forall j \\ & \sum_{i \neq j} \underline{w}_i + \overline{w}_j \leq 1 \ \forall j \\ & \dfrac{\underline{w}_i}{\overline{w}_j} \leq a_{ij} \leq \dfrac{\overline{w}_i}{\underline{w}_j} \ \forall (i,j) \\ & \underline{w}_i \geq \epsilon \ \forall i. \end{array} \tag{4}$$

Since comparisons are ratio measures, in case of crisp weights, their sum is constrained to be one; $\sum_i w_i = 1$. In (4), for the normalization of interval weights, the definition of interval probabilities [8], [9] at the 1st and 2nd constraints are used. They exclude any redundancy in the intervals in order for their sum to be one. The 3rd constraint is the inclusion relation;

$$a_{ij} \in \frac{W_i}{W_j} = \left[ \frac{\underline{w}_i}{\overline{w}_j}, \frac{\overline{w}_i}{\underline{w}_j} \right] \ \forall i, \tag{5}$$

where the given comparisons are included in the ratio of the corresponding interval weights with the maximum range. The inconsistency among comparisons is reflected in the obtained interval weights. When the comparisons are perfectly consistent, i.e., (3) is satisfied, the crisp weights are obtained, i.e., the optimal solutions of (4) are $\underline{w}_i = \overline{w}_i = w_i$ and $a_{ij} = w_i/w_j$. The more inconsistent the comparisons are given, the larger the widths of interval weighs are obtained. In other words, although crisp weights without uncertainty are preferred, the interval weights are obtained to reflect and include inconsistency among the given comparisons. The interval weights reflect the uncertainty of the decision problem and how uncertain they are is represented as the sum of their widths. Then, the uncertainty of interval weights is minimized in the objective function in (4). In the following, the less uncertain the interval weights are, the more preferable the decision is.

This paper handles a group of decision makers. The comparison matrix is given by member $k$ as

$$A_k = \begin{bmatrix} 1 & \cdots & a_{1nk} \\ \vdots & a_{ijk} & \vdots \\ a_{n1k} & \cdots & 1 \end{bmatrix}, \tag{6}$$

where the decision of member $k$ is obtained as interval weights $W_{ik} = [\underline{w}_{ik}, \overline{w}_{ik}] \ \forall i$ by (4).

Two models of obtaining group decisions for criteria and alternatives under each criterion from the corresponding more than two matrices are proposed. Then, the final decision in AHP, which is the total score of an alternative, is obtained as the sum of multiplications of the group-local weights under the criteria by the group-importance of the criteria.

## III. Group of Judgments and Group of Decisions

The group decision is reached by bringing together individually different judgments to agree a single action. It is investigated that there are two strategies, such as verdict-driven and evidence-driven, for juries to reach a consensus, guilty or innocent [10]. In the verdict-driven strategy, each individual states his/her judgment and tries to persuade the others to change. In the evidence-driven strategy, at first members reviewing evidence together and the common understanding leads a judgment. The final judgment by two strategies may be different, even if the given matrices are the same, so that the different decision support systems are needed.

In AHP in Fig.1, there are also two kinds of comparison matrices, one is by comparing criteria and the other is by comparing alternatives. The following sections propose two new approaches of performing Interval AHP for hierarchical clustering, in which inter-cluster similarity is measured by the uncertainty of the group decision. For instance, two individuals $k_1$ and $k_2$ give their judgments, where items are alternatives or criteria depending on the hierarchy in Fig. 1, and they are denoted as $A_{k_1} = [a_{ijk_1}]$ and $A_{k_2} = [a_{ijk_2}]$. When they try to determine importance of criteria in evaluation at the middle hierarchy, they may discuss and exchange their judgments on how much more important one criterion is than the other. This is similar to evidence-driven strategy, i.e., the common understanding of evidence for juries are the group comparisons of criteria in AHP. Such a discussion is modeled by aggregating individually given comparisons at first, and by obtaining the importance from them. While, they try to determine the scores of alternatives under a criterion at the bottom hierarchy, they may focus on their initial scores which are the decisions induced from their judgments and compromise to some extent or take all individual scores into account. This is similar to verdict-driven strategy, i.e., the judgments for juries are the initial individual scores of alternatives in AHP. Such a settlement is modeled by aggregating individual scores obtained from individually given comparisons. In this way, our behavior for criteria and alternatives is modeled by group of comparisons and group of weights, respectively. One of the advantage of group-weights is that a decision maker can realize his/her standpoint in the group decision [11]. In this paper, also in case of group-comparisons, the difference of an individual from the group is shown.

### A. Group of criteria-comparisons model

When a group determines the importance of criteria, the individuals work together to find a common understanding as in evidence-driven strategy. At first, individual judgments denoted as comparisons given by members $k_1$ and $k_2$ are aggregated from the possibility view by taking their minimum and maximum as

$$\begin{aligned} A_{ij} &= [\underline{a}_{ij}, \overline{a}_{ij}] \\ &= [\min\{a_{ijk_1}, a_{ijk_2}\}, \max\{a_{ijk_1}, a_{ijk_2}\}] \quad \forall (i,j). \end{aligned} \tag{7}$$

Since the aggregated comparisons are intervals, the inclusion relation (5) in (4) is rewritten as follows;

$$\begin{aligned} & a_{ijk_1}, a_{ijk_2} \in A_{ij} \in \frac{W_i^c}{W_j^c} = \left[\frac{\underline{w}_i^c}{\overline{w}_j^c}, \frac{\overline{w}_i^c}{\underline{w}_j^c}\right] \\ \Leftrightarrow \; & \frac{\underline{w}_i^c}{\overline{w}_j^c} \leq \underline{a}_{ij}, \overline{a}_{ij} \leq \frac{\overline{w}_i^c}{\underline{w}_j^c} \quad \forall (i,j), \end{aligned} \tag{8}$$

where the aggregated interval of the given comparisons is included in the ratios of the corresponding obtained interval importance. The inconsistency among individual judgments may be offset by the other's judgments.

Then, by (4) with (8), the group importance of criteria is obtained as interval $W_i^c = [\underline{w}_i^c, \overline{w}_i^c]$. Since the obtained intervals surely include all the individually given judgments denoted as comparisons $a_{ijk_1}, a_{ijk_2}$ as in (8), they are considered to come to a conclusion after a discussion. Since group importance $W_i^c$ is feasible solution of (4) for each $k_1$ or $k_2$, the objective function is more than that with individual ones $W_{ik_1}$ and $W_{ik_2}$; $\sum_i (\overline{w}_i^c - \underline{w}_i^c) \geq \sum_i (\overline{w}_{ik_1} - \underline{w}_{ik_1}), \sum_i (\overline{w}_{ik_1} - \underline{w}_{ik_2})$. Though, the individual importance is not always included in the group one; $W_{ik_1}, W_{ik_2} \not\subseteq W_i^c \; \exists i$. This is a result of finding common understanding of each comparison. They may accept the group importance of criteria, since their given judgments are considered but their initial importance is not included in it.

### B. Group of alternative-weights model

When a group determines the scores of alternatives under a criterion, the individuals tend to stick to their initial decisions as in verdict-driven strategy. Therefore, at first the individual decisions are obtained as interval scores of alternatives $W_{ik_1}$ and $W_{ik_2}$ from $A_{k_1}$ and $A_{k_2}$, respectively. Then, by taking both individuals' initial scores into account, they are aggregated from the possibility view to be a group score;

$$\begin{aligned} W_i^a &= [\underline{w}_i^a, \overline{w}_i^a] \\ &= [\min\{\underline{w}_{ik_1}, \underline{w}_{ik_2}\}, \max\{\overline{w}_{ik_1}, \overline{w}_{ik_2}\}] \; \forall i, \end{aligned} \tag{9}$$

which also satisfy the interval normalization constraints denoted as the 1st and 2nd constraints in (4) since the individual scores $W_{ik_1}, W_{ik_2}$ satisfy them [11].

The group score surely includes both individual ones; $W_{ik_1}, W_{ik_2} \subseteq W_i^a$, so that the uncertainty of the group decision is more than that of each individual's. The approximating comparison of alternatives $i$ and $j$, $A_{ij}^*$, by the obtained group score includes individually given comparisons, $a_{ijk_1}$, and possibly aggregated comparisons in (7) as

$$a_{ijk_1}, a_{ijk_2} \in A_{ij} \subseteq A_{ij}^* = \frac{W_i^a}{W_j^a} = \left[\frac{\underline{w}_i^a}{\overline{w}_j^a}, \frac{\overline{w}_i^a}{\underline{w}_j^a}\right] \quad \forall (i,j).$$

In this sense, the group of weights model is based on common understanding on comparisons more roughly than the group of comparisons model.

## IV. Sequence of being a group

In real situations, it is natural that a pair of individuals whose decisions are similar becomes a group and such a group decision tends not to be uncertain. While, when a pair of individuals whose decisions are different is forced to become a group, such a group decision tends to be uncertain. Therefore, the uncertainty of the group decision is used as the measurement of similarity of the individuals in the group. In the sense of Interval AHP (4), the uncertainty is denoted as the sum of widths of interval weights;

$$I_{k_1k_2} = \textstyle\sum_i (\overline{w}_i - \underline{w}_i),$$

where $W_i = [\underline{w}_i, \overline{w}_i]$ is group decision of members $k_1$ and $k_2$.

In case of $m$ individuals, there are $m(m-1)/2$ possible pairs and one of them whose decision is the least uncertain, i.e., its sum of widths is smaller than the other pairs', becomes a group primarily. It is acceptable for the individuals to be replaced their decisions into the less uncertain group decision than the other pairs. For instance $k_1$ and $k_2$ become a group, the uncertainty of the group decision is more than individual ones; $I_{k_1k_2} \geq I_{k_1}, I_{k_2}$, since two individuals seldom have exactly the same decisions. In case that several people get together and decide something, the group decision may be more uncertain than the individual decisions reflecting their varieties. When the sub-group decision by individuals $k_1$ and $k_2$ are open to all individuals, individuals $k_1$ and $k_2$ recognize how their initial decisions are replaced into the group decision and the other individuals $k \neq k_1, k_2$ consider the difference of their decisions from the sub-group decision supported by individuals $k_1, k_2$. Then, there are $(m-2)$ individuals and the sub-group left so that there are $(m-1)(m-2)/2$ possible pairs and the least uncertain pair becomes a group and the new sub-group decision becomes more uncertain than the previous one. It is repeated to make a group of two until all individuals become a group. Observing the sequence of being a group, it is often noticed that a group may consist of several sub-groups. Since the group decision becomes more uncertain as being a larger group, the increase of uncertainty of group decisions shows the divisions.

## V. Numerical example

We have shown two models, one is a group of comparisons in comparing criteria and the other is group of weights in comparing scores of alternatives. The suitable model to support decision making depends on the decision problem. In order to compare which and how a pair of decisions into a group by two models, we apply both models to the pairwise comparison matrices of four items, $i = 1, 2, 3, 4$, by four members, $k = 1, 2, 3, 4$ assumed in Table I. In Table II, the interval weights obtained from them by (4) and their uncertainty as the optimal function values of (4) are shown. They are explained briefly by comparing the row comparisons in each matrix. When the comparisons of the $i$th row are almost more than those of the $j$th row, item $i$ is more preferred than item $j$. Then, the rough ranking may be $1 > 2 > 3 > 4$ and it does not contradict to interval weights of items shown in Table II. Since the 1st rows of $A_1$, $A_2$ and $A_3$ are apparently more than the other rows, they strongly prefer item 1 to the others. While, as for $A_4$, its 2nd row is more than its 1st one so that item 2 is equally preferred to item 1. The comparisons of $A_2$ satisfy (3), i.e., perfectly consistent, the certain decision denoted as crisp weights is obtained and items are linearly ordered. While, comparisons of items 3 and 4 of $A_3$ and $A_4$, which are less than one, seem to be inconsistent with the other comparisons so that the uncertain decisions with some widths are obtained.

### A. Group of comparisons model

At the upper two rows of Table III, six possible pairs of four matrices are shown. The uncertainty of the sub-group decision, i.e., the interval weights from each matrix by (4) with (8), is shown at the bottom of each matrix. At the 1st step, because of the minimum uncertainty among six matrices, $A_1$ and $A_2$ become a group and the uncertainty of its group decision is larger than both individual ones; $I_{12} > I_1, I_2$. Repeating the same procedure with the left three matrices $A_{12}, A_3, A_4$, at the 2nd step, one of three possible pairs, $A_{12}$ and $A_3$, becomes a group and its uncertainty increases by 0.150 from the 1st step. At the 3rd step, $A_{12,3}$ and $A_4$

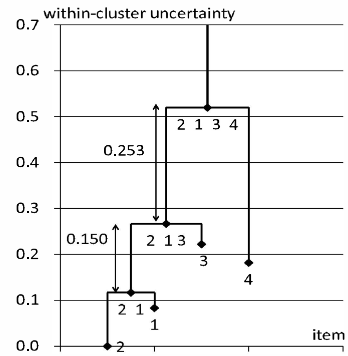


Fig. 2. Sequence by group of comparisons model

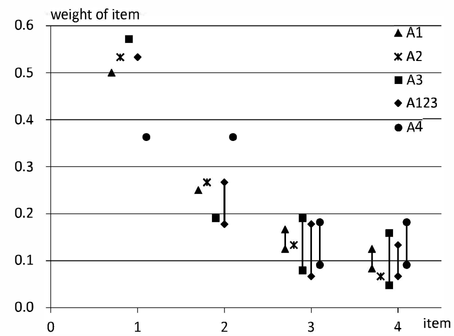


Fig. 3. Individual decisions and group decision $A_{123}$

become a group so that the group interval weights $W_{12,3,4} = ([0.316, 0.474], [0.158, 0.316], [0.105, 0.158], [0.059, 0.211])^t$ and its uncertainty $I_{12,3,4} = 0.520$ increased by 0.253 from the 2nd step. The uncertainty of the group decision is increased step by step as in Fig. 2. Comparing the increases of uncertainty, we find that $A_4$ is different from the group of $A_{123}$. These four decision makers might be potentially divided into two sub-groups, one is $A_{12,3}$ and the other is $A_4$. In Fig. 3, the decisions of $A_{12,3}$, on its left $A_1, A_2$ and $A_3$ and on its right $A_4$ are illustrated. $A_{12,3}$ prefers extremely item 1 to the others, on the other hand, $A_4$ prefers both items 1 and 2. If decision makers check the tentative results as in Fig. 3 before reaching their final decision, they may have chance to reconsider comparisons of items 1 and 2.

### B. Group of weights model

The initial individual decisions denoted as interval weights of items are shown in Table II. They are possibly aggregated by (9) and six possible pairs are shown at the left three columns of Table IV. Similarly to Section V-A, at the 1st step, $W_1$ and $W_2$ are aggregated into $W_{12}$ because of the least uncertain among six pairs. The smaller sum of widths represents that the two individual decisions are more similar. At the 2nd step, there are three pairs such as $W_{12,3}, W_{12,4}$ and $W_{34}$, and $W_3$ is added to $W_{12}$ because of $I_{12,3} < I_{12,4} < I_{34}$. It is noted that the uncertainty of the group decision increases by 0.220 from the 1st step.

TABLE I
GIVEN INDIVIDUAL JUDGMENTS AS COMPARISON MATRICES

$$A_1 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ & 1 & 2 & 3 \\ & & 1 & 2 \\ & & & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 2 & 4 & 8 \\ & 1 & 2 & 4 \\ & & 1 & 2 \\ & & & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 3 & 3 & 4 \\ & 1 & 2 & 4 \\ & & 1 & 1/2 \\ & & & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} 1 & 1 & 2 & 2 \\ & 1 & 3 & 4 \\ & & 1 & 1/2 \\ & & & 1 \end{bmatrix}$$

TABLE II
OBTAINED INDIVIDUAL DECISIONS AS INTERVAL WEIGHTS OF ITEMS

$$W_1 = \begin{bmatrix} 0.500 \\ 0.250 \\ [0.125, 0.167] \\ [0.083, 0.125] \end{bmatrix} \quad W_2 = \begin{bmatrix} 0.533 \\ 0.267 \\ 0.133 \\ 0.067 \end{bmatrix} \quad W_3 = \begin{bmatrix} 0.571 \\ 0.190 \\ [0.079, 0.190] \\ [0.048, 0.159] \end{bmatrix} \quad W_4 = \begin{bmatrix} 0.364 \\ 0.364 \\ [0.091, 0.182] \\ [0.091, 0.182] \end{bmatrix}$$

$I_1 = 0.083 \quad I_2 = 0 \quad I_3 = 0.222 \quad I_4 = 0.182$

TABLE III
GROUP OF COMPARISONS FOR THE 1ST AND 2ND STEPS

$$A_{12} = \begin{bmatrix} 1 & 2 & [3,4] & [4,8] \\ & 1 & 2 & [3,4] \\ & & 1 & 2 \\ & & & 1 \end{bmatrix} \quad A_{13} = \begin{bmatrix} 1 & [2,3] & 3 & 4 \\ & 1 & 2 & [3,4] \\ & & 1 & [1/2,2] \\ & & & 1 \end{bmatrix} \quad A_{14} = \begin{bmatrix} 1 & [1,2] & [2,3] & [2,4] \\ & 1 & [2,3] & [3,4] \\ & & 1 & [1/2,2] \\ & & & 1 \end{bmatrix}$$

$I_{12} = 0.117 \quad I_{13} = 0.267 \quad I_{14} = 0.389$

$$A_{23} = \begin{bmatrix} 1 & [2,3] & [3,4] & [4,8] \\ & 1 & 2 & 4 \\ & & 1 & [1/2,2] \\ & & & 1 \end{bmatrix} \quad A_{24} = \begin{bmatrix} 1 & [1,2] & [2,4] & [2,8] \\ & 1 & [2,3] & 4 \\ & & 1 & [1/2,2] \\ & & & 1 \end{bmatrix} \quad A_{34} = \begin{bmatrix} 1 & [1,3] & [2,3] & [2,4] \\ & 1 & [2,3] & 4 \\ & & 1 & 1/2 \\ & & & 1 \end{bmatrix}$$

$I_{23} = 0.267 \quad I_{24} = 0.417 \quad I_{34} = 0.500$

$$A_{12,3} = \begin{bmatrix} 1 & [2,3] & [3,4] & [4,8] \\ & 1 & 2 & [3,4] \\ & & 1 & [1/2,2] \\ & & & 1 \end{bmatrix} \quad A_{12,4} = \begin{bmatrix} 1 & [1,2] & [2,4] & [2,8] \\ & 1 & [2,3] & [3,4] \\ & & 1 & [1/2,2] \\ & & & 1 \end{bmatrix}$$

$I_{12,3} = 0.267 \quad I_{12,4} = 0.417$

Finally, at the 3rd step, all become a group $W_{12,3,4} = ([0.364, 0.571], [0.190, 0.364], [0.079, 0.190], [0.048, 0.182])^t$ and $I_{12,3,4} = 0.626$ increased by 0.256 from the 2nd step. The whole group interval weights are different from and more uncertain than those by group of comparisons model in Section V-A, since they only include individually given comparisons roughly. The uncertainty of group decisions in progress is illustrated in Fig. 4. Although the sequence of grouping is the same as that in Section V-A, the increase of uncertainty at the 2nd step by adding $W_3$ is greater than that in Fig. 2 and is as much as that at the 3rd step by adding $W_4$. By this model, four decision makers might be potentially divided into three groups as $W_{12}$, $W_3$ and $W_4$. In Fig. 5, group decision $W_{12}$ and individual decisions are shown. $W_3$ prefers item 1 extremely more than the others, while $W_4$ prefers item 2, either. When $W_4$ checks the tentative results, he/she might change the comparisons on items 1 and 2 in $A_4$ from 1 to more than 1.

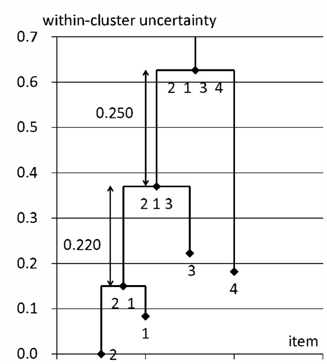


Fig. 4. Sequence by group of weights model

## VI. CONCLUSION

In this paper, two models for the sequence of group decision based on a hybrid of interval AHP and hierarchical clustering have been proposed and compared. In AHP sense, one is group of comparisons model for criteria and the other is group of weights model for alternatives under a criterion. Inter-cluster similarity is measured by the uncertainty of the group decision represented by intervals. It is based on the idea that if two similar thinking individuals get together, their possibly aggregated decision cannot be uncertain by each strategy. A pair of individuals and/or sub-groups whose group decision is

TABLE IV
GROUP OF WEIGHTS FOR THE 1ST AND 2ND STEPS

$$W_{12} = \begin{bmatrix} [0.500, 0.533] \\ [0.250, 0.267] \\ [0.125, 0.167] \\ [0.067, 0.125] \end{bmatrix} \quad W_{13} = \begin{bmatrix} [0.500, 0.571] \\ [0.190, 0.250] \\ [0.079, 0.190] \\ [0.048, 0.159] \end{bmatrix} \quad W_{14} = \begin{bmatrix} [0.364, 0.500] \\ [0.250, 0.364] \\ [0.091, 0.182] \\ [0.083, 0.182] \end{bmatrix} \quad W_{12,3} = \begin{bmatrix} [0.500, 0.571] \\ [0.190, 0.267] \\ [0.079, 0.190] \\ [0.048, 0.159] \end{bmatrix}$$

$I_{12} = 0.150$ $\quad I_{13} = 0.353$ $\quad I_{14} = 0.439$ $\quad I_{12,3} = 0.370$

$$W_{23} = \begin{bmatrix} [0.533, 0.571] \\ [0.190, 0.267] \\ [0.079, 0.190] \\ [0.048, 0.159] \end{bmatrix} \quad W_{24} = \begin{bmatrix} [0.364, 0.533] \\ [0.267, 0.364] \\ [0.091, 0.182] \\ [0.067, 0.182] \end{bmatrix} \quad W_{34} = \begin{bmatrix} [0.364, 0.571] \\ [0.190, 0.364] \\ [0.079, 0.190] \\ [0.048, 0.182] \end{bmatrix} \quad W_{12,4} = \begin{bmatrix} [0.364, 0.533] \\ [0.250, 0.364] \\ [0.091, 0.182] \\ [0.067, 0.182] \end{bmatrix}$$

$I_{23} = 0.337$ $\quad I_{24} = 0.473$ $\quad I_{34} = 0.626$ $\quad I_{12,4} = 0.489$

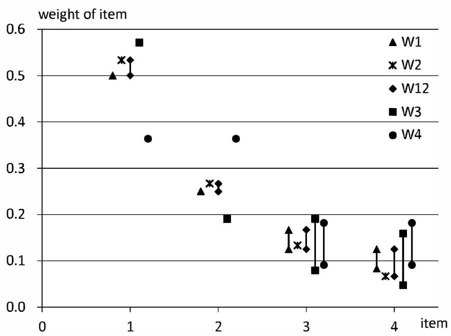


Fig. 5. Individual decisions and group decision $W_{12}$

the least uncertain becomes a group. It is repeated until all individuals become a group and the group decision becomes more uncertain by each step. Focusing on the increase of uncertainty, the divisions of a group can be noticed. The divisions and the whole group decisions where all individuals are considered by two models are different. The group of comparisons model reaches less uncertain decision than group of weights model since inconsistency among individual comparisons are offset by the others'. The group decision at each step is open to all individuals for giving a chance to an individual to reconsider his/her initial judgments in progress and releasing him/her from being discouraged by facing the difference at the end.

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