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Robust Roll and Yaw control systems using fuzzy model of the vehicle dynamics

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***Abstract*—This paper introduces a methodology for robust control of vehicle dynamics. The vehicle lateral behavior with roll dynamics is described by Takagi-Sugeno (TS) model. A fuzzy observer is first developed to estimate the vehicle sideslip angle as well as the roll parameters using the yaw rate measurement, while taking into account the road bank angle and the unmeasurable premise variables of the TS model. The objective of this study is the design of an observer-based controller to improve the stability and vehicle safety with perturbations related to the nonlinearities of the contact forces, the variations of the adhesion coefficients, the road bank angle. The proposed observer based controller is represented in the linear matrix inequality (LMI) form. The simulation results show the effectiveness of the proposed control method when different cornering maneuvers are applied to the vehicle.**
Key-words Takagi-Sugeno fuzzy model, lateral dynamics, roll dynamics, LTRd, vehicle skid control, unmeasurable premise functions, sideslip angle estimation, LMI.

I. Introduction

The active control systems perform an important role in improving the vehicle safety and comfort. Although, several active safety systems exist (ABS, ASR, TCS, DYC . . .) and some of them have already been commercialized and becoming as standard equipment in many vehicles, the increasing demands in safety and comfort terms encourage the manufacturers and the researchers to develop even more efficient security systems. In this context, considerable efforts continue to be made to improve the safety and comfort of passengers in dangerous driving situations [1],[10],[17],[23]. Thus, in order to take the nonlinearities tire contact forces, the TS fuzzy representation is often used. However, most of these works do not consider roll dynamics and unmeasurable premise variables. In this paper, the observer based control problem for a three freedom degrees model including sideslip, yaw and roll dynamics is considered. The vehicle dynamics nonlinear model is approximated by a Takagi-Sugeno (TS) fuzzy model. Then, a robust observer-based fuzzy controller with unmeasurable premise is designed to ensure the vehicle stability and to prevent the rollover.

The paper is organized as follows: in section 2, we present the considered nonlinear vehicle dynamics model and its representation by a T-S fuzzy model. Section 3 presents active control objectives and design methodology of robust observer based controller with unmeasurable premise variables. In section 4, simulation results are given to highlight the effectiveness of the design procedure of the observer. Section 5 concludes this paper.

II. Mathematical vehicle model and LTR_d

A. Sideslip Yaw and roll dynamics

The model used in this work describes the vehicle yaw and roll dynamics, which is obtained by considering the bicycle model with a roll degree of freedom (Fig. 1). The vehicle three

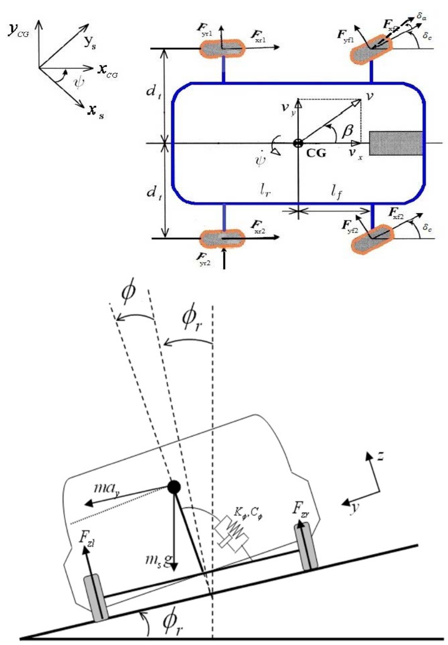

Fig. 1. Vehicle model

dimensional model with the road bank angle consideration and nonlinear tire characteristics of the four wheels can be described by the following differential equations:

$$(\Sigma:)\begin{cases} mv_x\left(\dot{\beta}+\dot{\psi}\right)=-mh_s\ddot{\phi}+2\left(F_{yf}+F_{yr}\right)-m_sg\phi_r \\ J_z\ddot{\psi}=2\left(a_fF_{yf}-a_rF_{yr}\right) \\ J_x\ddot{\phi}=m_sh_sa_y+m_sgh_ssin(\phi)-k_\phi\phi-c_\phi\dot{\phi}+m_sgh_s\phi_r \end{cases}$$

Where β denotes the side slip angle, whereas ψ, ϕ and ϕ_r are respectively the vehicle yaw, the roll angle and the road bank angle, F_{yf} is the cornering force of the two front tires, F_{yr} is the cornering force of the two rear tires. a_y is the lateral acceleration of the C.G. of sprung mass system. This acceleration can be expressed as:

$$a_y = a_{uy} - h_s\ddot{\phi} \tag{1}$$

where a_{uy} is the lateral acceleration of the center of unsprung mass. In this work, we use Pacejka's formulation [7] to express the cornering forces F_{yf} and F_{yr}. These forces are given as functions of tire slip angles as follows:

$$\begin{array}{l} F_f = D_f(\lambda) sin\left[C_f(\lambda)U_{1f}\right], \\ F_r = D_r(\lambda) sin(C_r(\lambda)U_{1r}). \end{array} \tag{2}$$

with

$$\begin{array}{rcl} U_{1i} & = & arctan\left[U_{2i}(\lambda)\alpha_i + E_i(\lambda) arctan\left\{B_i(\lambda)\alpha_i\right\}\right], \\ U_{2i}(\lambda) & = & B_i(\lambda)(1-E_i(\lambda)), i=f,r. \end{array}$$

Table I : Vehicle model parameters

Var.	Name	Unit
β	Sideslip angle at CG	$[rad]$
$\dot{\psi}$	Yaw rate	$[rad/s]$
ϕ	Roll angle	$[rad]$
$\dot{\phi}$	Roll rate	$[rad/s]$
δ_f	Front ster angle	$[rad]$
δ_r	Rear ster angle	$[rad]$
m_s	Sprung vehicle mass	$[kg]$
m	Vehicle mass	$[kg]$
v_x	Vehicle speed	$[m/s]$
J_x	Roll moment of inertia at CG	$[kgm2]$
J_z	Yaw moment of inertia at CG	$[kgm2]$
a_r	Distance from CG to rear axle	$[m]$
a_f	Distance from CG to front axle	$[m]$
T	Vehicle track width	$[m]$
h_s	CG height from roll axis	$[m]$
c_ϕ	Combined roll damping coefficient	$[Nms/rad]$
k_ϕ	Combined roll stifness coefficient	$[Nm/rad]$

The slip angles at the front and rear tires, denoted by α_f and α_r respectively, are given by:

$$\left\{ \begin{array}{l} \alpha_f = \delta_f - \beta - \frac{a_f}{v_x}\dot{\psi} \\ \alpha_r = \delta_r - \beta + \frac{a_r}{v_x}\dot{\psi} \end{array} \right. \tag{3}$$

Under assumption of small angle ϕ, the nonlinear vehicle model (Σ) can be written as:

$$(\Sigma_1 :) \left\{ \begin{array}{l} mv_x\left(\dot{\beta}+\dot{\psi}\right) = -mh_s\ddot{\phi} + 2\left(F_{yf}+F_{yr}\right) - m_s g\phi_r \\ J_z\ddot{\psi} = 2\left(a_f F_{yf} - a_r F_{yr}\right) \\ J_{xq}\ddot{\phi} = m_s h_s v_x\left(\dot{\beta}+\dot{\psi}\right) + \left(m_s g h_s - k_\phi\right)\phi - c_\phi\dot{\phi} + \\ \qquad m_s g h_s \phi_r \end{array} \right.$$

with, $J_{xq} = J_x + m_s h_s^2$. The model's parameters (2)-(3) are summarized in Table I.

B. TS model of the vehicle dynamics

The goal of this section is to obtain a TS model of (Σ_1) Thus, we consider two slip regions: a high slip region, where the absolute value of the slip angle of the tires is greater than $0.09rad$, and a low slip region, where the absolute value of the slip angle is less than $0.09rad$. We can also suppose that the cornering stiffness parameters of the tires are constant in each region [1]. Then, the front and rear cornering forces can be described by the following rule based system:

If $|\alpha_f|$ is M_1 then $F_{yf} = C_{f1}(\lambda)\alpha_f$, $F_{yr} = C_{r1}(\lambda)\alpha_r$
If $|\alpha_f|$ is M_2 then $F_{yf} = C_{f2}(\lambda)\alpha_f$, $F_{yr} = C_{r2}(\lambda)\alpha_r$

Where M_1 and M_2 are the fuzzy sets defined by the two slip regions as explained previously (M_1 is the symbol for the high slip region; M_2 for the low slip region). The membership functions parameters and the stiffness coefficients C_{fi}, $C_{ri}, i = 1, 2$, depend on the adhesion coefficient λ and are obtained using the identification method based on the Levenberb-Marquardt algorithm. For road friction coefficient $\lambda = 0.7$ (wet road), the membership functions are defined as follows:

$$h_1(\alpha_f) = \frac{w_1(\alpha_f)}{w_1(\alpha_f) + w_2(\alpha_f)}, \; h_2(\alpha_f) = 1 - h_1(\alpha_f)$$

where

$$w_1(\alpha_f) = \frac{1}{\left(1 + \left|\frac{|\alpha_f| - a_1}{b_1}\right|\right)^{c_1}}, \; w_2(\alpha_f) = \frac{1}{\left(1 + \left|\frac{|\alpha_f| - a_2}{b_2}\right|\right)^{c_2}}$$

and

$$\left\{ \begin{array}{l} a_1 = 3.1893\,,\ b_1 = 0.5077\,,\ c_1 = 0.9496 \\ a_2 = 0.5633\,,\ b_2 = 5.3907\,,\ c_2 = 0.8712 \end{array} \right.$$

The overall cornering forces are approximated by the following fuzzy systems:

$$\left\{ \begin{array}{l} F_{yf} = h_1(\alpha_f)C_{f1}(\lambda)\alpha_f + h_2(\alpha_f)C_{f2}(\lambda)\alpha_f \\ F_{yr} = h_1(\alpha_f)C_{r1}(\lambda)\alpha_r + h_2(\alpha_f)C_{r2}(\lambda)\alpha_r \end{array} \right. \tag{4}$$

Thus, from (4) and (Σ_1), the overall system is written as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\alpha_f)\left(A_i x(t) + B_i u(t) + F_i\delta_f(t) + L\phi_r(t)\right) \tag{5}$$

with,

$$x(t) = \left[\begin{array}{cccc} \beta & \dot{\psi} & \dot{\phi} & \phi \end{array}\right]^T, \; u(t) = \delta_r$$

Cornering forces change according to the road environment caused by the variation of the friction between the tires and the road. The stiffness coefficients thus vary according to the road adhesion coefficient λ. Taking into account these variations, we describe the stiffness coefficients by:

$$C_{fi}(\lambda) = C_{fi0}(1 + z_i\Delta_{fi}), \; C_{ri}(\lambda) = C_{ri0}(1 + z_i\Delta_{ri}), i = 1, 2$$

where $|\Delta_{fi}| \leq 1$, $|\Delta_{ri}| \leq 1$ and z_i depend on the road environment and describe the derivation magnitude from the nominal value. Therefore, taking into account these uncertainties, TS model (5) can be rewritten as:

$$\left\{ \begin{array}{l} \dot{x}(t) = \displaystyle\sum_{i=1}^{2} h_i(\alpha_f)\left[A_{di}x(t) + B_{di}u(t) + F_{di}\delta_f(t) + L\phi_r(t)\right] \\ y(t) = Cx(t) \end{array} \right. \tag{6}$$

with, $A_{di} = A_i^0 + \Delta A_i, B_{di} = B_i^0 + \Delta B_i, F_{di} = F_i^0 + \Delta F_i$. The output matrix is given by: $C = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array}\right]$.
The structured uncertainties considered here are norm-bounded in the form:

$$\Delta A_i = H_{ai}\Delta_i(t)E_{ai}$$

where $\Delta_i(t)$, for $i = 1, \ldots, 4$, are unknown matrix function satisfying:

$$\Delta_i^T(t)\Delta_i(t) \leq \mathbb{I}$$

H_{ai}, E_{ai} are known real constant matrices. The expressions of these matrices are omitted due to lack of space.

C. The Load Transfer Ratio, LTR_d

Traditionally, some estimate of the vehicle load transfer ratio (LTR) has been used as a basis for the design of rollover prevention systems. The quantity LTR can be simply defined as the load (i.e., vertical force) difference between the left and right wheels of the vehicle, normalized by the weight of the car [6]-[20]. In other words,

$$LTR = \frac{\text{Load on Right Tires} - \text{Load on Left Tires}}{\text{Total Weight}} \tag{7}$$

The LTR varies in the interval $[-1\ 1]$, its value is zero for a perfectly symmetrical vehicle moving on a straight line and reaches extremes when one side of the vehicle leaves the ground where the LTR is -1 or 1 depending on the direction of the rollover.

In order to obtain a Rollover estimation while taking into account the roll dynamics, we write a torque balance equation.We assume that the unsprung mass is insignificant and that the main body of the vehicle rolls about an axis along the centerline of the body at the ground level. We have,

$$F_r\frac{T}{2} - F_l\frac{T}{2} - k_\phi\phi - c_\phi\dot{\phi} = 0$$

Then, the expression of the LTR_d is given by,

$$LTR_d = \frac{2}{mgT}\left(k_\phi\phi + c_\phi\dot{\phi}\right) \tag{8}$$

This quantity can be expressed in terms of the state variable x, in the following form,

$$\begin{aligned} LTR_d &= \begin{bmatrix} 0 & 0 & \frac{2}{mgT}c_\phi & \frac{2}{mgT}k_\phi \end{bmatrix} x & (9)\\ &= C_1 x & (10)\end{aligned}$$

The following lemmas are useful to establish our main results.

Lemma 2.1: [12] Given constant matrices D and E, symmetric constant matrix S and unknown constant matrix F of appropriate dimension satisfying the constraint $F^TF < R$. The following two propositions are equivalent:

(i) $S + DFE + E^TF^TD^T < 0$;

(ii) $S + \begin{bmatrix} E^T & D \end{bmatrix}\begin{bmatrix} \varepsilon^{-1}R & 0 \\ 0 & \varepsilon\mathbb{I}\end{bmatrix}\begin{bmatrix} E \\ D^T\end{bmatrix} < 0$ for some $\varepsilon > 0$.

Lemma 2.2: [18] Considering a negative definite matrix $\Xi < 0$, a given matrix Z and a scalar $\mu > 0$, the following holds:

$$Z^T\Xi Z < -\mu\left(Z^T + Z\right) - \mu^2\Xi^{-1}.$$

III. Main results

This section is devoted to the observer-based controller design.

A. Observer based H_∞ controller design

The main idea of this part is to design an observer in order to estimate the sideslip angle and the roll angle of the vehicle. We remark from (3) that the premise variable $\alpha_f(t)$ depends on the lateral velocity v_y. Thus, let us denote by $\hat{z}(t)$ the estimation of $z(t)$. The system (6) can be expressed in the following form:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\hat{\alpha}_f)\begin{bmatrix} A_{di}x(t) + B_{di}u(t) + F_{di}\delta_f(t) + \\ L\phi_r(t) + \nu(t)\end{bmatrix} \tag{11}$$

with,

$$\nu(t) = \sum_{i=1}^{2}\left(h_i(\alpha_f) - h_i(\hat{\alpha}_f)\right)\left[A_{di}x(t) + B_{di}u(t) + F_{di}\delta_f(t)\right]$$

Based on the structure of the TS model (11), the following fuzzy state observer is proposed,

$$\dot{\hat{x}}(t) = \sum_{i=1}^{2} h_i(\hat{\alpha}_f)\begin{bmatrix} A_i^0\hat{x}(t) + B_i^0u(t) + F_i^0\delta_f(t) + \\ G_iC\left(x(t) - \hat{x}(t)\right)\end{bmatrix} \tag{12}$$

The matrices $G_i, i = 1, 2$ are observer gains to be determined.

To stabilize this class of systems, we use the PDC observer-based controller [15] defined as:

$$u(t) = -\sum_{i=1}^{2} h_i(\hat{\alpha}_f(t))K_i\hat{x}(t) \tag{13}$$

Let us define the estimation error $e(t)$ by:

$$e(t) = x(t) - \hat{x}(t)$$

and the perturbation terms on the membership functions by:

$$\begin{aligned} \mu_{1i}(t) &= h_i(\alpha_f)x(t) - h_i(\hat{\alpha}_f)\hat{x}(t), & (14)\\ \mu_{2i}(t) &= \left(h_i(\alpha_f) - h_i(\hat{\alpha}_f)\right)u(t), \\ \tilde{h}_i = &= h_i(\alpha_f) - h_i(\hat{\alpha}_f). \end{aligned}$$

Assumption 3.1: We assume that, for each $i = 1, 2$, the functions μ_{1i} and μ_{2i} are Lipschitz and verify:

- $\| \mu_{1i}(t) \| \leq \theta_{1i} \| x(t) - \hat{x}(t) \|$
- $\| \mu_{2i}(t) \| \leq \theta_{2i} \| x(t) - \hat{x}(t) \|$

Remark 3.1: Under the assumption 3.1, we can easily verify that the disturbance vector $\upsilon(t)$ is bounded. In fact, the term $\upsilon(t)$ can be rewritten as:

$$\upsilon(t) = \sum_{i=1}^{2}\left[A_{di}\mu_{1i}(t) + B_{di}\mu_{2i}(t) - h_i(\hat{\alpha}_f)A_{di}e(t) + \tilde{h}_iF_{di}\delta_f(t)\right]$$

and using the membership function characteristics, we have:

$$-1 \leq h_i(\alpha_f) - h_i(\hat{\alpha}_f) \leq 1.$$

From the expression of the control law (13), the estimation error dynamics are given by:

$$\begin{aligned} \dot{e}(t) = & \sum_{i,j=1}^{2} h_i(\hat{\alpha}_f)h_j(\hat{\alpha}_f)\left[\left(A_i^0 - G_iC + \Delta B_iK_j\right)e(t)\right. \\ & \left. + \left(\Delta A_i - \Delta B_iK_j\right)x(t) + \Delta F_i\delta_f(t) + L\phi_r(t) + \upsilon(t)\right] \end{aligned}$$

The augmented system $\tilde{x} = \begin{bmatrix} x(t) \\ e(t)\end{bmatrix}$ can be written as follows:

$$\begin{cases} \dot{\tilde{x}}(t) = \sum\limits_{i=1}^{2}\sum\limits_{j=1}^{2} h_i(\hat{\alpha}_f)h_j(\hat{\alpha}_f)\left(\tilde{A}_{ij}\tilde{x}(t) + \tilde{H}_i\tilde{w}(t)\right) \\ y_1(t) = LTR_d = C_1\tilde{x}(t) \end{cases} \tag{15}$$

with,

$$\tilde{A}_{ij} = \begin{bmatrix} A_i^0 - B_i^0K_j + \Delta A_i - \Delta B_iK_j & \left(B_i^0 + \Delta B_i\right)K_j \\ \Delta A_i - \Delta B_iK_j & A_i^0 - G_iC + \Delta B_iK_j \end{bmatrix},$$

$$\tilde{H}_i = \begin{bmatrix} F_i^0 + \Delta F_i & L & \mathbb{I} \\ \Delta F_i & L & \mathbb{I}\end{bmatrix},\ \tilde{w}(t) = \begin{bmatrix} \delta_f(t) \\ \phi_r \\ \upsilon(t)\end{bmatrix},\ y_1(t) = LTR_d$$

The objective is to design the observer-based controller i,e. to find $K_i, G_i, i = 1, \ldots, 4$ so that: first, the performances output should be, in order to prevent the vehicle rollover, satisfied, i.e.

$$\| y_1(t) \|_2 \leq \xi_1 \tag{16}$$

where $\xi_1 < 1$, and second, the estimation error converges to zero. The observer convergence is studied taking into account the dependence between the estimation error and the exogenous signals of the system (15). We thus seek to satisfy the following robust performance under zero initial conditions:

$$\frac{\| e(t) \|_2}{\| \tilde{\omega}(t) \|_2} \leq \gamma,\ \text{for}\ \tilde{\omega}(t) \neq 0 \tag{17}$$

where γ is the desired disturbance attenuation parameter.
The following theorem presents the stabilization conditions to guarantee the robust asymptotic stability as well as the previous control performances.

Theorem 3.1: Assuming that the initial condition x_0 is known. For given scalars γ, μ and $\varepsilon_{ij}, i, j = 1, 2$, if there exist symmetric matrices $X > 0, Y > 0$, matrices V_j and $W_j, j = 1, 2$, solution of the following LMIs:

$$\left\{\begin{array}{ll} \Psi_{ii} \leq 0, & i = 1,2, \\ \Psi_{ii} + \frac{1}{2}\left(\Psi_{ij} + \Psi_{ji}\right) \leq 0, & i \neq j \leq 2. \end{array}\right. \quad \left\{\begin{array}{l} \begin{bmatrix} 1 & x_0^T \\ * & X \end{bmatrix} \geq 0 \\ \begin{bmatrix} X & XC_1^T \\ * & \eta^2 \mathbb{I} \end{bmatrix} \geq 0 \end{array}\right. \tag{18}$$

where $\Psi_{ij} = \begin{bmatrix} \Gamma_{1ij} & \Upsilon_{1ij} & \mathbf{0} \\ * & \Gamma_{2ij} & \Upsilon_{2ij} \\ * & * & \Gamma_{3ij} \end{bmatrix}$,

and,

$$\Gamma_{1ij} = \begin{bmatrix} \mathbb{N}_{ij} & XE_{ai}^T & V_j^T E_{bi}^T \\ * & -0.5\varepsilon_{ij}\mathbb{I} & \mathbf{0} \\ * & * & -0.5\varepsilon_{ij}\mathbb{I} \end{bmatrix}$$

$$\Gamma_{2ij} = \begin{bmatrix} -2\mu X & 0 & 0 & 0 & V_j^T E_{bi}^T \\ * & -2\mu\mathbb{I} + \varepsilon_{ij}^{-1} E_{fi}^T E_{fi} & 0 & 0 & 0 \\ * & * & -2\mu\mathbb{I} & 0 & 0 \\ * & * & * & -2\mu\mathbb{I} & 0 \\ * & * & * & * & -0.5\varepsilon_{ij}\mathbb{I} \end{bmatrix},$$

$$\Gamma_{3ij} = \begin{bmatrix} \mathbb{N}_{2ij} & YH_{ai} & YH_{bi} & YH_{fi} & \mathbf{0} & YL & Y \\ * & -\varepsilon_{ij}^{-1}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -0.5\varepsilon_{ij}^{-1}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\varepsilon_{ij}^{-1}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\gamma^2\mathbb{I} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -\gamma^2\mathbb{I} & \mathbf{0} \\ * & * & * & * & * & * & -\gamma^2\mathbb{I} \end{bmatrix}$$

$$\Upsilon_{1ij} = \begin{bmatrix} B_i^0 V_j & F_i^0 & L & \mathbb{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\Upsilon_{2ij} = \begin{bmatrix} \mu\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mu\mathbb{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mu\mathbb{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbb{N}_{1ij} = A_i^0 X - B_i^0 V_j + XA_i^{0T} - V_j^T B_i^{0T} + \varepsilon_{ij} H_{ai} H_{ai}^T + 2\varepsilon_{ij} H_{bi} H_{bi}^T + \varepsilon_{ij} H_{fi} H_{fi}^T,$$
$$\mathbb{N}_{2ij} = YA_i^0 - W_i C + A_i^{0T} Y - C^T W_i^T + \mathbb{I}.$$

then the uncertain fuzzy system (15) is asymptotically stable via the observer-based controller (12-13), with the controller and observer gains are given by : $K_j = V_j X^{-1}$ and $G_i = Y^{-1} W_i$, respectively, for $i, j = 1, 2$.

Proof 1: See Appendix 1.

IV. Simulation results

The proposed observer based controller is evaluated through the computer simulations for a vehicle travelling at a constant velocity $v = 18m/s$ and doing a lane change manoeuver with a front steering angle pattern as shown in Fig. 2. The road bank angle is considered equal to 7°. The considered nominal stiffness coefficients are given in Table II, for the dry road friction coefficient $\lambda = 0.7$. For the simulation purpose, we consider $z_i = 0.1$, which means that the designed controller should ensure stability although if the stiffness coefficients change until 10% from their nominal values.

TABLE I
NOMINAL STIFFNESS COEFFICIENTS

Stiffness coefficient	C_{f10}	C_{f20}	C_{r10}	C_{r20}
Nominal values (N/rad)	71946	11213	67847	7868

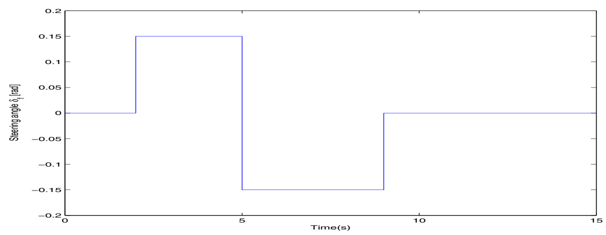

Fig. 2. Front steering angle δ_f

Using the toolbox YALMIP for the resolution of (18) with $\gamma = 0.1$, we obtain:

$$K_1 = \begin{bmatrix} 1.762 & -0.266 & -0.014 & -0.057 \end{bmatrix}, \; G_1 = \begin{bmatrix} 248.895 \\ 997.608 \\ 1638.001 \\ 0.526 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.943 & -0.132 & -0.0053 & -0.0258 \end{bmatrix}, \; G_2 = \begin{bmatrix} 111.515 \\ 467.581 \\ 728.786 \\ 1.628 \end{bmatrix}.$$

The estimated vehicle states using the designed observer based controller are shown in Fig.3. We can see the effectiveness of the proposed fuzzy observer-based controller, i.e, the estimated state variables are compared to the vehicle state variables for the same front steering angle given in Fig.2. Fig.4 shows the rear steering angle δ_r evolution considered as the control vector. From Fig.5, we observe that the proposed approach is effective in guaranteing the performance output (16) with $\xi_1 = 0.55$.

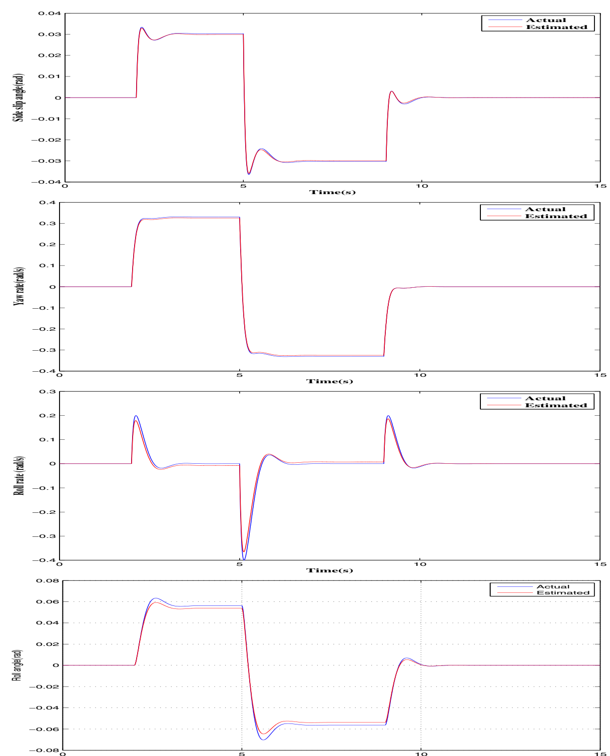

Fig. 3. Simulation results of the vehicle state estimator

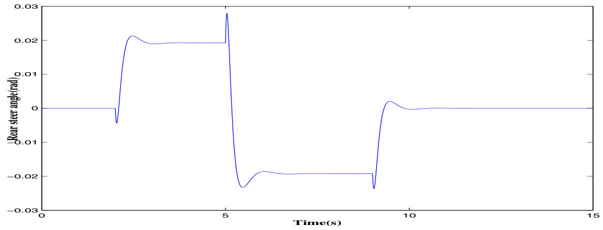

Fig. 4. Rear steering angle δ_r

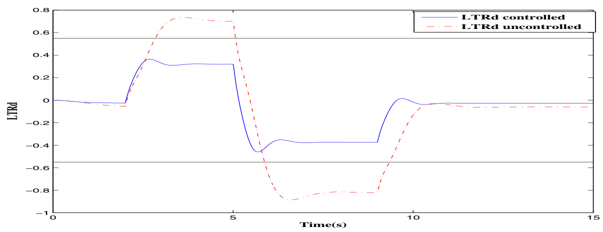

Fig. 5. Comparaison of the controlled and uncontrolled vehicle Load transfer ratio(LTRd)

V. CONCLUSION

In this study, a robust fuzzy observer-based controller for the lateral vehicle dynamics has been developed. The main objectives are the vehicle stabilization as well as the rollover prevention. The vehicle lateral dynamics including the roll dynamics have been approximated by a fuzzy TS model. The controller stabilizes the vehicle although the road changes (wet or dry road). The fuzzy observer is designed to estimate the system state, especially the sideslip angle, while considering unmeasurable premise variables. The robustness with respect to the variations of the adhesion coefficients and the road bank angle, the rollover prevention, the convergence of the estimation error as well as the vehicle stabilization, are ensured. The simulation results have shown the effectiveness of this proposed control scheme.

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VI. APPENDIX

The matrices $A_i^0, B_i^0, F_i^0, i = 1, 2$ are given by:

$$A_i^0 = \begin{bmatrix} -\frac{\sigma_i J_{xq}}{m v_x J_x} & \frac{\rho_i J_{xq}}{m I_x v_x^2} & -\frac{h_s c_\phi}{J_x v_x} & \frac{h_s(m_s g h_s - k_\phi)}{J_x v_x} \\ \frac{\rho_i}{J_z} & -\frac{\tau_i}{J_z v_x} & 0 & 0 \\ -\frac{m_s h_s \sigma_i}{m J_x} & \frac{m_s h_s \rho_i}{m J_x v_x} & -\frac{c_\phi}{J_x} & \frac{m_s g h_s - k_\phi}{J_x} \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_i^0 = \begin{bmatrix} \frac{2C_{r0i} J_{xq}}{m v_x J_x} \\ -\frac{2C_{r0i} a_r}{J_z} \\ \frac{2m_s h_s C_{r0i}}{m J_x} \\ 0 \end{bmatrix}, \; F_i^0 = \begin{bmatrix} \frac{2C_{f0i} J_{xq}}{m v_x J_x} \\ \frac{2C_{f0i} a_f}{J_z} \\ \frac{2m_s h_s C_{f0i}}{m J_x} \\ 0 \end{bmatrix}$$

Theorem 3.1: Proof 1

Consider the following Lyapunov function candidate:

$$V(\tilde{x}(t)) = \tilde{x}^T(t) P \tilde{x}(t), \quad P = P^T > 0. \tag{19}$$

The time derivative of $V(\tilde{x}(t))$ along the trajectory of (15) is given by:

$$\dot{V}(\tilde{x}(t)) = \sum_{i=1}^{2}\sum_{i=1}^{2} h_i(\hat{\alpha}_f) h_j(\hat{\alpha}_f) \Big[\tilde{x}^T(t)\left(P\tilde{A}_{ij} + \tilde{A}_{ij}^T P\right)\tilde{x}(t) + \tilde{x}^T(t) P \bar{H}_i \tilde{\omega}(t) + \tilde{\omega}^T(t) \bar{H}_i^T P \tilde{x}(t)\Big].$$

The objective (17) is guaranteed by ensuring the following inequality:

$$\dot{V}(\tilde{x}) + e^T(t) e(t) - \gamma^2 \tilde{\omega}^T(t) \tilde{\omega}(t) < 0 \tag{20}$$

Therefore, we have,

$$\dot{V}(\tilde{x}) + e^T(t)e(t) - \gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t) = \\ = \sum_{i,j=1}^{2} h_i(\hat{\alpha}_f)h_j(\hat{\alpha}_f) \begin{bmatrix} \tilde{x}(t) \\ \tilde{\omega}(t) \end{bmatrix}^T \left(\frac{\Omega_{ij}+\Omega_{ji}}{2}\right) \begin{bmatrix} \tilde{x}(t) \\ \tilde{\omega}(t) \end{bmatrix}$$

with,

$$\Omega_{ij} = \begin{bmatrix} P\tilde{A}_{ij} + \tilde{A}_{ij}^T P + L^T L & P\tilde{H}_i \\ * & -\gamma^2\mathbb{I} \end{bmatrix}, \quad L = \begin{bmatrix} \mathbf{0} & \mathbb{I} \end{bmatrix}.$$

By using the relaxation scheme from [11], (20) holds if the following conditions are satisfied:

$$\Omega_{ii} < 0, \ i = 1,2 \tag{21}$$

$$\Omega_{ii} + \frac{1}{2}(\Omega_{ij} + \Omega_{ji}) < 0, \ i \neq j \leq 2 \tag{22}$$

Let us consider the particular form of P

$$P = \begin{bmatrix} P_1 & \mathbf{0} \\ \mathbf{0} & P_2 \end{bmatrix}, \ P_1 = P_1^T > 0, \ P_2 = P_2^T > 0 \tag{23}$$

By substituting (16), the matrix Ω_{ij} can be expressed by:

$$\Omega_{ij} = S_{ij} + \mathbb{W}_{ij} + \mathbb{W}_{ij}^T.$$

with,

$$\mathbb{W}_{ij} = \begin{bmatrix} P_1\Delta\Gamma_{ij} & P_1\Delta B_i K_j + \Delta\Gamma_{ij}^T P_2 & P_1\Delta F_i & \mathbf{0} \\ \mathbf{0} & K_j^T \Delta B_i^T P_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Delta F_i^T P_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$S_{ij} = \begin{bmatrix} \Gamma_{1ij} & P_1 B_i K_j & P_1 F_i & P_1 \\ * & \Gamma_{2ij} & P_2 F_i & P_2 \\ * & * & -\gamma^2\mathbb{I} & \mathbf{0} \\ * & * & * & -\gamma^2\mathbb{I} \end{bmatrix}$$

with,

$$\begin{aligned} \Gamma_{1ij} &= P_1(A_i - B_i K_j) + (A_i - B_i K_j)^T P_1 \\ \Gamma_{2ij} &= P_2(A_i - G_i C) + (A_i - G_i C)^T P_2 + \mathbb{I} \\ \Delta\Gamma_{ij} &= (\Delta A_i - \Delta B_i K_j) \end{aligned}$$

According to the definition of matrices ΔA_i, ΔB_i, ΔF_i, and using the lemma 2.1, the inequalities (21)-(22) hold if there exist real scalars ε_{ij} satisfying the following condition:

$$\widetilde{\Omega}_{ii} < 0, i = 1,2 \tag{24}$$

$$\widetilde{\Omega}_{ii} + \frac{1}{2}\left(\widetilde{\Omega}_{ij} + \widetilde{\Omega}_{ji}\right) < 0, i \neq j \leq 2 \tag{25}$$

with

$$\widetilde{\Omega}_{ij} = \begin{bmatrix} \Pi_{ij} & \mathbb{M}_{ij} \\ * & \Phi_{ij} \end{bmatrix} \tag{26}$$

and,

$$\begin{aligned} \Phi_{ij} &= \begin{bmatrix} \Theta_{ij} & P_2 F_i & P_2 \\ * & -\gamma^2\mathbb{I} + 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi} & \mathbf{0} \\ * & * & -\gamma^2\mathbb{I} \end{bmatrix}, \\ \mathbb{M}_{ij} &= \begin{bmatrix} P_1 B_i K_j & P_1 F_i & P_1 \end{bmatrix}, \\ \Pi_{ij} &= P_1(A_i - B_i K_j) + (A_i - B_i K_j)^T P_1 + \varepsilon_{ij} P_1 H_{ai} H_{ai}^T P_1 + \\ &\quad 2\varepsilon_{ij} P_1 H_{bi} H_{bi}^T P_1 + \varepsilon_{ij} P_1 H_{fi} H_{fi}^T P_1 + 2\varepsilon_{ij}^{-1} E_{ai}^T E_{ai} + \\ &\quad 2\varepsilon_{ij}^{-1} K_j^T E_{bi}^T E_{bi} K_j, \\ \Theta_{ij} &= P_2(A_i - G_i C) + (A_i - G_i C)^T P_2 + \varepsilon_{ij} P_2 H_{ai} H_{ai}^T P_2 + \\ &\quad 2\varepsilon_{ij} P_2 H_{bi} H_{bi}^T P_2 + \varepsilon_{ij} P_2 H_{fi} H_{fi}^T P_2 + \\ &\quad 2\varepsilon_{ij}^{-1} K_j^T E_{bi}^T E_{bi} K_j + \mathbb{I}. \end{aligned}$$

Pre-post multiplying the equality (26) by the matrix $diag(X, X, \mathbb{I}, \mathbb{I})$, with $X = P_1^{-1}$, we obtain:

$$diag(X, X, \mathbb{I}, \mathbb{I})\widetilde{\Omega}_{ij} diag(X, X, \mathbb{I}, \mathbb{I}) = \begin{bmatrix} \bar{\Pi}_{ij} & \bar{\mathbb{M}}_{ij} \\ * & \bar{\Phi}_{ij} \end{bmatrix} \tag{27}$$

with,

$$\begin{aligned} \bar{\Pi}_{ij} &= A_i X - B_i V_j + X A_i^T - V_j^T B_i^T + \varepsilon_{ij} H_{ai} H_{ai}^T + 2\varepsilon_{ij} H_{bi} H_{bi}^T + \\ &\quad 2\varepsilon_{ij} H_{fi} H_{fi}^T + \varepsilon_{ij}^{-1} X E_{ai}^T E_{ai} X + 2\varepsilon_{ij}^{-1} V_j^T E_{bi}^T E_{bi} V_j, \\ \bar{\mathbb{M}}_{ij} &= \begin{bmatrix} B_i V_j & F_i & \mathbb{I} \end{bmatrix}, \\ \bar{\Phi}_{ij} &= \begin{bmatrix} X\Theta_{ij} X & X P_2 F_i & X P_2 \\ * & -\gamma^2\mathbb{I} + 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi} & \mathbf{0} \\ * & * & -\gamma^2\mathbb{I} \end{bmatrix}, \\ \Theta_{ij} &= P_2 A_i - W_i C + A_i^T P_2 - C^T W_i^T + \varepsilon_{ij} P_2 H_{ai} H_{ai}^T P_2 + \\ &\quad 2\varepsilon_{ij} P_2 H_{bi} H_{bi}^T P_2 + \varepsilon_{ij} P_2 H_{fi} H_{fi}^T P_2 + 2\varepsilon_{ij}^{-1} K_j^T E_{bi}^T E_{bi} K_j + \mathbb{I}. \end{aligned}$$

with the matrices V_j, W_i are given by:

$$V_j = K_j X, \ W_i = Y G_i \text{ and } Y = P_2.$$

Matrices $\bar{\Phi}_{ij}$ can be rewritten in the following form:

$$\bar{\Phi}_{ij} = diag(2\varepsilon_{ij}^{-1} V_j^T E_{bi}^T E_{bi} V_j, \ 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi}, \ \mathbf{0}) + diag(X, \mathbb{I}, \mathbb{I})\mathfrak{T}_{ij} diag(X, \mathbb{I}, \mathbb{I}),$$

with, $\mathfrak{L}_{ij} = Y A_i - W_i C + A_i^T Y - C^T W_i^T + \varepsilon_{ij} Y H_{ai} H_{ai}^T Y + 2\varepsilon_{ij} Y H_{bi} H_{bi}^T Y + \varepsilon_{ij} Y H_{fi} H_{fi}^T Y + \mathbb{I}$, $\mathfrak{T}_{ij} = \begin{bmatrix} \mathfrak{L}_{ij} & Y F_i & Y \\ * & -\gamma^2\mathbb{I} & \mathbf{0} \\ * & * & -\gamma^2\mathbb{I} \end{bmatrix}$.

Using the lemma 2.2, there exists a positif scalar $\mu > 0$, such that:

$$\bar{\Phi}_{ij} \leq diag(2\varepsilon_{ij}^{-1} V_j^T E_{bi}^T E_{bi} V_j, 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi}, \mathbf{0}) - 2\mu diag(X, \mathbb{I}, \mathbb{I}) - \mu^2 \mathfrak{T}_{ij}^{-1}$$

Finally, the expression (27) can be rewritten in the following form:

$$diag(X, X, \mathbb{I}, \mathbb{I})\widetilde{\Omega}_{ij} diag(X, X, \mathbb{I}, \mathbb{I}) \leq \begin{bmatrix} \bar{\Pi}_{ij} & \bar{\mathbb{M}}_{ij} \\ * & \mathfrak{S}_{ij} - \mu^2 \mathfrak{T}_{ij}^{-1} \end{bmatrix}$$

with,

$$\mathfrak{S}_{ij} = \begin{bmatrix} 2\varepsilon_{ij}^{-1} V_j^T E_{bi}^T E_{bi} V_j - 2\mu X & \mathbf{0} & \mathbf{0} \\ * & -2\mu\mathbb{I} + 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi} & \mathbf{0} \\ * & * & -2\mu\mathbb{I} \end{bmatrix}$$

Using the Schur's complement several times, sufficient conditions of Theorem 3.1 are established. ▲